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Mr Goldingham

1811

A
COMPLETE SYSTEM
OF
ASTRONOMY;

BY THE

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THIS Volume contains the PHYSICAL PRINCIPLES OF ASTRONOMY, with their application to all the phænomena which ariſe from the mutual attraction of the bodies in our ſyſtem. The author has endeavoured to render this difficult ſubject eaſy to be underſtood, by fully explaining all the principles, and omitting no material ſteps in the inveſtigations; he truſts therefore that what he has here done may prove a conſiderable help to ſtudents in phyſical Aſtronomy, and tend to diffuſe a more general knowledge of that ſubject. The Tables and Catalogues of the fixed Stars will be found very uſeful to the practical Aſtronomer.

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A COMPLETE

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CHAPTER XXXI.

ON THE GENERAL PRINCIPLES OF CENTRIPETAL FORCES.

Art. 805. IF a body revolve about an immoveable center of force, and be conſtantly attracted to it, it will always move in the ſame plane, and deſcribe areas about that center proportional to the times. For let S be the center of force, and ſuppoſe a body to be projected at P in the direction PQc, and take $PQ=Qc$, then by the firſt law of motion, the body would move uniformly in the direction PQc, and deſcribe PQ, Qc in the ſame time, if no other force acted upon it But when the body comes to Q let a *ſingle* impulſe act at S, ſufficient to draw the body through QV in the time it *would* have deſcribed Qc, or *did* deſcribe PQ, and complete the parallelogram $VQcC$, and the body, in the *ſame* time, will deſcribe QC, therefore PQ, QC are deſcribed in the *ſame* time. Now by EUCLID, B. 1. p 37, the triangle $SCQ=ScQ$, and by B. 1. p 38, $ScQ=SPQ$, therefore $SCQ=SPQ$, or equal areas are deſcribed in equal times For the ſame reaſon, if a *ſingle* impulſe act at C, D, E, &c. at equal intervals of time, then $SCQ=SCD=SED=$&c. Now as this is true whatever be theſe equal intervals, let them be diminiſhed *ſine limite*, and the limit gives a force which acts conſtantly; Fig. 187

and as the reaſoning reſpecting the equal deſcription of areas in equal times holds true up to the limit (as no point can be aſſumed before it comes to the limit when it is not true), it muſt be true in the limit, hence, when the force acts conſtantly, equal areas will be deſcribed in equal times (219, 641), and the body will deſcribe a curve about S. And as no force acts *out* of the plane of SPQ, the whole curve muſt lie in that plane.

806. Draw Sy perpendicular to QP produced, then the area $SPQ = \frac{1}{2} PQ \times SY$, which varies as $PQ \times SY$, therefore PQ varies as $\frac{\text{area } SPQ}{SY}$, but PQ varies as the velocity V, when the time is given, hence, V varies as $\frac{\text{area } SPQ}{SY}$. Now in that curve which is the limit of $PQCDE$ &c. SY becomes a perpendicular to a tangent to the curve. Hence, V varies as $\frac{\text{area } SPQ \text{ deſ. in a given time}}{\text{perpendicular on the tan.}}$, but in the *ſame* curve, the area SPQ is given (805) when the time is given, therefore in the *ſame* curve V varies as $\frac{1}{SY}$, conſidering SY as a perpendicular on a tangent to that point of the curve where the body is.

807. If equal areas be deſcribed about S in equal times, the force muſt tend to S. For let $SPQ = SQC$, now $SPQ = SQc$, therefore $SQC = SQc$, hence, by Euclid, B. 1. p. 39, Cc is parallel to QS, therefore $QcCV$ is a parallelogram, now, by ſuppoſition, the body deſcribes QC in conſequence of the impulſe at Q, and it would have deſcribed Qc if no impulſe had acted, therefore QV muſt repreſent that motion impreſſed at Q, which, in conjunction with the motion Qc, can make the body deſcribe QC, and QV is directed to S.

808. Draw Qv parallel to $zCxPw$ in it's limiting ſtate. Now (Newton's Prin. Book I. Sect. 1. Lem. 10. Cor.) QV being the ſpace deſcribed in a given time by the impulſe of the force acting at Q, the limiting ratio of QV in one point of the curve to QV in another, will expreſs the ratio of the forces in theſe two points, but $QV = 2Qx = 2Cv$, hence, the limiting ratio of Cv to Qv in two points will expreſs the ratio of the forces. Now by making P and C approach to Q as their limit, they will arrive at Q at the ſame time, becauſe PQ, QC are deſcribed in the ſame time; but when P and C arrive at Q, the line zw ceaſes to *cut* the curve, and only *touches* it at Q, and therefore it becomes a tangent, conſequently Qv (which is the ultimate direction of PC) is a tangent to the curve at Q. Hence, to find the proportion of the forces in Fig. 188. any two points P, p, of a curve, draw the tangents PX, px, to thoſe points, take two arcs PQ, pq deſcribed in the *ſame* time, and draw QR parallel

to

to PS, and qr parallel to pS, then diminiſh this time, and conſequently the arcs PQ, pq, and make them vaniſh, and that ratio to which PQ, pq approach as their limit, is the ratio of the forces at P and p.

Fig. 187.

809. If the areas be *accelerated*, the force tends in *conſequentia*, for if SQC be greater than SQP or SQc, then C muſt fall *above* a line drawn from c parallel to QS, hence, the other ſide QV, of the parallelogram $QcCV$, muſt fall *above* QS, and therefore if QV be produced, it muſt fall *above* S, hence S muſt have moved up into that line, or the ſame way the body has moved, becauſe the motion QV ariſes from the force at S, and therefore S muſt have been in the line QV. For the ſame reaſon, if the areas be *retarded*, the force tends in *antecedentia*, for then, as SQC is leſs than SPQ or SQc, C muſt fall *below* a line drawn from c parallel to QS, hence V will fall *below* QS, and therefore QV produced will fall *below* S, conſequently S muſt have moved in a direction contrary to the motion of the body.

Fig. 189.

810 Def. If the circle PAB touch the curve WPZ at P, and RQT be drawn, making any finite angle with the tangent PF, then if RQT move up to P, and the limiting ratio of $RQ \cdot RT$ be a ratio of equality, that circle is called a *circle of curvature* to the curve at P.

811. Let PV be a chord of that circle paſſing through the center of force S, draw SY perpendicular to the tangent, and RQT parallel to PV, and join TP, TV, then, by the Definition, the limit of RQ RT is a ratio of equality. Now the angle RTP = the alternate angle TPV, and the angle TPR between the chord and tangent = the angle PVT in the alternate ſegment, therefore the triangles RPT, TVP are ſimilar; hence $PV : PT$ PT $TR = \frac{PT^2}{PV}$, now when the time is given, TR becomes ultimately proportional to the force (808); alſo (Newt Prin Lem. 7. Lib 1 Sect 1) the limit of the chord PT to the arc PT is a ratio of equality, but as the time is given, the arc PT is ultimately proportional to the velocity V, hence, the force varies as $\frac{V^2}{PV}$. In the *ſame* curve (806) V varies as $\frac{1}{SY}$, hence, the force varies as $\frac{1}{SY^2 \times PV}$. Now this being proved true for the circle, it muſt be true for the curve, as the limit of the ſagittas RQ, RT is (810) a ratio of equality, and the limit of the two arcs is alſo a ratio of equality, as follows from Newton's Prin Lib. 1. Lem 7 Sect 1 Hence, in different parts of the *ſame* curve, the force varies as $\frac{1}{SY^2 \times PV}$.

812. As $\frac{V^2}{PV}$ varies as the force F in general, therefore V varies as $\sqrt{F \times PV}$;

that

that is, the velocity in the ſame or different curves varies in the ſubduplicate ratio of the force and chord of curvature conjointly.

FIG 190. 813. To find the force tending to the focus S of an *ellipſe*. Let P be the place of the body, H the other focus, AC, BC the ſemi-axis major and minor, SY, HZ, perpendiculars to a tangent at P, and DC parallel to the tangent Put s = the ſine of the angle SPY or HPZ, radius being unity, then $s=\frac{SY}{SP}$, and $s=\frac{HZ}{HP}$, hence, $s^2=\frac{SY\times HZ}{SP\times HP}$ = (as, by Conics, $SY\times HZ=BC^2$, and $SP\times HP=CD^2$) $\frac{BC^2}{CD^2}$, but $s^2=\frac{SY^2}{SP^2}$, therefore $\frac{SY^2}{SP^2}=\frac{BC^2}{CD^2}$, hence, $SY^2=\frac{SP^2\times BC^2}{CD^2}$, alſo, by Conics, the *chord of curvature*$=\frac{2CD^2}{AC}$, hence, (811) the force tending to S varies as $\frac{CD^2}{SP^2\times BC^2}\times\frac{AC}{2CD^2}$, which varies as $\frac{1}{SP^2}$, AC and BC being conſtant. The ſame proof holds for the *hyperbola* If the curve be a *parabola*, SY^2 varies as SP, alſo the *chord of curvature*$=4SP$, and therefore it varies as SP, hence, the force tending to the focus varies as $\frac{1}{SP^2}$. The force therefore tending to the focus of every conic ſection varies *inverſely as the ſquare of the diſtance*.

814 Draw QT perpendicular to SP, then by ſimilar triangles, $QT^2 : QP^2 :: SY^2 : SP^2$ (from the equation in the laſt Article) $BC^2 : CD^2$, hence, $\frac{QT^2}{QR} : \frac{QP^2}{QR} :: \frac{2BC^2}{AC} : \frac{2CD^2}{AC}$, but (811) the chord of curvature $=\frac{QP^2}{QR}$, and it alſo $=\frac{2CD^2}{AC}$ by Conics; hence, $\frac{QT^2}{QR}=\frac{2BC^2}{AC}$ = the latus rectum L. This is for the *ellipſe* and *hyperbola*. Now if the major axis of the ellipſe be increaſed *ſine limite*, it approaches the *parabola* as it's *limit*, and as all the above reaſoning holds up to the limit, it muſt be true for the parabola. Hence, in every conic ſection $L=\frac{QT^2}{QR}$, when the force tends to the focus.

815 Hence, if the force vary inverſely as the ſquare of the diſtance, the body muſt deſcribe a conic ſection having the center of force in it's focus. For let SP, the angle SPY, the velocity and force be given; then SY is given, alſo as the force is given, QR is given (808) when the time is given, and the time being given, the ſpace PQ deſcribed with the given velocity is given; hence QT is given, therefore $\frac{QT^2}{QR}$ is given; make $\frac{QT^2}{QR}=L$ the latus rectum

of

of a conic ſection, theſe quantities having been proved (814) to be equal in all the conic ſections. But if we know SP, SY and L, we have data ſufficient to deſcribe a conic ſection, therefore the body *may* revolve in the conic ſection deſcribed with theſe data, and if it *may*, it *muſt*, as a body cannot, with all the ſame data, deſcribe two different curves.

816. Take PQ indefinitely ſmall, then the area SPQ varies as $QT \times SP$, hence, SPQ^2 varies as $QT^2 \times SP^2$, or (814) as $L \times QR \times SP^2$, but (the time being given) QR varies (808) as the force, and about the *ſame* common center the force varies as $\frac{1}{SP^2}$, hence, QR varies as $\frac{1}{SP^2}$, therefore $QR \times SP^2$ is conſtant, hence, when the time is given, SPQ^2 varies as L in *different* conic ſections about the *ſame* common focus. About *different* foci, where the abſolute forces are different, if A = the abſolute force, then the force varies as $\frac{A}{SP^2}$, therefore QR varies as $\frac{A}{SP^2}$, hence, SPQ^2 varies as $A \times L$.

817. As (812) V varies as $\sqrt{\text{force} \times \text{ch. curv}}$ therefore at the ſame diſtance the force being the ſame, we have the velocity (V) in the conic ſection at P : velocity (v) in a circle at the ſame diſtance :: $\sqrt{\frac{2CD^2}{AC}} : \sqrt{2SP}$ (in the ellipſe and hyperbola) $\sqrt{\frac{2SP \times HP}{AC}} : \sqrt{2SP} :: \sqrt{HP} : \sqrt{AC}$. In the *ellipſe*, $HP = 2AC - SP$; hence $V : v :: \sqrt{2AC - SP} : \sqrt{AC}$, which is always leſs than $\sqrt{2} : 1$. If the major axis of the ellipſe be increaſed *ſine limite*, the ellipſe approaches to the parabola as it's limit, and the above ratio approaches to $\sqrt{2} : 1$ as it's limit, hence, in a *parabola*, $V : v :: \sqrt{2} : 1$ (645). In the *hyperbola*, $HP = 2AC + SP$, hence, $V : v :: \sqrt{2AC + SP} : \sqrt{AC}$, which is always greater than $\sqrt{2} : 1$. At the mean diſtance in the ellipſe, $SP = AC$, hence $V = v$ at that point.

818. Let M = the major axis of an ellipſe, N = the minor, P = the periodic time about the focus, m = the whole area, n = the area SPQ deſcribed in a given time, then $m = n \times$ *the number* of theſe areas, but *the number* of theſe areas muſt vary as P, becauſe equal areas are deſcribed in equal times, and therefore the greater P is, the greater will be *the number* of areas in proportion; hence, m varies as $n \times P$; therefore P varies as $\frac{m}{n}$. Now by the property of the

ellipſe,

ellipſe, m varies as $M \times N$, and $L = \frac{N^2}{M}$, therefore (816) n varies as $\frac{N \times A^{\frac{1}{2}}}{M^{\frac{1}{2}}}$, conſequently P varies as $\frac{M^{\frac{3}{2}}}{A^{\frac{1}{2}}}$. Hence, in different ellipſes about the *ſame* common focus, P varies as $M^{\frac{3}{2}}$. This law was diſcovered by KEPLER (218). If the ellipſe become a circle, M becomes the diameter, which varies as the radius R, therefore in *different* circles about *different* centers, P varies as $\frac{R^{\frac{3}{2}}}{A^{\frac{1}{2}}}$, and about the *ſame* center, P varies as $R^{\frac{3}{2}}$.

819. If, by the ſame law of force, bodies revolve in *different* circles about *different* centers, and A = the abſolute force, R = the radius, then the force varies as $\frac{A}{R^2}$, therefore (812) V varies as $\sqrt{\frac{A}{R^2} \times 2R}$, or as $\frac{A^{\frac{1}{2}}}{R^{\frac{1}{2}}}$. Hence about the *ſame* common center, V varies as $\frac{1}{R^{\frac{1}{2}}}$.

820. The angle QSP varies as $\frac{QT}{SP}$, but QT varies as $\frac{\text{area } SPQ}{SP}$, therefore the angle QSP varies as $\frac{\text{area } SPQ}{SP^2}$. Hence, in the *ſame* orbit, the angular velocity varies inverſely as the ſquare of the diſtance, the area SPQ being conſtant (805) when the time is given, let the orbit be what it will.

821. By Art. 817. we have, in the ellipſe, $V : v :: \sqrt{HP} : \sqrt{AC}$, hence, $V = v \times \frac{\sqrt{HP}}{\sqrt{AC}}$; now this is true whatever be the minor axis of the ellipſe; if therefore we diminiſh the minor axis and make it vaniſh, the body will move in a right line, and as all the reaſoning holds true up to that limit, it muſt be true in the limit.

822. Hence, draw Pp perpendicular to AM, and make the minor axis of the ellipſe vaniſh, then S coincides with A, H with M, P with p, HP becomes Mp, and the body deſcends in a right line to p from reſt at M, hence, (821) V (the velocity acquired from M to p) $= v \times \frac{\sqrt{pM}}{\sqrt{AC}}$, but (819) v varies as $\frac{1}{\sqrt{Ap}}$, let $v = \frac{a}{\sqrt{Ap}}$, and $V = \frac{a\sqrt{pM}}{\sqrt{Ap \times AC}}$, that is, the velocity acquired in falling from a ſtate of reſt, is equal to a certain conſtant quantity a multiplied into the ſquare root of the ſpace deſcribed divided by the ſquare root of the ſpace to be deſcribed multiplied into half the whole ſpace.

823. Produce

823. Produce SP to L, and take $PL=PH$, then (822) the velocity (m) of a body falling from L to $P=\frac{a\sqrt{PL}}{\sqrt{SP\times\frac{1}{2}SL}}=\frac{a\sqrt{PH}}{\sqrt{SP\times AC}}$, alſo let $n=\frac{a}{\sqrt{SP}}$ = the velocity in the circle at P, and p = the velocity in the ellipſe, then

$$m : n :: \frac{\sqrt{PH}}{\sqrt{AC}} : 1 : \sqrt{PH} : \sqrt{AC}, \text{ alſo}$$

$$n : p :: \sqrt{AC} : \sqrt{PH}\ (817)$$

$$\therefore m : p :: \sqrt{AC}\times\sqrt{PH} : \sqrt{PH}\times\sqrt{AC}, \text{ hence, } m=p,$$

That is, a body revolving in an ellipſe, muſt fall externally through a ſpace equal to the diſtance of the body from the other focus, to acquire the velocity in the ellipſe.

824. Let $PL=\frac{1}{4}PV$, then a body, with the force at P continued *conſtant*, muſt fall down PL to acquire the velocity in the curve. For let m = the velocity down RQ with the force at P, n = the velocity down PL by the ſame force, p = the velocity in the curve PQ, then in the time RQ is deſcribed by a falling body, PQ is deſcribed in the curve, and the velocity through RQ is (by Mechanics) repreſented by $2RQ$, hence $m : p :: 2RQ : PQ$, therefore

FIG. 189.

$$m^2 : p^2 :: 4RQ^2 : PQ^2,$$

$$\text{Alſo } m^2 : n^2 :: RQ : PL, \text{ by Mechanics;}$$

but $n=p$, therefore $PL=\frac{PQ^2}{4QR}=(811)\ \frac{1}{4}PV$.

825. If the curve be a circle, and the force be in the center, then PL = half the radius. Hence, as (817) the velocity in an ellipſe at the mean diſtance = the velocity in a circle, a body at that point of an ellipſe muſt fall down half the diſtance to acquire the velocity in the ellipſe, the force remaining conſtant.

826. Let $PE=\frac{1}{4}$ of the chord of curvature at the point P of an ellipſe; then (by Conics) $PE=\frac{CD^2}{2AC}$. Now $CD^2=SP\times PH=SP\times\overline{2AC-SP}=2AC\times SP-SP^2$, hence, $\frac{SP^2}{2AC}=\frac{2AC\times SP-CD^2}{2AC}$, alſo $SE=SP-PE=SP-\frac{CD^2}{2AC}=\frac{2AC\times SP-CD^2}{2AC}$, therefore $SE=\frac{SP^2}{2AC}=\frac{SP^2}{SL}$, hence, $SE\times SL=SP^2$.

827. When

FIG. 191

827. When a body revolves about a center of force, that part of it's motion which is perpendicular to the radius vector gives the body a tendency to recede from the center, and the force with which the body thus recedes is called a *centrifugal* force. Let S be the center of force, PK the curve described, PT a tangent to it, SY perpendicular to PT, and PQ an indefinitely ſmall arc, draw Qw perpendicular to SP, and with the center S deſcribe the circular arc Qx, and let RQ be parallel to SP, and PV be the chord of the circle of curvature. Let PQ repreſent the motion of the body in the curve, in a given time, then Pw repreſents that part of the motion which is towards the center, and by which alone the body would be found, at the end of the given time, at the diſtance Sw, but on account of the motion wQ, it is found at the end of the ſame time at the diſtance SQ or Sx, the perpendicular motion wQ has therefore made the body recede from S through a ſpace equal to wx, which therefore repreſents the *centrifugal* force. Alſo the centripetal force is repreſented (808) by QR. Now $wx = \frac{xQ^2}{2xS}$ ultimately, but xQ^2 varies as $\frac{\text{area } SxQ^2}{xS^2}$; therefore wx varies as $\frac{\text{area } SxQ^2}{2xS^3}$ which varies as $\frac{\text{area } SPQ^2}{SP^3}$ ultimately. Hence, in the *ſame* curve, the centrifugal force varies as $\frac{1}{SP^3}$, the area SPQ deſcribed in a given time being given (805). And in *different* curves, if the diſtance be the ſame, the centrifugal forces are as the ſquares of the areas deſcribed in a given time.

828. Hence, the centripetal force in the curve : the centrifugal force :: $QR : xw$ :: (becauſe (811) $QR = \frac{QP^2}{PV}$, and $xw = \frac{xQ^2}{2QS} = \frac{xQ^2}{2PS}$) $\frac{QP^2}{PV} : \frac{xQ^2}{2PS}$ :: (as by ſim. tria. $QP^2 : Qx^2 :: SP^2 : SY^2$) $\frac{SP^2}{PV} : \frac{SY^2}{2SP} :: 2SP^3 : SY^2 \times PV$.

829. Let the curve be an ellipſe whoſe major axis is $2a$, and the excentricity $= w$, and the body be at the greateſt diſtance from the center of force, which is ſuppoſed to be in the focus, then the centripetal force : centrifugal :: $SP : \frac{1}{2}PV :: a+w : \frac{a^2-w^2}{a} :: a : a-w$.

830. If the ratio of $a+mx : b+nx$ be conſtant, x only being variable, then $a : b :: m : n$. For if $x=0$, the ratio becomes $a : b$; hence, $a+mx : b+nx :: a : b$, therefore (alternando and dividendo) $mx : a :: nx : b$, conſequently $a : b :: m : n$.

FIG. 192

831. Let VPA be an ellipſe whoſe focus is S, and center C, KW a curve ſo conſtructed, that Sp may be always equal to SP, and the angle VSp to VSP in a given ratio $G : F$, then the areas VSp, VSP will be in the ſame given ratio.

Now

Now let a body revolve from V to P about the center of force S, in the ſame time in which another body revolves from V to p. Then as the area VSP varies as VSp, and (805) the area VSP varies as the time, the area VSp varies as the time, conſequently the body deſcribing Vp is (807) urged by a force tending to S. Let F be the chord of curvature. Now (827) the centrifugal forces of the two bodies are as $G^2 : F^2$, or as $\frac{G^2}{SP^3} : \frac{F^2}{SP^3}$, let them therefore be repreſented by theſe quantities; hence, the difference of the centrifugal forces is $\frac{G^2-F^2}{SP^3}$. Now if p recede from the center by a centrifugal force which is greater by $\frac{G^2-F^2}{SP^3}$ than that by which P recedes, it is manifeſt that p muſt be acted upon by a centripetal force which is greater by the ſame quantity, in order to deſtroy it, ſo that the bodies may keep at the ſame diſtance. Now (828) $SY^2 \times Pv : 2SP^3 :: \frac{F^2}{SP^3}$ (the centrifugal force in the ellipſe at P) : $\frac{2F^2}{SY^2 \times Pv}$ the centripetal force in the ellipſe at P, hence, the force in the orbit VW at p $= \frac{2F^2}{SY^2 \times Pv} + \frac{G^2-F^2}{SP^3}$. But $SY^2 = \frac{AC \times R \times SP^2}{CD^2}$, and $Pv = \frac{2CD^2}{AC}$, R being the latus rectum, and CD the ſemi-conjugate diameter to PC; hence, the force at $P = \frac{F^2}{R \times SP^2}$, and at $p = \frac{F^2}{R \times SP^2} + \frac{G^2-F^2}{SP^3}$, therefore the ratio of theſe forces is $\frac{F^2}{SP^2} : \frac{F^2}{SP^2} + \frac{RG^2-RF^2}{SP^3}$.

832. If the orbit VPA be very near to a circle, the force $\frac{F^2}{SP^2} + \frac{RG^2-RF^2}{SP^3}$ may be made to vary very nearly as any power of SP, or as SP^{n-3}. For $\frac{F^2}{SP^2} + \frac{RG^2-RF^2}{SP^3} : SP^{n-3} :: F^2 \times SP + RG^2 - RF^2 : SP^n ::$ (putting $T - x = SP$, where T is the greateſt diſtance, and neglecting all the terms where the powers of x enter above the firſt, as being very ſmall when compared with the reſt) $F^2T - F^2x + RG^2 - RF^2 : T^n - nT^{n-1}x$; now (830) this ratio will be conſtant, if we aſſume the conſtant terms on each ſide in the ſame ratio to each other as the coefficients of the variable terms, but if the ratio of the two laſt quantities be conſtant, the ratio of the two firſt muſt be very nearly ſo, as we have only neglected terms which are very ſmall in reſpect to thoſe which are retained; aſſume therefore $F^2T + RG^2 - RF^2 : T^n$ as $-F^2x : -nT^{n-1}x$ or as $F^2 : nT^{n-1}$, and the ratio of the two firſt terms above becomes very nearly conſtant;

conſtant; but as R and T are very nearly equal, the proportion becomes $G^2 : F^2 :: 1 : n$, very nearly, hence $G = \frac{F}{\sqrt{n}}$. Now the two bodies at P, p, being always at the ſame diſtance from S, muſt come to an apſide at the ſame time; but the body in the ellipſe comes to an apſide when $F = 180°$, hence, $G = \frac{180°}{\sqrt{n}}$ the angle deſcribed between the apſides by the body in the curve KW, when the force varies as the $n-3$ power of the diſtance. If $n-3=-2$, the diſtance of the apſides is 180°, if $n-3$ be a negative number *greater* than 2, the diſtance of the apſides is *greater* than 180°, but if it be *leſs* than 2, the diſtance is *leſs* than 180°. That is, if the force vary inverſely as the ſquare of the diſtance, the apſides of the orbit deſcribed are at reſt; if the force vary in a greater or leſs ratio than the inverſe ſquare of the diſtance, the apſides are progreſſive or regreſſive.

833. Let the force vary as $\frac{b \times SP^m \pm c \times SP^n}{SP^3}$, and the greateſt diſtance be aſſumed unity, then by exactly the ſame proceſs, we find $G = 180° \times \sqrt{\frac{b \pm c}{bm \pm cn}}$ the angle between the apſides.

834. In the *former* caſe, if the diſtance ($d°$) of the apſides be given, we can find the law of force, for if $d° = \frac{180°}{\sqrt{n}}$, then $n = \frac{180^2}{d^2}$, conſequently the law of force is $SP^{\frac{180^2}{d^2}-3}$.

835. The concluſions hitherto deduced when the force varies in the inverſe ſquare of the diſtance, have been upon ſuppoſition that the bodies were of indefinitely ſmall magnitudes, we muſt therefore conſider what will be the conſequence if the bodies be ſpherical, and of finite magnitudes.

FIG. 193. 836 Let O be the center of the circle $ABCD$, draw OP perpendicular to the plane, and let P be a corpuſcle attracted to the circle, deſcribe about O the circle vw, and let the attraction of P to any particle $v = \frac{1}{Pv^2}$. Put $PO=a$, $Pv=x$, $p=3,14159$, then $Ov^2 = x^2 - a^2$, and $p \times \overline{x^2 - a^2}$ = the area of the circle vw, hence, the fluxion of that area is $2px\dot{x}$, and by the reſolution of forces, $x : a :: \frac{1}{x^2} : ax^{-1}$ the attraction of P to v in the direction PO; hence, the fluxion of the attraction of the corpuſcle P to the circle vw will be $2pax$, whoſe fluent is $2pax$; but when $x=a$, $vO=0$, and the attraction vaniſhes;

vaniſhes, hence, the fluent corrected, or the attraction of P to the circle vw is $2pax - 2pa^2$, which varies as $1 - \frac{a}{x}$, and when $x = PA$, the attraction to the whole circle varies as $1 - \frac{PO}{PA}$.

837. Let $ABCD$ be a ſphere, P a corpuſcle, draw $PAOC$ through the center O, and let $BvDw$ be a ſection perpendicular to it. Put $AO = a$, $OP = b$, $AP = b - a = c$, $PK = y$, and let $PB = c + x$, then $AK = y - c$, and $CK = 2a - y + c$, therefore $\overline{y-c} \times \overline{2a-y+c} = BK^2 = BP^2 - PK^2 = \overline{c+x}^2 - y^2$, hence, (as $b = a + c$) $y = \frac{2bc + 2cx + x^2}{2b}$; therefore (836) the attraction of P to the circle $BvDw$ is as $1 - \frac{2bc + 2cx + x^2}{2b \times \overline{c+x}}$ or as $\frac{2ax - x^2}{b \times \overline{c+x}}$; alſo $y = \frac{cx + xx}{b}$, hence, the fluxion of the attraction of P to the ſphere is as $\frac{2axx - x^2x}{b^2}$, whoſe fluent is $\frac{ax^2 - \frac{1}{3}x^3}{b^2}$, the attraction to ABD, and when $x = 2a$, the attraction to the whole ſphere becomes $\frac{4a^3}{3b^2}$, which varies as $\frac{a^3}{b^2}$. Now if the denſity d of the ſphere ſhould vary, the attraction muſt, cæteris paribus, vary as d, hence for all ſpheres, the attraction varies as $\frac{da^3}{b^2}$. But the quantity of matter m varies as da^3; therefore the attraction varies as $\frac{m}{b^2}$. Hence, if the ſpheres were evaneſcent in magnitude, with the ſame quantity of matter, the attraction would be the ſame, conſequently the attraction of a corpuſcle to a ſphere is juſt the ſame as if all the matter of the ſphere were collected into it's center. If the corpuſcle be at the ſurface of the ſphere, then $a = b$, and the attraction varies as ad. Fig. 194.

838. Hence, if the particles of two ſpheres A, B, attract each other by a force varying in the inverſe ſquare of the diſtance, the attraction is the ſame as if the whole quantity of matter in each ſphere were collected into it's reſpective center, because the attraction of *each* corpuſcle of one ſphere A to the other ſphere B is the ſame as if the whole quantity of matter in B were concentrated into it's center, and therefore the attraction of the *whole* ſphere A to B muſt alſo be the ſame as if the whole quantity of matter in B were collected into it's center. Hence, what has been proved for two corpuſcles attracting each other when the force varies inverſely as the ſquare of the diſtance, holds true for two ſpheres, the particles of which attract each other ac-

cording to the ſame law. If therefore (815) the particles of two ſpheres attract each other with a force varying in the inverſe ſquare of the diſtance, one ſphere will deſcribe a conic ſection about the other in it's focus. Now all the planets are ſpherical, and (217) they revolve about the ſun in ellipſes having the ſun in the focus of each. Hence, we conclude, that each planet is attracted to the ſun by a force which varies inverſely as the ſquare of the diſtance of their centers, and that the conſtituent particles alſo attract each other by a force varying according to the ſame law.

FIG. 195. 839. A body P attracting another body Q, exerts it's influence equally upon every particle of Q, and therefore the acceleration of Q to P is the ſame whatever be the quantity of matter in Q, and will be in proportion to the quantity of matter in P, the magnitudes of the bodies being ſuppoſed to be indefinitely ſmall, ſo that every particle of matter in one body may be ſuppoſed to be equidiſtant from every particle in the other. In like manner it appears, that the acceleration of P towards Q, from the attraction of Q, is in proportion to the quantity of matter in Q.

840. Let therefore P and Q attract each other, and G be their center of gravity, then the acceleration of Q towards P from the action of P : the acceleration of P towards Q from the action of Q :: P : Q :: GQ : GP; hence, the ſpaces Qa, Pb, moved over by Q and P from their mutual attractions in an indefinitely ſmall time, will be as GQ : GP, conſequently Ga muſt be to Gb in the ſame ratio, therefore G continues to be their center of gravity. Hence, the center of gravity is not affected by the mutual attractions of the bodies.

841. Now let Q and P be projected in the directions Qr, Ps, oppoſite and parallel to each other, with velocities as GQ : GP, or as Ga : Gb, and let Qr, Ps be the ſpaces that would have been deſcribed in the time in which the bodies would have moved over Qa, Pb by their mutual attractions, and complete the parallelograms $Qrqa$, $Pspb$, then the bodies at the end of that time will be found at q and p. Now the ſpaces deſcribed in the ſame time being as the velocities, Qr, or aq : Ps, or bp :: Ga : Gb, alſo the angle $qaG = pbG$, hence, the triangles Gaq, Gbp are ſimilar, conſequently the angle $aGq = bGp$, and therefore pGq is a ſtraight line, alſo Gq : Gp :: Ga : Gb :: GQ : GP :: P : Q, hence, G is the center of gravity of P and Q when they come to p and q, conſequently the center of gravity will ſtill be at reſt, and the bodies will deſcribe ſimilar figures about G.

842. Now let us conceive each body to be acted upon, at the ſame time, by equal accelerative forces in the ſame direction, then the *relative* motions of the two bodies will not be altered, and they will ſtill continue to deſcribe

ſimilar

ſimilar figures about G which is now in motion. And by varying the motion of the ſyſtem, we may vary the abſolute initial velocities of P and Q as we pleaſe. Hence, if P and Q be projected with *any* velocities, they will continue to revolve about their center of gravity, and deſcribe ſimilar figures about it. And the center of gravity not being diſturbed by their mutual attractions (840), will continue to move on uniformly in a ſtraight line. Hence, the center of gravity of the ſolar ſyſtem remains at reſt, or moves uniformly in a ſtraight line, the latter of which is probably the caſe (729). Sir I. NEWTON has therefore concluded, that the earth and moon revolve about their center of gravity, but **FRISI** has maintained, that this will not be the caſe, unleſs the earth and moon had been at firſt projected in oppoſite directions, with velocities inverſely as their quantities of matter; it appears however, from what is proved above, that this is by no means neceſſary.

843. Let the bodies be ſpherical, and the particles attract each other by a force varying in the inverſe ſquare of the diſtance, then (838) the whole attraction will vary inverſely as the ſquare of the diſtance of their centers. Now as the attraction of Q to P varies as $\frac{1}{QP^2}$ it muſt vary as $\frac{1}{QG^2}$, becauſe QP is to QG in a given ratio; and as G is in the line QP, the acceleration of Q towards G muſt be the ſame as towards P, hence, Q is attracted towards G with a force which varies as $\frac{1}{QG^2}$, therefore (815) Q deſcribes about G a conic ſection having G in the focus. For the ſame reaſon, P will deſcribe a ſimilar figure about G in it's focus. Alſo, as $QG : QP$ in a given ratio, and G is always in the line QP, the angular velocity of Q about G muſt be equal to it's angular velocity about P, becauſe, in reſpect to any fixed line LM, QC and QP always make the ſame angle, therefore Q deſcribes a figure about P ſimilar to that which it deſcribes about G, and in the ſame periodic time. Now all the planets are attracted to the ſun by a force varying according to the above law, hence, each planet deſcribes about the center of gravity of itſelf and the ſun, an ellipſe having that center in their focus, except ſo far as they diſturb each other's motions by their mutual attractions.

844. Conceive a body Z to be placed at G, whoſe attraction upon Q ſhall be equal to that of P, then as the attraction varies as the quantity of matter directly and the ſquare of the diſtance inverſely, we have $\frac{Z}{QG^2} = \frac{P}{QP^2}$, hence $Z = P \times \frac{QG^2}{QP^2}$. Now (818) the ſquares of the periodic times of bodies revolving about the focus of an ellipſe vary as the cubes of the major axes directly and the abſolute forces inverſely, therefore, if the periodic time be given, the major

major axis muſt vary as the cube root of the abſolute force. Hence, the major axis of the ellipſe which Q deſcribes about Z : the major axis of the ellipſe which Q would deſcribe about P at reſt in the ſame periodic time :: $P^{\frac{1}{3}} \times \frac{QG^{\frac{2}{3}}}{QP^{\frac{2}{3}}} : P^{\frac{1}{3}} :: QG^{\frac{2}{3}} : QP^{\frac{2}{3}}$, alſo from ſimilar figures, the major axis of Q about P at reſt : the major axis of Q about Z :: $QP : QG$, hence, by compounding theſe ratios, we get the major axis of the ellipſe which Q deſcribes about P when they revolve about the center of gravity : the major axis of the ellipſe which Q would deſcribe about P at reſt in the ſame periodic time :: $QP^{\frac{1}{3}} : QG^{\frac{1}{3}} :: \overline{P+Q}^{\frac{1}{3}} : P^{\frac{1}{3}}$. This is the ſame concluſion as that deduced by Sir I. NEWTON in a very different manner, in his *Principia*, Lib. I. Sect. 2. Pr. 65. Hence, as the quantity of matter in the earth : that of the moon :: 78 : 1, the major axis of the ellipſe which the moon deſcribes about the earth (they revolving about their common center of gravity) : the major axis of the ellipſe which the moon would deſcribe about the earth at reſt in the ſame time :: $79^{\frac{1}{3}} : 78^{\frac{1}{3}}$ :: 429 : 427.

FIG. 196. 845. Let a body E revolve about a body S in a circle, and at the ſame time let a body M revolve about E in a circle, and be carried with E about S, to find the diſturbing force of S upon M revolving about E, ſuppoſing ME to be very ſmall when compared with SE, and the force to vary inverſely as the ſquare of the diſtance. As the relative ſituation of M to E is juſt the ſame as if E was at reſt, let us ſuppoſe E to be at reſt. Produce SAE to D, draw CEB perpendicular to AD, and MK to CB. Now let ES repreſent the attractive force of E to S, then $\frac{1}{SE^2} : \frac{1}{SM^2} :: SE$: the force of M to $S = \frac{SE^3}{SM^2}$; hence, by the reſolution of forces, $SM : SE :: \frac{SE^3}{SM^2}$: the force of S upon M in a direction parallel to ES, which therefore $= \frac{SE^4}{SM^3} = \frac{SE^4}{\overline{SE-MK}^3} = SE + 3MK$, omitting the other terms of the ſeries on account of their ſmallneſs. Hence, as the force of E to S is repreſented by SE, we have $3MK$ for the difference of the forces with which E and M are drawn in directions perpendicular to BC, and therefore it repreſents the diſturbing force of S upon M in that direction; produce therefore KM to r, and take $Mr = 3MK$, and Mr will repreſent this diſturbing force. This force is called the *ablatitious* force, becauſe it tends to draw M *from* E, and in the oppoſite ſemicircle BDC, the force is conceived to act in the contrary direction, becauſe M being there leſs attracted to S than E is, the effect is juſt the ſame as if M were drawn from

from E in the oppoſite direction. Hence, the effect of this force is the ſame in each ſemicircle CAB, BDC, or in oppoſite points of the circle $ABDC$. Alſo $SM : ME :: \frac{SE^3}{SM^2} : \frac{SE^3}{SM^3} \times ME = ME$ very nearly, the diſturbing force of S upon M in the direction ME, this is called the *addititious* force, becauſe it tends to draw M *to* E. Hence, the addititious force : the ablatitious force :: $ME : Mr$, or $3MK$, :: rad : three times the ſine of the angle MEC, the diſtance of M from quadratures. Hence, in ſyzygies, $Mr = 3EM$ or $3EA$, from which, if we take EA the addititious force, there remains $2EA$ for the whole force with which M is drawn from E in ſyzygies.

846. The diſturbing force ME compounded with the attraction of M to E, which varies as $\frac{1}{ME^2}$, makes a force which does not vary as $\frac{1}{ME^2}$, and therefore by altering the law of force it muſt alter the form of the orbit; but becauſe the force ME is directed to E, it will not (805) deſtroy the equal deſcription of areas about E in equal times. But as the diſturbing force Mr, or $3MK$, neither varies as $\frac{1}{ME^2}$, nor is it directed from M to E, it will both deſtroy the form of the orbit and the equal deſcription of areas in equal times. Therefore M will not continue to deſcribe a circle about E

847. Reſolve the addititious force ME into MK, KE, then MK acting in oppoſition to Mr $(= 3MK)$, we get $2MK$ for the whole force with which M is drawn from CB in the direction KM, hence, we may conſider M as acted upon by two diſturbing forces, one of which $= 2MK$ in the direction KM, and the other $= KE$ in a direction perpendicular to KM, the force of S to E being repreſented by SE. Hence, the addititious force at M, and theſe two diſturbing forces, will always be as ME, $2MK$ and KE.

848. Hitherto we have ſuppoſed the plane of the orbit of M to coincide with that of E, but if it do not, then as the force Mr acts *out* of the plane of the orbit of M (except when the nodes lie in the line SAD) it muſt continually draw M *from* the plane of it's orbit, the plane of the orbit therefore continually changing will cauſe a conſtant motion of the nodes, and a variation of the inclination of the orbit, the method of computing which will be afterwards ſhown. Theſe are, in general, the conſequences of the diſturbing forces, the particular effects of which we ſhall now proceed to conſider.

849. Reſolve the force Mr into Ms in the direction of the tangent to the point M, and Mw in the direction EM. Then as the body moves from C to A, the force Ms acting in the direction in which the body moves, muſt accelerate the body, hence, the velocity of the body is accelerated from quadratures

at

at C to ſyzygies at A But whilſt the body moves from A to D, the force Mr acts in a direction contrary to the motion of the body, and therefore it retards the body as much from A to B as it accelerated it from C to A, and the ſame is true for the other ſemicircle BDC. Hence, from quadratures to ſyzygies the body is accelerated, and from ſyzygies to quadratures it is as much retarded, conſequently the velocity is greateſt in ſyzygies and leaſt in quadratures

850. Hence alſo the areas will be accelerated from quadratures to ſyzygies, and retarded from ſyzygies to quadratures, for (809) when the force tends in *conſequentia* the areas are accelerated, and when it tends in *antecedentia* they are retarded. Hence, the areas deſcribed by M about E are greateſt in ſyzygies and leaſt in quadratures.

851 The ablatitious force Mr $(=3MK)$ vaniſhes in quadratures, and the addititious force remaining, the whole force of M to E in quadratures is there the greateſt, and therefore the whole gravity of M to E is increaſed, and in ſyzygies, Mr is the greateſt, and therefore the force of M to E is there the leaſt, and the whole gravity of M to E is diminiſhed; for as Mr is there $=3AE$, if we take from it the addititious force $=AE$, there remains $2AE$ for the whole force with which M is drawn from E. The gravity therefore of M to E is twice as much diminiſhed in ſyzygies as increaſed in quadratures; and in a whole revolution, the gravity of M to E is diminiſhed

852. As the force of M to E is greateſt in quadratures, the ſagitta rt, in a given time, is greateſt in thoſe points, and as the velocity is there leaſt, the arc Mt deſcribed muſt be the leaſt, hence, the curvature of the orbit is there the greateſt, the curvature being as the ſagitta directly and the ſquare of the arc inverſely. And as the force, and conſequently the ſagitta, is leaſt in ſyzygies, and the velocity, and conſequently the arc, is greateſt, the curvature of the orbit muſt there be the leaſt. Hence, the orbit muſt put on the form of an oval, whoſe longeſt axis paſſes through quadratures, and the ſhorteſt through ſyzygies; conſequently the body M muſt recede further from E in quadratures than in ſyzygies. It will afterwards be ſhown, that the orbit is very nearly an ellipſe. This elliptic form of the orbit, and the above-mentioned (849) acceleration and retardation of velocity, cauſe, when applied to the moon, an inequality in it's motion, called it's *variation*

853. If ES be diminiſhed, the action of S on E and M will be increaſed, and their difference, or the diſturbing forces, will be increaſed, and as the gravity of M to E is, in a whole revolution, diminiſhed by the diſturbing forces, that diminution muſt be increaſed as E approaches to S, conſequently the radius ME will be increaſed Now in different circles, the periodic time varies in the ſeſquiplicate ratio of the radius directly, and the ſquare root of the abſolute force inverſely (818), hence, as, when E approaches to S, the radius

is

is increaſed and the force diminiſhed, the periodic time muſt increaſe. Therefore if E revolve about S in an ellipſe having S in it's focus, when E is in the higher apſide, the radius ME and the periodic time will be the leaſt, and when E is in the lower apſide, ME and the periodic time will be the greateſt. How the periodic time is altered by the alteration of the orbit of E from a circle to an ellipſe, will be afterwards ſhown. Theſe are the effects of the diſturbing forces of S upon M, on ſuppoſition that the orbit of M would (independent of the diſturbing forces) have been a circle.

854. As the moon's orbit is very nearly a circle, ſimilar effects will take place upon it's orbit, from the action of the ſun; that is, when the earth is neareſt to the ſun, the moon's orbit will be dilated, and the periodic time ~~increaſed~~, and when the earth is furtheſt from the ſun, the orbit will be contracted, and the periodic time diminiſhed. Hence, the moon's periodic time and diſtance is ~~leaſt~~ in ſummer, and greateſt in winter. We come now to conſider the effect of the diſturbing forces upon an elliptic orbit *CABD*.

855. Let M move in an ellipſe about E in it's focus, put $a=ME$, and let a repreſent the addititious force, and let the natural gravity of M to $E=\frac{b}{a^2}$. Now, in quadratures, the whole force (845) of M to $E=\frac{b}{a^2}+a=\frac{ba+a^4}{a^3}$; hence, with that force (833) the diſtance of the apſides $=180° \times \sqrt{\frac{b+1}{b+4}}$, which is leſs than 180°, becauſe $\sqrt{\frac{b+1}{b+4}}$ is leſs than unity, therefore the apſides are regreſſive when the body is in quadratures. Now in ſyzygies, the whole force of M to $E=\frac{b}{a^2}-2a=\frac{ba-2a^4}{a^3}$ (845), therefore (833) the diſtance of the apſides $=180° \times \sqrt{\frac{b-2}{b-8}}$, which is greater than 180°, becauſe $\sqrt{\frac{b-2}{b-8}}$ is greater than unity; hence, the apſides are progreſſive. But as the force $2a$ which cauſes the progreſſive motion in ſyzygies is double of the force a which cauſes the regreſſive motion in quadratures, the progreſſive motion in ſyzygies is greater than the regreſſive motion in quadratures. In the points between quadratures and ſyzygies, it is manifeſt that theſe principles cannot be applied to find the motion of the apſides, becauſe the force does not vary as any power of the diſtance; and moreover, there is a force Ms acting perpendicularly to the radius, which alſo produces a motion of the apſides, we ſhall afterwards ſhow upon what principles we may compute the whole motion.

856. As

856. As the attractive force varies inverſely as the ſquare of the diſtance, we may repreſent the attractive force of E to S by $\frac{1}{SE^2}$, and of M to S by $\frac{1}{SM^2}$, hence, $SM \quad SE \quad \frac{1}{SM^2}$ that part of the force of S upon M, which acts parallel to $ES = \frac{SE}{SM^3} = \frac{SE}{\overline{SE - MK}^3} = \frac{1}{SE^2} + \frac{3MK}{SE^3}$, omitting the other terms of the ſeries on account of their ſmallneſs; hence, the difference of the forces of S upon M and E in a direction perpendicular to CB, or the ablatitious force, is $= \frac{3MK}{SE^3}$; if, therefore the poſition of M be given, and SE vary, the ablatitious force varies as $\frac{1}{SE^3}$; but if the poſition of M be given, the ratio of the ablatitious to the additious force is given (845). Hence, if the poſition of M be given, and SE vary, the diſturbing forces will vary as $\frac{1}{SE^3}$, and if the abſolute force (A) of S ſhould vary, the diſturbing forces will vary as $\frac{A}{SE^3}$. But if P = the periodic time of E about S, then (818) $\frac{1}{P^2}$ varies as $\frac{A}{SE^3}$. Alſo, if d = the diameter of the body S, and m = it's denſity, then A varies as $d^3 \times m$, hence, the diſturbing forces vary as $\frac{d^3}{SE^3} \times m$, or as the cube of the apparent diameter of S ſeen from E, and it's denſity conjointly. Hence, if the abſolute force of S and the diſtance SE vary, the diſturbing forces, and conſequently the errors produced by them, vary as $\frac{1}{P^2}$, or they vary as the cube of the apparent diameter of S ſeen from E; and the denſity of S conjointly. Now theſe are the *linear* errors of M ſeen from E; and as ME is conſtant, the *angular* errors will vary in the ſame ratio.

857. Now let us ſuppoſe ME only to vary, or which is the ſame, conceive two orbits to be deſcribed by M, and M to be ſimilarly ſituated in them; then the additious force varies as ME (845), and in any given poſition of M, the additious force being to the ablatitious in a given ratio (845), both the diſturbing forces, and conſequently the linear errors generated by them in a given time, will vary as ME. Hence, conſidering two different radii ME, if in any given poſition of M, we ſuppoſe the bodies to deſcribe a given indefinitely ſmall angle about E, the linear errors generated in that time will be as the force ME × the ſquare of the time, but the times of deſcribing equal angles about E will be as the whole periodic times (p), hence, the linear errors will be as ME

$ME \times p^2$, and as the ſame is true for every indefinitely ſmall ſimilar parts of the circles, the linear errors in a whole revolution will vary as $ME \times p^2$, but the angular errors are as the linear errors directly and the radius ME inverſely, therefore the angular errors vary as p^2. Hence, from this and the laſt Article, if ME, SE, and the abſolute force of S vary, the angular errors will vary as $\frac{p^2}{P^2}$. Or if, for $\frac{1}{P^2}$ we ſubſtitute $\frac{A}{SE^3}$, the angular errors vary as $\frac{A \times p^2}{SE^3}$, and if A be given, they vary as $\frac{p^2}{SE^3}$ Now the error generated in any given time $\times$ the number of thoſe times in a revolution, or $\times\, p$, (for that number is in proportion to the time of a revolution,) muſt be as the whole error in the time of a revolution, or as $\frac{p^2}{P^2}$, hence, by dividing by p, the errors in a given time will vary as $\frac{p}{P^2}$ Hence, the mean motion of the apſides of the orbit deſcribed by M will vary as the mean motion of the nodes, and each will vary as $\frac{p}{P^2}$, the excentricity and inclination being ſmall, and remaining the ſame.

858. By Art. 845 the addititious force ME : the force of E to S :: $ME : SE$, and the forces of bodies revolving in different circles being as their radii directly and the ſquares of the periodic times inverſely[*], we have, the force of E to S : the mean force of M to E :: $\frac{SE}{P^2}$: $\frac{ME}{p^2}$, hence, by compounding theſe two proportions, the addititious force : the mean force of M to E :: $p^2 : P^2$. Hence, if $p = 27d.\ 7h\ 43'$, and $P = 365d.\ 6h\ 9'$, we get the addititious force of the moon : it's mean force towards the earth :: 1 : 178,725 Now (845) the ablatitious force : the addititious :: $3MK : ME$, hence, this, compounded with the laſt proportion, gives the ablatitious force : the mean force of the moon to the earth :: $3MK : ME \times 178{,}725$ :: $MK : ME \times 59{,}575$ But Ms : the ablatitious force Mr :: $KE : ME$, hence, Ms : the mean force of the moon to the earth :: $MK \times KE : ME^2 \times 59{,}575$. If $P : p :: 1 : n$, we have the addititious force : the mean force of the moon to the earth :: $n^2 : 1$, and the ablatitious force : that force :: $3n^2 MK : ME$.

859. It

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[*] For (810) F varies as $\frac{V^2}{R}$, becauſe in this caſe R = half the chord of curvature, but if P be the periodic time, V varies as $\frac{\text{circum.}}{P}$ which varies as $\frac{R}{P}$, hence, by ſubſtitution, F varies as $\frac{R}{P^2}$.

859. It will be proved in the next Chapter, that the nodes of the orbit deſcribed by the body M, have a retrograde motion. Hence, if inſtead of one body M, we ſuppoſe the whole circumference of the circle to be filled with bodies, the ſame effect will be produced on each, and the nodes of the orbit of each will have a retrograde motion, if therefore we ſuppoſe theſe bodies to be connected together, ſo as to form a ſolid ring, the nodes of this ring will have a retrograde motion. Hence, if this ring were joined to a ſpherical body, ſo that it's plane might paſs through it's center, the nodes of that plane would have a retrograde motion, but leſs than before, inaſmuch as the force which cauſes that motion would have a greater quantity of matter to move, and therefore the motion would be diminiſhed as the inertia was increaſed. Now the earth, from it's rotation about it's axis, has it's equatorial diameter greater than it's polar, from the centrifugal force of it's parts; hence, the exceſs of the quantity of matter in the earth about the equator above that in the ſphere whoſe diameter is the earth's polar diameter, anſwers to the aforementioned ring, conſequently the attraction of the ſun and moon upon this exceſs of matter at the equator muſt cauſe a motion of the equinoctial points upon the ecliptic, called the *Preceſſion of the Equinoxes*, the quantity of which will be afterwards inveſtigated.

860 Now let us ſuppoſe $CABD$ to be a globe revolving about an axis perpendicular to that ſection, and conceive a canal to be cut upon it's ſurface in this ſection, and to be filled with a fluid. Then this fluid will be accelerated from quadratures to ſyzygies, and retarded from ſyzygies to quadratures, by the force Ms Hence, there will be an accumulation of the fluid in ſyzygies, which will cauſe it to riſe higher in thoſe points, and the fluid being drawn from quadratures, it will there be depreſſed. But in conſequence of the motion acquired, the water will flow beyond ſyzygies, and will continue to riſe, until it's motion be deſtroyed by the force Ms in the next quadrant; the fluid will therefore continue to riſe to ſome diſtance beyond the ſyzygies, and the higheſt point of the fluid will be beyond the ſyzygies, and the loweſt point beyond the quadratures. Thus the fluid will be higheſt and loweſt at the ſame time, at the diſtance of about 90°. If we ſuppoſe the body $CABD$ to repreſent the earth, and S, the ſun or moon, it is manifeſt that their attraction will cauſe the ſame effect upon the water which covers it's ſurface, and the ſyzygies being that meridian on which the ſun or moon is, it follows that the time of the high tide from the ſun or moon will be ſome time after they have paſſed the meridian, and the low water about ſix hours after. When the ſun and moon are in conjunction or oppoſition, it is manifeſt that they will both tend to raiſe the water at the ſame place, and therefore there will then be the higheſt tide, and when they are in quadratures,

quadratures, one tends to raiſe the water where the other tends to depreſs it, and therefore the tides will then be the leaſt. Theſe are the general principles of the tides, which will be further explained in a future Chapter.

861. In Art. 845, &c. we have ſuppoſed that M revolves about E at reſt, whereas it is proved (842) that two bodies revolving and attracting each other, will revolve about their center of gravity; but the motions of M and E, (which we will ſuppoſe to repreſent the moon and earth) about that center are diſturbed by ſimilar forces by S repreſenting the ſun; and if the ſum of theſe forces be referred to the moon, and the earth be ſuppoſed at reſt, their effect in diſturbing the relative ſituation of the earth and moon will be the ſame as if the reſpective effects on each took place, and they revolved about their center of gravity G. For here the addititious force of S on M to the center G is MG, GS repreſenting the force of G to S, alſo EG will repreſent the addititious force of E to G. Hence, the ſum of theſe forces $=ME$, the whole addititious force by which the tendency of M and E towards each other is increaſed, which is the ſame quantity as when E was at reſt. Alſo the ſum of the ablatitious forces in the former caſe is equal to the ablatitious force in the latter, for the diſtance of each body from quadratures being the ſame, the ratio (845) of the ablatitious to the addititious forces are the ſame, and therefore the *ſum* of the ablatitious forces of M and E when revolving about G (for they act in oppoſite directions) is equal to the ablatitious force of M when revolving about E at reſt. In the theory of the moon therefore, we conſider the moon to revolve about the earth at reſt, and refer all the diſturbing force to the moon. If E and M repreſent the maſſes of the earth and moon, then $\frac{E}{EM^2}$ repreſents the attraction of M to E, and $\frac{M}{EM^2}$ repreſents the attraction of E to M, hence, M is ſuppoſed to be attracted to E at reſt by the force $\frac{M+E}{EM^2}$.

FIG. 197.

CHAP.

CHAP. XXXII.

ON THE THEORY OF THE MOON.

Art. 862. IN the laſt Chapter we explained the general principles of the diſturbing force of the ſun upon the moon, and the nature of the effects produced by it; but to enter into a computation of all theſe effects, would require an inveſtigation of the nature of the curve deſcribed by a body attracted to two points, called the *Problem of the Three Bodies.* This problem has been ſolved by M. Clairaut, in a Treatiſe, entitled, *Theorie de la Lune*, by M. Euler, in his *Theoria Motuum Lunæ*, by M. d'Alembert in his *Recherches ſur differens Points importans du Syſtême du Monde*, by Friſi in his *Coſmographiæ Phyſicæ et Mathematicæ, pars prior.*; and by T. Mayer in his *Theoria Lunæ, juxta Syſtema Newtonianum.* Mr. T. Simpson began a *Theory of the Moon*, but left it unfiniſhed. M. Clairaut at firſt objected to Sir I. Newton's law of gravity, that it would not account for the motion of the apſides of the moon's orbit; he afterwards, however, diſcovered his error, and found that it would account, not only for that motion, but for all the lunar irregularities, and he was the firſt perſon who gave a complete Theory of the moon. But Mayer's Tables, with the laſt corrections by C. Mason, under the direction of Dr. Maskelyne, are the moſt accurate of any, and are ſubject to no greater error than about 30″ in longitude. Sir I. Newton firſt gave a Theory of the moon from the principles of gravity, and by very ingenious artifices, he found ſome of the principal equations, but his indirect method did not carry him to many of the ſmaller equations, ſo that his computations could not be depended upon to give the moon's place nearer than 5′ or 6′. Others have attempted the ſame by indirect methods, of whom Friſi has been the moſt ſucceſsful. As it would not be conſiſtent with the plan of this Work to give a complete Theory of the moon, we ſhall explain ſuch parts thereof as can properly be here introduced, in which we ſhall principally follow the indirect methods of Newton and Friſi; and although the concluſions thus deduced are not always ſo accurate as thoſe which are derived from a direct ſolution of the problem, yet they give the true arguments, and their coefficients to a very conſiderable degree of accuracy. This method of treating the ſubject has this advantage, that it points out more clearly the particular cauſes of the ſeveral equations ſo deduced, which are not obvious

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in the equations derived from the general ſolution. They who wiſh to ſee more on this ſubject, muſt conſult the Treatiſes above-mentioned.

863. As the attractive force varies inverſely as the ſquare of the diſtance, let $\frac{1}{SM^2}$ repreſent the attraction of the moon M towards the ſun S, then $\frac{1}{SE^2}$ will be the attraction of the earth E to S. Reſolve the force $\frac{1}{SM^2}$ into the directions ME, ES, then, $SM : SE :: \frac{1}{SM^2} : \frac{SE}{SM^3} = \frac{SE}{\overline{SE-MK}^3} = \frac{1}{SE^2} + \frac{3MK}{SE^3}$, the force with which S acts upon M in a direction parallel to ES, from which ſubtract $\frac{1}{SE^2}$, the force of E to S, and we have $\frac{3MK}{SE^3}$ $(=Mr)$ the force with which S draws M from E in the direction KM. Hence, by ſimilar triangles, rMs, MEK, $ME : KE :: \frac{3MK}{SE^3} (=Mr) : \frac{3MK \times KE}{ME \times SE^3} = Ms$ the part of the force Mr which acts perpendicularly to EM; alſo, $ME : MK :: \frac{3MK}{SE^3} : \frac{3MK^2}{ME \times SE^3} = Mw$ the part of the force Mr which acts in the direction EM, but $SM : EM :: \frac{1}{SM^2}$ (the force of S upon M) : $\frac{EM}{SM^3}$ the additious force, or the force of S upon M in the direction ME, hence, the whole diſturbing force of S upon M in the direction ME is $\frac{EM}{SM^3} - \frac{3MK^2}{ME \times SE^3}$ $=$ (as $\frac{MK^2}{EM} = EM \times \frac{MK^2}{EM^2} = EM \times \overline{\text{ſin } MEC}^2$) $\frac{EM}{SM^3} - \frac{3EM}{SE^3} \times \overline{\text{ſin. } MEC}^2 = \frac{EM}{SM^3} - \frac{3EM}{SE^3} \times \overline{\tfrac{1}{2} - \tfrac{1}{2} \text{ coſ } 2MEC} = -\frac{EM}{2SE^3} + \frac{3EM}{2SE^3} \times \text{coſ } 2MEC$ very nearly.

864. Let the periodic time of M about E be to that of E about S as $n : 1$; then the orbits being ſuppoſed to be very nearly circular, we have (858) the additious force $\frac{EM}{SM^3}$: the mean force of M to $E :: n^2 : 1$. Let the mean force of M to E, at the mean diſtance unity, be repreſented by unity, and we get $n^2 = \frac{1}{SM^3} = \frac{1}{SE^3}$ nearly, hence, the diſturbing force of S upon M in the direction ME is $-\tfrac{1}{2} n^2 \times EM + \tfrac{3}{2} n^2 \times EM \times \text{coſ. } 2MEC$. Now in a whole revolution,

FIG. 196.

revolution, the laſt term is deſtroyed by the oppoſition of it's ſigns, and the mean value of EM being unity, we get $-\frac{1}{2}n^2$ for the mean diſturbing force of S upon M in the direction ME. Now in order to obtain the force of gravity of the moon towards the earth, upon the ſuppoſition already made, we muſt conſider, that when we aſſume the periodic time of the moon to be n, that is the periodic time correſponding to the *mean* force of the moon towards the earth, which force is equal to the natural gravity of the moon towards the earth, diminiſhed by the mean diſturbing force of S upon M in the direction EM, that is, the mean force of gravity of the moon towards the earth $-\frac{1}{2}n^2 = 1$, the mean force of M to E, hence, the mean force of gravity of the moon towards the earth, at the mean diſtance unity, $= 1 + \frac{1}{2}n^2$; conſequently, that force at any other diſtance EM is $\frac{1+\frac{1}{2}n^2}{EM^2}$*. Hence, the whole force of the moon towards the earth $= \frac{1+\frac{1}{2}n^2}{EM^2} - \frac{1}{2}n^2 \times EM + \frac{3}{2}n^2 \times EM \times \text{coſ. } 2MEC$. Put $\frac{n^2}{1+\frac{1}{2}n^2} = m$, and the whole force becomes proportional to $\frac{1}{EM^2} - \frac{1}{2}m \times EM + \frac{3}{2}m \times EM \times \text{coſ } 2MEC$, in which expreſſion, unity repreſents the force of gravity of the moon towards the earth at the mean diſtance, and the other quantities repreſent the proper proportional diſturbing forces.

865. If the diſturbing force be aſſumed $\frac{EM}{SE^3} - \frac{3EM}{SE^3} \times \overline{\text{ſin. } MEC}^2$, then, upon the ſame ſuppoſition, the whole force will be $\frac{1}{EM^2} + m \times EM - 3m \times EM \times \overline{\text{ſin } MEC}^2$. Alſo, the quantity $\frac{3MK \times KE}{ME \times SE^3} = \frac{3n^2 MK \times KE}{ME}$, which repreſented the force acting in the direction Mr, muſt now be repreſented by $\frac{1}{1+\frac{1}{2}n^2} \times \frac{3n^2 MK \times KE}{ME} = \frac{3m \times MK \times KE}{ME}$.

866. If the velocity with which the moon was projected at the mean diſtance unity, be u, then at any other diſtance EM, it would be $\frac{u}{ME}$ nearly, in an orbit very nearly a circle, ſuppoſing that there was no tangential force. Now the force at the mean diſtance $= 1 - \frac{1}{2}m$, and (825) a body muſt fall down half that diſtance $(\frac{1}{2})$ to acquire the velocity in the circle, hence, by the laws of falling bodies, $u = \sqrt{2 \times \overline{1-\frac{1}{2}m} \times \frac{1}{2}} = \sqrt{1-\frac{1}{2}m} = 1 - \frac{1}{4}m$ nearly.

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* Frisi, and other Writers, make this force $= \frac{1}{EM^2}$. For this correction we are indebted to Dr Maskelyne.

nearly. Let v be the velocity of the moon at M; then as the force (865) which acts upon the moon at M in the direction of the tangent is $\frac{3m \times MK \times KE}{ME}$, we have, by the principles of motion, $v\dot{v} = \frac{3m \times MK \times KE}{ME} \times \dot{\overline{CM}} =$ (if we put $MK = x$, and aſſume $ME = 1$, which we may here conſider conſtant, without producing any ſenſible error) $3mx\dot{x}$, hence $v^2 = 3mx^2 + Cor.$ But (866) at the mean diſtance, the velocity $v = 1 - \frac{1}{4}m = u$, hence, $v^2 = \frac{u^2}{ME^2} + 3mx^2 = \frac{u^2}{ME^2} + 3m \times ME^2 \times \frac{x^2}{ME^2} = \frac{u^2}{ME^2} + 3m \times ME^2 \times \overline{\text{ſin}\ MEC}^2 =$ (as $\overline{\text{ſin.}\ MEC}^2 = \frac{1}{2} - \frac{1}{2}$ coſ $2MEC$) $\frac{u^2}{ME^2} + \frac{3}{2}m \times ME^2 - \frac{3}{2}m \times ME^2 \times$ coſ. $2MEC$.

867 Thus far we have conſidered the velocity of M in reſpect to S as fixed, but as S is in motion, let us put $d : 1 ::$ ſynodic revolution of the moon : it's periodic time; then at any angular diſtance CEM from quadratures, the moon has actually deſcribed the angle $d \times CEM$ from the time it was in quadratures. Hence, we muſt write $d \times \dot{\overline{CM}}$ for $\dot{\overline{CM}}$, conſequently $v^2 = \frac{u^2}{ME^2} + 3m \times d \times ME^2 \times \overline{\text{ſin.}\ MEC}^2 =$ (if $s =$ ſine of MEC) $\frac{u^2}{ME^2} + 3m \times d \times ME^2 \times s^2$; and (as the ſecond term is very ſmall compared with the firſt) $v = \frac{u}{ME} + \frac{3}{2}m \times d \times ME^2 \times s^2$ very nearly.

On the Radius Vector of the Moon's Orbit, and the Equation of it's Center.

868. Let AGQ be a ſemi-ellipſe, F it's center, E, L, it's foci, FG it's ſemi-axis minor, and draw EM, EG, and MN perpendicular to AQ. Put $FA = 1$; $\angle AEM = z$, $FG = c$, $FE = w$. By the property of the ellipſe, $AF^2 : FG^2 = EG^2 - EF^2 = AF^2 - EF^2 :: AN \times NQ = \overline{AF - FN} \times \overline{AF + FN} = AF^2 - FN^2 : NM^2 :: AF^2 - \overline{EN \mp EF}^2 : NM^2$, to the value of NM^2 found from hence, add EN^2, and extract the ſquare root, and we get $EM = FA \pm \frac{EF \times EN}{FA} - \frac{EF^2}{FA}$; but $\pm EN = EM \times$ coſ. MEA, hence, $EM - \frac{EF \times EM \times \text{coſ}\ MEA}{FA} = \frac{FA^2 - EF^2}{FA} = \frac{FG^2}{FA}$, therefore $EM =$ FIG. 198.

$\frac{FG^2}{\overline{FA - FE \times \text{cof. } AEM}} = \frac{c^2}{\overline{1 - w \times \text{cof. } z}} =$ (by diviſion) $c^2 \times \overline{1 + w \times \text{cof. } z + w^2 \times \overline{\text{cof. } z}^2 + w^3 \times \overline{\text{cof. } z}^3 + \&c.}$ but $\overline{\text{cof. } z}^2 = \frac{1}{2} + \frac{1}{2}$ cof. $2z$, $\overline{\text{cof. } z}^3 = \frac{3}{4}$ cof. $z + \frac{1}{4}$ cof. $3z$, &c. alſo $c^2 = 1 - w^2$, hence, $EM = 1 - \frac{1}{2} w^2 + \overline{w - \frac{3}{4} w^3} \times \text{cof. } z + \frac{1}{2} w^2 \times \text{cof. } 2z + \frac{1}{4} w^3 \times \text{cof. } 3z + \&c.$ If the excentricity be very ſmall, $EM = 1 + w \times \text{cof. } z$ nearly, and the variation of EM is nearly in proportion to cof. AEM.

869. Draw Ev indefinitely near to EM, and with radius $Ez = \sqrt{AF \times FG}$, deſcribe the circle zv, and take the area $zEw = AEM$, and ſuppoſe a body to revolve from A to M about E, then the angle AEM is the true anomaly, and AEw (Note to Art. 227.) is the mean anomaly; hence, the mean anomaly : the true anomaly :: $\angle zEw$: $\angle zEv$ :: ſector zEw : ſector zEv :: area AEM : area zEv, therefore the increment of the mean anomaly $(\dot{Z})$: the increment of the true anomaly $(\dot{z})$ :: area EMm : area Evn :: EM^2 : $Ev^2 = Ez^2 = c$, hence, $\dot{Z} = \frac{EM^2}{c} \times \dot{z} =$ (by ſubſtituting for EM it's value $1 + w \times \text{cof. } z + w^2 \times \overline{\text{cof. } z}^2 + w^3 \times \overline{\text{cof. } z}^3 + \&c.$ by Article 868.) $c^3 \times \dot{z} \times \overline{1 + 2w \times \text{cof. } z + 3w^2 \times \overline{\text{cof. } z}^2 + 4w^3 \times \overline{\text{cof. } z}^3 + \&c.} =$ (by ſubſtituting for c^3 it's value $1 - \frac{3}{2} w^2$ nearly, and putting for $\overline{\text{cof. } z}^2$, $\overline{\text{cof. } z}^3$, &c. their values as before) $\dot{z} + 2w \times \text{cof. } z \times \dot{z} + \frac{3}{2} w^2 \times \text{cof. } 2z \times \dot{z} + w^3 \times \text{cof. } 3z \times \dot{z} + \&c.$ whoſe fluent is $Z = z + 2w \times \text{ſin. } z + \frac{3}{4} w^2 \times \text{ſin. } 2z + \frac{1}{3} w^3 \times \text{ſin. } 3z + \&c.$ the mean anomaly in terms of the true.

870. Given the mean anomaly, to find the true. By the laſt Article, $z = Z - 2w \times s.\, z - \frac{3}{4} w^2 \times s.\, 2z - \frac{1}{3} w^3 \times s.\, 3z$, omitting the terms which come after; hence, $z = Z - 2w \times s.\, z$ nearly, $= Z - 2w \times s.\, Z$ nearly, alſo $z = Z - 2w \times s.\, z - \frac{3}{4} w^2 \times s.\, 2z$ more nearly; ſubſtitute in the ſecond term of this equation, the above value of z, and in the third term, ſubſtitute Z for z, and $z = Z - 2w \times \text{ſin. } (Z - 2w \times s.\, Z) - \frac{3}{4} w^2 \times s.\, 2Z$. Now in the given equation, ſubſtitute in the ſecond term $(-2w \times s.\, z)$ the laſt value of z, and in the third term $(-\frac{3}{4} w^2\, s.\, 2z)$ ſubſtitute $Z - 2w \times \text{ſin. } Z$ for z, alſo in the laſt term $(-\frac{1}{3} w^3 \times s.\, 3z)$ ſubſtitute Z for z, and we get $z = Z - 2w \times \text{ſin. } (Z - 2w \times s.\, \overline{Z - 2w \times s.\, Z} - \frac{3}{4} w^2 \times s.\, 2Z) - \frac{3}{4} w^2 \times \text{ſin. } (2Z - 4w \times s.\, Z) - \frac{1}{3} w^3 \times \text{ſin. } 3Z$. Now as $4w \times \text{ſin. } Z$ is very ſmall compared with $2Z$, we have $-\frac{3}{4} w^2 \times \text{ſin. } (2Z - 4w \times s.\, Z) = -\frac{3}{4} w^2 \times \text{ſin. } \overline{2Z - 4w \times \text{ſin. } Z \times \text{cof. } 2Z} = -\frac{3}{4} w^2 \times \text{ſin. } 2Z + 3w^3 \times \text{ſin. } Z \times \text{cof. } 2Z =$ (as $\text{ſin. } Z \times \text{cof. } 2Z = \frac{1}{2} \text{ſin. } 3Z - \frac{1}{2} \text{ſin. } Z$) $- \frac{3}{4} w^2 \times \text{ſin. } 2Z + \frac{3}{2} w^3 \times \text{ſin. } 3Z - \frac{3}{2} w^3 \times \text{ſin. } Z$. In like manner, $-2w \times \text{ſin. } (Z - 2w \times \text{ſin. } Z) =$ (by conſidering cof. $2w \times \text{ſin. } Z = 1$) $-2w \times \text{ſin. } Z + 4w^2 \times \text{ſin. } Z \times \text{cof. } Z = -2w \times \text{ſin. } Z + 2w^2 \times \text{ſin. } 2Z$. Hence, $-2w \times \text{ſin. } (Z - 2w \times \text{ſin. } \overline{Z - 2w \times \text{ſin. } Z} - \frac{3}{4} w^2 \times \text{ſin. } 2Z) = -2w \times \text{ſin. } (Z - 2w$

× ſin.

$\times\,\text{ſin}\,Z+\frac{5}{4}w^2\times\text{ſin.}\,2Z)=-2w\times\text{ſin}\,(Z-[2w\times\text{ſin.}\,Z-\frac{5}{4}w^2\times\text{ſin}\,2Z])=-2w\times\text{ſin}\,Z\times\text{coſ.}(2w\times\text{ſin.}\,Z-\frac{5}{4}w^2\times\text{ſin.}\,2Z)-\text{coſ.}\,Z\times\text{ſin.}(2w\times\text{ſin.}\,Z-\frac{5}{4}w^2\times\text{ſin}\,2Z)$ $=$ [as coſ $(2w\times\text{ſin.}\,Z-\frac{5}{4}w^2\times\text{ſin.}\,2Z)=1-2w^2\times\overline{\text{ſin}\,Z}^2$ nearly, and ſin. $(2w\times\text{ſin.}\,Z-\frac{5}{4}w^2\times\text{ſin}\,2Z)=2w\times\text{ſin}\,Z-\frac{5}{4}w^2\times\text{ſin.}\,2Z$ nearly] $-2w\times\text{ſin}\,Z+4w^3\times\overline{\text{ſin.}\,Z}^3+4w^2\times\text{ſin}\,Z\times\text{coſ}\,Z-\frac{5}{2}w^3\times\text{coſ}\,Z\times\text{ſin}\,2Z=$ [as $\text{ſin}\,Z^3=\frac{3}{4}\,\text{ſin}\,Z-\frac{1}{4}\,\text{ſin.}\,3Z$, $\text{ſin.}\,Z\times\text{coſ}\,Z=\frac{1}{2}\,\text{ſin}\,2Z$, and $\text{coſ.}\,Z\times\text{ſin.}\,2Z=\frac{1}{2}\,\text{ſin.}\,3Z+\frac{1}{2}\,\text{ſin}\,Z$] $-2w\times\text{ſin.}\,Z+\frac{7}{4}w^3\times\text{ſin.}\,Z+2w^2\times\text{ſin}\,2Z-\frac{9}{4}w^3\times\text{ſin.}\,3Z$. Theſe quantities being ſubſtituted into the firſt equation, and the like terms being collected together, we get $z=Z-2w\times\text{ſin.}\,Z+\frac{5}{4}w^2\times\text{ſin.}\,2Z-$
$+\frac{1}{4}w^3\times\text{ſin.}\,Z$
$\frac{13}{12}w^3\times\text{ſin}\,3Z$ the true anomaly Thus we may find the true anomaly from the mean, for all the planets; but this rule is not ſo well adapted for calculation, as thoſe given in the *tenth* Chapter If the excentricity be very ſmall, the equation of the center becomes nearly $=-2w\times\text{ſin}\,Z=-2w\times\text{ſin}\,z$ nearly, hence, the equation is in proportion to the ſine of the true anomaly nearly.

871. The firſt of theſe equations becomes a maximum at 90° from the apſides; the ſecond, at the octants from the apſides, and the third, at the diſtance of 30° from the apſides. As the firſt equation is the principal one, the whole equation muſt be a maximum when Z is nearly 90°, or $90°+e$, e being very ſmall, hence, $\text{ſin.}\,Z=1-\frac{1}{2}e^2$ nearly, $\text{ſin.}\,2Z=-2e$ nearly and $\text{ſin.}\,3Z=-1$ very nearly; by ſubſtitution therefore we get the equation when a maximum $=-2w+we^2+\frac{w^3}{4}-\frac{5w^2e}{2}-\frac{13w^3}{12}$ very nearly, make the fluxion of this $=0$, and we find $e=\frac{5w}{4}$, hence, the greateſt equation is $-2w-\frac{115w^3}{48}$. For the moon, $w=,05505$, hence, the greateſt equation is $-6°\ 18'\ 30''-1'.\ 22''=-6°.\ 19'\ 52''$.

872. For w ſubſtitute 0,05505 the mean excentricity of the moon's orbit, and the true anomaly of the moon $z=Z-6°.\ 18'.\ 30''\times\text{ſin}\,Z+13'\times\text{ſin.}\,2Z-$
$+8''\times\text{ſin.}\,Z$
$37''\times\text{ſin}\,3Z=Z-6°\ 18'.\ 22''\times\text{ſin}\,Z+13'\times\text{ſin.}\,2Z-37''\times\text{ſin.}\,3Z$.

873. By Art. 868. the radius vector $=1-\frac{1}{2}w^2+w\times\text{coſ}\,z+\frac{1}{2}w^2\times\text{coſ.}\,2z+\&c.$ in which, if we ſubſtitute for z it's value $Z-2w\times\text{ſin}\,Z$ nearly, and neglect all thoſe terms where any powers of w greater than the ſecond enter, and for coſ. z we put $\text{coſ.}\,(Z-2w\times\text{ſin}\,Z)=$ (as $2w\times\text{ſin.}\,Z$ is very ſmall) $\text{coſ.}\,Z+2w\times\overline{\text{ſin}\,Z}^2=\text{coſ.}\,Z+w-w\times\text{coſ.}\,2Z$, we get the radius vector $=1+\frac{1}{2}w^2+w\times\text{coſ.}\,Z-\frac{1}{2}w^2\times\text{coſ.}\,2Z$.

On

On the Effect of increaſing or diminiſhing by a very ſmall Quantity, the Force or Velocity of a Body moving in an Ellipſe about the Focus.

FIG. 199. 874. Let *APM* be an ellipſe deſcribed by a body *P* revolving about the focus *S*, *H* the other focus, and conceive the force at *P* to be augmented by a very ſmall quantity, in the ratio of $1 : 1+r$, and let it ſtill continue to vary inverſely as the ſquare of the diſtance, to find the new ellipſe which it deſcribes, and it's major axis, the excentricity of the ellipſe being ſuppoſed to be very ſmall. Let *PE*, *Pe* be the ſpaces through which a body muſt fall with the forces $1+r$ and 1 at *P*, to acquire the velocity at *P*, then $PE : Pe :: 1 : 1+r$, therefore $PE : Ee :: 1 : r$, and $Ee = r \times PE$. Now let *PL*, *Pl* be the ſpaces fallen through to acquire the velocity at *P* in theſe reſpective caſes, then (826) $SP^2 = SE \times SL = Se \times Sl$, hence, $SE : Se :: Sl : SL$, and $SE : Ee$, or $r \times PE :: Sl$, or SL nearly, $Ll = \frac{r \times PE \times SL}{SE}$. But (868) $SP = \frac{c^2}{1 - w \times \text{cof. } z} = \frac{1 - w^2}{1 - w \times \text{cof. } z}$, and (826) $SE = \frac{SP^2}{SL} = \frac{1}{2} SP^2$, therefore $PE = SP - SE = SP - \frac{1}{2} SP^2$, hence, $Ll = \frac{4r}{SP} - 2r = \frac{4 - 4w \times \text{cof. } z}{1 - w^2} \times r - 2r =$ (by dividing and neglecting the powers of w above the firſt) $2r - 4rw \times \text{cof. } z$ the variation of the major axis. Now in a whole revolution, $4rw \times \text{cof. } z$ is deſtroyed by the oppoſition of it's ſigns, hence, the major axis is diminiſhed by $2r$; therefore the ſemi-major axis $= 1 - r$, or the ſemi-major axis is diminiſhed in the ratio of $1 : 1 - r$. If the ſame force be ſubtracted inſtead of added, the ſemi-major axis will be increaſed in the ratio of $1 : 1 + r$. If the ellipſe become a circle, it's radius muſt vary in the ſame ratio, by the addition or ſubtraction of this ſmall new force. Now if this new force *r* be conſtant, inſtead of varying inverſely as the ſquare of the diſtance, and be ſuppoſed to be very ſmall, and the orbit nearly a circle, the whole force will ſtill vary nearly in the inverſe ſquare of the diſtance *, and therefore *P* will ſtill deſcribe an ellipſe very nearly, the variation of whoſe major axis will be very nearly the ſame.

874. As

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* That this is true, appears from hence. Let x and r be very ſmall when compared with unity; then $\frac{1}{\overline{1+x}^2} = 1 - 2x$ very nearly, add r to each, and $\frac{1}{\overline{1+x}^2} + r = 1 - 2x + r = \frac{1}{\overline{1 - \frac{1}{2}r + x}^2}$ very nearly. The ſame is true, if r be ſubtracted. Hence, with the force $\frac{1}{\overline{1+x}^2} \pm r$, a body will deſcribe an ellipſe very nearly.

875. As r is very ſmall, the periodic time will (818) vary as $1 : \frac{\overline{1 \mp r}|^{\frac{3}{2}}}{\overline{1 \pm r}|^{\frac{1}{2}}}$ $= 1 \mp 2r$ very nearly. But the mean motion is inverſely as the periodic time; hence, when the force varies in the ratio of $1 : 1 \pm r$, the mean motion varies as $1 : \frac{1}{1 \mp 2r}$, or as $1 : 1 \pm 2r$, and the firſt mean motion : the difference of the mean motions : $1 : \pm 2r$.

876. If by the continual addition of ſome ſmall new force which varies inverſely as the ſquare of the diſtance, the ellipſe continually changes it's figure into a new ellipſe, but that, upon account of the ſmall variation of the ellipſe, the computation of the variation of the tranſverſe axis may be conſidered as made for the ſame ellipſe, then, if r repreſent the ſum of all the forces added in a whole revolution, after a whole revolution $Ll = 2r$, in an orbit nearly circular, hence, the ſemi-axis major $= 1 - r$ very nearly. If the forces be ſubtracted, the major axis $= 1 + r$. If the orbit be a circle, the radius of the circle will be increaſed or diminiſhed in the ratio of $1 : 1 \mp r$.

877. If by the addition or ſubtraction of this new force, the axis major be diminiſhed or increaſed by Ll, and we take $Pv = Pl$, v will be the focus of the new ellipſe; if the force be increaſed, then Pv is leſs than PH, and the apſides are progreſſive in the deſcent of the body from the higher to the lower apſide, but from the lower to the higher apſide, they will be as much regreſſive. If the force be diminiſhed, the contrary takes place. Hence, in a whole revolution, the addition or ſubtraction of any new force which varies inverſely as SP^2, will cauſe no motion of the apſides. The progreſs or regreſs of the apſides therefore depend upon the increment or decrement of the force being in a greater or leſs ratio than the inverſe ſquare of the diſtance.

878. With the center S deſcribe the circular arc vr, and $\frac{1}{2} rH$ will be the variation of the excentricity. Now $\frac{1}{2} rH = \frac{1}{2} vH \times$ coſ. z nearly $=$ (874) $r \times$ coſ. $z - 2rw \times \overline{\text{coſ. } z}^2 = r \times$ coſ. $z + rw \times$ coſ. $2z - rw$, and of theſe three quantities, the two firſt will be deſtroyed in a whole revolution by the oppoſition of their ſigns, and the third is conſtant. If therefore r denote the ſum of all the forces added, the variation of excentricity in a whole revolution $= -rw$. And if r and w be very ſmall, the variation of excentricity becomes extremely ſmall, hence, the variation of the excentricity principally depends upon the increment or decrement of the force being in a greater or leſs ratio than the inverſe ſquare of the diſtance.

879. If the gravity remain the ſame, and the velocity be increaſed in the ratio of $1 : 1 + v$, v being very ſmall, then $PE : Pe :: 1 : \overline{1+v}^2 :: 1 : 1+2v$, therefore $PE : Ee :: 1 : 2v$, hence, $Ll = 4v - 4vw \times$ coſ. z. The major axis

is therefore increaſed in the ratio of $1 : 1+2v$, hence, the periodic time is increaſed in the ratio of $1 : 1+3v$. If the velocity be diminiſhed in the ratio of $1 : 1-v$, then the major axis will be diminiſhed in the ratio of $1 : 1-2v$, and the periodic time in the ratio of $1 : 1-3v$. And in reſpect to the motion of the apſides, whilſt P moves from the higher to the lower apſide, or through FIG. 200. $MPCA$, if the velocity be increaſed or diminiſhed, the diſtance Pw, Px from the other focus will be increaſed or diminiſhed, and the apſides will move backwards or forwards to m or n, but in the aſcent from the lower apſide A to the higher M, if the velocity be increaſed or diminiſhed, then the diſtance Pv, Py from the other focus will be increaſed or diminiſhed, and the apſides will move forwards or backwards.

On the Alteration of the Figure of the Moon's Orbit, ſuppoſed to have no Excentricity, and the Variation *of the Moon.*

FIG. 196. 880. The velocity in quadratures : velocity in ſyzygies $= \frac{u}{ME} : \frac{u}{ME}+\frac{3}{2}m\times d\times ME^2$ (867) $: 1 : 1+\frac{3}{2}\times\frac{m}{u}\times d$ nearly $:$ (as $m=0{,}00557959$, $u=0{,}9986095$, $d=1{,}080853$) $1 : 1+\frac{100}{11039}$, hence, if we aſſume the mean velocity $=11039$, the velocity in quadratures $=10989$, and in ſyzygies $=11089$. Alſo, from quadratures to ſyzygies, the increment of the velocity, and conſequently of the area, in a circular orbit, varies as s^2.

FIG. 201. 881. Let $ABDC$ be an ellipſe, whoſe major axis CB is very nearly equal to it's minor axis AD, E it's center, draw any diameter MET, and QEK it's conjugate diameter, on which let fall a perpendicular MO, and draw MHI perpendicular to CB. Now by the property of the ellipſe, $HE^2=\frac{EC^2}{EA^2}\times\overline{EA^2-MH^2}$, hence ME^2 $(=HE^2+MH^2)=EC^2-\overline{EC^2-EA^2}\times\frac{HM^2}{EA^2}=$ (as the ellipſe is very nearly a circle) $EC^2-\overline{EC^2-EA^2}\times\frac{HM^2}{EM^2}$ nearly, and by taking the ſquare root, $ME=EC-\overline{EC-EA}\times\frac{HM^2}{EM^2}$ nearly, $=EC-\overline{EC-EA}\times\overline{\text{ſin. }CEM}^2$ nearly; hence, in going from C to A, the diminution of EM is in proportion to the ſquare of the ſine of the angle CEM.

882. If the mean diſtance be repreſented by unity, and e be the difference between the mean and the greateſt or leaſt diſtances, then $EC = EA + 2e$, and $EM = EA + 2e - 2e \times \overline{\text{ſin. } CEM}^2 =$ (as $EA + e = 1$, and $\overline{\text{ſin. } CEM}^2 = \frac{1}{2} - \frac{1}{2}$ coſ. $2CEM$) $1 + e \times$ coſ. $2CEM = 1 - e \times$ coſ. $2AEM$.

883. If we ſuppoſe the moon's orbit at firſt to have been a circle, the diſturbing forces will make the orbit an oval (852), whoſe major axis CB lies in quadratures, and the minor axis AD in ſyzygies; and as the minor axis DEA is always directed to the ſun, whilſt the earth E revolves about the ſun, we muſt conceive this oval figure to revolve in a moveable plane about E, ſo as to keep DEA always directed to the ſun. This oval is very nearly an ellipſe. For take a very ſmall arc Mm, draw the ordinate mr, and mt perpendicular to ME. Now if v be the abſolute velocity of M, then (867) $v^2 = \frac{1 - \frac{1}{2} m}{ME^2} + 3m \times d \times ME^2 \times s^2$, but the abſolute velocity of M : it's velocity in reſpect to the revolving plane :: d : 1; therefore in reſpect to the revolving plane, we have the ſquare of the velocity $= \frac{1}{d^2} \times \overline{\frac{1 - \frac{1}{2} m}{ME^2} + 3m \times d \times ME^2 \times s^2} = Mm^2$. Alſo the force (865) at $M = \frac{1}{ME^2} + m \times ME - 3m \times ME \times s^2 = 2Mr$, the force being repreſented by the ſpace through which the body is drawn by that force, or by twice the ſagitta (808). But by ſimilar triangles, $mr^2 : mt^2 :: ME^2 : MO^2$, hence, $\frac{mr^2 \times MO^2}{ME^2} = mt^2 = Mm^2$ very nearly, as the orbit is nearly a circle; alſo, by the property of the ellipſe, $MO = \frac{EC \times CA}{EK}$, and $Mr = \frac{rm^2 \times ME^2}{2ME \times EK^2}$, hence, $\frac{Mr}{Mm^2} = \frac{\frac{1}{2} EM^3}{EC^2 \times EA^2} = \frac{1}{2} EM^3$ very nearly. Let $EM = x$, $EC = 1 + e$, $EA = 1 - e$, and neglecting thoſe terms where all the powers of e above the firſt enter, and ſubſtituting for Mr and Mm^2 their before-mentioned values, we get from the laſt equation, $x^3 = d^2 \times \frac{1 + mx^3 - 3mx^3s^2}{1 - \frac{1}{2} m + 3mdx^4s^2} = \frac{d^2}{1 - \frac{1}{2} m} \times \overline{1 + mx^3 - 3mx^3s^2} \times \overline{1 + \frac{dx}{1 - \frac{1}{2} m}}$, and putting $\frac{d^2}{1 - \frac{1}{2} m} = t^2$, we get $x^3 = t^2 + mt^4 \times \overline{1 - 3s^2} \times \overline{1 + \frac{dx}{1 - \frac{1}{2} m}}$, and taking the cube root, we get $x = t^{\frac{2}{3}} + \frac{1}{3} mt^{\frac{8}{3}} \times \overline{1 - 3s^2} \times \overline{1 + \frac{dx}{1 - \frac{1}{2} m}} =$ (as $x = t^{\frac{2}{3}}$ nearly) $t^{\frac{2}{3}} + \frac{1}{3} mt^{\frac{8}{3}} - mt^{\frac{8}{3}} \times s^2 \times \overline{1 + \frac{dt^{\frac{2}{3}}}{1 - \frac{1}{2} m}}$; hence, it appears that the diminution of EM (from the force and

and velocity) is very nearly in proportion to the ſquare of the ſine of *CEM*, conſequently (881) the point *M* deſcribes very nearly an ellipſe upon the moveable plane, the major axis of which : the difference of the axes :: $t^{\frac{2}{3}} + \frac{1}{3}mt^{\frac{8}{3}} : mt^{\frac{8}{3}} \times 1 + \frac{dt^{\frac{2}{3}}}{1-\frac{1}{2}m}$ :: (as $m=0,0055796$, $d=1,080853$) 71,6 : 1; hence, the ratio of the diameters is as 71,6 : 70,6, but as we have here neglected ſmall quantities, this ratio is not ſo correct as that which we ſhall now proceed to determine.

884. By Art. 865. the force at *A* : the force at *C* :: $\frac{1}{AE^2} - 2m \times AE : \frac{1}{CE^2} + m \times CE$, alſo, (880) from the acceleration of velocity by the ablatitious force, the velocity at *A* : velocity at *C* :: 1,0090588 (v) : 1, but independent of this acceleration, the velocity at *A* : the velocity at *C* :: *CE* : *AE* (806); hence, the velocity at *A* : velocity at *C* :: $v \times CE : AE$. Now the increment of the arc deſcribed in a given time is as the velocity, and the ſagitta is as the force; alſo the curvature is as the ſagitta directly and the ſquare of the arc inverſely, hence, the curvature at *A* : curvature at *C* ::

$$\frac{\frac{1}{AE^2} - 2m \times AE}{v^2 \times CE^2} : \frac{\frac{1}{CE^2} + m \times CE}{AE^2} :: 1 - 2m \times AE^3 : v^2 + v^2 m \times CE^3.$$

FIG. 202. 885. When the moon is in quadratures at *C* with the ſun *S*, it will deſcribe above 90° before it comes into conjunction, owing to the motion *SS′* of the ſun in the ſame time. Now for the *mean* motions, the angle *CEA* : *CEa* :: 27*d* 7*h*. 43′ : 29*d*. 12*h* 44′, or as the periodic : the ſynodic revolution of the moon, which put as 1 : *d*, and take $Ea=EA$, and *a* is the place of the moon at the next conjunction, and the curve deſcribed by the moon will be found, by taking $Em=EM$, and the angle *CEm* : *CEM* :: *d* : 1. To find the curvature of the orbit *Ca* at *C* and *a*, with the center *E* deſcribe the circle *Cu*, draw the tangent *Ch*, and *Ervt*, and take $Es=Er$, and the angle *CEs* : *CEr* :: *d* : 1, and produce *Es* to *y*, and conſider the angles *CEv*, *CEt* in their evaneſcent ſtate; now as $Er=Es$, and $Ev=Ez$, therefore $rv=sz$; and as the ſagitta repreſents the curvature when the tangent is the ſame, we have at the point *C*, curv. of *Cr* : curv. of *Cv* :: *tr* : *tv*, and curv. of *Cs* : curv. of *Cv* :: *ys* : *yz*; hence, FIG. 203.

Curv. of *Cr* − curv. of *Cv* : curv. of *Cv* :: *rv* or *zs* : *vt*
Curv. of *Cv* : curv. of *Cs* − curv. of *Cv* :: *zy* : *zs*

∴ curv. of *Cr* − curv. of *Cv* : curv. of *Cs* − curv. of *Cv* :: $zy : vt :: tz^2 : tv^2 :: d^2 : 1$.

Now

Now the curvature being inverſely as the radius of curvature, the curvature of Cu may be repreſented by $\frac{1}{CE}$, and then (by the property of the ellipſe) the curvature of $Cr = \frac{CE}{AE^2}$; hence, $\frac{C}{AE^2} - \frac{1}{CE}$: curv. of $Ca - \frac{1}{AE}$ :: $d^2 : 1$; conſequently the curvature of Ca at $C = \frac{CE^2 + rAE^2}{d^2AE^2 \times CE}$, putting $r = d^2 - 1$. For the ſame reaſon, the curvature of Ca at $a = \frac{AE^2 + rCE^2}{d^2CE^2 \times AE}$. Hence, the curvature in conjunction : the curvature in quadratures :: $\frac{AE^2 + rCE^2}{d^2CE^2 \times AE}$: $\frac{CE^2 + rAE^2}{d^2AE^2 \times CE}$ :: $AE^3 + rCE : CE^3 + rAE$, aſſuming $AE \times CE = 1$, becauſe the mean diſtance is unity, and the orbit is very nearly a circle. Hence, from this and the laſt Article, we have, $1 - 2m \times AE^3 : v^2 + v^2m \times CE^3 :: AE^3 + r \times CE : CE^3 + r \times AE$. Put $1 + x = CE$ and $1 - x = AE$, and ſubſtituting for CE and AE theſe values, and neglecting all the powers of x above the firſt, on account of the ſmallneſs of x, and multiplying extremes and means, we ſhall get a ſimple equation, from which $x = ,00716$; hence $CE = 1,00716$ and $AE = ,99284$, conſequently $CE : AE :: 1,00716 : ,99284 :: 70 : 69$ in whole numbers. This therefore is the variation of the form of the orbit ariſing from the force of the ſun, ſuppoſing that the orbit would have been a circle without that diſturbing force. And as the orbit of the moon is an ellipſe having the the earth in it's focus, and this ellipſe is nearly a circle, the ſame cauſe muſt produce very nearly the ſame effect in the moon's orbit. Dr. HALLEY, firſt took notice of this contraction of the lunar orbit in ſyzygies, from the phænomena of the moon's motion, and made the ratio of the diameters as 44,5 : 45,5, from obſervation. See his remarks upon the lunar Theory, at the end of his catalogue of the ſouthern ſtars. Hence (882), $EM = 1 - \frac{1}{140} \times$ Fig. 201. cos $2AEM$.

886. From the alteration of the form of the orbit, as explained in the laſt Article, and from the acceleration of the areas (880), there will ariſe two corrections to be applied to the mean motion of the moon, in order to give the true motion; the joint effect of theſe two, conſtitute an equation called the *Variation*. We ſhall conſider each ſeparately, beginning with that ariſing from the figure of the orbit. Let CA be the quadrant of a circle which the moon would have deſcribed from quadratures to ſyzygies without any diſturbing Fig. 204. force; take $Ea : EA :: 69 : 70$, and deſcribe the ellipſe aC; draw MmK

perpendicular to EC, and join EM, Em. Then the moon in the quadrant CA revolving uniformly, the angle CEM will repreſent the mean motion, but it's true place will be in the ellipse at m, for not here conſidering the acceleration of the areas, the true and mean place muſt be both in the ſame perpendicular to EC, for the area CEA : area CEa :: area CEM : area CEm, and as CEA, CEa are ſuppoſed to be deſcribed in the ſame time, and the areas are (805) proportional to the times, therefore CEM, CEm are deſcribed in the ſame time. Hence, the angle MEm (being the difference of the two angles deſcribed by the true and mean motion of the moon about E) is the correction from this cauſe.

887. Now to conſider the effect from the acceleration of the areas, it appears by Art. 880. that if the mean area, or the area deſcribed in the octants, be 11039, the areas at a and C will be 11089 and 10989 in the ſame time. Take therefore $Ea : E\alpha :: \sqrt{11089} : \sqrt{10989}$, or as 11039 : 10989, or as 69 : 68,6875, and deſcribe the ellipſe αnC, and draw Enr; and n will be the true place of the moon correſponding to the mean place M. For as the area $CEm : CEn$ in the conſtant ratio of $mK : nK$, $CEm : CEn$ in a given ratio, but CEm is conſtant, therefore CEn is conſtant. Now $CEr : CEn :: Er^2 : En^2$, therefore $CEr - CEn : CEn :: Er^2 - En^2 : En^2 :: 2En \times nr : En^2$ very nearly, $:: 2nr : En$, and conſidering En as conſtant on account of it's very ſmall variation, the ſecond and fourth terms being conſtant, $CEr - C\dot{E}n$ varies as $2nr$, or as nr. But at C, $C\dot{E}n = C\dot{E}r$, put therefore the fluxion of $C\dot{E}n = v$, and $C\dot{E}r - v$ varies as nr. Now $nr : mn :: nK : nE$, or αE nearly, and $mn : a\alpha :: nK : \alpha E$, hence, $nr : a\alpha :: nK^2 : \alpha E^2$, conſequently nK^2 varies as nr, or as $C\dot{E}r - v$, hence, the increment of the area CEr at r above the increment at C, increaſes as nK^2, or very nearly as the ſquare of the ſine of the angle rEC, and therefore (880) it increaſes as the areas deſcribed by the moon from quadratures to ſyzygies increaſe. Now bringing mnK to bew, and r to s, indefinitely near to C, the area $CEs : CEb :: we : wb :: E\alpha : Ea :: \sqrt{10989} : \sqrt{11089} :: 10989 : 11039 ::$ (880) the area deſcribed by the moon at C : the mean area deſcribed by the moon. As therefore the area CEs deſcribed by the moon at C has it's proper ratio to the mean area CEb that would have been deſcribed in the ſame time, and it increaſes in it's proper ratio, at any point r the moon has it's true poſition in reſpect to the place m where it would have been if there had been no acceleration of areas, and m is the place, if there had been no acceleration of areas, correſponding to the mean place M, therefore the angle MEn is the whole equation, or the *Variation*.

888. If

888. If we conſider EK as radius, Kn, KM will be the tangents of the true and mean motions, and the difference of the two angles will be greateſt in the octants*, therefore if the angle $MCE = 45°$, $MK : Kn$ (:: 70 : 68,6875) :: tan 45° : tan. 44° 27′. 28″, conſequently the greateſt variation is 32′. 32″. This would be the caſe if the moon deſcribed 90° from quadratures to ſyzygies; but as it deſcribes a greater angle in the proportion of a periodic to a ſynodic revolution, or in the proportion of 27*d* 7*h*. 43′ to 29*d*. 12*h* 44′, therefore if 32′. 32″ be increaſed in this proportion, it gives 35′. 10″ for the greateſt variation. But $nE : EK :: nm : mr$ (nearly) $= \frac{EK \times nm}{nE}$ which (becauſe nm varies as nK) varies as $EK \times nK$ very nearly, or nearly as the ſine of $2 \times EC$, or ſin. $2 \times EA$ twice the diſtance of the moon from the ſun; hence, the *variation* = 35′. 10″ × ſine of twice the diſtance ($2X$) of the ſun from the moon. In the tranſit of the moon from quadratures to ſyzygies, the mean place M is *before* the true place r, and from ſyzygies to quadratures, the mean place is *behind* the true.

889. This is the variation at the mean diſtance of the ſun from the earth; but at other diſtances it (857) varies as the ſquare of the ſynodic† time of the moon directly and the cube of the diſtance of the earth from the ſun inverſely; let therefore $d : 1 ::$ the time of the ſynodic : the time of the periodic revolution of the moon at the mean diſtance CS (unity) of the ſun from the earth, and let SCs be the angle deſcribed by the ſun in the time $d-1$, or by which the ſynodic exceeds the periodic revolution; and let TCt be the angle deſcribed by the ſun in the difference of thoſe times, at any other diſtance CT of the ſun from the earth. Now if c = the excentricity of the earth's orbit, Υ the ſun's true anomaly, (868) the diſtance of the ſun from the earth $= 1 + c \times \text{coſ}.\,\Upsilon$, and the angular motion of the ſun in the ſame time varies (820) inverſely as the ſquare of the diſtance, and the angles SCs, TCt deſcribed by the ſun may be conſidered as deſcribed in the ſame time, the difference being only the angle deſcribed by the ſun in the difference of the times of de[illegible]ing theſe angles by the moon, hence, $\frac{1}{1^2} : \frac{1}{1 + c \times \text{coſ}.\,\Upsilon^2} = 1 - 2c \times \text{coſ}.\,\Upsilon : d-1$ [illegible]

FIG. 205.

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* [illegible]his is not accurately true, but ſufficiently nearly ſo for the purpoſe for which we here [illegible] Be[illegible] early.

† [illegible]e period of the variation being a *ſynodic* revolution, it appears, by a like reaſoning as in [illegible]7. that the error will be in proportion to the ſquare of the *ſynodic* time of a revolution and the cube of the diſtance of the ſun inverſely

:: $d-1$: the time of deſcribing $TCt=\overline{d-1}\times\overline{1-2c\times \text{coſ.}\ \Upsilon}$, therefore the difference of the times of deſcribing SCs, TCt, is $-\overline{d-1}\times 2c\times \text{coſ}\ \Upsilon$, conſequently the time of a ſynodic revolution of the moon at it's mean diſtance : the time at any other diſtance : $1 . 1-2c\times\frac{d-1}{d}\times \text{coſ}\ \Upsilon = 1-{,}1496c\times \text{coſ}\ \Upsilon$. Hence, the variation becomes $35'\ 10''\times\frac{\overline{1-{,}1496c\times \text{coſ}\ \Upsilon}^2}{\overline{1+c\times \text{coſ.}\ \Upsilon}^3}\times \text{ſin.}\ 2X = 35'.\ 10''\times\overline{1-3{,}299c\times \text{coſ.}\ \Upsilon}\times \text{ſin}\ 2X$, and if we take the excentricity of the earth's orbit to be ,01681, when the ſun is in it's apogee the variation will be 33′. 13″, and in it's perigee, it becomes 37′ 7″. All this is upon ſuppoſition that the moon's orbit has no excentricity, and that the ratio of CE . aE continues conſtant, whereas that ratio depends (885) upon the time of a ſynodic revolution; the orbit alſo has an excentricity, the variation therefore will differ a little from what is here determined. TYCHO firſt obſerved this irregularity of the moon FIG. 204.

890. The annual equation of the ſynodic revolution is the difference of the times of deſcribing SCs, TCt, or $-0{,}1496\times c\times \text{coſ}\ \Upsilon$, the mean ſynodic time being unity. When $\text{coſ}\ \Upsilon=1$, this equation becomes $-1h\ 46'\ 5$[illegible]$''$ for it's maximum. Hence, the equation at any other time will be $-1h.\ 46'\ 5$[illegible]$''\times \text{coſ.}\ \Upsilon$.

To find the Equation of the mean Motion of the Moon, ariſing from the different Diſtances of the Earth from the Sun.

891. As the earth approaches the ſun, the moon's orbit is (854) dilated, and as the earth recedes from the ſun, the orbit is contracted, and this ariſes from the diminution and augmentation of the diſturbing force of the [illegible] on the moon in the direction ME; but the equal deſcription of areas [illegible] times will not (805) be affected by this force, and in reſpect to the acceleration and retardation of velocity from the ablatitious force, the acceleration [illegible] one quadrant very nearly deſtroys the retardation in the next, we may conſider the mean area deſcribed to be always in proportion to the time [illegible] at the mean diſtance unity of the moon from the earth, the natural [illegible] the moon to the earth being unity, the mean force of the moon [illegible] at that diſtance of the moon $=1-\frac{1}{2}m$, but if $\Upsilon=$ the true ano[illegible] ſun, and the diſturbing force of the ſun at the mean diſtance unity, [illegible] the FIG 196.

the diſturbing force at any other diſtance $1 + c \times \text{coſ.}\ \Upsilon$ will (856) be $\frac{1}{1 + c \times \text{coſ}\ \Upsilon} = 1 - 3c \times \text{coſ}\ \Upsilon$ very nearly, therefore the mean force of the moon to the earth at that diſtance of the ſun $= 1 - \frac{1}{2}m \times \overline{1 - 3c \times \text{coſ}\ \Upsilon}$. Hence the force is altered in the ratio of $1 - \frac{1}{2}m : 1 - \frac{1}{2}m + \frac{3}{2}mc \times \text{coſ}\ \Upsilon$, or as $1 : 1 + \frac{3}{2}mc \times \text{coſ}.\ \Upsilon$ very nearly, therefore (874) the radius of the moon's orbit is changed in the ratio of $1 : 1 - mc \times \text{coſ}\ \Upsilon$, and (875) the mean motion of the moon at the mean diſtance of the earth from the ſun : the mean motion at the diſtance $1 + c \times \text{coſ.}\ \Upsilon :: 1 : 1 + 3mc \times \text{coſ.}\ \Upsilon$, therefore in the time in which the ſun has deſcribed the arc Υ, the mean motion of the moon : the equation of the mean motion :: fluent of $\dot{\Upsilon}$: fluent of $3mc \times \text{coſ.}\ \Upsilon \times \dot{\Upsilon} :: \Upsilon : 3mc \times \text{ſin}\ \Upsilon$, but in the time in which the ſun deſcribes Υ, the mean motion of the moon is $\frac{2139}{160}$* $\times \Upsilon$, hence, $\frac{2139}{160} \times \Upsilon$: the equation of the mean motion $:: \Upsilon : 3mc \times \text{ſin}\ \Upsilon$, therefore the annual equation of the mean motion is $\frac{2139}{160} \times 3mc \times \text{ſin}\ \Upsilon = 12' \ 55'' \times \text{ſin.}\ \Upsilon$.

[illegible] is poſitive from aphelion to perihelion, and negative from [illegible] aphelion, the annual equation is to be added to the mean motion [illegible] former caſe, and ſubtracted in the latter. When the earth is in the [illegible]on, the orbit of the moon is dilated, and the moon moves ſlower; [illegible] earth is in the aphelion, the orbit of the moon is contracted, and the mean moves faſter; and the annual equation, by which this inequality is compenſated, is nothing in aphelion and perihelion, and at the mean diſtance of the ſun it is 12′ 55″ according to our determination. Sir I. NEWTON makes it 11′. 50″, according to MAYER it is 11′ 16″, M D'ALEMBERT makes it 12′. 57″; and HALLEY makes it about 13′; according to M. de la [illegible] is 11′. 8″,6. This annual equation being in proportion to ſin Υ, is in proportion to the equation of the ſun's center, that equation being in proportion to the ſame quantity (870), the excentricity of the earth's orbit being very ſmall.

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* Becauſe the mean motion of the ſun is to the mean motion of the moon as 160 : 2139 very nearly.

To find the Equation ariſing from the Inclination of the Orbit to the Ecliptic.

FIG. 207. 893. Let $CABD$ be the plane of the ecliptic, NMn the plane of the moon's orbit, Nn the line of the nodes, M the moon, E the earth, S the ſun, draw Ss, Av, perpendicular to the plane NMn, alſo [illegible] perpendicular to Nn, and MK perpendicular to CB; join vw, [illegible] svE, sM, and on EM produced let fall the perpendicular sI. Put $a = SE$, s = the ſine of Awv the inclination of the orbit, and conſider the mean value of $ME = 1$, $v = Aw$ the ſine of SEN, x = MK, [illegible] $vA = vs$, and $Ss = avs$, hence, $Es^2 = a^2 - a^2v^2s^2$; and Es [illegible]; alſo, $sM^2 = sE^2 + EM^2 - 2sE \times MK = a^2 - a^2v^2s^2 + 1^2 - 2ax \times \overline{1 - v^2 s^2}|^{\frac{1}{2}}$; hence, $SM^2\ (= Ss^2 + sM^2) = a^2 + 1^2 - 2ax \times \overline{1 - v^2 s^2}|^{\frac{1}{2}}$, therefore $SM^3 = \overline{a^2 + 1^2 - 2ax \times \overline{1 - v^2 s^2}|^{\frac{1}{2}}}|^{\frac{3}{2}} = a^3 - 3a^2x \times \overline{1 - \frac{1}{2}v^2s^2}$ nearly, conſequently $\frac{1}{SM^3} = \frac{1}{a^3} + \frac{3x}{a^4} \times \overline{1 - \frac{1}{2}v^2s^2}$ nearly. Now if $\frac{1}{ES^2}$, $\frac{1}{MS^2}$ repreſent the attractions of E and M to S, then $ES : Es :: \frac{1}{ES^2} : \frac{Es}{ES^3}$ the attraction [illegible] direction Es, and $SM : sM :: \frac{1}{SM^2} : \frac{sM}{SM^3}$ the attraction of M in [illegible] rection Ms, hence, $ES : EI :: EM\ (=1) : MK :: \frac{Es}{ES^3} : \frac{MK \times Es}{ES^3}$ the [illegible] attraction of E in the direction EM, and $sM : MI :: \frac{sM}{SM^3} : \frac{MI}{SM^3}$ the attraction of M in the direction EM, hence, the difference of the forces of M and E in that direction is $\frac{MI}{SM^3} - \frac{MK \times Es}{ES^3}$; but $MI = EI - 1 = Es \times MK - 1$; hence, the diſturbing force in the direction EM is $\overline{Es \times MK - 1} \times \frac{1}{SM^3} - \frac{MK \times Es}{ES^3} = Es \times \overline{x - 1} \times \overline{\frac{1}{a^3} + \frac{3x}{a^4} \times \overline{1 - \frac{1}{2}v^2s^2}} - \frac{Es \times x}{a^3} = -\frac{1}{a^3} + \frac{3x^2}{a^3} - \frac{3x^2v^2s^2}{a^3}$, but $\frac{1}{a^3} = n^2$, and as $x^2 = \frac{1}{2} - \frac{1}{2}$ coſ. $2MEC$, for a whole revolution of the moon we may aſſume $x^2 = \frac{1}{2}$; alſo, $v^2 = \frac{1}{2} - \frac{1}{2}$ coſ. $2SEN$; hence, in any given poſition of the nodes in reſpect to the ſun, the mean diſturbing force is $\frac{1}{2}$ [illegible] $\frac{3}{4}n^2s^2 + \frac{3}{4}n^2s^2 \times$ coſ $2SEN$, and as the attraction of M to $E = 1 + \frac{1}{2}n^2$, the [illegible] force of M to $E = 1 + \frac{1}{2}n^2 - \frac{1}{2}n^2 + \frac{3}{4}n^2s^2 - \frac{3}{4}n^2s^2 \times$ coſ $2SEN$ [illegible]

$\frac{3}{4}n^2s^2\times$ coſ $2SEN$. Hence, the mean force : the mean force in different ſituations of the ſun in reſpect to the node .: $1+\frac{3}{4}n^2s^2$. $1+\frac{3}{4}n^2s^2-\frac{3}{4}n^2s^2\times$ coſ. $2SEN$ [illegible] $\frac{3n^2s^2}{4+3n^2s^2}\times$ coſ. $2SEN=1-\frac{3}{4}n^2s^2\times$ coſ. $2SEN$ nearly, therefore (874) the mean radius of the orbit will be increaſed in the ratio of 1 $1+\frac{3}{4}n^2s^2\times$ coſ. $2SEN$, and (875) the mean motion is diminiſhed in the ratio of [illegible] $1-\frac{3}{2}n^2s^2\times$ coſ $2SEN$, conſequently the mean motion of the moon in the time the ſun departed from the node through the angle SEN . the equation of the mean motion in the ſame time the fluent of $\dot{\overline{SEN}}$, or SEN, the fluent of $-\frac{3}{2}n^2s^2\times$ coſ $2SEN\times\dot{\overline{SEN}}$, or $-\frac{3}{4}n^2s^2\times$ ſin $2SEN$. Now when the ſun leaves the node, it comes to the node again after it has deſcribed an angle of about 341,3°, therefore in the time in which the ſun has departed from the node through angle SEN, it's motion has been $\frac{341,3}{360}\times SEN$, and the mean motion of the moon in that time has been $\frac{2139}{160}\times\frac{341,3}{360}\times SEN$; hence, $\frac{2139}{160}\times\frac{341,3}{360}\times SEN$ · the equation of the mean motion : [illegible] $-\frac{3}{4}n^2s^2\times$ ſin. $2SEN$, conſequently the equation of the mean motion [illegible] $\times\frac{341,3}{360}\times-\frac{3}{4}n^2s^2\times$ ſin. $2SEN=-$ [illegible] $28''\times$ ſin. $2SEN$. But if S [illegible] diſtance of the moon from the node, and X the mean diſtance of [illegible] from the ſun, then $2SEN=2S-2X$ very nearly, hence, the equation is $-$ [illegible] $28''\times$ ſin $\overline{2S-2X}$, this equation is therefore to be ſubtracted from the mean motion of the moon, in the tranſit of the ſun from the nodes to quadratures from thence, and added, in the tranſit from quadratures to the nodes.

To find the [illegible] of the Periodic Time of the Moon, by the diſturbing Forces.

894. Let the mean diſtance of the moon from the earth be unity, $u=$ the velocity of projection at that point, and $1:p$ the ratio of radius to the circumference of a circle, then $\frac{p}{u}=$ the periodic time of a body revolving in a circle at that diſtance, and (889) $\frac{dp}{u}=$ the ſynodic revolution. Now (867) [illegible] $=\frac{u}{ME}$ [illegible] dm [illegible] $E^2\times$ ſin. $\overline{MEC}^2$ and the orbit being without excentricity, it

it becomes (883) elliptical, whoſe minor axis lies in ſyzygies. Let $\dot z$ repreſent the fluxion of the arc of the angle MEC to radius unity, then $ME\times\dot z$ is the fluxion to radius ME, and the correſponding ſynodic fluxion is $d\times ME\times\dot z$. Hence, if t repreſent the correſponding fluxion of the time, we have $t = \frac{d\times ME\times\dot z}{\frac{u}{ME}+\frac{3}{2}d\times m\times ME^2\times\overline{\text{ſin. }MEC}^2}$ [illegible] $\frac{d\times ME^2\times\dot z}{u+\frac{3}{2}d\times m\times ME^3\times\overline{\text{ſin. }MEC}^2}=\frac{d}{u}\times ME^2\times\dot z-\frac{d^2}{u^2}\times\frac{3}{2}m\times ME^5\times\overline{\text{ſin }MEC}^2\times\dot z$. Put $e=$ the difference between the mean diſtance (1) [illegible] greateſt or leaſt diſtance, then (882) $ME=1+e\times\text{coſ. }2MEC$, therefore [illegible] $ME^2=1+2e\times\text{coſ. }2MEC$, and $ME^5=1+5e\times\text{coſ. }2MEC$, very nearly; by ſubſtitution therefore we get $t=\frac{d}{u}\times\overline{\dot z+2e\times\text{coſ. }2MEC\times\dot z}-\frac{d^2}{u^2}\times\overline{\frac{3}{2}m\times\overline{\text{ſin. }MEC}^2\times\dot z+\frac{15}{2}em\times\overline{\text{ſin }MEC}^2\times\text{coſ. }2MEC\times\dot z}=\frac{d}{u}\times\dot z+\frac{d}{u}\times 2e\times\text{coſ. }2MEC\times\dot z-\frac{d^2}{u^2}\times\frac{3}{2}m\times\overline{\frac{1}{2}-\frac{1}{2}\text{coſ. }2MEC}\times\dot z-\frac{d^2}{u^2}$ [illegible] $\times\frac{15}{2}\times em\times\text{coſ. }2MEC\times\overline{\frac{1}{2}-\frac{1}{2}\text{coſ. }2MEC}\times\dot z=\frac{d}{u}\times\dot z+\frac{d}{u}\times 2e\times\text{coſ.}$ [illegible] $\times\dot z-\frac{d^2}{u^2}\times\frac{3}{4}m\times\dot z+\frac{d^2}{u^2}\times\frac{3}{4}m\times\text{coſ. }2MEC\times\dot z$, neglecting the [illegible] other two terms on account of their ſmallneſs, hence, $t=\frac{d}{u}\times z-\frac{d^2}{u^2}\times\frac{3}{4}m$ [illegible] $z+\frac{d}{u}$ [illegible] $\overline{e}+\frac{d}{u}\times\frac{3}{8}m\times\text{ſin. }2MEC$; therefore the time of a *ſynodic* revolution $=\frac{d}{u}\times\overline{p-\frac{d^2}{u^2}\times\frac{3}{4}mp}$; and the equation of the mean time is $\frac{d}{u}\times e+\frac{d}{u}\times\frac{3}{8}m\times\text{ſin. }2MEC$. Hence, the time of a *periodic* revolution is $\frac{1}{u}\times p-\frac{3}{4}\times\frac{1}{u}\times m$ [illegible]

If the moon deſcribe a circle with the mean radius $ME=1$, the periodic ti[illegible] is the ſame, with the ſame diſturbing force, as appears by making $e=$ [illegible]. Now (818) the periodic time in this circle (there being no diſturbing for[illegible]) = the periodic in the ellipſe which the moon would deſcribe if it we[illegible] diſturbed, the radius of the circle being the mean diſtance, and this cu[illegible] being nearly a circle, the ſame diſturbing forces muſt produce very nea[illegible] ſame effect in each caſe; we may therefore conſider the periodic time cir[illegible] moon to be the ſame as that which we have here determined.

895. If

895. If there had been no diſturbing force, the time of a ſynodic revolution would have been $\frac{d}{u} \times p$; conſequently the time of a *ſynodic* revolution is diminiſhed by the quantity $\frac{3}{4} mp \times \frac{d^2}{u^2}$, in conſequence of the diſturbing forces. The time of a *periodic* revolution is diminiſhed by the quantity $\frac{3}{4} mp \times \frac{1}{u}$.

On the Motion of the Moon's Apogee, and the Variation of the Excentricity of its Orbit.

896. The principle upon which we here propoſe to find the motion of the apogee, is, to find in what time the whole force by which the moon is urged towards the earth will draw it through a ſpace in the direction of the radius vector, equal to the difference between the greateſt and leaſt diſtances, the moon ſetting off from the apogee; and comparing twice that time with the time of a revolution.

897. To find the whole force with which the moon is urged towards E. The force of M to E (864) $= \frac{1}{EM^2} - \frac{1}{2} m \times EM + \frac{3}{2} m \times EM \times$ coſ. $2MEC$, and if N be the higher apſide, and lie in quadratures, the force in that point $= \frac{1}{EN^2} - \frac{1}{2} m \times EN$, the coſ $2MEC$ being then $= 0$, and if this force at N : the centrifugal force :: c : 1, then the centrifugal force at $N = \frac{1}{c \times EN^2} - \frac{1}{2} m \times \frac{EN}{c}$, but * the centrifugal force varies as the ſquare of the velocity perpendicular to the radius directly, and the radius inverſely; and the ſquare of the velocity at $M = \frac{u^2}{EM^2} + \frac{3}{2} m \times EM^2 - \frac{3}{2} m \times EM^2 \times$ coſ. $2MEC$ (866); alſo the ſquare of the velocity at N in the octants $= \frac{u^2}{EN^2} + \frac{3}{2} m \times EN^2$, hence, the

Fig. 206.

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* For in Art. 827. the centrifugal force wx varies as $\frac{xQ^2}{xS}$, and as the time is given, xQ varies as the velocity in a direction perpendicular to the radius.

the centrifugal force $\frac{1}{c \times EN^2} - \frac{1}{2} m \times \frac{EN}{c}$ at N in the octants : the centrifugal force at M :: $\frac{u^2}{EN^3} + \frac{1}{2} m \times EN : \frac{u^2}{EM^3} + \frac{3}{2} m \times EM - \frac{3}{2} m \times EM \times$ coſ. $2MEC$ :: $1 : \frac{EN^3}{EM^3} - \frac{3mNE^3}{2u^2} \times$ coſ. $2MEC$ nearly, therefore the centrifugal force at $M = \frac{EN}{c \times EM^3} \times \overline{1 - \frac{1}{2} m \times EN^3} - \frac{3m \times EN}{2c \times u^2} \times$ coſ. $2MEC$; hence, the whole force of the moon towards $E = \frac{1}{EM^2} - \frac{1}{2} m \times EM + \frac{3}{2} m \times EM \times$ coſ. $2MEC - \frac{EN}{c \times EM^3} \times \overline{1 - \frac{1}{2} m \times EN^3} + \frac{3m \times EN}{2c \times u^2} \times$ coſ. $2MEC$.

898 Now in eſtimating the *mean* motion of the apſides, all the terms where coſ. $2MEC$ enters, may be neglected, for the following reaſons. The coſ. $2MEC =$ coſ. $2CEN \times$ coſ. $2MEN -$ ſin. $2CEN \times$ ſin. $2MEN$, hence, the force $\frac{3m \times EN}{2c \times u^2} \times$ coſ. $2MEC$ is compoſed of two parts, the firſt of which, having given the angle CEN the diſtance of the apogee from quadratures, varies as the coſ $2MEN$, which, in the tranſit of the moon from the higher apſide to it's octants decreaſes, and therefore makes the moon recede further from the focus, and thence to it's quadratures it increaſes and makes the moon approach as much to the focus, and four times in every revolution this acceſs and receſs will deſtroy each other The other part, which is as ſin. $2MEN$, increaſes from the apogee to it's octants, and decreaſes from it's octants to it's quadratures, and this alſo deſtroys itſelf four times in a revolution If the excentricity be w, then (868, as $EM = 1 + w \times$ coſ. MEN) $\frac{3}{2} m \times EM \times$ coſ $2MEC = \frac{3}{2} m \times \overline{1 + w \times \text{coſ. } MEN} \times$ coſ. $2MEC$; of which, $\frac{3}{2} m \times$ coſ. $2MEC$ is deſtroyed, as before ſhown, alſo $\frac{3}{2} m \times w \times$ coſ $MEN \times$ coſ. $2MEC$ is very ſmall, and in the oppoſite ſituation of the apſides has a different ſign; the acceſs and receſs therefore from this part of the force, will deſtroy each other in a whole revolution of the apſides. In eſtimating therefore the *mean* motion of the apſides, the terms in the force where coſ $2MEC$ enters, may be neglected, the whole force therefore in the direction of the radius may be here taken $= \frac{1}{EM^2} - \frac{1}{2} m \times EM - \frac{EN}{c \times EM^3} + \frac{m \times EN^4}{2c \times EM^3}$.

899. Let v be the velocity of the moon at M in the direction ME, $x = EM$, then by the principles of motion, $v\dot{v} = -\frac{\dot{x}}{x^2} + \frac{1}{2} m x\dot{x} + \frac{EN}{c} \times$

1—

$\overline{1-\frac{m\times EN^3}{2}}\times\frac{\dot{x}}{x^3}$, whoſe fluent corrected is $\frac{1}{2}v^2=\frac{1}{x}-\frac{1}{EN}-\frac{1}{4}m\times\overline{EN^2-x^2}$ $-\overline{1-\frac{1}{2}m\times EN^3}\times\overline{\frac{EN}{2c\times x^2}-\frac{1}{2c\times EN}}$, or $v^2=\frac{1}{x^2}\times\ldots$ $\overline{2x\times\overline{1-\frac{x}{EN}}-\frac{1}{2}m\times\overline{EN^2\times x^2-x^4}-\overline{1-\frac{1}{2}m\times EN^3}\times\frac{EN^2-x^2}{c\times EN}}$. But if we ſuppoſe the moon to be projected from it's apogee in the octants, with the velocity $\sqrt{\frac{u^2}{EN^2}+\frac{3}{2}m\times EN^2}$, and force of gravity $\frac{1}{EN^2}-\frac{1}{2}m\times EN$, and which varies from that point in the inverſe duplicate ratio of the diſtance, then if a repreſent the ſemi-axis major of this ellipſe, and w it's excentricity, we have (829) $c:1::a:a-w$, if therefore we put $EN-x=z$, and $EN^2-x^2=2EN\times z-z^2$, we get $2x\times\overline{1-\frac{x}{EN}}=2z-\frac{2z^2}{a+w}$; alſo $-\frac{1}{2}m\times\overline{EN^2x^2-x^4}=-m\times EN^3\times z+\frac{5}{2}m\times EN^2\times z^2$, and $-\overline{1-\frac{1}{2}m\times EN^3}\times\frac{EN^2-x^2}{c\times EN}=-\overline{1-\frac{1}{2}m\times EN^3}\times\frac{2EN\times z-z^2}{c\times EN}=-2z+\frac{2wz}{a}+\frac{z^2}{a+w}+\frac{wz^2}{a^2+aw}$ $+m\times EN^3\times z-\frac{mw\times EN^3z}{a}-\frac{1}{2}m\times EN^2\times z^2$; hence, $v^2=\frac{1}{x^2}\times$ $\overline{\frac{2w-mw\times EN^3}{a}\times z-\overline{\frac{1}{a}-2m\times EN^2}\times z^2}$, and if $\frac{1}{a}-2m\times EN^2=d^2$, and $\frac{\frac{w}{a}-\frac{mw\times EN^3}{2a}}{\frac{1}{a}-2m\times EN^2}=e$, we have $v=\frac{d}{x}\times\sqrt{2ez-z^2}$.

900. The body beginning to deſcend from it's apogee, when it comes to it's perigee, $v=0$, therefore $z=2e=\frac{2w-mw\times EN^3}{1-2ma\times EN^2}=2w+3mw$; hence, twice the excentricity of the ellipſe deſcribed without any diſturbing force : the whole approach of the moon to the earth in it's paſſage from it's apogee to it's perigee by the diſturbing forces ∴ $2w:3mw::1:\frac{3}{2}m$. And as the difference between $2e$ and $2w$ is only the ſmall quantity $3mw$, the mean diſtance of the moon from the earth may be conſidered very nearly equal to the ſemi-axis major of the ellipſe which would be deſcribed by the moon projected from the apogee in the octants, by a force varying in the inverſe duplicate ratio of the diſtance.

901. Let $Er=EN-En$, then $Nr=2e$; take the indefinitely ſmall arc Mn, and with the center E deſcribe the circular arcs Mt, ns, then $ts=$ the deſcent

deſcent of the body towards E in the time of deſcribing Mm; put $Nt=z$, $ts=\dot z$, $\dot t=$the time down ts, and draw tv, sw perpendicular to rN, and wx to tv. Then by the principles of motion, $\dot t = \frac{\dot x z}{d\sqrt{2ez-z^2}} = \frac{\overline{EN-e}\times \dot z}{d\sqrt{2ez-z^2}} + \frac{\overline{e-z}\times \dot z}{d\sqrt{2ez-z^2}} = \frac{EN-e}{d} \times \frac{vw}{e} + \frac{xv}{d}$, whoſe fluent is $t = \frac{EN-e}{d} \times \frac{Nv}{e} + \frac{tv}{d}$, and when M comes to n, the whole time of deſcent from the higher to the lower apſide becomes $\frac{EN-e}{d} \times \frac{Nvr}{e} =$ (if $1 : p :: $ radius : circumference) $\frac{1}{2}p \times \frac{EN-e}{d} = \frac{1}{2}p \times \frac{a+w-\frac{\frac{w}{a}-\frac{mw\times EN^3}{2a}}{\frac{1}{a}-2m\times EN^2}}{\sqrt{\frac{1}{a}-2m\times EN^2}} = \frac{\frac{1}{2}p\times\overline{1-2m\times EN^3}}{\overline{\frac{1}{a}-2m\times EN^2}\big|^{\frac{3}{2}}}$, and if the mean diſtance be unity, then $EN=1+e$, and e being very ſmall, $EN^3=1+3e$, $EN^2=1+2e$ nearly; hence, the time from the higher apſide till the return to the ſame apſide becomes $\frac{p\times\overline{1-2m\times\overline{1+3e}}}{\overline{\frac{1}{a}-2m\times\overline{1+2e}}\big|^{\frac{3}{2}}} = \frac{p\times\overline{1-2m}}{\overline{\frac{1}{a}-2m}\big|^{\frac{3}{2}}}$, and the orbit being nearly circular, a is nearly $=1$; hence, the time $= \frac{p}{\sqrt{1-2m}}$ very nearly.

902. By Art. 894. the moon's periodic time is $\frac{p}{u} - \frac{3}{4}m \times \frac{p}{u^2}$; hence, the mean time from the moon leaving it's apogee till it returns to it : the mean periodic time :: $\frac{p}{\sqrt{1-2m}} : \frac{p}{u} - \frac{3}{4}m \times \frac{p}{u^2} :: \frac{1}{\sqrt{1-2m}} : \frac{1}{u} - \frac{3}{4}m \times \frac{1}{u^2}$. Now $m=,0055796$, and $u=\sqrt{1-\frac{1}{2}m}=,9986041$, hence, the above ratio becomes $1 : ,9916199$, therefore $,9916199 : 1 :: 360° : 363°. 2'. 32'',3916$; conſequently the motion of the apogee in one mean periodic revolution of the moon, is 3°. 2′. 32″,3916, hence, 27*d*. 7*h*. 43′ : 365*d*. 6*h*. 9′ :: 3°. 2′. 32″,3916 : 40° 40′. 20″ the mean progreſſive motion of the apogee in a year. According to MAYER's Tables, it is 40°. 41′. 33″.

903. If

903 If for u we put $\sqrt{1-\frac{1}{2}m}$, we have the mean interval of time between the paſſages of the moon through it's apogee : it's periodic time · $\frac{1}{\sqrt{1-2m}}$: $\frac{1}{\sqrt{1-\frac{1}{2}m}} - \frac{3}{4}m \times \frac{1}{1-\frac{1}{2}m}$:. $\frac{1}{1-m}$ $\frac{1}{1-\frac{1}{4}m} - \frac{3}{4}m \times \frac{1}{1-\frac{1}{2}m}$ nearly, · $1+m$: $1+\frac{1}{4}m - \frac{3}{4}m \times \overline{1+\frac{1}{2}m}$ nearly, $1+m$ $1-\frac{1}{2}m$:. $\frac{1+m}{1-\frac{1}{2}m}$: 1 : $1+\frac{3}{2}m$ · 1 very nearly. Hence, the mean motion of the moon . the mean progreſſive motion of the apſides :: 1 · $\frac{3}{2}m$. But this is the mean motion of the moon, and anſwers to the mean diſtance of the earth from the ſun; but at any other diſtance $1+c \times$ coſ Υ of the earth from the ſun, the diſturbing force of the ſun (891) is $1-3c \times$ coſ Υ, hence, in general, the mean motion of the moon the mean progreſſive motion of the apſides, including the annual variation, 1 $\frac{3}{2} \times m \times \overline{1-3c \times \text{coſ.}\ \Upsilon}$, therefore the mean progreſſive motion of the apſides in the time the ſun deſcribes the angle Υ : equation of the mean motion in that time .. fluent of $\dot{\Upsilon}$. fluent of $-3c \times$ coſ. $\Upsilon \times \dot{\Upsilon}$ Υ · $-3c \times$ ſin. Υ, but in the time the ſun deſcribes the angle Υ, the mean motion of the apſides is $\frac{40°.\ 41'\ 33''}{360°} \times \Upsilon$, hence, $\frac{40°\ 41'.\ 33''}{360°} \times \Upsilon$: the equation of the mean motion :: Υ : $-3c \times$ ſin. Υ, therefore the *annual* equation of the apſides $= -3c \times \frac{40°.\ 41'.\ 33''}{360°} \times$ ſin. $\Upsilon = -19'.\ 35'' \times$ ſin. Υ. Sir I. NEWTON makes it 19′. 43″.

904 By Art. 864. the force of M to $E = \frac{1}{ME^2} - \frac{1}{2}m \times ME + \frac{3}{2}m \times ME \times$ coſ $2MEC$; and at the higher apſide N, it becomes $\frac{1}{NE^2} - \frac{1}{2}m \times NE + \frac{3}{2}m \times NE \times$ coſ. $2NEC$, now if the force from N were to vary inverſely as the ſquare of the diſtance, at M it would become $\frac{1}{ME^2} - \frac{m \times NE^3}{2ME^2} \times \overline{1-3\ \text{coſ}\ 2NEC}$; and by changing the angle NEC, the ellipſe is continually changing into a new one, but if the orbit be very nearly a circle, the excentricity and poſition of the apſides will (877, 878) not ſenſibly be altered in a whole revolution Hence, the motion of the apſides, and the variation of the excentricity, depend upon $-\frac{1}{2}m \times ME \times \overline{1-3.\ \text{coſ.}\ 2MEC} + \frac{m \times NE^3}{2ME^2} \times \overline{1-3.\ \text{coſ}\ 2NEC}$, which is the difference between the real force at M, and what would have been the force if it had varied inverſely as the ſquare of the diſtance from N.

905. The

905. The forces $-\frac{1}{2}m \times ME + \frac{m \times NE^3}{2ME^2}$ are the ſame, very nearly, in every ſituation of the apſides, alſo the force $3m \times ME \times$ coſ. $2MEC$ (in orbits nearly circular) deſtroys itſelf in every revolution, by the oppoſition of it's ſigns Hence, the variation of the excentricity and the inequality of the motion of the apſides, ſo far as they depend upon the ſituation of the apſides, ariſes from the force $-\frac{3m \times NE^3}{2ME^2} \times$ coſ. $2NEC$, which varies as coſ. $2NEC$ nearly, in an orbit which is very nearly a circle. Hence, the whole inequality of the motion of the apſides in their tranſit from quadratures C to N is in proportion to the fluent of coſ. $2NEC \times \dot{\overline{2NEC}}$, or to ſin. $2NEC$, or ſin $2NES$ the diſtance of the apſides from ſyzygies. By obſervation, this greateſt variation of the true from the mean place of the apſide is about 12°. 18′, hence, this equation of the motion of the apſides $= 12°.\ 18' \times$ ſin. $2NES$.

906. Let the abſolute gravity in the higher apſide $N = \frac{b}{EN^2}$, and let it vary from that point in the inverſe duplicate ratio of the diſtance, and let w be the excentricity of the ellipſe ſo deſcribed, and a be it's ſemi-axis major, then (829) the centrifugal force at $M = \frac{a + w}{a} \times \frac{b \times EN}{ME^3}$. Now (905) the variation of the excentricity depends upon the force $-\frac{3m \times EN^3}{2ME^2} \times$ coſ. $2NEC$. Hence, if $x = ME$, $v =$ the velocity in direction of the radius, then $v\dot{v} = \frac{-b\dot{x}}{x^2} + \frac{3m \times EN^3}{2} \times \frac{\dot{x}}{x^3} \times$ coſ. $2NEC + b \times \overline{1 - \frac{w}{a}} \times EN \times \frac{\dot{x}}{x^2}$, whoſe correct fluent (if $z = EN - x$) is $v^2 = \frac{b}{EM^2} \times \frac{2wz - z^2}{a} - 3m \times EN^3 \times$ coſ. $2NEC \times \overline{\frac{1}{EM} - \frac{1}{EN}}$, now the laſt term is equal $-\frac{3m \times EN^3}{EM^2} \times$ coſ. $2NEC \times \overline{EM - \frac{EM^2}{EN}} = -\frac{3m \times EN^3}{EM^2} \times$ coſ. $2NEC \times \overline{z - \frac{z^2}{EN}}$, hence, $v^2 = \frac{1}{EM^2} \times \left(\frac{2bwz - bz^2}{a} - 3m \times EN^3 \times \text{coſ. } 2NEC \times z + 3m \times EN^2 \times \text{coſ. } 2NEC \times z^2;\right)$ but the body deſcending from the higher to the lower apſide, this velocity at the lower apſide vaniſhes; therefore if we make $v = 0$, we get $z = \frac{\frac{2bw}{a} - 3m \times EN^3 \times \text{coſ. } 2NEC}{\frac{b}{a} - 3m \times EN^2 \times \text{coſ. } 2NEC}$ the whole ſpace through which the body has

deſcended in the direction of the radius vector, in it's motion from the higher to the lower apſide, or twice the excentricity of the orbit deſcribed. Now (864) the force of gravity $\frac{b}{EN^2}$ in the higher apſide $= \frac{1}{EN^2} - \frac{1}{2} m \times EN + \frac{3}{2} m \times EN \times \text{coſ.}\ 2NEC$, and if unity repreſent the ſemi-axis major of that orbit which would be deſcribed by the mean force $\frac{1}{EN^2} - \frac{1}{2} m \times EN$ in the higher apſide, and that force be increaſed in the ratio of $\frac{1}{EN^2} - \frac{1}{2} m \times EN : \frac{1}{EN^2} - \frac{1}{2} m \times EN + \frac{3}{2} m \times EN \times \text{coſ.}\ 2NEC$, or as $1 : 1 + \frac{3}{2} m \times EN^3 \times \text{coſ}\ 2NEC$, then (874) by this increaſe of force, the ſemi-axis major is diminiſhed in the ratio of $1 \quad 1 - \frac{3}{2} m \times EN^3 \times \text{coſ}\ 2NEC$, therefore unity being the ſemi-axis major by the former force, the ſemi-axis major a by the latter force $= 1 - \frac{3}{2} m \times EN^3 \times \text{coſ}\ 2NEC$. But $b = 1 - \frac{1}{2} m \times EN^3 + \frac{3}{2} m \times EN^3 \times \text{coſ}\ 2NEC$, hence, $\frac{b}{a} = 1 - \frac{1}{2} m \times EN^3 + 3m \times EN^3 \times \text{coſ}\ 2NEC$, conſequently $z = 2w - \frac{3m \times EN^3 \times \text{coſ}\ 2NEC}{1 - \frac{1}{2} m \times EN^3 + 3m \times EN^3 \times \text{coſ}\ 2NEC - 3m \times EN \times \text{coſ}\ 2NEC} = 2w - \frac{3m \times EN^3 \times \text{coſ}\ 2NEC}{1 - \frac{1}{2} m \times EN^3}$ nearly, therefore the variation of the excentricity $= - \frac{3m \times EN^3 \times \text{coſ.}\ 2NEC}{2 - m \times EN^3}$. Hence, the variation is in proportion to the coſine of twice the diſtance of the apogee from quadratures, or twice the diſtance of the apogee from the ſun.

907 When the apſides are in the octants, $\text{coſ.}\ 2NEC = 0$, and the excentricity is there the mean When the apſides are in ſyzygies, $\text{coſ.}\ 2NEC = -1$, and the variation of excentricity $= \frac{3m \times EN^3}{2 - m \times EN^3}$, the mean excentricity being then increaſed by this quantity, on account of it's being poſitive When the apſides are in quadratures, $\text{coſ.}\ 2NEC = 1$, and the variation $= - \frac{3m \times EN^3}{2 - m \times EN^3}$, which being negative, ſhows that the mean excentricity is there diminiſhed by this quantity. Hence, the excentricity is increaſed whilſt the apſides move from quadratures to ſyzygies, and decreaſed whilſt they move from ſyzygies to quadratures.

908. As $m = 0{,}0055796$, and $EN = 1{,}05505$, the mean diſtance being unity, and the mean excentricity $= {,}05505$, we have $\frac{3m \times EN^3}{2 - m \times EN^3} = 0{,}00986$ the greateſt variation of the excentricity from the mean excentricity. By obſervation

obſervation it is 0,01168 Hence, if $X =$ the mean diſtance of the moon from the ſun, $r = 0{,}01168$, the excentricity $= w + r \times$ coſ. $\overline{2Z - 2X}$ very nearly.

909. By Article 905. the principal inequality of the motion of the apſides varies as the ſine of double the diſtance of the apſides from ſyzygies, and is equal to 12° 18′ × ſin. $2NES$. Now if we make the mean excentricity (,05505) radius, then ,01168 will be the ſine of 12° 18′ very nearly. Hence, Sir I. NEWTON gives the following conſtruction for finding the excentricity Fig. 208. of the orbit, and the equation of the apogee Let E be the focus of the elliptic orbit deſcribed by the moon M, and $EKFI$ the mean place of the apogee, corrected by the annual equation (903), $EF = {,}05505$ the mean excentricity, $EK = {,}04387$ the leaſt excentricity of the moon's orbit, and $EI = {,}06671$ the greateſt, biſect KI in F, and with the center F deſcribe the circle IHK, take the angle HFI equal to twice the diſtance of the apogee from ſyzygies, and EH will be the excentricity of the orbit, and the angle HEF the equation of the apogee, nearly. For the angle HEG being ſmall, we may conſider $EH = EG$, therefore $EH - EF = FG =$ coſ $HFI =$ coſ twice the diſtance of the apogee from ſyzygies, to the radius FI, but (906) the variation of the excentricity varies as that coſine, therefore if the coſine expreſſes the value in one caſe, it muſt always expreſs it, now when H comes to I (or the apogee is in ſyzygies), FG becomes FI, and expreſſes the variation in that caſe, by conſtruction, hence, FG expreſſes, in general, the variation, conſequently EH will be the excentricity. Alſo (on account of the ſmall variation of EH) HG may be conſidered as meaſuring the angle HEG, and HG is the ſine of HFI twice the diſtance of the apogee from ſyzygies, and (905) the equation of the apogee varies as that quantity; therefore if HEG expreſs the true variation in one caſe, it muſt in all, now when G coincides with F, $GH = FD$ (FD being perpendicular to KI), and the angle $DEF =$ 12° 15′ which is very near to 12° 18′ the greateſt equation, hence, the angle HEG always expreſſes very nearly the equation of the apogee

910. Produce EF to A the apogee of the orbit, and let L be the other focus, join MF, ML, MH, and on MF let fall the perpendicular Hv; draw Ew parallel to LM, and ES towards the ſun. Now (227) $2EMF$ is the mean equation of the center, and $2EMH$ is the equation at the given time; hence, $2HMF$ is the difference of the two equations, or the *Evection*, whoſe ſine is $\frac{2Hv}{MH}$, or nearly $= \frac{2Hv}{MF} = \frac{2HF}{MF} \times$ ſin. HFM. Let $Z =$ the angle MLA the mean anomaly of the moon; alſo let $X =$ the diſtance of the mean place of the moon from the ſun, or the angle SEw, and $z = LME = MEw$; then $Z - z = AEM$ the true anomaly of the moon, and $X - z = SEM$, and

as

as the moon moves quicker from the apogee than from the ſun, we have $Z-X=SEN$ the true diſtance of the ſun from the apogee of the moon, hence, $2Z-2X=HFI$. Now MFA is very nearly $=MLA=Z$, therefore $MFH=\overline{2X-Z}$, hence, if the mean radius of the lunar orbit be aſſumed unity, we have the evection $=2HF\times$ ſin $\overline{2X-Z}$. Now when $2X-Z=90°$, the evection $=2HF=0{,}02336$, which is an arc of 1°. 20′. 18′ the greateſt evection; hence, the evection at any other time is $-1°.\ 20'.\ 18''\times$ ſin $\overline{2X-Z}$, it being manifeſt from the conſtruction of the figure, that when $2X-Z$ is leſs than 180° the equation is ſubtractive, and additive when it is greater.

On the Equation which depends upon the Place of the Apogee of the Moon in reſpect to the Sun

FIG. 206.

911. By Art 864 the diſturbing force of S upon $M=-\frac{1}{2}m\times EM+\frac{3}{2}m\times EM\times$ coſ. $2MEC=-\frac{1}{2}m\times EM-\frac{3}{2}m\times EM\times$ coſ. $2MES=-\frac{1}{2}m\times EM\times\overline{1+3\times \text{coſ.}\ 2MES}$ But (868) $EM=1-\frac{1}{2}w^2+w\times$ coſ $z+\frac{1}{2}w^2\times$ coſ. $2z$, neglecting the higher powers of w, hence, the diſturbing force $=-\frac{1}{2}m\times\overline{1-\frac{1}{2}w^2}\times\overline{1+3\times\text{coſ}\ 2MES}-\frac{1}{2}mw\times$ coſ $z-\frac{1}{4}mw^2\times$ coſ $2z-\frac{3}{2}mw\times$ coſ. $2MES\times$ coſ $z-\frac{3}{4}mw^2\times$ coſ. $2MES\times$ coſ. $2z=$ (as coſ $2MES\times$ coſ. $2z=\frac{1}{2}$ coſ $\overline{2MES+2z}+\frac{1}{2}$ coſ $\overline{2z-2MES}$) $-\frac{1}{2}m\times\overline{1-\frac{1}{2}w^2}\times\overline{1+3\times\text{coſ}\ 2MES}-\frac{1}{2}mw\times$ coſ. $z-\frac{1}{4}mw^2\times$ coſ. $2z-\frac{3}{2}mw\times$ coſ. $2MES\times$ coſ. $z-\frac{3}{8}mw^2\times$ coſ. $\overline{2MES+2z}-\frac{3}{8}mw^2\times$ coſ. $\overline{2z-2MES}$ Now $z-MES=SEN$ the diſtance of the ſun from the apogee N of the moon, which is conſtant in a whole revolution of the moon, when the poſition of the apogee in reſpect to the ſun is given, alſo, $-\frac{1}{2}m$ is conſtant, and if we neglect $\frac{1}{4}mw^2$ on account of it's ſmallneſs, all the other terms will be deſtroyed by the oppoſition of their ſigns, in a revolution of the moon Therefore when the poſition of the ſun in reſpect to the apogee of the moon's orbit is given, the *mean* diſturbing force in one revolution of the moon is $-\frac{1}{2}m-\frac{3}{8}mw^2\times$ coſ. $\overline{2z-2MES}$ Hence, the diſturbing force is greateſt when the line of the apſides paſs through the ſun, and leaſt, when it paſſes through quadratures in reſpect to the ſun.

912. Hence, the mean force of the moon to the earth the mean force in one revolution of the moon in any given ſituation of the apſides $1-\frac{1}{2}m$: $1-\frac{1}{2}m-\frac{3}{8}mw^2\times$ coſ. $\overline{2z-2MES}$ $(2Z-2X)$ nearly, :: 1 $1-\frac{3}{8}mw^2\times$ coſ. $\overline{2Z-2X}$ nearly. But if $w=$ the mean excentricity, and $r=,01168$, then will the true excentricity be equal to $w+r\times$ coſ. $\overline{2Z-2X}$ (908), and

conſequently it's ſquare is $w^2 + 2rw \times \text{coſ.}\ \overline{2Z-2X}$ nearly, ſubſtitute this therefore for w^2 in the above proportion, and we get the mean force of the moon to the earth : the mean force in any given ſituation of the apſides : 1 :: $1 - \frac{3}{8}m \times \overline{w^2 + 2rw \times \text{coſ}\ \overline{2Z-2X}} \times \text{coſ}\ \overline{2Z-2X} =$ (as $\text{coſ.}\ \overline{2Z-2X} \times \text{coſ}\ \overline{2Z-2X} = \frac{1}{2} + \frac{1}{2}\text{coſ.}\ \overline{4Z-4X}$) $1 - \frac{3}{8}mw^2 \times \text{coſ}\ \overline{2Z-2X} - \frac{3}{8}mrw \times \text{coſ}\ \overline{4Z-4X}$ nearly, hence (875), the mean motion of the moon is changed in the ratio of $1 : 1 - \frac{3}{4}mw^2 \times \text{coſ}\ \overline{2Z-2X} - \frac{3}{4}mrw \times \text{coſ}\ \overline{4Z-4X}$; conſequently the mean motion of the moon in the time the ſun departed from the apogee through the angle $Z-X$: the equation of the mean motion in that time :: the fluent of $\overline{Z-X}$, or $Z-X$, : the fluent of $-\frac{3}{4}mw^2 \times \text{coſ.}\ \overline{2Z-2X} \times \overline{Z-X} - \frac{3}{4}mrw \times \text{coſ}\ \overline{4Z-4X} \times \overline{Z-X}$, or $-\frac{3}{8}mw^2 \times \text{ſin}\ \overline{2Z-2X} - \frac{3}{16}mrw \times \text{ſin.}\ \overline{4Z-4X}$. Now when the ſun leaves the apogee, it comes to it again after it has deſcribed an angle of 405,7°, as appears from the mean motion of the apogee found in Art 902. Therefore in the time in which the ſun has departed from the apogee through an angle $Z-X$, it's motion has been $\frac{405,7}{360} \times \overline{Z-X}$; hence, the mean motion of the moon in that time has been $\frac{2139}{160} \times \frac{405,7}{360} \times \overline{Z-X}$, we have therefore $\frac{2139}{160} \times \frac{405,7}{360} \times \overline{Z-X}$: the equation of the mean motion :: $Z-X : -\frac{3}{8}mw^2 \times \text{ſin}\ \overline{2Z-2X} - \frac{3}{16}mrw \times \text{ſin.}\ \overline{4Z-4X}$, conſequently the equation of the mean motion is $-\frac{2139}{160} \times \frac{405,7}{360} \times \frac{3}{8}mw^2 \times \text{ſin.}\ \overline{2Z-2X} - \frac{2139}{160} \times \frac{405,7}{360} \times \frac{3}{16}mrw \times \text{ſin.}\ \overline{4Z-4X} = -20'' \times \text{ſin.}\ \overline{2Z-2X} - 2,1'' \times \text{ſin.}\ \overline{4Z-4X}$.

To correct the Equation of the Center of the Moon.

913. If w be the mean excentricity of the moon's orbit, Z the correſponding mean anomaly, the true anomaly $= Z - 2w \times \text{ſin}\ Z + \frac{5}{4}w^2 \times \text{ſin}\ 2Z$ (870) omitting the other terms, but (908) the true excentricity $= w + r \times \text{coſ.}\ \overline{2Z-2X}$, and the ſine of the aberration of the apſide from the mean place $= \frac{r}{w} \times \text{ſin.}\ \overline{2Z-2X}$ (924). Now (903) the annual equation of the mean motion of the apogee is $-19'.\ 35'' \times \text{ſin.}\ \Upsilon$, and therefore Z corrected by applying

plying this correction of the apogee, is $Z+19'\ 35''\times\text{ſin. }\Upsilon$, alſo, there is (891) an annual equation of the mean motion of the moon $=12'.\ 55''\times\text{ſin. }\Upsilon$, applying therefore this correction alſo to Z, it becomes $Z+32'.\ 30''\times\text{ſin. }\Upsilon$ Now in the ſecond term $(-2w\times\text{ſin. }Z)$, ſubſtitute for ſin. Z it's more correct value, $\text{ſin. }(Z+32'.\ 30''\times\text{ſin. }\Upsilon)=\text{ſin. }Z+32'.\ 30''\times\text{ſin }\Upsilon\times\text{coſ }Z$, and a new term $-2w\times 32'.\ 30''\times\text{ſin }\Upsilon\times\text{coſ }Z$ is introduced, which is $=-32'\ 30''\,w\times\text{ſin. }\overline{Z+\Upsilon}+32'.\ 30''\,w\times\text{ſin. }\overline{Z-\Upsilon}$. If we were to correct this ſecond term, by ſubſtituting for w and Z their corrected values $w+r\times\text{coſ. }\overline{2Z-2X}$ and $Z-\frac{r}{w}\times\text{ſin. }\overline{2Z-2X}$, as we do in the correction of the third term, it would produce the evection, which we have already determined (910). Next, in the third term $(\frac{5}{4}w^2\times\text{ſin. }2Z)$, for w put $w+r\times\text{coſ. }\overline{2Z-2X}$ (908), and for ſin $2Z$ write the value of it corrected for the principal equation of the apogee, that is, $\text{ſin. }(2Z-\frac{2r}{w}\times\text{ſin. }\overline{2Z-2X})=\text{ſin. }2Z-\frac{2r}{w}\times\text{ſin. }\overline{2Z-2X}\times\text{coſ }2Z$ nearly, $=\text{ſin. }2Z+\frac{r}{w}\times\text{ſin. }2X-\frac{r}{w}\times\text{ſin. }\overline{4Z-2X}$, and we get theſe new terms $\frac{5}{4}rw\times\text{ſin }2X-\frac{5}{4}rw\times\text{ſin. }\overline{4Z-2X}+\frac{5}{2}rw\times\text{ſin. }2Z\times\text{coſ. }\overline{2Z-2X}$ nearly, $=$ (as the laſt term $=\frac{5}{4}rw\times\text{ſin. }\overline{4Z-2X}+\frac{5}{4}rw\times\text{ſin. }2X$) $\frac{5}{2}rw\times\text{ſin. }2X$. The preceding correction $Z+32'.\ 20''\times\text{ſin. }\Upsilon$ of Z was not here introduced, on account of the ſmallneſs of the equations which it would have produced. Hence, we have theſe new equations, $-32'\ 30''\,w\times\text{ſin }\overline{Z+\Upsilon}+32'.\ 30''\,w\times\text{ſin. }\overline{Z-\Upsilon}+\frac{5}{2}rw\times\text{ſin. }2X=-1'.\ 47''\times\text{ſin. }\overline{Z+\Upsilon}+1'.\ 47''\times\text{ſin. }\overline{Z-\Upsilon}+5'\ 31''\times\text{ſin. }2X$.

To correct the Equation of the Variation of the Moon

914. The variation of the moon as already determined (889) is $35'\ 10''\times\overline{1-3{,}299c\times\text{coſ }\Upsilon}\times\text{ſin. }2X=35'.\ 10''\times\text{ſin. }2X-3{,}299c\times\text{coſ. }\Upsilon\times\text{ſin. }2X$. Now (870) the difference of the true and mean places of the moon is $-2w\times\text{ſin }Z$, omitting the other terms of the ſeries, but this is without conſidering the annual equation $+12'\ 55''\times\text{ſin. }\Upsilon$, or $,003758\times\text{ſin. }\Upsilon$, of the moon, hence, the difference becomes $-2w\times\text{ſin. }Z+,003758\times\text{ſin. }\Upsilon$; and conſidering X as the mean diſtance of the moon from the ſun, we have $X-2w\times\text{ſin. }Z+,003758\times\text{ſin. }\Upsilon$ for a corrected diſtance of the ſun from the

the moon; hence, ſin. $2X$ becomes $\text{ſin }(2X-4w\times\text{ſin }Z+,007516\times\text{ſin }Y)=\text{ſin. }2X-4w\times\text{ſin. }Z\times\text{coſ }2X+,007516\times\text{ſin. }Y\times\text{coſ }2X=\text{ſin }2X-2w\times\text{ſin }\overline{2X+Z}+2w\times\text{ſin }\overline{2X-Z}+,003758\times\text{ſin. }\overline{2X+Y}-,003758\times\text{ſin. }\overline{2X-Y}$. Hence, the corrected variation of the moon becomes $35'\ 10''\times(\text{ſin }2X-2w\times\text{ſin }\overline{2X+Z}+2w\times\text{ſin }\overline{2X-Z}+,003758\times\text{ſin. }\overline{2X+Y}-,003758\times\text{ſin. }\overline{2X-Y})-3,299\times35'\ 10''\times c\times\text{coſ. }Y\times(\text{ſin }2X-2w\times\text{ſin. }\overline{2X+Z}+2w\times\text{ſin }\overline{2X-Z})$ nearly, omitting the other two terms on account of their ſmallneſs But the laſt term, after actually multiplying each part of it into coſ Y, may be further reſolved into $-3,299\times35'.\ 10''\times c\times(\tfrac{1}{2}\text{ſin. }\overline{2X+Y}+\tfrac{1}{2}\text{ſin }\overline{2X-Y}-w\times\text{ſin }\overline{2X+Z+Y}-w\times\text{ſin. }\overline{2X+Z-Y}+w\times\text{ſin. }\overline{2X-Z+Y}+w\times\text{ſin. }\overline{2X-Z-Y}$. Hence, beſides the equation $35'.\ 10''\times2X$, we get the following, $35'.\ 10''\times(-2w\times\text{ſin }\overline{2X+Z}+2w\times\text{ſin. }\overline{2X-Z}+,003758\times\text{ſin. }\overline{2X+Y}-,003758\times\text{ſin. }\overline{2X-Y})-3,299\times35'\ 10''\times c\times(\tfrac{1}{2}\text{ſin }\overline{2X+Y}+\tfrac{1}{2}\text{ſin }\overline{2X-Y}-w\times\text{ſin }\overline{2X+Z+Y}-w\times\text{ſin. }\overline{2X+Z-Y}+w\times\text{ſin }\overline{2X-Z+Y}+w\times\text{ſin }\overline{2X-Z-Y})=-3'\ 52''\times\text{ſin. }\overline{2X+Z}+3'\ 52''\times\text{ſin }\overline{2X-Z}+7'',9\times\text{ſin. }\overline{2X+Y}-7'',9\times\text{ſin. }\overline{2X-Y}-58'',4\times\text{ſin }\overline{2X+Y}-58'',4\times\text{ſin. }\overline{2X-Y}+6'',4\times\text{ſin }\overline{2X+Z+Y}+6'',4\times\text{ſin }\overline{2X+Z-Y}-6'',4\times\text{ſin }\overline{2X-Z+Y}-6'',4\times\text{ſin. }\overline{2X-Z-Y}=-3'.\ 52''\times\text{ſin }\overline{2X+Z}+3'\ 52''\times\text{ſin. }\overline{2X-Z}-50'',5\times\text{ſin }\overline{2X+Y}-1'\ 6'',3\times\text{ſin }\overline{2X-Y}+6'',4\times\text{ſin. }\overline{2X+Z+Y}+6'',4\times\text{ſin. }\overline{2X+Z-Y}-6'',4\times\text{ſin. }\overline{2X-Z+Y}-6'',4\times\text{ſin }\overline{2X-Z-Y}$

915 The principal part of the variation is $35'.\ 10''\times\text{ſin. }2X$ Now let $2X$ (double the diſtance of the moon from the ſun) be corrected, by taking two terms (870) of the equation of the center of the moon, that is $-2w\times\text{ſin. }Z+\tfrac{5}{4}w^2\times\text{ſin. }2Z$, and then $2X$ becomes $2X-4w\times\text{ſin. }Z+\tfrac{5}{2}w^2\times\text{ſin. }2Z$, ſubſtitute this therefore for $2X$ into $35'.\ 10''\times\text{ſin. }2X$, and we get $35'.\ 10''\times\text{ſin }(2X-4w\times\text{ſin. }Z+\tfrac{5}{2}w^2\times\text{ſin }2Z)=35'\ 10''\times(\text{ſin }2X-4w\times\text{ſin. }Z\times\text{coſ }2X+\tfrac{5}{2}w^2\times\text{ſin. }2Z\times\text{coſ. }2X)$, and the two firſt having been already conſidered, there ariſes a new equation $35'.\ 10''\times\tfrac{5}{2}w^2\times\text{ſin. }2Z\times\text{coſ }2X=35'.\ 10''\times\tfrac{5}{4}w^2\times\text{ſin. }\overline{2Z+2X}+35'.\ 10''\times\tfrac{5}{4}w^2\times\text{ſin. }\overline{2Z-2X}=8''\times\text{ſin }\overline{2Z+2X}+8''\times\overline{2Z-2X}$.

916. The principal part of the variation may alſo be corrected, if X be corrected by the equation of the evection, or if for X we ſubſtitute $X-2r\times\text{ſin. }\overline{2X-Z}$ (910), in which caſe we get $35'.\ 10''\times\text{ſin. }(2X-4r\times\text{ſin. }\overline{2X-Z})=35'\ 10''\times\text{ſin. }2X-35'\ 10''\times4r\times\text{coſ }2X\times\text{ſin }\overline{2X-Z}$, hence, there ariſes a new equation $-35'.\ 10''\times4r\times\text{coſ. }2X\times\text{ſin. }\overline{2X-Z}=-35'\ 10''$

$\times\ 2r$

$\times 2r \times$ ſin. $\overline{4X-Z}+35'.\ 10''\times 2r\times$ ſin $Z=-49'',3\times$ ſin. $\overline{4X-Z}+49'',3$ $\times$ ſin. Z

917 The variation will alſo vary in proportion to the diſturbing force of the ſun. Now (893) the mean diſturbing force : that diſturbing force which ariſes from the different ſituations of the node of the lunar orbit :: 1 : $\frac{3}{2}s^2\times$ coſ $\overline{2S-2X}$; hence, there ariſes another correction of the variation $=35'.\ 10''\times\frac{3}{2}s^2\times$ ſin. $2X\times$ coſ. $\overline{2S-2X}=35'.\ 10''\times\frac{3}{4}s^2\times$ ſin. $2S+35'.\ 10''\times\frac{3}{4}s^2\times$ ſin. $\overline{4X-2S}=12'',5\times$ ſin. $2S-12'',5\times$ ſin. $\overline{2S-4X}$.

To correct the Equation of the Evection of the Moon.

918. The evection (910) is $-2r\times$ ſin. $\overline{2X-Z}$, and $r=HF$, which (906) is proportional to m, or to the diſturbing force of the ſun, therefore, in general, the evection $=-\overline{1-3c\times\text{coſ}\ \Upsilon}\times 2r\times$ ſin. $\overline{2X-Z}$; there ariſes therefore a new equation $=6cr\times$ coſ. $\Upsilon\times$ ſin $\overline{2X-Z}=3cr\times$ ſin. $\overline{2X-Z+\Upsilon}+3cr$ $\times$ ſin $\overline{2X-Z-\Upsilon}=+2'.\ 1''\times$ ſin. $\overline{2X-Z+\Upsilon}+2'\ 1''\times$ ſin. $\overline{2X-Z-\Upsilon}$

919. On account of the various ſituations of the node, the diſturbing force (893) varies as 1 : $\overline{1+\frac{3}{2}s^2\times\text{coſ}\ \overline{2S-2X}}$, hence, the evection becomes $-\overline{1+\frac{3}{2}s^2\times\text{coſ.}\ \overline{2S-2X}}\times 2r\times$ ſin $\overline{2X-Z}$, there ariſes therefore a new equation $=-3rs^2\times$ coſ. $\overline{2S-2X}\times$ ſin. $\overline{2X-Z}=-\frac{3}{2}rs^2\times$ ſin. $\overline{2S-Z}-\frac{3}{2}rs^2\times$ ſin. $\overline{4X-2S-Z}=-29''\times$ ſin $\overline{2S-Z}-29''\times$ ſin $\overline{4X-2S-Z}$

920 If double the diſtance of the moon from the ſun ($2X$) be corrected by the annual equation (891), it becomes $2X+25'.\ 50''\times$ ſin. Υ, and if the mean motion Z of the moon from the apogee be corrected by the ſame equation $12'.\ 55''\times$ ſin. Υ, and by the annual equation $-19'\ 35''\times$ ſin Υ, then Z becomes $Z+32'\ 30''\times$ ſin. Υ, hence, $\overline{2X-Z}$ becomes $\overline{2X-Z}-6'\ 40''\times$ ſin. Υ, therefore the evection becomes $-2r\times$ ſin. $(\overline{2X-Z}-6'.\ 40''\times$ ſin. $\Upsilon)=-2r$ $\times$ ſin. $\overline{2X-Z}-2r\times 6'.\ 40''\times$ ſin. $\Upsilon\times$ coſ $\overline{2X-Z}$, hence, there ariſes a new equation $-2r\times 6'\ 40''\times$ ſin. $\Upsilon\times$ coſ. $\overline{2X-Z}=-r\times 6'\ 40''\times$ ſin. $\overline{2X-Z+\Upsilon}$ $+r\times 6'.\ 40''\times$ ſin. $\overline{2X-Z-\Upsilon}=-4\frac{2}{3}''\times$ ſin $\overline{2X-Z+\Upsilon}+4\frac{2}{3}''\times$ ſin. $\overline{2X-Z-\Upsilon}$

921. Hence, we have the following equations of the moon's longitude (reckoned upon it's orbit), to be applied to the mean in order to obtain the true longitude, having collected into one term, all thoſe which have the ſame argument:

I.	−	6°	17′.	32″,7	ſin. Z.
II	+		13	0	ſin. $2Z$.
III.	−		0	37	ſin $3Z$
IV.	−	1.	16.	26	ſin $\overline{2X-Z}$
V.	−		3	52	ſin. $\overline{2X+Z}$.
VI	+		40	41	ſin $2X$
VII.	+		12	55	ſin Y.
VIII.	+		1.	47	ſin $\overline{Z-Y}$.
IX	−		1	47	ſin $\overline{Z+Y}$
X	−		1	6,3	ſin $\overline{2X-Y}$.
XI.	−		0.	50,5	ſin $\overline{2X+Y}$.
XII.	−		0.	12	ſin $\overline{2Z-2X}$
XIII	+		0.	2,1	ſin. $\overline{4Z-4X}$.
XIV.	−		0.	49,3	ſin. $\overline{4X-Z}$.
XV.	+		0.	12,5	ſin $2S$
XVI	−		1.	28	ſin. $\overline{2S-2X}$.
XVII	−		0.	29	ſin. $\overline{2S-Z}$.
XVIII.	+		1.	59,3	ſin. $\overline{2X-Z-Y}$
XIX.	+		1.	49,9	ſin. $\overline{2X-Z+Y}$
XX	+		0.	8	ſin. $\overline{2Z+2X}$
XXI	−		0	12,5	ſin. $\overline{2S-4X}$.
XXII.	−		0	29	ſin. $\overline{4X-2S-Z}$.
XXIII.	+		0	6,4	ſin. $\overline{2X+Z-Y}$
XXIV.	+		0.	6,1	ſin $\overline{2X+Z+Y}$

922. Theſe are the equations of the moon's longitude upon it's orbit, deduced from the principles which Sir I. NEWTON has given in his *Principia*; and it is manifeſt, that we might have proceeded in the ſame manner, and found many more ſmaller equations. By this method of treating the ſubject, the ſources of all the equations become manifeſt, and every one is calculated directly from the cauſe which produces it. Comparing theſe with the equations deduced from the direct method, no greater difference is found, than what is obſerved to take place amongſt thoſe which are computed from the direct ſolution, by different Authors, and MAYER, after his moſt laborious calculations, founded upon a very elegant theory, was obliged to correct many of his equations from obſervations. We ſhall afterwards ſhow the reduction neceſſary to give the longitude upon the ecliptic.

923. The equation 1′ 49″,9 × ſin $\overline{2X-Z+Y}$, Sir I. NEWTON thus repreſents The conſtruction of FIG. 208. remaining, make the angle $MHr=2X-Z+Y$, and take $Hr=,000352$, which quantity ſubtends an angle of 1′. 12½″ at M, the whole equation being 2′ 25″ according to Sir I NEWTON, and not 1′ 49″,9 as we make it, and Hmr will be the correction for this equation. For conceive the center of the lunar orbit to be tranſported from H to r, and to deſcribe a circle about H, and draw rs perpendicular to MH, then $rs=,000352\times\text{ſin.}\,\overline{2X-Z+Y}$, therefore the angle $rMH=1'.\ 12\tfrac{1}{2}''\times\text{ſin.}\,\overline{2X-Z+Y}$, which varies as ſin. $\overline{2X-Z+Y}$, and the whole quantity of variation is 2′ 25″, taking Hr on oppoſite ſides of H, and at right angles to MH. As therefore the whole variation of rMH is (from this conſtruction) the true quantity, and it varies in it's proper ratio, it muſt always repreſent the true correction. FIG. 208.

On the Equations of the horizontal Parallax of the Moon.

924 By Art. 868. the diſtance of the moon from the center of the earth is $1+\frac{1}{2}w^2+w\times\text{coſ.}\,Z-\frac{1}{2}w^2\times\text{coſ.}\,2Z$, but the horizontal parallax of the moon is inverſely as the diſtance, or as $\frac{1}{1+\frac{1}{2}w^2+w\times\text{coſ}\,Z-\frac{1}{2}w^2\times\text{coſ}\,2Z}$ $=1-\frac{1}{2}w^2-w\times\text{coſ.}\,Z+w^2\times\overline{\text{coſ.}\,Z}^2+\frac{1}{2}w^2\times\text{coſ.}\,2Z=$ (as $\overline{\text{coſ.}\,Z}^2=\frac{1}{2}\,\text{coſ}\,2Z+\frac{1}{2}$) $1-w\times\text{coſ}\,Z+w^2\times\text{coſ.}\,2Z$ Now (908) $w+r\times\text{coſ.}\,\overline{2Z-2X}$ $=EH$ the true excentricity at the given time, and (909) the ſine of HEG (the aberration of the line of the apſides from the mean place) $=\frac{r\times\text{ſin}\,\overline{2Z-2X}}{w+r\times\text{coſ.}\,\overline{2Z-2X}}$ $=\frac{r}{w}\times\text{ſin.}\,\overline{2Z-2X}$ nearly. Hence, if in the above expreſſion for the radius vector, we put $w+r\times\text{coſ.}\,\overline{2Z-2X}$ for w, and $Z-\frac{r}{w}\times\text{ſin}\,\overline{2Z-2X}$ for Z; and if for coſ. $(Z-\frac{r}{w}\times\text{ſin.}\,\overline{2Z-2X})$ we write coſ. $Z+\frac{r}{w}\times\text{ſin.}\,Z\times\text{ſin.}\,\overline{2Z-2X}$ (the coſ of $\frac{r}{w}\times\text{ſin}\,\overline{2Z-2X}$ being conſidered unity), we get the horizontal parallax of the moon proportional to $1-w\times\text{coſ.}\,Z-r\times\text{ſin}\,Z\times\text{ſin}\,\overline{2Z-2X}-r\times\text{coſ}\,Z\times\text{coſ}\,\overline{2Z-2X}-\frac{r^2}{w}\times\text{ſin.}\,Z\times\text{ſin}\,\overline{2Z-2X}\times\text{coſ.}\,\overline{2Z-2X}+w^2\times\text{coſ.}\,2Z+2wr\times\text{coſ.}\,2Z\times\text{coſ.}\,\overline{2Z-2X}$. But ſin. $Z\times$ ſin.

$\overline{2Z-2X}$

$\overline{2Z-2X} = \frac{1}{2}\,\text{coſ.}\,\overline{2X-Z} - \frac{1}{2}\,\text{coſ.}\,\overline{3Z-2X}$, alſo, $\text{coſ.}\,Z \times \text{coſ}\,\overline{2Z-2X} = \frac{1}{2}\,\text{coſ.}\,\overline{2X-Z} + \frac{1}{2}\,\text{coſ.}\,\overline{3Z-2X}$, laſtly, $\text{ſin}\,Z \times \text{ſin}\,\overline{2Z-2X} \times \text{coſ.}\,\overline{2Z-2X} = \frac{1}{2}\,\text{ſin.}\,Z \times \text{ſin.}\,\overline{4Z-4X} = \frac{1}{4}\,\text{coſ.}\,\overline{4X-3Z} - \frac{1}{4}\,\text{coſ}\,\overline{5Z-4X}$, hence, by ſubſtitution, we get the horizontal parallax of the moon proportional to $1 - w \times \text{coſ.}\,Z + w^2 \times \text{coſ}\,2Z + wr \times \text{coſ.}\,2X - \frac{r^2}{4w} \times \text{coſ.}\,\overline{4X-3Z} - r \times \text{coſ.}\,\overline{2X-Z} + wr \times \text{coſ.}\,\overline{4Z-2X} + \frac{r^2}{4w} \times \text{coſ}\,\overline{5Z-4X}$, the mean horizontal parallax being unity.

925. This would be the horizontal parallax of the moon in an elliptic orbit having the earth in it's focus, but on account of the change of the form of the orbit, at the diſtance X of the moon from the ſun, the radius (882) varies in the ratio of $1 - \frac{1}{140} \times \text{coſ.}\,2X : 1$, hence, the parallax varies in the ratio of $\frac{1}{1 - \frac{1}{140} \times \text{coſ.}\,2X} : 1$, or as $1 + \frac{1}{140} \times \text{coſ.}\,2X : 1$; there ariſes therefore another equation $+\frac{1}{140} \times \text{coſ.}\,2X$ of the parallax. The horizontal parallax of the moon is therefore $1 - w \times \text{coſ.}\,Z + w^2 \times \text{coſ}\,2Z + \overline{wr + \frac{1}{140}} \times \text{coſ.}\,2X - \frac{r^2}{4w} \times \text{coſ.}\,\overline{4X-3Z} - r \times \text{coſ}\,\overline{2X-Z} + wr \times \text{coſ}\,\overline{4Z-2X} + \frac{r^2}{4w} \times \text{coſ.}\,\overline{5Z-4X}$, the mean horizontal parallax being unity. If therefore we aſſume $r = ,01168$, $w = ,05505$, and the *mean* equatorial parallax to be 57′ 11″,4, as MAYER makes it, we have the equatorial horizontal parallax $= 57'\ 11'',4 - 3'.\ 8'',9 \times \text{coſ.}\,Z + 10'',4 \times \text{coſ.}\,2Z + 26'',7 \times \text{coſ.}\,2X - 2'',1 \times \text{coſ}\,\overline{4X-3Z} - 40'',1 \times \text{coſ}\,\overline{2X-Z} + 2'',2 \times \text{coſ.}\,\overline{4Z-2X} + 2'',1 \times \text{coſ.}\,\overline{5Z-4X}$. Hence, we find (173) the horizontal parallax for any other latitude.

On

On the Motion of the Nodes of the Moon's Orbit.

926. If z be the coſine of any angle x, then $z^2 = \frac{1}{2} + \frac{1}{2}$ coſ. $2x$, $z^4 = \frac{3}{8} + \frac{1}{2}$ coſ. $2x + \frac{1}{8}$ coſ. $4x$, $z^6 = \frac{5}{16} + \frac{15}{32}$ coſ $2x + \frac{3}{16}$ coſ. $4x + \frac{1}{32}$ coſ $6x$, &c. radius being unity. This appears from the principles of plane Trigonometry.

927. Let E be the earth, $CADB$ be the plane of the ecliptic, NMn the moon's orbit inclined to it, and ſuppoſed to be a circle, Nn the line of the nodes, AEB the line of the ſyzygies perpendicular to CED, MK perpendicular to CD, Av a perpendicular to the plane of the moon's orbit, and vt, At perpendicular to NEn. Now the ablatitious force acting on the moon at M acts parallel to AE, and conſequently it makes the ſame angle with the plane of the moon's orbit as AE does, and if we conceive AE to be a force, Av is that part of it which acts perpendicular to the plane of the moon's orbit; and if we put s = the ſine of the angle Atv the inclination of the orbit, to radius unity, then $1 : s :: At : Av = s \times At$, hence, AE : the ablatitious force $3MK$ (845) $:: s \times At : \frac{3s \times At \times MK}{AE}$* that part of the ablatitious force which acts perpendicular to the plane of the moon's orbit. Let the periodic time of the ſun : that of the moon :: $1 : n$; then the part of the ablatitious force acting perpendicular to the moon's orbit : the ablatitious force :: $\frac{3s \times At \times MK}{AE} : 3MK :: s \times At : AE$, and (858) the ablatitious force : the gravity of the moon :: $3n^2 MK : ME$ or AE, hence, the force acting perpendicular to the moon's orbit : the moon's gravity :: $3n^2 s \times At \times MK : AE^2$. Now the velocity of the moon being repreſented by the ſmall arc Mw deſcribed in a given time, the velocity generated by the gravity of the moon to the earth in the ſame time is repreſented by twice the ſagitta of Mw which $= \frac{Mw^2}{ME} = \frac{Mw^2}{AE}$, hence, the velocity generated in a given time being as the force, $AE^2 : 3n^2 s \times At \times MK :: \frac{Mw^2}{AE} : \frac{3n^2 s \times Mw^2 \times At \times MK}{AE^3}$ the

FIG. 209.

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* On account of the inclination of the moon's orbit, $3MK$ does not accurately expreſs the ablatitious force; the difference however which this conſideration would make in the reſult is ſo very ſmall, that we ſhall here omit it.

the velocity generated by the ablatitious force in a direction perpendicular to the moon's orbit; draw therefore wz perpendicular to the orbit and equal to this quantity, and deſcribe the circle $N'Mzn'$, and NN', or nn', will be the correſponding motion of the nodes. Draw np perpendicular to Mn. Now Mw : wz :: ſin. Mp or Mn, which is Mk : $n'p = \frac{wz \times Mk}{Mw}$, and ſin. pnn' (s) : $n'p = \frac{wz \times Mk}{Mw}$ :: 1 : $nn' = \frac{wz \times Mk}{s \times Mw}$; hence, the angle $nEn' = \frac{wz \times Mk}{s \times ME \times Mw} = 3n^2 \times \frac{Mw}{ME} \times \frac{At \times MK \times Mk}{AE^3} = 3n^2 \times \frac{Mw}{ME} \times$ ſin. AEN × ſin. MEC × ſin. MEN the motion of the node, whilſt the moon deſcribes Mw, $\frac{Mw}{ME}$ denoting the angle which Mw ſubtends at E. This is the caſe when the orbit is a circle. Let us conſider therefore what will be the motion of the nodes, if we diminiſh the diameter AB by a very ſmall quantity, and ſuppoſe the curve to become an ellipſe, the periodic time remaining the ſame.

Fig 210

928. Let $CA'DB'$ be the ellipſe; draw MQK perpendicular to CD, and mqk parallel and indefinitely near to it; join EQ, EM, Eq, Em, with the center E deſcribe the circular arc qct, let nEN be the line of the nodes, and draw Mr and Qs perpendicular to En. Let one body deſcribe the circle, and the other the ellipſe, and let them ſet out together from C, then they will come to M, Q, at the ſame time, for (805) the area CEM : CEA :: the time through CM : the time through CA, and CEQ : CEA' :: the time through CQ : the time through CA; but CEM : CEA :: CEQ : CEA', therefore the time through CM : time through CA :: time through CQ : the time through CA', but by ſuppoſition, the ſecond and fourth terms are equal, therefore the firſt and third are equal, and the bodies come to M and Q at the ſame time, and Mm, Qq will be the cotemporary arcs deſcribed, and the cotemporary areas deſcribed about E being in proportion to the whole areas, we have $Mm \times ME$: $qc \times QE$ :: EA : EA' :: ME : EA', therefore Mm : qc :: QE : EA'. But (845) the diſturbing forces at M and Q are as MK : QK very nearly*, and therefore (927) the motions perpendicular to the planes of the orbits (repreſented in Art. 927 by wz) will be as MK : QK. Now the force acting perpendicular to the plane of the orbit at Q turns the plane about QE, and thereby produces a motion of the nodes; and the motion produced by this force is compounded with the motion Qq, or with the two motions Qc, cq, of which Qc lies in the line about which the plane turns.

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* They are not accurately ſo, as before obſerved, becauſe the orbits are a little inclined to the ecliptic $CABD$.

turns; therefore the motion of the nodes is the ſame as it would be in a body deſcribing the circular arc cq in the ſame time. Hence, from the firſt expreſſion in the laſt Article for the angle nEn', we get the motion of the nodes of the circle the cotemporary motion of the nodes of the ellipſe :. $\frac{MK\times Mr}{ME\times Mm}$ $\frac{QK\times Qs}{QE\times qc}$ ∷ $\frac{MK\times Mr}{ME\times QE}$ $\frac{QK\times Qs}{QE\times EA'}$, but $\frac{Mr}{ME}$ = ſin nEM = ſin. $\overline{nEC+CEM}$ = ſin. $nEC \times$ coſ. CEM + ſin. $CEM \times$ coſ. nEC, alſo, $\frac{Qs}{QE}$ = ſin nEQ = ſin. $\overline{nEC+CEQ}$ = ſin. $nEC\times$ coſ. CEQ + ſin $CEQ\times$ coſ nEC; but in a whole revolution, coſ. CEM, and coſ CEQ are deſtroyed by the oppoſition of their ſigns, and therefore to get the *mean* motions, the terms where they enter may be neglected, alſo, ſin $CEM = \frac{MK}{ME}$, and ſin. $CEQ = \frac{QK}{QE}$, hence, the *mean* motion of the nodes of the circle : that of the ellipſe $\frac{MK^2}{ME\times QE} : \frac{QK^2}{EA'\times QE}$:. $\frac{AE^2}{AE} : \frac{EA'^2}{EA'} : AE \cdot EA'$.

929. With the center E and radius Ea deſcribe the circle ab, and let that be the circle which the moon would have deſcribed if there had been no diſturbing forces, ſuppoſing $CA'BD'$ to be the ellipſe which it would deſcribe without excentricity, from the diſturbing forces, and let the periodic time in this circle be equal to the periodic time in the circle $CABD$, and conſequently equal to the periodic time in the ellipſe, according to the foregoing ſuppoſition. Now the periodic time in the circles being the ſame, the mean motion of the nodes (927) are the ſame; hence, the motion of the nodes in the circular orbit ab the motion of the nodes in the elliptic orbit $CA'BD'$ $EA : EA'$. (885) 70 · 69.

930. Now let us ſuppoſe the moon to deſcribe it's true orbit $CABD$, in which CD is the line of quadratures, AEB that of ſyzygies, aEb the major axis, E the earth in the focus, Fe the ſemi-minor axis; M the place of the moon, Mm a very ſmall arc, and with the center E, deſcribe the circles mn, Ft, cutting the line Nn of the nodes in n, t, draw MK, Qk, perpendicular to CD, and Mr, Qs, perpendicular to Nn; and let a body revolving in the circle Ft, deſcribe Qq in the time in which Mm is deſcribed. Now (818) the periodic time in the circle tF is equal to the periodic time in the ellipſe, hence (805), on account of the equality of the periodic times, the area $MEm \cdot QEq$, or $ME\times md$ $QE\times Qq$, the whole area of the ellipſe : the whole area of the circle . eF ea, therefore md . Qq : $QE\times eF : ME\times ea$. But (845) the diſturbing forces at M and Q are as MK, Qk very nearly, and therefore FIG. 211.

(827) the motions perpendicular to the plane of the orbits will be in the ſame ratio; hence, (as explained in the laſt Article) the motion of the nodes of the ellipſe in the time *Mm* is deſcribed : the motion of the nodes in the circle whilſt Qq is deſcribed :: $\frac{MK \times Mr}{ME \times md} : \frac{Qk \times Qs}{QE \times Qq} :: \frac{ME}{md} : \frac{QE}{Qq} :: \frac{ME^2}{\text{area } EMm} : \frac{QE^2}{\text{area } EQq} :: \frac{ME^2}{\text{area ellip}} : \frac{QE^2}{\text{area circ.}} :: \frac{\text{area } MEm}{eF} : \frac{\text{area } QEc}{ea}$; hence, the mean motion of the nodes of the ellipſe : the mean motion of the nodes of the circle :: $\frac{\text{ſum of all the areas } MEm}{eF} : \frac{\text{ſum of all the areas } QEc}{ea} :: \frac{\text{area of ellipſe}}{eF} : \frac{\text{area of circle}}{ea} :: \frac{eF}{eF} : \frac{ea}{ea}$, therefore the mean motion of the nodes of the ellipſe is equal to the mean motion of the nodes of the circle. Hence, if (929) we find the mean motion of the nodes for a circular orbit, and diminiſh it in the ratio of 70 : 69, we get the mean motion of the nodes of the true lunar orbit. It appears therefore that the motion of the nodes is not affected by the excentricity of the orbit, as Sir I NEWTON ſuppoſed.

931. The true motion of the nodes of the ellipſe at any time : the cotemporary motion of the nodes of the circle :: $\frac{ME^2}{eF} : \frac{QE^2}{ea} :: ME^2 : eF \times ea$; hence, when *ME* is a mean proportional between the ſemi-major and ſemi-minor axes, the true motion of the nodes of the ellipſe and circle are equal.

FIG. 209. 932. If we take the nodes in quadratures and the moon in ſyzygies, *At*, *MK*, *Mk* become each equal to *AE*, and the motion of the nodes $= 3n^2 \times \frac{Mw}{ME}$, and they are regreſſive. Hence, if we take $\frac{Mw}{ME} = 32'.\ 56''.\ 27'''.\ 12\frac{1}{2}''''$ the mean horary motion of the moon, the horary motion of the nodes in this ſituation will be $33''\ 10'''.\ 33''''\ 12'''''$ for a circular orbit, and if we diminiſh this in the ratio of 70 : 69 on account of the elliptic form of the orbit, we have (930) $32'\ 42''.\ 7'''$ for the true horary motion of the nodes when the moon is in ſyzygies and the nodes in quadratures. And the horary motion of the nodes at any other time : this quantity :: ſin. *AEN* × ſin. *MEC* × ſin. *MEN* : the cube of radius.

933. As often as any of the ſines of *AEN*, *MEC*, *MEN* changes from poſitive to negative, the regreſſive motion of the nodes becomes progreſſive, and the contrary. Let the ſituation of the nodes be given, then ſin *MEC* becomes negative when the moon paſſes through quadratures *D*, and the motion of the nodes becomes progreſſive, when the moon paſſes the node *n*, the ſin. *MEN* becomes negative, and the nodes become again regreſſive.

The

Thus it appears, that the nodes are progreſſive when the moon is paſſing between quadratures and the neareſt node, and regreſſive for the other part of the revolution. Hence, in one revolution of the moon the nodes are regreſſive. When the ſine of either of theſe three angles AEN, MEC, MEN, becomes $=0$, the nodes are ſtationary. Hence, they are ſtationary when the ſun is in the node, when the moon is in quadratures, and when the moon is in the node.

934 The ſine of $MEN = \text{ſin}\ \overline{MEC \mp CEN} = \text{ſin.}\ MEC \times \text{coſ.}\ CEN \mp \text{coſ}\ MCE \times \text{ſin.}\ CEN = \text{ſin.}\ MEC \times \text{ſin.}\ NEA \mp \text{coſ.}\ MEC \times \text{coſ.}\ NEA$; hence, if $ME = 1$, the true horary motion of the nodes $= 3n^2 \times Mw \times \overline{\text{ſin.}\ MEC}^2 \times \overline{\text{ſin.}\ NEA}^2 \mp 3n^2 \times Mw \times \text{ſin}\ MEC \times \text{coſ}\ MEC \times \text{ſin.}\ NEA \times \text{coſ}\ NEA$. Now in a whole revolution of the moon, the poſition of the ſun and node remaining the ſame, the effect of the laſt term will be deſtroyed by the oppoſition of the ſigns of ſin MEC and coſ MEC, the poſitive and negative ſigns of their product deſtroying one another in a whole ſynodic revolution, hence, we get the mean horary motion of the node in one ſynodic revolution of the moon $= 3n^2 \times Mw \times \overline{\text{ſin.}\ MEC}^2 \times \overline{\text{ſin.}\ NEA}^2$. Now if the poſition of the node be given, and we ſubſtitute $\frac{1}{2} - \frac{1}{2}\,\text{coſ.}\ 2MEC$ for the $\overline{\text{ſin.}\ MEC}^2$, we have the mean horary motion of the node $= 3n^2 \times Mw \times \overline{\text{ſin.}\ NEA}^2 \times \overline{\tfrac{1}{2} - \tfrac{1}{2}\,\text{coſ.}\ 2MEC}$, and neglecting $-\frac{1}{2}\,\text{coſ.}\ 2MEC$, the effect of which will be deſtroyed in one revolution of the moon by the oppoſition of it's ſigns, we get the mean horary motion of the nodes in one revolution of the moon $= \frac{3}{2} n^2 \times Mw \times \overline{\text{ſin}\ NEA}^2$. Now when the moon is in ſyzygies, the $\overline{\text{ſin}\ MEC}^2 = 1$, and $\text{coſ}\ MEC = 0$, therefore the horary motion of the nodes is then $3n^2 \times Mw \times \overline{\text{ſin}\ NEA}^2$. Hence, the mean horary motion of the nodes in one ſynodic revolution of the moon is equal to half their horary motion when the moon is in ſyzygies, whatever be the poſition of the node. Now when the nodes are in quadratures and the moon in ſyzygies, their horary motion (932) is 32′. 42″ 7‴, hence, the mean horary motion of the nodes when in quadratures is 16′. 21″ 3½‴ in the elliptic orbit. In the circular orbit it is 16″ 35‴. 16⁗. 36′′′′′.

935 As $\overline{\text{ſin}\ NEA}^2 = \frac{1}{2} - \frac{1}{2}\,\text{coſ}\ 2NEA$, the mean horary motion of the nodes in one ſynodic revolution of the moon $= \frac{3}{2} n^2 \times Mw \times \overline{\tfrac{1}{2} - \tfrac{1}{2}\,\text{coſ}\ 2NEA} = \frac{3}{4} n^2 \times Mw - \frac{3}{4} n^2 \times Mw \times \text{coſ}\ 2NEA$, and as in one revolution of the ſun in reſpect to the nodes, the laſt term will be deſtroyed by the oppoſition of it's ſigns, we get the mean horary motion of the nodes $= \frac{3}{4} n^2 \times Mw$. Hence, the mean motion of the moon : the mean motion of the nodes :: $Mw : \frac{3}{4} n^2 \times Mw :: 1 : \frac{3}{4} n^2$.

936. To get the mean annual motion of the nodes, we muſt firſt get their mean

mean motion, upon ſuppoſition that they had been fixed for one revolution of the ſun, and then we ſhall get nearly the motion of the ſun in reſpect to the node, and by repeating the operation, we may apply corrections and arrive at any degree of accuracy. Now $1 : n :: Mw : z'$ the arc deſcribed by the ſun in the time the moon deſcribes Mw, hence, $Mw = \frac{z'}{n}$, ſubſtitute therefore $\frac{z'}{n}$ for Mw, in $\frac{3}{4} n^2 \times Mw$ (which expreſſes the mean motion of the nodes), and we have $\frac{3}{4} n \times z'$ for the mean horary motion of the nodes, hence, if $z' = 360°$, we have $\frac{3}{4} n \times 360° = 20° \ 11\frac{3}{4}'$ for the mean annual motion of the nodes, upon ſuppoſition that the ſun had retained the ſame ſituation in reſpect to the nodes through the whole revolution, which it would have had if the nodes had been fixed, or if it's motion in reſpect to the nodes had been z'; we muſt therefore correct this ſuppoſition.

937 The mean horary motion of the nodes for one revolution of the moon is $3n^2 \times Mw \times \overline{\text{ſin. } MCE}^2 \times \overline{\text{ſin. } NEA}^2$; ſubſtitute $\frac{z'}{n}$ for Mw, and put $\frac{1}{2} - \frac{1}{2}$ coſ. $2MCE$ for ſin. MCE^2, neglecting the coſ. $2MCE$ as being deſtroyed by the oppoſition of ſigns in one revolution of the moon, hence, the mean motion of the nodes for one revolution of the moon is $\frac{3}{2} n z \times \overline{\text{ſin. } NEA}^2$, on ſuppoſition that the ſun's motion from the node had continued the ſame all the time as if the node had been fixed.

938 Now let v' = the motion of the ſun in reſpect to the node in the time the ſun deſcribes z', and the nodes deſcribe $\frac{3}{2} n z' \times \overline{\text{ſin. } NEA}^2$ nearly, then as the nodes are retrograde, $v' = z' + \frac{3}{2} n z' \times \overline{\text{ſin. } NEA}^2$, therefore $z' = v' - \frac{3}{2} n z' \times \overline{\text{ſin. } NEA}^2 = v' - \frac{3}{2} n v' \times \overline{\text{ſin. } NEA}^2$ very nearly. Subſtitute this for z' in $\frac{3}{2} n z' \times \overline{\text{ſin. } NEA}^2$, and we have $\frac{3}{2} n v' \times \overline{\text{ſin. } NEA}^2 - \frac{9}{4} n^2 v' \times \overline{\text{ſin. } NEA}^4$ for the mean motion of the nodes more nearly. Put now w' = the motion of the ſun in reſpect to the nodes, then $w' = v' + \frac{3}{2} n v' \times \overline{\text{ſin. } NEA}^2 - \frac{9}{4} n^2 v' \times \overline{\text{ſin. } NEA}^4$, therefore $v' = w' - \frac{3}{2} n v' \times \overline{\text{ſin. } NEA}^2 + \frac{9}{4} n^2 v' \times \overline{\text{ſin. } NEA}^4 = w' - \frac{3}{2} n w' \times \overline{\text{ſin. } NEA}^2 + \frac{9}{4} n^2 w' \times \overline{\text{ſin. } NEA}^4$ nearly, the motion of the ſun in reſpect to the nodes more correctly, ſubſtitute this quantity therefore for z' in $\frac{3}{2} n z' \times \overline{\text{ſin. } NEA}^2$, and we get $\frac{3}{2} n w' \times \overline{\text{ſin. } NEA}^2 - \frac{9}{4} n^2 w' \times \overline{\text{ſin. } NEA}^4 + \frac{27}{8} n^3 w' \times \overline{\text{ſin. } NEA}^6$ for the mean motion of the node ſtill more nearly; hence, the motion of the ſun in reſpect to the node becomes $w' + \frac{3}{2} n w' \times \overline{\text{ſin. } NEA}^2 - \frac{9}{4} n^2 w' \times \overline{\text{ſin. } NEA}^4 + \frac{27}{8} n^3 w' \times \overline{\text{ſin. } NEA}^6$, thus we may pro-

ceed,

ceed, correcting the mean motion of the node and the motion of the ſun in reſpect to it, as far as we pleaſe. Now (926) $\overline{\text{ſin. } NEA}^2 = \frac{1}{2} + \frac{1}{2}$ coſ. $2NEA$, the $\overline{\text{ſin. } NEA}^4 = \frac{3}{8} + \frac{1}{2}$ coſ. $2NEA + \frac{1}{8}$ coſ. $4NEA$, the $\overline{\text{ſin. } NEA}^6 = \frac{5}{16} + \frac{15}{32}$ coſ $2NEA + \frac{3}{16}$ coſ $4NEA + \frac{1}{32}$ coſ. $6NEA$, &c. and in a whole revolution of the ſun in reſpect to the node, the coſines of $2NEA$, $4NEA$, $6NEA$, &c. will be deſtroyed by the oppoſition of ſigns, therefore if for the powers of NEA we ſubſtitute $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$, &c we ſhall get the mean motion of the nodes for one revolution of the ſun in reſpect to the nodes, equal to $w' \times \overline{\frac{3}{4} n - \frac{27}{32} n^2 + \frac{135}{128} n^3}$, where w' repreſents the ſun's motion in reſpect to the nodes; hence, if we make $w' = 360°$, we have $360° \times \overline{\frac{3}{4} n - \frac{27}{32} n^2 + \frac{135}{128} n^3} =$ 18°. 40′ for the motion of the nodes between two conjunctions of the ſun and the node. If we had gone to another term of the ſeries, the motion would have been found = 18° 39′. Now in this time, the motion of the ſun muſt have been 360° − 18° 39′ = 341° 21′, hence, 341°. 21′ : 360° :: 18°. 39′ : 19°. 40′ the mean regreſſive motion of the nodes in a year, upon ſuppoſition that the orbit is a circle Let us therefore ſuppoſe the orbit to be an ellipſe deſcribed on CD, whoſe axis minor AE :: 69 : 70, then (930), 70 : 69 :: 19°. 40′ : 19°. 23′ the mean annual regreſſion of the nodes of the true lunar orbit. If we had conſidered the inclination of the orbit, it would have made the motion about 4′ leſs, for it is manifeſt, that the inclination will diminiſh, by a very ſmall quantity, the value of MK, we may therefore ſuppoſe the mean annual motion to be 19°. 19′

To find the Equations of the mean Motion of the Nodes.

939. Aſſuming $ME = 1$, and putting $Mw = \dot{z}$, we have the fluxion of the motion of the nodes $= -3n^2\dot{z} \times$ ſin. $AEN \times$ ſin $MEN \times$ ſin. MEC, but ſin. $MEC =$ coſ MEA, and ſin $AEN =$ ſin $\overline{MEN \mp MEA} =$ ſin $MEN \times$ coſ. $MEA \mp$ coſ. $MEN \times$ ſin MEA, hence, the motion of the nodes $= -3n^2\dot{z} \times (\overline{\text{ſin. } MEN}^2 \times \overline{\text{coſ } MEA}^2 \mp$ ſin. $MEN \times$ coſ. $MEN \times$ ſin. $MEA \times$ coſ $MEA = -3n^2\dot{z} \times (\overline{\frac{1}{2} - \frac{1}{2} \text{coſ } 2MEN} \times \overline{\frac{1}{2} + \frac{1}{2} \text{coſ } 2MEA} \mp \frac{1}{2}$ ſin. $2MEN \times \frac{1}{2}$ ſin. $2MEA) = -\frac{3}{4} n^2\dot{z} \times (\overline{1 - \text{coſ. } 2MEN} \times \overline{1 + \text{coſ } 2MEA}$

∓ ſin.

$\mp$ ſin $2MEN \times$ ſin. $2MEA) = -\frac{3}{4}n^2\dot{z} \times (1 +$ coſ $2MEA -$ coſ $2MEN -$ coſ. $2MEN \times$ coſ $2MEA \mp$ ſin $2MEN \times$ ſin $2MEA)$ But $-$ coſ $2MEN \times$ coſ. $2MEA \mp$ ſin $2MEN \times$ ſin $2MEA = -$ coſ $\overline{2MEN \mp 2MEA} = -$ coſ $2AEN$; hence, the motion of the nodes $= -\frac{3}{4}n^2\dot{z} \times (1 +$ coſ $2MEA -$ coſ. $2MEN -$ coſ $2AEN)$. Now the mean motion of the moon is to that of the ſun as $1 : n$, and let the mean motion of the moon be to that of the node as $1 : m$, then whilſt the moon deſcribes the angle $\dot{Z}$, the ſun deſcribes $n\dot{Z}$, and the node $m\dot{Z}$; and as the node is retrograde, $\dot{Z} + m\dot{Z}$ expreſſes the moon's motion from the node, or the angle MEN, alſo $\dot{Z} - n\dot{Z} =$ the motion of the moon from the ſun, or the angle MEA, and $n\dot{Z} + m\dot{Z} =$ the motion of the ſun from the node, or the angle AEN Hence, the motion of the nodes $= -\frac{3}{4}n^2\dot{Z} \times (1 +$ coſ $\overline{2Z - 2nZ} -$ coſ $\overline{2Z + 2mZ} -$ coſ. $\overline{2nZ + 2mZ}) = -\frac{3}{4}n^2\dot{Z} - \frac{3}{4}n^2\dot{Z} \times$ coſ. $\overline{2Z - 2nZ} + \frac{3}{4}n^2\dot{Z} \times$ coſ. $\overline{2Z + 2mZ} + \frac{3}{4}n^2\dot{Z} \times$ coſ $\overline{2nZ + 2mZ}$, whoſe fluent is $-\frac{3}{4}n^2Z - \frac{3}{8} \times \frac{n^2}{1-n} \times$ ſin $\overline{2Z - 2nZ} + \frac{3}{8} \times \frac{n^2}{1+m} \times$ ſin. $\overline{2Z + 2mZ} + \frac{3}{8} \times \frac{n^2}{n+m} \times$ ſin $\overline{2nZ + 2mZ}$ the motion of the nodes. And if $X =$ the diſtance $Z - nZ$ of the ſun from the moon, and for the diſtance $Z + mZ$ of the moon for the node we put S, then $2nZ + 2mZ = S - X$; hence, the motion of the nodes becomes $-\frac{3}{4}n^2Z + \frac{3}{8} \times \frac{n^2}{n+m} \times$ ſin. $\overline{2S - 2X} + \frac{3}{8} \times \frac{n^2}{1+m} \times$ ſin. $2S - \frac{3}{8} \times \frac{n^2}{1-n} \times$ ſin. $2X$ But (935) $m = \frac{3}{4}n^2$; and the firſt term expreſſes the mean motion of the nodes in a circular orbit, hence, we get theſe equations of the mean motion of the nodes, $\frac{3}{8} \times \frac{n}{1 + \frac{3}{4}n} \times$ ſin. $\overline{2S - 2X} + \frac{3}{8} \times \frac{n^2}{1 + \frac{3}{4}n^2} \times$ ſin. $2S - \frac{3}{8} \times \frac{n^2}{1-n} \times$ ſin. $2X = 1°. 31'. 18'' \times$ ſin. $\overline{2S - 2X} + 7'. 11'' \times$ ſin. $2S - 7'. 49'' \times$ ſin. $2X$, and if theſe be diminiſhed in the ratio of 70 : 69, we get the true equations, $1°. 30'. 1'' \times$ ſin. $\overline{2S - 2X} + 7'. 5'' \times$ ſin $2S - 7'. 42'' \times$ ſin $2X$

940. The moon's motion being here aſſumed Z, if for Z we ſubſtitute (870) $Z - 2w \times$ ſin. Z, which is very nearly the true motion, and for $\dot{Z}$ we put $\dot{Z} - 2w \times$ coſ $Z \times \dot{Z}$, then the fluxion of the mean motion $-\frac{3}{4}n^2\dot{Z}$ becomes $-\frac{3}{4}n^2\dot{Z} + \frac{1}{2}n^2w \times$ coſ. $Z \times \dot{Z} =$ (as $Y = nZ$) $- \frac{3}{4}n \times \dot{Y} + \frac{3}{2}n^2w \times$ coſ. $Z \times \dot{Z}$, and to find the annual effect, we muſt (891) multiply this quantity by $1 - 3c \times$ coſ. Y, hence, the fluxion of the motion of the nodes from theſe cauſes $= -\frac{3}{4}n\dot{Y} + \frac{3}{2}n^2w \times$ coſ. $Z \times \dot{Z} + \frac{9}{4}nc \times$ coſ. $Y \times \dot{Y}$ nearly, whoſe fluent $= -\frac{3}{4}nY + \frac{3}{2}n^2w \times$ ſin. $Z + \frac{9}{4}nc \times$ ſin. Y; and the mean motion being

$-\frac{3}{4}nY$,

$-\frac{3}{8}nY$, the equations become $\frac{3}{2}n^2w\times\text{ſin.}\,Z+\frac{9}{4}nc\times\text{ſin.}\,Y=1'.\,35''\times\text{ſin.}\,Z+9'.\,43''\tfrac{1}{2}\times\text{ſin.}\,Y$; and theſe diminiſhed in the ratio of 70 : 69, give $1'.\,34''\times\text{ſin.}\,Z+9'.\,35''\times\text{ſin.}\,Y$ for the true equations.

941. To correct the three equations found in Art. 939. *Firſt*, to correct $\frac{3}{4}n^2\dot{Z}\times\text{coſ.}\,\overline{2nZ+2mZ}$. For Z write $Z-2w\times\text{ſin}\,Z$, and for $\dot{Z}$ ſubſtitute $\dot{Z}-2w\times\text{coſ.}\,Z\times\dot{Z}$, alſo, for the motion of the ſun nZ ſubſtitute a more correct value $n\dot{Z}-2c\times\text{ſin.}\,nZ$; and multiply the whole by $1-3c\times\text{coſ.}\,nZ$, being the diſturbing force at any diſtance $1+c\times\text{coſ.}\,Y$ (891); hence, the above equation becomes $\frac{3}{4}n^2\dot{Z}\times\overline{1-2w\times\text{coſ.}\,Z}\times\overline{1-3c\times\text{coſ.}\,nZ}\times\text{coſ.}\,\overline{2nZ+2mZ-4c\times\text{ſin.}\,nZ}$; multiply theſe quantities together, and neglect thoſe terms in which the product wc enters as being very ſmall, and we ſhall get the following quantity, $\frac{3}{4}n^2\dot{Z}\times\overline{1-2w\times\text{coſ.}\,Z-3c\times\text{coſ}\,nZ}\times\text{coſ.}\,\overline{2nZ+2mZ-4c\times\text{ſin}\,nZ}$; but $\text{coſ.}\,\overline{2nZ+2mZ-4c\times\text{ſin.}\,nZ}=\text{coſ.}\,\overline{2nZ+2mZ}+4c\times\text{ſin.}\,nZ\times\text{ſin.}\,\overline{2nZ+2mZ}$ nearly, becauſe the coſ. of $4c\times\text{ſin.}\,nZ$ is nearly unity, hence, (neglecting $\frac{3}{4}n^2\dot{Z}\times\text{coſ.}\,\overline{2nZ+2mZ}$ found in Art. 939.), the correction becomes $\frac{3}{4}n^2\dot{Z}\times\overline{-2w\times\text{coſ.}\,Z-3c\times\text{coſ.}\,nZ}\times\text{coſ}\,\overline{2nZ+2mZ}+3n^2c\dot{Z}\times\text{ſin.}\,nZ\times\text{ſin.}\,\overline{2nZ+2mZ}$ nearly, $=-\frac{3}{2}n^2w\times\dot{Z}\times\text{coſ.}\,Z\times\text{coſ.}\,\overline{2nZ+2mZ}-\frac{9}{4}n^2c\times\dot{Z}\times\text{coſ.}\,Z\times\text{coſ.}\,\overline{2nZ+2mZ}+3n^2c\times\dot{Z}\times\text{ſin}\,nZ\times\text{ſin.}\,\overline{2nZ+2mZ}$; but $\text{coſ.}\,Z\times\text{coſ.}\,\overline{2nZ+2mZ}=\frac{1}{2}\,\text{coſ.}\,\overline{2nZ+2mZ+Z}+\frac{1}{2}\,\text{coſ.}\,\overline{2nZ+2mZ-Z}$; and $\text{coſ.}\,nZ\times\text{coſ.}\,\overline{2nZ+2mZ}=\frac{1}{2}\,\text{coſ.}\,\overline{3nZ+2mZ}+\frac{1}{2}\,\text{coſ.}\,\overline{nZ+2mZ}$, alſo $\text{ſin}\,nZ\times\text{ſin.}\,\overline{2nZ+2mZ}=\frac{1}{2}\,\text{coſ.}\,\overline{nZ+2mZ}-\frac{1}{2}\,\text{coſ.}\,\overline{3nZ+2mZ}$; ſubſtitute theſe quantities, and collect the like terms together, and the fluxional correction becomes, $-\frac{3}{4}n^2w\dot{Z}\times\text{coſ.}\,\overline{2nZ+2mZ+Z}-\frac{3}{4}n^2w\dot{Z}\times\text{coſ.}\,\overline{2nZ+2mZ-Z}-\frac{21}{8}n^2c\dot{Z}\times\text{coſ.}\,\overline{3nZ+2mZ}+\frac{3}{8}n^2c\times\dot{Z}\times\text{coſ.}\,\overline{nZ+2mZ}$, the fluent of which is $-\frac{3}{4}n^2w\times\frac{\text{ſin.}\,\overline{2nZ+2mZ+Z}}{2n+2m+1}+\frac{3}{4}n^2w\times\frac{\text{ſin.}\,\overline{2nZ+2mZ-Z}}{1-2n-2m}-\frac{21}{8}n^2c\times\frac{\text{ſin.}\,\overline{3nZ+2mZ}}{3n+2m}+\frac{3}{8}n^2c\times\frac{\text{ſin.}\,\overline{nZ+2mZ}}{n+2m}=$ (as $Y=nZ$) $-\frac{3}{4}n^2w\times\frac{\text{ſin.}\,\overline{2S-2X+Z}}{1+2n}+\frac{3}{4}n^2w\times\frac{\text{ſin.}\,\overline{2S-2X-Z}}{1-2n}-\frac{7}{8}nc\times\text{ſin.}\,\overline{2S-2X+Y}+\frac{3}{8}nc\times\text{ſin.}\,\overline{2S-2X-Y}$ very nearly, $=-41''\times\text{ſin.}\,\overline{2S-2X+Z}+55''\times\text{ſin.}\,\overline{2S-2X-Z}-3'.\,48'\times\text{ſin.}\,\overline{2S-2X+Y}+1'.\,38''\times\text{ſin.}\,\overline{2S-2X-Y}$, which two laſt diminiſhed in the ratio of 70 : 69, give $-3'.\,45''\times\text{ſin.}\,\overline{2S-2X+Y}+1'.\,37''\times\text{ſin.}\,\overline{2S-2X-Y}$; the ſmaller equations it is unneceſſary thus to reduce.

942. *Secondly*, to correct $\frac{3}{4}n^2Z \times \text{coſ}\ \overline{2Z+2mZ}$. For Z write $Z-2w\times \text{ſin.}\ Z$, and for Z ſubſtitute $Z-2w\times\text{coſ.}\ Z\times Z$, and multiply the whole by $1-3c\times\text{coſ}\ nZ$, and the equation becomes $\frac{3}{4}n^2Z\times\overline{1-2w\times\text{coſ.}\ Z}\times\overline{1-3c\times\text{coſ.}\ nZ}\times\text{coſ.}\ \overline{2Z+2mZ-4w\times\text{ſin.}\ Z}$, proceed now exactly the ſame as in the laſt operation, and we ſhall get the correction $=-\frac{9}{4}n^2w\times\frac{\text{ſin}\ \overline{2S+Z}}{3+2m}+\frac{3}{4}n^2w\times\frac{\text{ſin}\ \overline{2S-Z}}{1+2m}-\frac{9}{8}n^2c\times\frac{\text{ſin}\ \overline{2S+Y}}{2+n+2m}-\frac{9}{8}n^2c\times\frac{\text{ſin}\ \overline{2S-Y}}{2-n+2m}=-45''\times\text{ſin.}\ \overline{2S+Z}+45''\times\text{ſin}\ \overline{2S-Z}-11''\times\text{ſin}\ \overline{2S+Y}-11''\times\text{ſin}\ \overline{2S-Y}$.

943. *Thirdly*, to correct $-\frac{3}{4}n^2Z\times\text{coſ}\ \overline{2Z-2nZ}$. For Z write $Z-2w\times\text{ſin.}\ Z$, and for Z ſubſtitute $\dot{Z}-2w\times\text{coſ.}\ Z\times Z$, and multiply the whole by $1-3c\times\text{coſ.}\ nZ$, and the equation becomes $-\frac{3}{4}n^2\dot{Z}\times\overline{1-2w\times\text{coſ.}\ Z}\times\overline{1-3c\times\text{coſ.}\ nZ}\times\text{coſ.}\ \overline{2Z-2nZ-4w\times\text{ſin}\ Z}$, proceed now in the ſame manner as before, and we get the correction $=-\frac{3}{4}n^2w\times\frac{\text{ſin}\ \overline{2X-Z}}{1-2n}+\frac{9}{4}n^2w\times\frac{\text{ſin}\ \overline{2X+Z}}{2-3n}+\frac{9}{8}n^2c\times\frac{\text{ſin}\ \overline{X+Z}}{2-n}+\frac{9}{8}n^2c\times\frac{\text{ſin}\ \overline{X-Z}}{2-3n}=-53''\times\text{ſin.}\ \overline{2X-Z}+47''\times\text{ſin}\ \overline{2X+Z}+12''\times\text{ſin.}\ \overline{X+Z}+12''\times\text{ſin}\ \overline{X-Z}$.

944. The principal equation of the nodes is $\frac{3}{8}n\times\text{ſin}\ \overline{2nZ+2mZ}$ (939); therefore a more correct value of $2mZ$ (twice the motion of the nodes) is $2mZ-\frac{3}{4}n\times\text{ſin.}\ \overline{2nZ+2mZ}$, the equation being ſubtracted, becauſe the motion $2nZ$ of the nodes is retrograde, and this equation is additive, or to be applied according to the order of the ſigns Subſtitute therefore this value for $2mZ$ in $\frac{3}{4}n^2\dot{Z}\times\text{coſ.}\ \overline{2nZ+2mZ}$ (which is the fluxion of the principal equation) and it becomes $\frac{3}{4}n^2\dot{Z}\times\text{coſ}\ (2nZ+2mZ-\frac{3}{4}n\times\text{ſin.}\ \overline{2nZ+2mZ})$, and omitting the equation already found, the reſult is $\frac{3}{4}n^2Z\times\frac{3}{4}n\times\text{ſin}\ \overline{2nZ+2mZ}^2$ $=\frac{9}{16}n^3\dot{Z}\times\overline{\tfrac{1}{2}-\tfrac{1}{2}\text{coſ}\ \overline{4nZ+4mZ}}=\frac{9}{32}n^3Z-\frac{9}{32}n^3Z\times\text{coſ.}\ \overline{4nZ+4mZ}$, whoſe fluent is $\frac{9}{32}n^3Z-\frac{9}{32}n^3\times\frac{\text{ſin.}\ \overline{4nZ+4mZ}}{4n+4m}=\frac{9}{32}n^3Z-\frac{9}{128}n^2\times\text{ſin.}\ \overline{4nZ+4mZ}$, hence, the equation is $-\frac{9}{128}n^2\times\text{ſin.}\ \overline{4nZ+4mZ}=-1'\,23''\times\text{ſin.}\ \overline{4S-4X}$, which diminiſhed in the ratio of 70 : 69, gives $-1'.\ 22''\times\text{ſin.}\ \overline{4S-4X}$.

945. The quantity $\frac{9}{32}n^3Z$, found in deducing the laſt equation, continually increaſes as Z increaſes, and is therefore a correction of the mean motion

$-\frac{3}{4}n^2Z$

$-\frac{3}{4}n^2Z$ before found; hence, a more correct value of the mean motion is $-\frac{3}{4}n^2Z + \frac{9}{32}n^3Z$.

Hence, the equations of the mean motion of the nodes are,

I.	+	1°. 30′. 1″	× ſin. $\overline{2S-2X}$.
II.	+	7. 5	× ſin. $2S$.
III.	−	7. 42	× ſin $2X$.
IV.	+	1. 34	× ſin Z.
V.	+	9 35	× ſin Υ.
VI.	−	0. 41	× ſin $\overline{2S-2X+Z}$.
VII.	+	0. 55	× ſin $\overline{2S-2X-Z}$.
VIII.	−	3. 45	× ſin. $\overline{2S-2X+\Upsilon}$
IX.	+	1. 37	× ſin. $\overline{2S-2X-\Upsilon}$.
X.	−	0 45	× ſin. $\overline{2S+Z}$.
XI.	+	0. 45	× ſin $\overline{2S-Z}$.
XII.	−	0. 11	× ſin. $\overline{2S+\Upsilon}$.
XIII.	−	0. 11	× ſin. $\overline{2S-\Upsilon}$.
XIV.	−	0. 53	× ſin. $\overline{2X-Z}$.
XV.	+	0 47	× ſin. $\overline{2X+Z}$.
XVI.	+	0. 12	× ſin $\overline{X+Z}$.
XVII.	+	0. 12	× ſin. $\overline{X-Z}$.
XVIII.	−	1. 22	× ſin. $\overline{4S-4X}$.

On the Variation *of the Inclination of the Moon's Orbit.*

946. Let NN' be the horary motion of the moon's nodes, draw Mk perpendicular to Nn; and Mw perpendicular to the ecliptic, and draw wks, join Ms, and draw kv perpendicular to Ms; then the angle sMk is the cotemporary variation of the inclination. Now the angle NEN' : sMk :: $\frac{sk}{sE}$: $\frac{kv}{kM}$ :: $\frac{1}{sE}$: $\frac{1}{kM}$ :: ſin. MEN : s × coſ MEN. Now if the angle $MEw = Z$, the motion of the nodes (927) is $3n^2 \times \dot{Z} \times$ ſin. AEN × ſin. MEC × ſin. MEN; hence, the cotemporary variation of the inclination is

$3n^2s\dot{Z}$ × ſin. AEN × coſ. MEN × coſ MEA. Now coſ. MEN = coſ. $\overline{MEC \mp CEN}$ = coſ MEC × coſ CEN ± ſin. MEC × ſin. CEN = coſ MEC × ſin. AEN ± ſin MEC × coſ. AEN, hence, the fluxion of the variation of the inclination is $3n^2sZ$ × ſin. AEN × ſin. MEC × (coſ. MEC × ſin AEN ± ſin. MEC × coſ AEN), now in one ſynodic revolution of the moon, coſ. MEC will be deſtroyed, therefore the fluxion of the mean variation for that time $= 3n^2s\dot{Z}$ × ſin AEN × coſ. AEN × $\overline{\text{ſin } MEC}^2 = 3n^2Z$ × ſin. AEN × coſ. AEN × $\overline{\tfrac{1}{2} - \tfrac{1}{2}\text{ coſ. } 2MEC}$ = (for the mean of one ſynodic revolution) $\tfrac{3}{2}n^2s\dot{Z}$ × ſin. AEN × coſ. AEN.* Now the mean motion of the moon ($\dot{Z}$) : that of the ſun :: 2139 : 160, and (938) the mean motion of the ſun : it's mean motion $\dot{\overline{AN}}$ in reſpect to the nodes :: 341,3 : 360, hence, $\dot{Z} = \frac{2139 \times 341,3}{160 \times 360} \times \dot{\overline{AN}} = 12,68 \times \dot{\overline{AN}}$, therefore the fluxion of the mean variation of the inclination, as the nodes move from quadratures to ſyzygies, is $\tfrac{3}{2} \times 12,68 \times n^2s \times \dot{\overline{AN}}$ × ſin. AEN × coſ $AEN = 19,02 \times n^2s$ × ſin AEN × $\dot{\overline{\text{ſin } AEN}}$, whoſe fluent is $9,51 \times n^2s \times \overline{\text{ſin } AEN}^2 = 16'.\ 24'' \times \overline{\text{ſin } AEN}^2$, taking $s = 0,896$ it's mean value. This is for a circular orbit, diminiſh it in the ratio of 70 : 69, and it becomes $16'\ 10'' \times \overline{\text{ſin. } AEN}^2$. Hence, the mean variation of the inclination varies as the ſquare of the ſine of the ſun's diſtance from the node. When $AEN = 90°$, the mean diminution is 16′. 10″. This is the mean for one revolution of the moon when the nodes come into quadratures, it's half therefore 8′ 5″ is the mean variation from the mean inclination of the orbit. Now $\overline{\text{ſin } AEN}^2 = \tfrac{1}{2} - \tfrac{1}{2}$ coſ $2AEN$. Hence, the mean variation = 8′. 5″ − 8′. 5″ × coſ $2AEN$, which ſubtracted from 8′ 5″ the mean variation from the mean inclination gives 8′. 5″ × coſ. $2AEN$, the quantity to be applied to the mean inclination in order to get the mean inclination for one revolution of the moon at any time.

947. Now when the nodes are in quadratures, the fluxion of the variation is $3n^2s\dot{Z}$ × ſin. MEC × coſ. MEC (the ſin. AEN being unity) $= 3n^2s$ × ſin. MEC × $\dot{\overline{\text{ſin. } MEC}}$, whoſe fluent is $\tfrac{3}{2}n^2s \times \overline{\text{ſin } MEC}^2$ = (aſſuming $s = ,0871$ the ſine of the inclination at this time) $2'\ 31'' \times \overline{\text{ſin } MEC}^2$; but this muſt be increaſed in the ratio of the periodic to the ſynodic revolution for the reaſon in Art 867 hence, the variation $= 2'.\ 43'' \times \overline{\text{ſin. } MEC}^2$ for a circular orbit, which diminiſhed in the ratio of 70 : 69, gives $2'.\ 40'',7 \times \overline{\text{ſin. } MEC}^2$ for the variation of the inclination. Hence, when the moon is

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* Hence, the mean variation of the inclination is s × ſin. AEN × coſ AEN, or as s × ſin. $2AEN$, from which Sir I NEWTON, Pr. 35. Lib. 3. deduces his very elegant Conſtruction for finding the inclination at any time.

in ſyzygies, the variation = 2′. 40″,7 is the diminution of the inclination in the tranſit of the moon from the nodes (in quadratures) to ſyzygies, the half of which 1′. 20″ is the variation from the mean inclination in that time. Hence (946), in the tranſit of the nodes from ſyzygies to quadratures, when the moon is in quadratures the variation of the inclination has been 16′. 10″ − 1′. 20″ = 14′. 50″, and when the moon is in ſyzygies, the variation has been 16′. 10″ + 1′. 20″ = 17′. 30″, therefore if the inclination be 5° 17′ 20″ when the nodes are in ſyzygies, the leaſt inclination becomes 4°. 59′. 50″, and the mean = 5°. 8′. 35″.

To find the Equations of the Variation of the Inclination of the Lunar Orbit.

948. By Art 927 the fluxion of the motion of the nodes is $3n^2\dot{Z} \times \text{ſin. } AEN \times \text{ſin } MEN \times \text{coſ. } MEA$, but (946) the motion of the nodes the cotemporary variation of the inclination . ſin. $MEN : s \times \text{coſ. } MEN$, hence, the fluxion of the variation of the inclination is $3n^2s\dot{Z} \times \text{ſin. } AEN \times \text{coſ } MEN \times \text{coſ } MEA$. Now, writing the ſines and coſines with their proper ſigns, when the inclination is diminiſhing this fluxion is poſitive, and when the inclination is increaſing the fluxion is negative, therefore that it may be properly applied, it's ſign muſt be changed. But $AEN = MEN \mp MEA$, therefore ſin AEN = ſin $MEN \times \text{coſ. } MEA \mp \text{ſin. } MEA \times \text{coſ. } MEN$, hence, the fluxion of the variation of the inclination $= -3n^2s\dot{Z} \times \overline{\text{coſ } MEA}^2 \times \text{ſin. } MEN \times \text{coſ } MEN \mp \overline{\text{coſ. } MEN}^2 \times \text{ſin } MEA \times \text{coſ } MEA = -3n^2s\dot{Z} \times (\overline{\tfrac{1}{2} + \tfrac{1}{2}\text{coſ } 2MEA} \times \tfrac{1}{2}\text{ſin } 2MEN \mp \overline{\tfrac{1}{2} + \tfrac{1}{2}\text{coſ } 2MEN} \times \tfrac{1}{2}\text{ſin } 2MEA) = -\tfrac{3}{4}n^2s\dot{Z} \times (\text{ſin } 2MEN \mp \text{ſin } 2MEA + \text{ſin } \overline{2MEN \mp 2MEA}) = -\tfrac{3}{4}n^2s\dot{Z} \times (\text{ſin } 2MEN \mp \text{ſin. } 2MEA + \text{ſin. } 2AEN)$ Now this is the ſame expreſſion as that for the motion of the nodes, except that we have here the ſines of the angles inſtead of the coſines, therefore making the ſame ſubſtitution here as in Art 939. we get the fluxion of the variation of the inclination $= -\tfrac{3}{4}n^2s\dot{Z} \times (\text{ſin. } \overline{2nZ + 2mZ} + \text{ſin. } \overline{2Z + 2mZ} - \text{ſin. } \overline{2Z - 2nZ})$, whoſe fluent is

$$-\tfrac{3}{4}n^2s \times \left\{\frac{\text{ver. ſin } \overline{2nZ + 2mZ}}{2n + 2m} + \frac{\text{ver. ſin. } \overline{2Z + 2mZ}}{2 + 2m} - \frac{\text{ver ſin } \overline{2Z - 2nZ}}{2 - 2n}\right\} = -\tfrac{3}{4}n^2s \times \left\{\frac{1 - \text{coſ } \overline{2nZ + 2mZ}}{2n + 2m} + \frac{1 - \text{coſ. } \overline{2Z + 2mZ}}{2 + 2m} - \frac{1 - \text{coſ. } \overline{2Z - 2nZ}}{2 - 2n}\right\} = (\text{as } m = \tfrac{3}{4}n^2) - \frac{3ns}{8 + 6n} - \frac{3n^2s}{8 + 6n^2} + \frac{3n^2s}{8 - 8n} + \frac{3ns}{8 + 6n}$$

× coſ.

$\times$ coſ. $\overline{2S-2X}+\frac{3n^2s}{8+6n^2}\times$ coſ. $2S-\frac{3n^2s}{8-8n}\times$ coſ $2X$, and the three equations are $8'.\ 12''\times$ coſ. $\overline{2S-2X}+39''\times$ coſ. $2S-42''\times$ coſ. $2X$, which diminiſhed in the ratio of $70:69$, gives $8'\ 5''\times$ coſ. $\overline{2S-2X}+38\frac{1}{2}''\times$ coſ $2S-41\frac{1}{2}''\times$ coſ. $2X$ for the true equations. And the three conſtant quantities ſhow that the mean inclination of the orbit is leſs than it would have been if there had been no diſturbing force.

949. To correct theſe three equations. *Firſt*, let us take $-\frac{3}{4}n^2s\dot{Z}\times$ ſin $\overline{2nZ+2mZ}$, in which, for $\dot{Z}$ write $\dot{Z}-2w\times$ coſ $Z\times\dot{Z}$, for nZ write $nZ-2c\times$ ſin nZ, and for $2mZ$ write $2mZ-\frac{3}{4}n\times$ ſin $\overline{2nZ+2mZ}$, which is the correction of the motion of the node by it's principal equation (939), and multiply the whole by $1-3c\times$ coſ. nZ, for the reaſons already given in Art 941. and the equation becomes $-\frac{3}{4}n^2s\dot{Z}\times\overline{1-2w\times\text{coſ } Z}\times\overline{1-3c\times\text{coſ } nZ}\times$ ſin $(2nZ+2mZ-4c\times$ ſin. $nZ-\frac{3}{4}n\times$ ſin. $\overline{2nZ+2mZ})$; but ſin. $(2nZ+2mZ-4c\times$ ſin. $nZ-\frac{3}{4}n\times$ ſin. $\overline{2nZ+2mZ})=$ ſin. $\overline{2nZ+2mZ}-$ coſ. $\overline{2nZ+2mZ}\times\overline{4c\times\text{ſin } nZ+\frac{3}{4}n\times\text{ſin } \overline{2nZ+2mZ}}$ nearly, becauſe $4c\times$ ſin. $nZ+\frac{3}{4}n\times$ ſin. $\overline{2nZ+2mZ}$ being ſmall, it's coſine is nearly $=1$, and we may put that quantity itſelf for it's ſine; make therefore this ſubſtitution, and actually multiply the quantities, and reject $-\frac{3}{4}n^2s\dot{Z}\times$ ſin. $\overline{2nZ+2mZ}$ which is the equation to be corrected, and we get the fluxional correction $=\frac{3}{2}n^2sw\dot{Z}\times$ coſ. $Z\times$ ſin. $\overline{2nZ+2mZ}+\frac{9}{4}n^2sc\dot{Z}\times$ coſ. $nZ\times$ ſin. $\overline{2nZ+2mZ}+3n^2sc\dot{Z}\times$ ſin. $nZ\times$ coſ. $\overline{2nZ+2mZ}+\frac{9}{16}n^3s\dot{Z}\times$ ſin. $\overline{2nZ+2mZ}\times$ coſ. $\overline{2nZ+2mZ}$ nearly, $=\frac{3}{4}n^2sw\dot{Z}\times$ ſin. $\overline{2nZ+2mZ+Z}+\frac{3}{4}n^2sw\dot{Z}\times$ ſin. $\overline{2nZ+2mZ-Z}+\frac{21}{8}n^2sc\dot{Z}\times$ ſin. $\overline{3nZ+2mZ}-\frac{3}{8}n^2sc\dot{Z}\times$ ſin. $\overline{nZ+2mZ}+\frac{9}{32}n^3s\dot{Z}\times$ ſin. $\overline{4nZ+4mZ}$, whoſe fluent is $\frac{3}{4}n^2sw\times\frac{\text{ver ſin } \overline{2nZ+2mZ+Z}}{1+2n+2m}+\frac{3}{4}n^2sw\times\frac{\text{ver ſin } \overline{2nZ+2mZ-Z}}{2n+2m-1}+\frac{21}{8}n^2sc\times\frac{\text{ver ſin } \overline{3nZ+2mZ}}{3n+2m}-\frac{3}{8}n^2sc\times\frac{\text{ver ſin. } \overline{nZ+2mZ}}{n+2m}+\frac{9}{32}n^3s\times\frac{\text{ver ſin. } \overline{4nZ+4mZ}}{4n+4m}=$ (by ſubſtituting $1-$ coſ. for ver. ſin) $\frac{3}{4}\times\frac{n^2sw}{1+2n}-\frac{3}{4}\times\frac{n^2sw}{1-2n}+\frac{7}{8}nsc-\frac{3}{8}nsc+\frac{9}{128}n^2s-\frac{3}{4}\times\frac{n^2sw}{1+2n}\times$ coſ. $\overline{2nZ+2mZ+Z}+\frac{3}{4}\times\frac{n^2sw}{1-2n}$

$\frac{n^2 s w}{1-2n} \times \text{coſ.}\ \overline{2nZ+2mZ-Z} - \frac{7}{8} nsc \times \text{coſ.}\ \overline{3nZ+2mZ} + \frac{3}{8} nsc \times \text{coſ.}\ \overline{nZ+2mZ} - \frac{9}{128} n^2 s \times \text{coſ}\ \overline{4nZ+4mZ}$, but the five firſt terms are conſtant, and therefore only affect the mean inclination, and the laſt five expreſs the corrections of the aſſumed equation Now $2nZ+2mZ+Z=2S-2X+Z$, $2nZ+2mZ-Z=2S-2X-Z$, $3nZ+2mZ=2S-2X+Y$, $nZ+2mZ=2S-2X-Y$, and $4nZ+4mZ=4S-4X$; and by ſubſtituting for n, w, c and s their values, we get theſe equations, $-3\frac{2}{3}'' \times \text{coſ}\ \overline{2S-2X+Z} + 5'' \times \text{coſ.}\ \overline{2S-2X-Z} - 20\frac{1}{3}'' \times \text{coſ}\ \overline{2S-2X+Y} + 8\frac{3}{4}'' \times \text{coſ.}\ \overline{2S-2X-Y} - 7\frac{1}{3}'' \times \text{coſ.}\ \overline{4S-4X}$ It is unneceſſary to reduce theſe ſmall equations in the ratio of 70 69

950 *Secondly*, to correct $-\frac{3}{4} n^2 s Z \times \text{ſin.}\ \overline{2Z+2mZ}$ For Z put $Z-2w \times \text{ſin}\ Z$, and for Z write $Z-2w \times \text{coſ}\ Z \times Z$, omitting the other corrections, as the effects which they here produce are very ſmall, hence, the quantity becomes $-\frac{3}{4} n^2 s Z \times \overline{1-2w \times \text{coſ}\ Z} \times \text{ſin}\ (2Z+2mZ-4w \times \text{ſin.}\ Z) = -\frac{3}{4} n^2 s Z \times \overline{1-2w \times \text{coſ}\ Z} \times (\text{ſin}\ \overline{2Z+2mZ} - 4w \times \text{ſin.}\ Z \times \text{coſ.}\ \overline{2Z+2mZ}) = -\frac{3}{4} n^2 s \dot{Z} \times \overline{1-2w \times \text{coſ}\ Z} \times (\text{ſin.}\ \overline{2Z+2mZ} - 2w \times \text{ſin}\ \overline{3Z+2mZ} + 2w \times \text{ſin}\ \overline{Z+2mZ})$, and neglecting $-\frac{3}{4} n^2 s Z \times \text{ſin}\ \overline{2Z+2mZ}$ which is the given quantity, and alſo thoſe terms where w^2 enters as being very ſmall, we get the fluxional correction $= \frac{3}{2} n^2 w s \dot{Z} \times \text{ſin.}\ \overline{3Z+2mZ} - \frac{3}{2} n^2 w s Z \times \text{ſin.}\ \overline{Z+2mZ} + \frac{3}{2} n^2 w s \dot{Z} \times \text{coſ.}\ Z \times \text{ſin.}\ \overline{2Z+2mZ} = \frac{3}{2} n^2 w s Z \times \text{ſin.}\ \overline{3Z+2mZ} - \frac{3}{2} n^2 w s Z \times \text{ſin.}\ \overline{Z+2mZ} + \frac{3}{4} n^2 w s Z \times \text{ſin}\ \overline{3Z+2mZ} + \frac{3}{4} n^2 w s Z \times \text{ſin.}\ \overline{Z+2mZ}$, whoſe fluent is $\frac{3}{2} n^2 w s \times \frac{1-\text{coſ.}\ \overline{3Z+2mZ}}{3+2m} - \frac{3}{2} n^2 w s \times \frac{1-\text{coſ}\ \overline{Z+2mZ}}{1+2m} + \frac{3}{4} n^2 w s \times \frac{1-\text{coſ}\ \overline{3Z+2mZ}}{3+2m} + \frac{3}{4} n^2 w s \times \frac{1-\text{coſ}\ \overline{Z+2mZ}}{1+2m} = \frac{1}{2} n^2 w s - \frac{3}{2} n^2 w s + \frac{1}{4} n^2 w s - \frac{3}{4} n^2 w s - \frac{1}{2} n^2 w s \times \text{coſ}\ \overline{3Z+2mZ} + \frac{3}{2} n^2 w s \times \text{coſ}\ \overline{Z+2mZ} - \frac{1}{4} n^2 w s \times \text{coſ.}\ \overline{3Z+2mZ} - \frac{3}{4} n^2 w s \times \text{coſ.}\ \overline{Z+2mZ} =$ (omitting the conſtant quantities which affect only the mean inclination) $-\frac{3}{4} n^2 w s \times \text{coſ}\ \overline{3Z+2mZ} + \frac{3}{4} n^2 w s \times \text{coſ}\ \overline{Z+2mZ} = -4\frac{1}{4}'' \times \text{coſ}\ \overline{3Z+2mZ} + 4\frac{1}{4}'' \times \text{coſ.}\ \overline{Z+2mZ} = -4\frac{1}{4}'' \times \text{coſ}\ \overline{2S+Z} + 4\frac{1}{4}'' \times \text{coſ.}\ \overline{2S-Z}$, the corrections required.

951. *Thirdly*, to correct $\frac{3}{4} n^2 s \dot{Z} \times \text{ſin.}\ \overline{2Z-2nZ}$. For Z put $Z-2w \times \text{ſin.}\ Z$, and for $\dot{Z}$ put $\dot{Z}-2w \times \text{coſ.}\ Z \times \dot{Z}$, and the given quantity becomes $\frac{3}{4} n^2 s \dot{Z} \times \overline{1-2w \times \text{coſ.}\ Z} \times \text{ſin}\ (2Z-2nZ-4w \times \text{ſin}\ Z) = \frac{3}{4} n^2 s \dot{Z} \times \overline{1-2w \times \text{coſ.}\ Z} \times (\text{ſin.}\ \overline{2Z-2nZ} - 4w \times \text{ſin.}\ Z \times \text{coſ.}\ \overline{2Z-2nZ}) = \frac{3}{4} n^2 s \dot{Z} \times$

1 —

therefore the inclination is the leaſt poſſible, conſequently the mean inclination has decreaſed from the time the ſun leaves the nodes till it comes into quadratures. Now from the time the ſun leaves quadratures till it comes to the next node, the coſine of $\overline{2S-2X}$ increaſes from -1 to $+1$, conſequently the inclination increaſes for that time, and then becomes the ſame as it was when the ſun left the preceding node. We have here conſidered the inclination without any reference to the place of the moon in it's orbit, which, as appears from the other equations, will affect the inclination, the inclination here mentioned muſt therefore be conſidered as nearly the mean inclination for a revolution of the moon. Hence, the whole variation in the tranſit of the ſun from the nodes to quadratures is the difference between $-8'.\ 5''$ and $+8'.\ 5''$, or $16'.\ 10''$ as before ſtated. Sir I. NEWTON makes it $16'.\ 23\frac{1}{2}''$ for a circular orbit, which diminiſhed in the ratio of 70 : 69, gives $16'.\ 9\frac{1}{2}''$. The other ſmall equations have alſo their periods, in which they return to the ſame quantity. Hence, the mean inclination remains conſtant.

To reduce the Place of the Moon in it's Orbit, to the Ecliptic.

957. By ſpherical Trigonometry we have, $\tan NV = \text{coſ.}\ N \times \tan NM = \sqrt{1-s^2} \times \tan NM = \overline{1-\frac{1}{2}s^2} \times \tan NM$, hence, $\tan.\ \overline{NV-NM} = \dfrac{-\frac{1}{2}s^2 \times \tan NM}{1+\overline{1-\frac{1}{2}s^2} \times \overline{\tan NM}^2}$ = (putting $\dfrac{\text{ſin}\ NM}{\text{coſ.}\ NM}$ for $\tan.\ NM$) $-\dfrac{\frac{1}{2}s^2 \times \text{ſin.}\ NM \times \text{coſ}\ NM}{1-\frac{1}{2}s^2\,\overline{\text{ſin}\ NM}^2} = -\frac{1}{2}s^2 \times \text{ſin.}\ NM \times \text{coſ.}\ NM$ nearly, $= -\frac{1}{4}s^2 \times \text{ſin.}\ 2NM$; and as $NV-NM$ is very ſmall, we may conſider this quantity as expreſſing the difference of the two arcs. Now the moon's diſtance from the node $= S - \frac{3}{8}n \times \text{ſin}\ \overline{2S-2X}$ nearly, the latter term being (939) the principal equation of the nodes, and if we correct the value of S by the principal equation of the center $-2w \times \text{ſin}\ Z$, we get a more correct value of $NM = S - 2w \times \text{ſin}\ Z - \frac{3}{8}n \times \text{ſin.}\ \overline{2S-2X}$, alſo, if we correct the value of s by the principal equation $\frac{3}{8}sn \times \text{coſ.}\ \overline{2S-2X}$ (948) of the inclination, it becomes $s + \frac{3}{8}sn \times \text{coſ}\ \overline{2S-2X}$, hence, $NV-NM$ becomes $= \text{ſin.}\ (2S - 4w \times \text{ſin}\ Z - \frac{3}{4}n \times \text{ſin}\ \overline{2S-2X}) \times (-\frac{1}{4}s^2 - \dfrac{3}{16}sn \times \text{coſ}\ \overline{2S-2X})$ nearly, $= (\text{ſin.}\ 2S - 4w \times \text{ſin.}\ Z \times \text{coſ.}\ 2S - \frac{3}{4}n \times \text{ſin}\ \overline{2S-2X} \times \text{coſ}\ 2S) \times (-\frac{1}{4}s^2 - \dfrac{3}{16}sn \times \text{coſ.}\ \overline{2S-2X})$ $= -\frac{1}{4}s^2 \times \text{ſin.}\ 2S + s^2w \times \text{ſin.}\ Z \times \text{coſ}\ 2S - \dfrac{3}{16}s^2n \times \text{ſin.}\ 2S \times \text{coſ.}\ \overline{2S-2X} +$

$\frac{3}{16}s^2n$

$\frac{3}{16}s^2n \times \text{coſ.}\ 2S \times \text{ſin.}\ \overline{2S-2X}$ nearly, $= -\frac{1}{4}s^2 \times \text{ſin.}\ 2S + \frac{1}{2}s^2w \times \text{ſin.}\ \overline{2S+Z}$ $-\frac{1}{2}s^2w \times \text{ſin.}\ \overline{2S-Z} - \frac{3}{16}s^2n \times \text{ſin.}\ 2X = -6'.\ 54'' \times \text{ſin.}\ 2S + 46'' \times \text{ſin.}$ $\overline{2S+Z} - 46'' \times \overline{2S-Z} - 23'' \times \text{ſin.}\ 2X$ the reduction.

958. Hence, we may reduce the equations (921) for the longitude upon the moon's orbit, to the longitude upon the ecliptic, by applying the firſt, third and fourth equations to the equations XV. XVII. and VI. having the ſame arguments, and adding the third equation, hence, we get the following equations of the moon's longitude upon the ecliptic.

I.	−	6°. 17′. 32″,7	ſin Z.
II . . .	+	13. 0	ſin $2Z$.
III	−	0. 37	ſin. $3Z$.
IV.	−	1. 16 26	ſin. $\overline{2X-Z}$.
V.	−	3. 52	ſin. $\overline{2X+Z}$.
VI.	+	40. 18	ſin $2X$.
VII.	+	12. 55	ſin. Y.
VIII.	+	1. 47	ſin. $\overline{Z-Y}$.
IX.	−	1. 47	ſin. $\overline{Z+Y}$.
X.	−	1. 6,3	ſin. $\overline{2X-Y}$.
XI.	−	0. 50,5	ſin. $\overline{2X+Y}$.
XII.	−	0 12	ſin. $\overline{2Z-2X}$.
XIII.	+	0. 2,1	ſin. $\overline{4Z-4X}$.
XIV. . . .	−	0. 49,3	ſin $\overline{4X-Z}$.
XV.	−	6. 41,5	ſin. $2S$
XVI.	−	1. 28	ſin. $\overline{2S-2X}$.
XVII.	−	1. 15	ſin. $\overline{2S-Z}$.
XVIII. . . .	+	0. 46	ſin. $\overline{2S+Z}$.
XIX.	+	2. 12	ſin. $\overline{2X-Z-Y}$.
XX.	+	2. 3	ſin. $\overline{2X-Z+Y}$.
XXI.	+	0. 8	ſin. $\overline{2Z+2X}$.
XXII. . .	−	0. 12,5	ſin $\overline{2S-4X}$.
XXIII. . . .	−	0. 29	ſin. $\overline{4X-2S-Z}$.
XXIV. . . .	−	0. 6,4	ſin. $\overline{2X+Z-Y}$.
XXV.	−	0. 6,4	ſin. $\overline{2X+Z+Y}$.

On the true Place and Motion of the Moon's Apogee.

FIG 213. 959 Let N be the *mean* place of the moon's apogee corrected by it's annual equation, Nn the major axis of the moon's orbit, E the earth, F the center of the orbit, EI, EK the greateſt and leaſt excentricity, and with the center F and radius FI deſcribe a circle $ICKB$, and draw AED perpendicular to Nn, draw ES towards the ſun, and make the angle $IFH = 2NES$, and draw EHN', and it gives (909) the ſituation of the apogee corrected by it's annual and it's great equation. Hence, when the ſun is in the apogee N, N' coincides with N, when EN' becomes a tangent to the circle at C, FHE becomes a right angle, and N' is then got to it's limit, when the ſun comes to quadratures at A, H coincides with K, and N' with N, as the ſun moves from quadratures A to the perigee n, when EN' becomes a tangent to the circle at B, N' gets to it's limit on that ſide of N, when the ſun comes into ſyzygies at n, H then coincides with I, and N' with N, and the period of the equation is completed. The ſame takes place whilſt the ſun moves from ſyzygies at n to N

960 By Art. 909. the principal equation of the apogee is the angle HEF, now in that triangle, we know EF, FH and the angle EFH, which is the ſupplement of HFI, or twice the diſtance of the ſun from the mean place of the apogee corrected by the annual equation (903), hence, $EF+FH$ $EF-FH$.: tan. $\frac{1}{2}$ $\overline{FHE+FEH}$ tan. $\frac{1}{2}$ $\overline{FHE-FEH}$, and if we take $EF \cdot FH$ :: 5505 · 1168 according to Sir I. NEWTON, the log of $\overline{EF-FH}$ − log of $\overline{EF+FH} = -0{,}1871317$, hence, if from the log. tan of $\frac{1}{2} . \overline{FHE+FEH}$ we ſubtract 0,1871317, we have the log. tan. of $\frac{1}{2} . \overline{FHE-FEH}$, and this ſubtracted from the third term leaves the angle HEF, and thus a Table of this equation may be very readily conſtructed. This equation of the apogee *added* to it's place as corrected above, whilſt the ſun moves from ſyzygies to quadratures of the apſides, or *ſubtracted* whilſt it moves from quadratures to ſyzygies, gives the place of the apogee corrected by the great equation The following Table of this equation and of the excentricity of the moon's orbit is taken from Dr. HALLEY's *Aſtronomical Tables*, the argument, called the *Annual Argument*, is the diſtance of the ſun from the mean place of the apogee corrected by it's annual equation.

A TABLE OF THE GREAT EQUATION OF THE MOON's APOGEE, AND OF THE ECCENTRICITY OF IT's ORBIT

	Sig O VI +				Sig I. VII +				Sig. II VIII +				
Ann Arg	Equation of ☽'s Apogee			Excentricity of the Moon's Orbit	Equation of ☽'s Apogee			Excentricity of the Moon's Orbit	Equation of ☽'s Apogee			Excentricity of the Moon's Orbit	Ann Arg
Deg	D	M	S		D	M	S		D	M	S		Deg
0	0	0	0	,066777	9.	27	57	,061754	11	40.	0	,050224	30
1	0	21	4	,066771	9	42.	12	,061434	11	30	39	,049838	29
2	0.	42	8	,066754	9	55	58	,061107	11	20	14	,049457	28
3	1.	3	10	,066724	10	9	14	,060772	11.	8.	44	,049082	27
4	1	24	9	,066683	10.	21	58	,060429	10	56.	8	,048714	26
5	1	45	5	,066630	10.	34	9	,060080	10.	42	26	,048354	25
6	2	5	57	,066566	10	45	47	,059725	10	27	38	,048001	24
7	2	26	44	,066489	10.	56	49	,059363	10	11.	45	,047656	23
8	2	47	25	,066402	11.	7	15	,058995	9	54.	47	,047321	22
9	3	8	0	,066302	11.	17	4	,058621	9	36.	44	,046995	21
10	3	28	27	,066192	11.	26	14	,058243	9	17.	37	,046679	20
11	3	48	46	,066070	11.	34	43	,057860	8	57	25	,046374	19
12	4	8.	55	,065936	11.	42	31	,057472	8.	36	11	,046081	18
13	4	28.	54	,065792	11.	49	36	,057080	8	13.	56	,045800	17
14	4	48	42	,065636	11.	55	57	,056684	7	50	42	,045531	16
15	5.	8.	19	,065469	12	1.	33	,056285	7	26	29	,045275	15
16	5	27	43	,065292	12.	6	22	,055884	7	1	21	,045033	14
17	5	46	53	,065103	12.	10	23	,055479	6	35	19	,044805	13
18	6	5	48	,064905	12	13	35	,055073	6.	8	26	,044592	12
19	6	24	27	,064695	12.	15	56	,054666	5	40.	45	,044394	11
20	6.	42	50	,064476	12	17	24	,054257	5	12	18	,044212	10
21	7	0	56	,064246	12.	17.	59	,053848	4.	43.	10	,044046	9
22	7	18	44	,064006	12	17	40	,053438	4	13.	23	,043896	8
23	7.	36	12	,063757	12	16	25	,053030	3	43	1	,043763	7
24	7	53	20	,063498	12	14	13	,052622	3.	12.	9	,043647	6
25	8.	10	6	,063230	12	11	2	,052215	2	40.	49	,043548	5
26	8.	26.	29	,062952	12	6	52	,051811	2	9	7	,043467	4
27	8	42.	29	,062665	12	1	42	,051409	1	37	6	,043404	3
28	8	58.	5	,062370	11.	55	31	,051010	1.	4	52	,043359	2
29	9	13.	15	,062066	11	48.	17	,050615	0.	32	28	,043332	1
30	9	27	57	,061754	11	40.	0	,050224	0.	0.	0	,043323	0
	Sig. V XI. –				Sig. IV. X. –				Sig III IX. –				

961. When EH becomes a tangent to the circle, the equation then becomes a maximum, and this is found by ſaying, $EF . FH$: rad : ſin. HEF, after that, as the point H approaches K the equation grows leſs, and the apogee, ſo far as it's motion ariſes from this equation, goes backward, and therefore when this velocity becomes equal to the mean progreſſive velocity, they will deſtroy each other, and the apogee becomes ſtationary. This poſition of the apogee might be determined by making the fluxions of theſe two angular motions equal, but ſuch a method would be very troubleſome in practice, and it can be done very readily from inſpection in the Table, by the method of trial and error. The mean motion of the ſun in a day is 59′. 8″, and the mean motion of the apogee in a day is 6′. 41″; hence, in a day the ſun recedes 52′ 27″ from the mean apogee, conſidering the ſun as moving with it's mean motion, which will be ſufficiently accurate for the purpoſe for which it is here wanted, in the time therefore that the ſun recedes 1° from the apogee, the apogee has moved 7′ 43″. Now from 1^s 28° to 1^s. 29° of argument, the motion of the apogee (from the equation) has been retrograde 7′. 14″, and from 1^s. 29° to 2^s the motion has been 8′. 17″, hence, the regreſſive motion muſt have been equal to the progreſſive motion ſomewhere between 1^s. 29° and 2^s of argument. By trial and error therefore it is readily found, that with the argument 1^s. 29°. 23′ the motions become equal, and this will appear by computing it's motion for a very ſmall increaſe of argument, and comparing it with the cotemporary mean motion. Now the equation to the argument 1^s 29°. 23′ is 11°. 42′, hence, 1^s. 29° 23′ − 11°. 42′ = 47°. 41′ the diſtance of the ſun from the *true* place of the apogee when it becomes ſtationary from the effect of it's greateſt equation, which muſt be very near the true diſtance, as the other equations which affect it's motion are very ſmall. In the paſſage therefore of the ſun from about 47°. 41′ before it comes to the apſide, to 47° 41′ beyond the apſide, the apogee is *progreſſive*, and from 42° 19′ before the ſun comes into quadratures, to 42°. 19′ beyond quadratures, the apogee is *regreſſive*

962 In half a day, the ſun, moving with it's mean motion, recedes 26′. 13″,5 from the mean apogee, and with that argument (the ſun being ſo far from the mean apogee) the equation is found to be 9′ 12″, hence, in one day whilſt the ſun paſſes through the apogee, it's progreſſive motion has been 18′ 24″ from the equation, to which add 6′ 41″ the mean motion of the apogee in the ſame time, and we have 25′. 5″ for the *progreſſive* motion of the apogee in a day, whilſt the ſun paſſes through the apogee. The ſame is true when the ſun paſſes through the perigee. When the ſun's diſtance from the apogee is 90°, the equation in half a day from thence is 14′. 11″, therefore in a day whilſt the ſun paſſes through quadratures, the equation is 28′. 22″, giving that regreſſive

motion to the apogee, hence, 28' 22" − 6'. 41" = 21'.41" * the *regreſſive* motion of the apogee in a day when the ſun paſſes it's quadratures. It appears then from theſe two Articles, that the progreſſive motion of the apogee in ſyzygies is greater than the regreſſive motion in quadratures, and that in a whole revolution of the ſun in reſpect to the apogee, it is progreſſive for a longer time than it is regreſſive. This is the meaning of what Sir I. NEWTON has delivered in Cor. 8. Pr. 66. Lib. I. He ſays, Manifeſtum eſt quod apſides in ſyzygiis ſuis, per vim ablatitiam, progredientur velocius, inque quadraturis ſuis tardius recedent per vim addititiam. Ob diuturnitatem temporis, quo velocitas progreſſus vel tarditas regreſſus continuatur, fit hæc inequalitas longè maxima. It muſt be acknowledged, however, that what he has here advanced is no immediate conſequence of any thing which he has before inveſtigated.

963. Some of the principal equations of the moon's motion (349 . . 354) were diſcovered by obſervation, but Sir I. NEWTON having found that the moon was retained in it's orbit by a force, which, at different diſtances from the earth, varied inverſely as the ſquare of the diſtance †, and concluding from analogy that the ſame law of attraction might take place between all the bodies in the ſyſtem, applied this theory (called the *Theory of Gravity*) to compute the effect of the ſun's attraction upon the earth and moon, ſo far as it might affect the relative ſituation of the latter as ſeen from the former, and hence he diſcovered, beſides the irregularities before obſerved, other ſmaller inequalities of the moon's motion, which alſo were found to agree with obſervations. From theſe,

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* Sir I. NEWTON, in the firſt edition of his *Principia*, page 462, has given the following Scholium. Hactenus de motibus lunæ quatenus excentricitas orbis non conſideratur. Similibus computationibus inveni, quod apogæum, ubi in conjunctione vel oppoſitione ſolis verſatur, progreditur ſingulis diebus 23' reſpectu fixarum, ubi verò in quadraturis eſt, regreditur ſingulis diebus 16$\frac{1}{3}$' circiter; quodque ipſius motus medius annuus ſit quaſi 40°. Per Tabulas Aſtronomicas à Cl. FLAMSTEDIO ad hypotheſin HORROXII accommodatas, apogæum in ipſius ſyzygiis progreditur cum motu diurno 24' 28", in quadraturis autem regreditur cum motu diurno 20' 12", et motu medio annuo 40° 41' fertur in conſequentia. Quod differentia inter motum diurnum progreſſivum apogæi in ipſius ſyzygiis, et motum diurnum regreſſivum in ipſius quadraturis, per Tabulas ſit 4' 16", per computationem verò noſtram 6$\frac{2}{3}$', vitio Tabularum tribuendam eſſe ſuſpicamur. Sed neque computationem noſtram ſatis accuratam eſſe putamus. Nam rationem quandam ineundo prodiere apogæi motus diurnus progreſſivus in ipſius ſyzygiis, et motus diurnus regreſſivus in ipſius quadraturis, paulo majores. Computationes autem, ut nimis perplexas et approximationibus impeditas, neque ſatis accuratas, apponere non lubet.

† Sir I. NEWTON found, that if the force with which bodies fall upon the earth's ſurface were extended to the moon, and to vary inverſely as the ſquare of the diſtance from the center of the earth, it would in one minute draw the moon through a ſpace which is equal to the verſed ſine of the arc which the moon deſcribes in a minute. He concluded therefore that the moon was retained in it's orbit by the ſame force as that by which bodies are attracted upon the earth.

theſe, and other applications of his theory, he was confirmed in his conjectures concerning the principle of univerſal gravitation, and the ſame principle having ſince been further applied, and found to produce concluſions conformable to obſervation, his theory of gravity is now firmly eſtabliſhed. In the year 1747, M. Clairaut, in a Memoir read before the Academy of Sciences at Paris, made an objection to this law, upon this ground, that it would not account for the motion of the moon's apogee, it giving, according to his calculations, that motion only one half of what it was found to be by obſervations; and he concluded, that it was neceſſary to change this law, by adding ſomething to correct it. He however ſoon afterwards diſcovered his miſtake, and was the firſt who gave a complete theory of the moon, in which he ſhowed that Sir I. Newton's law of gravity would not only account for the motion of the moon's apogee, but alſo for all the other irregularities of the moon. M. Euler has done great juſtice to M. Clairaut upon his ſolution of this important problem, in a letter to the Rev. Mr. Caspar Wetstein, in which he obſerves, that "this queſtion is of the laſt importance, and I muſt own, that, till now, I always believed, that this theory did not agree with the motion of the apogee of the moon. M. Clairaut was of the ſame opinion, but he has publickly retracted it, by declaring that the motion of the apogee is not contrary to the Newtonian theory. Upon this occaſion I have renewed my enquiries on this affair; and, after moſt tedious calculations, I have at length found to my ſatisfaction, that M. Clairaut was in the right, and that this theory is entirely ſufficient to explain the motion of the apogee of the moon. As this enquiry is of the greateſt difficulty, and as thoſe, who hitherto pretended to have proved this nice agreement of the theory with the truth, have been much deceived, it is to M. Clairaut that we are obliged for this important diſcovery, which gives quite a new luſtre to the theory of the Great Newton; and it is but now, that we can expect good Aſtronomical Tables of the moon." Sir I. Newton in his *Principia*, Lib. I. Prop. 45. Cor. 2 by aſſuming the mean force $\left(\frac{1}{EM^2} - \frac{1}{2} m \times EM\right)$ of the moon to the earth in the direction of the radius *ME*, and conſidering that force as acting for a whole revolution of the moon, finds (855) the motion of the apſides to be only one half of the real motion, for this force $\frac{1}{EM^2} - \frac{1}{2} m \times EM = \frac{EM - \frac{1}{2} m \times EM^4}{EM^3}$ gives the diſtance of the apſides $= 180° \times \sqrt{\frac{1 - \frac{1}{2} m}{1 - 2 m}} = 180° \times \sqrt{1 + \frac{3}{2} m} = 180° \times \overline{1 + \frac{3}{4} m}$, and conſequently the motion is $180° \times \frac{3}{4} m$, whereas (903) the true motion is $180° \times \frac{3}{2} m$. But in this he has neglected that part of the force in a direction perpendicular to *EM*, and which is found

to

to produce the other half of the motion; and Sir I. NEWTON, in this place, intended only to ſhow what part of the motion the mean force in the direction of the radius would produce. In Lib. III. Pr. 3. he ſays, that "the action of the ſun, ſo far as it draws the moon from the earth, is twice as great as he has aſſumed it above;" by which he does not mean that he has aſſumed the mean force ($-\frac{1}{2}m \times EM$) of the ſun too little by one half, but that, as it would require twice ſuch a force of the ſun to give the true motion of the apſides, the force which acts in a direction perpendicular to the radius *EM*, muſt, in it's effects upon this occaſion, be equivalent to the force $-\frac{1}{2}m \times EM$ in the direction of the radius. In the firſt edition of the *Principia*, in the Scholium to the Theory of the moon, he ſaid that he had found, by a very complex calculation, that the mean annual motion of the apogee was about 40°, and that the diurnal motion of the apogee, when in ſyzygies, was progreſſive about 23′, and when it was in quadratures, it was regreſſive about $16\frac{1}{3}'$, this he omitted in his future editions, as he was not perfectly ſatisfied with his computations. Mr. MACHIN, in his *Laws of the Moon's Motion according to Gravity*, makes the mean annual motion of the apogee to be $40°.\ 40'.\ 40\frac{1}{2}''$, upon a principle which he firſt ſuggeſted, and upon which we have in this Work computed that motion, according to the method given by FRISI. M. WALMSLEY, in his *Theory of the Motion of the Apſides*, has, upon the ſame principle, computed the mean motion of the moon's apogee, and his concluſion agrees very well with obſervation; but his principles are altogether wrong; for he has entirely omitted that part of the force which acts in a direction perpendicular to the radius, which, as we have ſhown, produces juſt one half of the motion; he alſo aſſumes the mean diſturbing force in the direction of the radius as acting conſtantly, inſtead of the real diſturbing force; and he has alſo wrongly computed the periodic time of the moon; it was by accident therefore that he obtained the mean motion, in reſpect to the true motion, his concluſions are erroneous. M. MACHIN has not given us his proceſs, we cannot therefore ſay how far it was juſt. In the *Phil. Tranſ.* 1751, Mr. P. MURDOCK has given a method of computing the mean motion of the moon's apogee, by firſt conſidering only that part of the diſturbing force, which acts in the direction of the radius, and then, inſtead of ſuppoſing the earth to be at reſt, by conceiving the earth and moon to revolve about their common center of gravity, he imputes about one half the motion to that cauſe, and thence deduces a concluſion agreeing nearly with obſervation. What we have already obſerved in Art. 861, is ſufficient to ſhow, that no part of the effect can ariſe from the latter circumſtance, and he has alſo (as we have already ſhown) omitted a cauſe which produces about one half of the motion; by two miſtakes he has therefore fallen upon a true concluſion.

964. Now to bring into one point of view, the ſources of all the equations which we have here found, we may conſider, *firſt*, the equation of the center, which ariſes from the elliptic form of the orbit. *Secondly*, we may conſider that the diſturbing forces would change a circular orbit into that of an ellipſe, having the earth in the center, with the minor axis lying in ſyzygies, and therefore the moon's orbit muſt ſuffer very nearly a ſimilar effect, the areas (849) are alſo accelerated and retarded, and from theſe cauſes, the mean place differs from the true, thus there is produced an equation, called the *Variation*. But at different diſtances of the earth from the ſun, the diſturbing forces vary, this equation therefore being firſt calculated for the mean diſtance of the earth from the ſun, will be ſubject to a variation, from the variation of that diſtance, and hence ariſe ſome new equations. *Thirdly*, the moon's orbit being dilated and contracted as the earth approaches to, or recedes from the ſun, it's motion will accordingly be diminiſhed or increaſed, and hence ariſes an annual equation, aſſigning the difference between the mean motion at the mean diſtance of the earth from the ſun, and the mean motion at any other diſtance of the ſun. *Fourthly*, the variation depending on the true diſtance of the ſun from the moon, if the mean diſtance at firſt uſed be corrected by the firſt term of the equation of the center, and by the annual equation, new equations of the variation will ariſe. And if the ſecond term of the equation of the center be alſo taken, and applied, there will thence ariſe further equations of the variation. Alſo, if the diſtance of the moon from the ſun be further corrected by the evection, new equations will be introduced into the variation. Again, another correction of the variation may be introduced, by conſidering the difference between the diſturbing forces ariſing from the ſituation of the nodes of the lunar orbit. Thus, by correcting the moon's diſtance from the ſun, from what was firſt aſſumed, we get new equations of the variation. *Fifthly*, the orbit of the moon being inclined to the ecliptic, the diſturbing force in the orbit is different from what it would be, if the orbit coincided with the ecliptic, and that difference of forces will produce another equation, depending on the ſituation of the nodes in reſpect to the ſun. *Sixthly*, the diſturbing forces cauſe a motion of the apogee, but that motion is not uniform, it being regreſſive when the ſun is in it's quadratures, and progreſſive, when in ſyzygies, but upon the whole it is progreſſive, there ariſes therefore an equation of the motion of the apogee, which depends upon it's diſtance from the ſun; there is alſo a ſmaller annual equation, ariſing from the diſturbing forces being different at different times of the year. *Seventhly*, the excentricity of the orbit is ſubject to a change, and that change cauſes a change of the equation of the center, called the evection, hence ariſe new equations to be applied, depending on the ſituation of the apogee, the change of excentricity depending on that circumſtance.

ſtance. *Eighthly*, the evection at the mean diſtance of the earth from the ſun being found, the diſturbing force of the ſun at different diſtances from the earth being different, equations of the evection will hence ariſe. Alſo, the diſturbing force varies as the ſituation of the nodes vary, and hence alſo ariſe new equations of the evection. *Laſtly*, if the diſtance of the moon from the ſun be corrected by the annual equation, and the motion of the moon from the apogee be corrected by the ſame equation of the mean motion, and by the annual equation of the apogee, further equations of the evection are found. Thus, by correcting the firſt aſſumed values of the quantities repreſenting the ſeveral ſources of the equations, new equations ariſe, which may be conſidered as corrections of the original equations, and hence we derive the equations of the moon's motion, which applied to the mean place, will give the true place. Sir I. NEWTON calls the equations *Menſtruæ*, *Semeſtres*, *Annuæ*, &c. according to the periods in which they return. The mean motion of the nodes, and the mean inclination of the orbit, are alſo corrected by equations found upon ſimilar principles, which applied to the mean place of the node, and mean inclination, give the true. The concluſions thus deduced will be found generally to agree very well with thoſe which are derived from a direct ſolution; the few caſes where they were not ſufficiently accurate for a ground of computation, we have pointed out, and corrected; but in whatever manner this ſubject is treated, ſome corrections are applied from obſervations, in order to render the equations more perfect; not that the principle of attraction is inſufficient to furniſh concluſions which ſhall agree with obſervations, but that the method of deducing thoſe concluſions being only by approximation, ſmall errors are introduced, which are eaſier to be corrected by obſervation, than by continuing ſuch laborious calculations in order to a further correction*. MAYER's Tables†, founded upon a very elegant theory which he corrected by obſervations, are the moſt correct and do not err more than half a minute in longitude. The method however by which we have here treated the ſubject has this advantage, that it ſhows the cauſes of all the equations, and thereby gives a very clear and comprehenſive view of the whole buſineſs. Thus I have given the reader

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* MAYER in his *Theoria Lunæ*, page 50, makes this obſervation. Præterea in eadem hac formula plures termini occurrunt, quos theoria, licet ſummo ſtudio tractata, accurate præbere non poteſt, ob rationes nulli non cognitas, qui in hac re vires ſuas ac patientiam exercuit. Hos igitur terminos malui ex obſervationibus definire, quam illorum gratia calculum tædioſiſſimum, à me jamdudum longe accuratius inſtitutum, quam a quoque hactenus factum, ulterius adhuc perſequi, noviſque ac forte inſuperabilibus augere difficultatibus. Denique etiam eos terminos, quos theoria ſatis manifeſte oſtendit, per obſervationes ita correxi, ut paucis ſecundis adjectis vel demtis, cum cœlo magis conſentirent.

† As corrected by Dr MASKELYNE. See the Preface to the *Nautical Almanac*, 1797, &c.

reader all the ſatisfaction I am able upon this difficult ſubject, without entering into a direct ſolution of the problem, which requires the integration of a fluxional equation of the ſecond order, and this can be done only by an approximation of a very intricate nature, and of great labour, to explain the whole of which ſo fully as to render it intelligible to the generality of readers, would of itſelf require a conſiderable volume, by ſo treating the ſubject, however, we obtain the following very important concluſion, which was firſt obſerved by Mr. SIMPSON: That as no terms enter into the equation of the orbit, but what conſiſt of the coſine of an arc, or of it's multiples, all the terms, by a regular increaſe and decreaſe, do after a certain time return again to their former values, and therefore the mean motion of the moon, and the greateſt quantities of the ſeveral equations, undergo no change from gravity.

CHAP.

CHAP. XXXIII.

ON THE FIGURE OF THE EARTH.

Art. 965. IF a fluid body had no motion about it's axis, and all it's parts were at reſt and kept together by the mutual attraction of it's conſtituent particles, the body would form itſelf into a ſphere, for the preſſures of all the columns upon the central particle could not be equal, unleſs they were all of the ſame length. Now Sir I. NEWTON having proved that the bodies in our ſyſtem attract each other with forces varying inverſely as the ſquares of their diſtances, concluded it to be an univerſal principle for every particle of matter, and that the parts of each body were kept together by the ſame power (838). On this ſuppoſition therefore, if the earth were a fluid body, and perfectly at reſt, it's form muſt be that of a ſphere. But as the earth revolves about an axis, each particle, beſides it's gravity, will be urged by a centrifugal force, by which the particles will all have a tendency to recede from the axis, and hence, M. HUYGENS and Sir I. NEWTON concluded that the earth muſt put on a ſpheroidical form, whoſe polar diameter is the ſhorteſt. For let POp FIG. be the axis, EQ the equator, draw abm perpendicular to Pp, and let bm repreſent 214. the centrifugal force of the point b, and reſolve it into two, one bn in the direction of the tangent, and the other mn perpendicular to it. Then the force bn draws the fluid from p towards E, and conſequently it will diminiſh the radius pO and increaſe the radius EO. We ſhall therefore firſt conſider, what will be the form of the curve PEp, and then determine the ratio of pO to EO, according to the principles of M. CLAIRAUT, in his Treatiſe, entitled, *Théorie de la Figure de la Terre*, who has followed MAC LAURIN in his inveſtigation of the attraction of a corpuſcle upon the ſurface of a ſpheroid, in directions parallel to each axis.

966. Let Pp, Qa be two concentric circles, O the center, draw $PQOap$, FIG. and mQM, ar, perpendicular to it, and join Mr, and make the angles aQb, 215. aQc; rMs, rMw equal, then $Qb+Qc=Ms+Mw$. For from the conſtruction, Mr is parallel to Qa, and hence $Mr=Qa$; draw rt perpendicular to Ms, and take $rv=rs$. Then as the angle $rMt=aQb$, $rtM=abQ$, and $Mr=Qa$, we have $Mt=Qb$. Now as the angle $sMr=wMr$, the chord $rw=$ chord $rs=rv$; therefore the triangles Mrv, Mrw are ſimilar and equal, hence, $Mw=Mv$; conſequently $Mw+Ms=Mv+Ms=2Mt=2Qb=Qb+Qc$.

967 Now

967. Now let us ſuppoſe this figure to be orthographically projected upon a plane paſſing through Mm, and the two circles will be projected into ſimilar ellipſes, and the lines Qb, Qc, Ms, Mw being all diminiſhed in the ſame ratio, we ſhall have in the ellipſes, $Qb+Qc=Ms+Mw$ When by increaſing the angle rMw it becomes greater than the angle rMx of the ſegment, Mw will lie on the other ſide of M, or w falls in the arc MPm, in which caſe, $Ms-Mw=Qb+Qc$

FIG. 216. 968 The attraction of a corpuſcle at A towards any pyramid $Avwzx$, the area $vwzx$ of whoſe baſe is indefinitely ſmall, varies as the length, the angle A being given, and the attraction to each particle varying inverſely as the ſquare of the diſtance. For put $a=$the area $zxvw$, $m=Az$, $x=Aa$, and let ab be a ſection parallel and therefore ſimilar to zv, then, $m^2 : x^2 :: a \quad \frac{ax^2}{m^2}=$the ſection ab, hence, $\frac{ax^2x}{m^2}=$the fluxion of Aab, conſequently the fluxion of the attraction of $Aab=\frac{ax}{m^2}$, whoſe fluent is $\frac{ax}{m^2}$, the attraction of any length x which therefore varies as x Hence, the attractions of corpuſcles at the vertices of ſimilar pyramids are in proportion to their lengths. If $x=m$, we get the attraction to the whole pyramid $=\frac{a}{m}$.

969. Hence, if two corpuſcles be ſimilarly ſituated in reſpect to two ſimilar ſolids, the attractions to the ſolids will be as their lengths For if the two ſolids be divided into ſimilar pyramids, having the corpuſcles in the vertices, the attractions to all the correſponding pyramids will be as their lengths, or as the lengths of the ſolids; for the pyramids being ſimilarly ſituated in the two ſimilar ſolids, their lengths muſt be in proportion to the lengths of the ſolids, and therefore the whole attractions will be in proportion to the lengths of the ſolids, or in proportion to any two lines ſimilarly ſituated in them.

FIG. 217. 970. Let FIG 217 be the projection deſcribed in Art 967 and make the angles bQv, cQg, sMx, wMz indefinitely ſmall and equal, and conceive the whole figure to revolve about Mm through an indefinitely ſmall angle, then the pyramids generated by bQv, cQg, sMx, wMz being ſimilar, the attractions of the corpuſcles at Q and M towards them will (968) be as their lengths. But $Qb+Qc=Ms+Mw$, or $Ms-Mw$ when w falls on the other ſide of M, hence, the attraction of Q to the pyramids generated by bQv, cQg is equal to the attraction of M towards the pyramids generated by sMx, wMz, the attraction to wMz being reckoned negative when w lies on the other ſide of M Hence, if each attraction be divided into two, one perpendicular to Mm, as bn, cq, sr, wy, and the other parallel to Mm, then from ſimilar triangles,

$bn+cq=sr+wy$, conſequently the attractions of the corpuſcles at Q and M in a direction perpendicular to Mm are equal. And as this reaſoning holds for every ſuch correſponding poſitions of pyramids about Q and M, it follows that the attraction of the corpuſcles at Q and M to the whole ſolids generated by $Qead$, $Pupt$ about Mm through an indefinitely ſmall angle, in directions perpendicular to Mm, will be equal.

971. Now let $Qead$, $Pupt$ be two ſimilar ſpheroids, and conceive the line Pp to paſs through the center, and Mm to be perpendicular to it. Then every plane which paſſes through Mm, cutting the two ſpheroids, will be ſimilar ellipſes, hence, by the laſt Article, if we conceive any two of theſe planes to be inclined to each other at an indefinitely ſmall angle, the attraction of the corpuſcles at Q and M in a direction which is perpendicular to Mm, towards the ſolids between theſe planes in the two reſpective ſpheroids, will be equal Hence, by the reſolution of forces, the whole attraction of a corpuſcle at Q in the direction QO towards the ſpheroid $Qead$ is equal to the attraction of a corpuſcle at M in a direction parallel to QO towards the ſpheroid $Pupt$. In like manner it appears, that if Tef be drawn parallel to the axis Pp, the attraction of a corpuſcle at e in the direction eO towards the ſpheroid $eadQ$ is equal to the attraction of a corpuſcle at T in a direction parallel to eO towards the ſpheroid $uptP$

972 The attraction of a corpuſcle at P towards the ſpheroid $Pupt$ is to the attraction of a corpuſcle at Q towards the ſpheroid $Qead$. $PO : QO$. For conceive two ſimilar pyramids, whoſe baſes are indefinitely ſmall, to be ſimilarly ſituated in the two ſpheroids, having their vertices in P and Q, then (968) the attractions of the corpuſcles at P and Q will be as their lengths, or as PO to QO from their ſimilar ſituations. Hence, if we reſolve the attraction of each into the direction PO, QO, the attractions in this direction will ſtill be as PO to QO, from the pyramids being equally inclined to PO and QO. Therefore by dividing the whole of each ſpheroid into ſimilar pyramids, it follows that the attraction of the corpuſcles at P and Q to the centers of their reſpective ſpheroids will be as $PO \quad QO$.

973 Let $PEpU$ be an ellipſe, Pp the minor and EU the major axis, draw MG perpendicular to the curve, MT a tangent to it, and MQ, MR perpendicular to OP and OE Now $QT \quad QM \quad QM \cdot QG=\frac{QM^2}{QT}$, and by the property of the ellipſe, $QT \quad OP \quad OP \quad OQ=\frac{OP^2}{OT}$, hence, $QG \quad OQ \cdot \frac{QM^2}{QT} \quad \frac{OP^2}{OT} \cdot QM \times \frac{OT}{QT} : OP^2$; but from the ſecond proportion, $OT \quad OQ \quad OP^2 \quad OQ^2$, hence, $OT : TQ :: OP^2 . OP^2-OQ^2=\overline{OP+OQ}\times\overline{OP-OQ}=pQ\times PQ ::$ OE^2 FIG 218.

$OE^2 : QM^2$, therefore $OE^2 = QM^2 \times \frac{OT}{QT}$, hence, $QG : OQ :: OE^2 : OP^2$, conſequently $QG = \frac{OE^2}{OP^2} \times OQ$.

974 A fluid body will preſerve it's figure, if the direction of it's gravity at every point be perpendicular to it's ſurface, for then gravity cannot put it's ſurface in motion.

975. If the particles of an homogeneous fluid body attract each other with forces varying inverſely as the ſquares of their diſtances, and it revolve about an axis, it will put on the form of a ſpheroid. For if $PEpU$ be a fluid, Pp it's axis about which it revolves, then may the ſpheroid revolve in ſuch a time, that the centrifugal force of any particle M combined with it's attraction to the ſpheroid may make this whole force act perpendicular to the ſurface. For let $E =$ the attraction at the equator E, $P =$ that at the pole P, $F =$ the centrifugal force at the equator. Now (971) the attraction of M in the direction $MR =$ the attraction of Q to a ſpheroid ſimilar to $PUpE$ whoſe ſemi-axis is QO, and (972) the attractions at P and Q upon theſe ſimilar ſpheroids are as $QO : PO$; hence, the attraction of M in the direction MR is $P \times \frac{QO}{PO}$; take $Mr =$ this quantity. For the ſame reaſon, the attraction of M in the direction MQ is $E \times \frac{OR}{OE}$. But the centrifugal forces of bodies revolving in equal times being as their radii[*], we have $OE : QM$, or OR, $:: F : F \times \frac{OR}{OE}$ the centrifugal force of M; hence, $\overline{E-F} \times \frac{OR}{OE} =$ the whole force of M in the direction MQ; take $Mg =$ this quantity. Complete the parallelogram $Mrqg$, and Mq will be the direction of the whole force acting on the particle at M. Produce Mq to G. It remains therefore only to be proved, that OE, OP may have ſuch a ratio to each other, that MG ſhall be every where perpendicular to the curve. Now by ſimilar triangles, gq, or Mr, $: Mg :: QG : QM$, that is, $P \times \frac{QO}{PO} : \overline{E-F} \times \frac{OR}{OE} :: \frac{OE^2}{OP^2} \times OQ : QM$, or $P : E-F :: OE : OP$, in which there is no line concerned but the two axes; therefore to a ſpheroid having two axes in ſuch a ratio, the whole attractive force will at every point be perpendicular to it's ſurface; hence (974), the fluid will be at reſt. And

F may

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[*] For (827) the centrifugal force varies as $\frac{Qx^2}{Sx}$, and in this caſe Qx varies as Sx; hence, the centrifugal force varies as Sx, or as the radius.

F may always have ſuch a value as will ſatisfy this proportion, by adjuſting the time of revolution. Or having given F together with P and E, the ſpheroid whoſe axes are as $P : E-F$ muſt be that into which the fluid will form itſelf.

976. The attraction at any point M in the direction MR is as $P\times\frac{QO}{PO} = P\times\frac{MR}{PO}$, let therefore P (the attraction at P) be repreſented by PO, and MR will repreſent the attraction at M in the direction MR, conſequently Mv will repreſent the whole attraction acting in the direction perpendicular to the ſurface. Draw vc perpendicular to MO, then $MO : Ma :: Mv : Mc$ ∴ the attraction in the direction Mv $(=Mv)$: the attraction in the direction $MO = \frac{Ma\times Mv}{MO}$ which varies as $\frac{1}{MO}$, becauſe $Ma\times Mv$ is conſtant, by the property of the ellipſe.

977. To determine the attraction of a corpuſcle at P the pole of a ſpheroid, to the ſpheroid. Draw Pm, PM, indefinitely near each other, and MQ, Mr, perpendicular to Pp, Pm, and conceive the plane PEp to revolve about Pp through an indefinitely ſmall angle whoſe arc is equal a, radius being unity, put $PO=1$, $OE=m$, $PQ=z$, $QM=u$, the coſine of $MPQ=s$, and $\sqrt{m^2-1}=n$. Now $ua=$ the indefinitely ſmall arc deſcribed by M about Pp, conſequently $ua\times Mr=$ the baſe of the pyramid generated by PMr; hence (968), the attraction of P to this pyramid $=\frac{ua\times Mr}{PM}$, but conſidering the angle MPm as the increment of MPQ, and Mr the increment of the arc to the radius PM, we have $Mr : s :: PM : \sqrt{1-s^2}$, hence, the attraction of the pyramid in the direction $PO=\frac{uass}{\sqrt{1-s^2}}$. But $u^2=2m^2z-m^2z^2$, also $s=\frac{PQ}{PM}$ (radius being unity) $=\frac{z}{\sqrt{z^2+u^2}}$, hence, $\frac{us}{\sqrt{1-s^2}}=z=1+\sqrt{1-\frac{u^2}{m^2}}$ from the firſt equation, from which $u=\frac{2m^2\sqrt{1-s^2}\times s}{1+n^2s^2}$; hence, the attraction of the pyramid in the direction $PO=\frac{2am^2s^2\dot{s}}{1+n^2s^2}=\frac{2am^2}{n^3}\times\overline{n\dot{s}-\frac{n\dot{s}}{1+n^2s^2}}$, whoſe fluent is $\frac{2am^2}{n^3}\times\overline{ns-z}$ where z is a circular arc whoſe radius $=1$, tangent $=ns$, hence, when $s=1$, we have $\frac{2am^2}{n^3}\times\overline{n-z}$ for the attraction of P towards the ſolid generated by the revolution of PEp through

Fig. 219.

an indefinitely ſmall angle, where z is the arc whoſe tangent is n. Hence, as this is the attraction for every indefinitely ſmall ſolid thus cut, if we put c = the circumference of a circle whoſe radius is unity, and ſubſtitute c inſtead of a, we ſhall get $P = \frac{2cm^2}{n^3} \times \overline{n - z}$ for the attraction to the whole ſpheroid. But $z = n - \frac{1}{3}n^3 + \frac{1}{5}n^5 - \frac{1}{7}n^7 + \&c.$ put alſo $m = 1 + d$, then $n^2 = m^2 - 1 = 2d + d^2$, hence, by ſubſtitution, $P = \frac{2c}{3} \times \overline{1 + \frac{4}{5}d - \frac{2}{7}d^2 + \frac{8}{105}d^3\ \&c.}$ If $d = 0$, we get $\frac{2}{3}c$ for the attraction to a ſphere whoſe radius $= 1$.

FIG. 220. 978. To find the attraction of a corpuſcle at the equator E of a ſpheroid $EPDp$. Draw EK parallel to Pp, and ſuppoſe a ſphere EH to be inſcribed in the ſpheroid, whoſe radius is equal to unity. Now if we conceive any plane to paſs through EK cutting the ſpheroid and ſphere, the ſection of the ſpheroid will be ſimilar to $EPDp$, and the ſection of the ſphere a circle, hence, if we find the attraction to the two ſolids between any two planes paſſing through EK and forming an indefinitely ſmall angle e with each other, we ſhall get the ratio of the attractions to the two whole ſolids. Draw EN, En indefinitely near, and NL perpendicular to En, and NK to EK. Put $EO = m$, $OP = 1$, $m^2 - 1 = n^2$, $EK = u$, $NK = z$, the arc of the angle e to radius unity $= a$, and the ſine of $NEO = s$, then az = the arc deſcribed by N, hence, the baſe of the pyramid deſcribed by $ENL = az \times LN$, and (968) the attraction of the pyramid $= \frac{az \times LN}{EN}$, and the attraction in the direction $EO = \frac{az \times LN}{EN} \times \sqrt{1 - s^2} = az\dot{s}$, becauſe, conſidering NL as a circular arc whoſe radius is EN, and the angle NEL as the increment of NEO, $LN = \dot{s}\,EN \div \sqrt{1 - s^2}$. But by the property of the ellipſe, $u^2m^2 = 2mz - z^2$, alſo $s = \frac{u}{\sqrt{u^2 + z^2}}$, make the two values of u^2 from theſe two equations equal to each other, and thence we get $z = \frac{2m \times \overline{1 - s^2}}{1 + n^2s^2}$, hence, the attraction $= \frac{2ma \times \overline{1 - s^2} \times \dot{s}}{1 + n^2s^2} = 2a \times \overline{\frac{m^3}{n^3} \times \frac{n\dot{s}}{1 + n^2s^2} - \frac{m\dot{s}}{n^2}}$, whoſe fluent is $2a \times \overline{\frac{m^3}{n^3} \times z - \frac{ms}{n^2}}$, where z is a circular arc whoſe radius $= 1$, tangent $= ns$, and when $s = 1$, we get $2a \times \overline{\frac{m^3}{n^3} \times z - \frac{m}{n^2}}$ for the whole attraction to the part cut off from the ſpheroid. If we make $m = 1$, and conſequently $n = 0$, we have $2a\dot{s} - 2as^2\dot{s}$ for the attraction to the pyramid of the ſphere generated by Ewv, whoſe fluent, when $s = 1$, is $\frac{4a}{3}$ for the attraction to the correſponding part of the ſphere cut off by the two planes. Hence,

Hence, the ratio of theſe attractions is as 1 : $\frac{3}{2}\times\overline{\frac{m^3}{n^3}\times z-\frac{m}{n^2}}$. But (977) the attraction to a ſphere whoſe radius $=1$, is $\frac{2}{3}c$, hence, $E=c\times\overline{\frac{m^3}{n^3}\times z-\frac{m}{n^2}}$ is the attraction to the ſpheroid at E. Put $m=1+d$, then $n^2=2d+d^2$; alſo $z=n-\frac{1}{3}n^3+\frac{1}{5}n^5-\frac{1}{7}n^7+$ &c. hence, by ſubſtitution, $E=\frac{2c}{3}\times\overline{1+\frac{3}{5}d-\frac{9}{35}d^2+\frac{11}{105}d^3}$ &c. Now (975) when the fluid is in equilibrio, $P : E-F :: 1+d : 1$, hence, $F=E-\frac{P}{1+d}$; ſubſtitute for E and P their values, and we get $F=\frac{2}{3}c\times\overline{\frac{4}{5}d-\frac{6}{35}d^2-\frac{2}{35}d^3}$ &c.

979 Let v expreſs the centrifugal force of a body at the equator compared with it's weight, then $v=\frac{F}{E-F}$, the weight being unity, ſubſtitute for E and F their values, and $v=\frac{4}{5}d-\frac{2}{175}d^2+\frac{8}{875}d^3$ &c. hence, by reverting the ſeries, $d=\frac{5}{4}v+\frac{5}{224}v^2-\frac{135}{6272}v^3$ &c. conſequently $1+d=1+\frac{5}{4}v+\frac{5}{224}v^2-\frac{135}{6272}v^3$ &c is the equatorial radius, the polar radius being unity

980. To determine from hence the actual ratio of the diameters of the earth. By Art. 965. mn is that part of the centrifugal force bm which acts in oppoſition to gravity, now $Ob : ba :: bm : mn=\frac{ba\times bm}{Ob}$, but bm (975) varies as ba, and Ob is conſtant, therefore mn varies as ba^2 the ſquare of the coſine of latitude. Now according to Sir I NEWTON, if the earth be a ſphere, it's radius is 19615800 Paris feet. In the latitude of Paris, a body falls 2174 lines the firſt ſecond. The verſed ſine of the arc deſcribed by the equator in 1″ is 7,54064 lines, which therefore repreſents the centrifugal force at the equator. Hence, the force of gravity at Paris : centrifugal force at the equator :: 2174 : 7,54064. But rad^2 : $\overline{\text{coſ lat Paris}}^2$ :: 7,54064 : 3,267 that part of the centrifugal force at Paris which is oppoſite to gravity, hence, 2174+3,267=2177,267 is the force of gravity at Paris, therefore the force of gravity at Paris : centrifugal force at the equator :: 289 : 1, and this ratio may be taken for the gravity at the equator to the centrifugal force. Hence, $v=\frac{1}{288}$, FIG. 214.

conſequently

conſequently $d = \frac{5}{1152} = \frac{1}{230{,}4}$ neglecting all the terms after the firſt on account of their ſmallneſs. Hence, the ratio of the diameters is as $1 : 1 + \frac{1}{230{,}4} :: 230{,}4 : 231{,}4$, or as $230 : 231$ without any ſenſible difference. Sir I. NEWTON makes it $229 : 230$, from which ratio the above does not ſenſibly differ.

981. If the whole denſity of the body ſhould vary, then the gravity E at the equator will vary as the denſity, but the centrifugal force F will not be altered; now $v = \frac{F}{E - F} = \frac{F}{E}$ nearly, hence, v varies inverſely as the denſity nearly, conſequently d will vary nearly in the inverſe ratio of the denſity, that is, by increaſing the whole denſity, the body approaches nearer to a ſphere.

982. To find the ratio of the diameters of *Jupiter*. Let t = the time of it's rotation, T = the time of revolution of one of it's ſatellites, h = the diſtance of that ſatellite from the center of Jupiter, the radius of Jupiter (here ſuppoſed a ſphere) being $= 1$. Then (977) as $\frac{2c}{3}$ expreſſes the attraction at the ſurface of Jupiter, $\frac{2c}{3h^2}$ = the attraction of the ſatellite. Now the centrifugal forces being as the radii directly and ſquares of the periodic times inverſely [*], and the centrifugal force of the ſatellite being equal to it's centripetal force (theſe forces in a circle being always equal), we have, $\frac{h}{T^2} : \frac{1}{t^2} :: \frac{2c}{3h^2} : F = \frac{2cT^2}{3h^3t^2}$. Hence (978), $\frac{4}{5}d - \frac{6}{35}d^2 - \frac{2}{35}d^3$ &c. $= \frac{T^3}{h^3t^2} = w$, and by the reverſion of ſeries, $d = \frac{5}{4}w + \frac{75}{224}w^2 + \frac{125}{392}w^3$ &c. conſequently $1 + d = 1 + \frac{5}{4}w + \frac{75}{224}w^2 + \frac{125}{392}w^3$ &c. Now according to CASSINI, the time of the rotation of Jupiter is 596′, and the diſtance of the fourth ſatellite, according to Mr POUND, is $26{,}63 = h$, and the mean time of it's revolution $= 24032' = T$. Hence, the ratio of the diameters becomes $100{,}5 : 90{,}5$. By obſervation, Mr. POUND found

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[*] For by Art. 827. the centrifugal force varies as $\frac{Qx^2}{Sx}$. Let P = the periodic time in the circle Qx, v = the velocity in the circle, then v varies as Qx, becauſe the time is given; but P varies as $\frac{\text{circum}}{v}$, or as $\frac{Sx}{v}$, therefore $\frac{Sx}{P}$ varies as $\frac{v}{Sx}$ which varies as $\frac{Qx^2}{Sx}$, or as the centrifugal force.

found the ratio of the diameters to be 12 : 13; and Dr. **BRADLEY** as 12,5 : 13,5.

983. Hence, the difference of the diameters being $\frac{5}{4}w$, taking the firſt term only for the value of d, it appears that that difference, or w, varies as $\frac{1}{t^2}$, or directly as the ſquare of the velocity of the planet about it's axis. Hence, and by Art. 981. if the denſity and time of rotation ſhould vary, the difference of the diameters will vary as the ſquare of the velocity directly and denſity inverſely.

984 Sir I. **NEWTON** determines the ratio of the diameters of the earth in the following manner*. He aſſumes the figure to be a ſpheroid, and finds the centrifugal force at the equator gravity 1 289 as in Art 980. He then aſſumes the ratio of the diameters to be as 100 101, and finds the gravity at the pole : gravity at the equator : 501 : 500 Now if we conceive two canals to be cut from the pole and equator down to the center of the earth, and filled with a fluid, they muſt balance each other But the force of gravity at different parts of the ſame canal varies directly as the diſtance from the center, and the centrifugal force of every point of the equatorial canal

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FIG. 221.

[*] Sir I. **NEWTON** in his *Principia*, Lib 1 Pr 91 Cor. 3. proves, that if a corpuſcle P be placed within a ſpheroid, it is attracted to the center O by a force proportional to PO For conceive the ſpheroid PpQ to be ſimilar to the given ſpheroid MTR, and draw the radii OpV, OPM, and draw $hpwv$, $npst$, making an indefinitely ſmall angle npm, then conſidering pmn, ptv as ſimilar pyramids, the attraction of p to them (968) will be as their lengths; but by the property of the ellipſes, $wv = pm$, therefore the attraction of p to the part $vstv$ = the attraction to pmn, hence, p is attracted only by the pyramid pws. Thus it appears, that the attraction of p is only to the ſpheroid PQ, hence, the attraction (969) at p . the attraction at V pO VO But this is not true for corpuſcles in different radii For PQ is a ſpheroid ſimilar to MIR; and (as above proved) the corpuſcles P, p, will not be diſturbed by the attraction of the matter exterior to PQ, conſequently (976) the attraction of p to O the attraction of P to O PO pO, but (969) conſidering rz ſimilar to PQ, the attraction of a corpuſcle P the attraction of a corpuſcle r . PO rO, therefore the attraction of p to O : the attraction of r to O . PO^2 $pO\times rO$ $pO\times PO^2$ $pO^2\times rO$: pO $rO\times\frac{pQ^2}{PO^2}$. pO $rO\times\frac{VO^2}{MO^2}$ Therefore for corpuſcles ſituated in different radii, this laſt proportion muſt be applied, and not that of Sir I. **NEWTON** From hence it alſo appears, that if VO, MO be two canals meeting at O and filled with a fluid, they will balance, for the fluxions of their preſſures will be as the forces of attraction multiplied into the fluxions of their lengths, or as $pO\times\overline{pO}$. $rO\times\overline{rO}\times\frac{VO^2}{MO^2}$, whoſe fluents are as pO^2 $rO^2\times\frac{VO^2}{MO^2}$, and when Op and Or become OV and OM, we get the whole preſſures as 1 1, or they balance each other. It is from aſſuming this principle, that Sir I **NEWTON** proves the ratio of the diameters of the earth, and that the attraction on the ſurface varies inverſely as the radius. Hence alſo, any ſimilar parts Op, OP will balance each other.

canal varying (975) in the ſame ratio, therefore the whole force to the center varies as the diſtance from the center, and if we take any two indefinitely ſmall parts of each canal, ſimilarly ſituated, their weights muſt be as the weights of the whole, and as the weights are as the magnitudes and gravities, they will be as 101 × 500 : 100 × 501, or as 505 : 501. Hence, that there may be an equilibrium, the centrifugal force muſt take off $\frac{4}{505}$ from the equatorial canal, and then the weights of each will be equal, and they will balance. But the centrifugal force of the earth at the equator takes off $\frac{1}{289}$ part of gravity, hence (266), $\frac{4}{505} : \frac{1}{289} :: \frac{1}{100} : \frac{1}{229}$ the exceſs of the equatorial above the polar radius, therefore the ratio of the radii is $1 : 1 + \frac{1}{229}$ or 229 : 230.

985. To find the ratio of the diameters of any other body, Sir Isaac proceeds thus. If the body be greater or leſs than the earth, the denſity and time of rotation being the ſame, the ratio of the centrifugal force to gravity, and therefore the ratio of the diameters, will remain the ſame. But if the time of rotation and the denſity vary, the difference of the diameters will (963) vary very nearly in a duplicate ratio of the velocity directly and the denſity inverſely. Now the earth revolves in 23*h* 56′, and *Jupiter* in 9*h* 56′, the ſquares of theſe are as 29 : 5 very nearly, and denſities (as will afterwards appear) are as 400 : 94,5, hence, the difference of the diameters of Jupiter : it's leaſt diameter :: $\frac{29}{5} \times \frac{400}{94{,}5} \times \frac{1}{229} : 1$, or as 1 : $9\frac{1}{3}$ very nearly, hence, the equatorial : the polar diameter of Jupiter :: $10\frac{1}{3} : 9\frac{1}{3}$, agreeing very nearly with Art. 982.

986. The ratio of the diameters of the earth and planets here determined from the principle of attraction, ſuppoſes the earth to be of an uniform denſity, but as it appears that this does not give accurately the ratio of the diameters of Jupiter, it will be proper to examine, how this determination agrees with the figure of the earth deduced from an actual menſuration. The method of performing this operation, I ſhall explain from the meaſurement of a degree of the meridian at the polar circle in Lapland*, by Clairaut, Camus, Le Monnier, Maupertuis, the Abbé Outhier, and M. Celsius of Upſal.

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* From the meaſurement of the degrees of the meridian in France, the longeſt degree appeared to be that which lay moſt to the ſouth, from which Cassini concluded that the earth was an oblong ſpheroid, or the polar diameter the greateſt. To ſettle therefore the figure of the earth, the meaſurement of a degree of the meridian at Lapland was undertaken.

Upſal. They ſet out together from Stockholm, and went to Tornea; from thence they departed on July 6, 1736, to ſurvey the country, and fix the proper ſtations. Fig. 222. repreſents the triangles, upon which the calculations of the degree of the meridian were founded. *T* repreſents Tornea, *n*, Niwa, *K*, Kakama, *C*, Cuitaperi, *A*, Avaſaxa, *P*, Pullingi, *Q*, Kittis, *N*, Niemi; and *H*, Horrilakero. Theſe are the ſtations from which they meaſured the angles, which they found to be as follows.

FIG. 222.

$CTK = 24^{\circ}.\ 22'.\ 54'',5$
$KTn = 19.\ 38.\ 17,8$
$TnK = 87.\ 44.\ 19,4$
$HnK = 73.\ 58\ \ 5,7$
$AnH = 21.\ 32\ \ 2,5$
$HnC = 31\ \ 57.\ 3,6$
$TKn = 72\ \ 37.\ 27,8$
$CKn = 45\ \ 50.\ 44,2$
$HKn = 89.\ 36\ \ 2,4$
$HKN = 9.\ 41.\ 47,7$
$KCn = 28\ \ 14.\ 54,7$
$KCT = 37\ \ 9.\ 12$
$KCH = 100\ \ 9.\ 56,8$
$HCA = 30.\ 56.\ 53,4$

$HAP = 53^{\circ}\ 45'\ 56'',7$
$HAC = 112.\ 21.\ 48,6$
$APH = 31.\ 19.\ 55,5$
$HPN = 37\ \ 22\ \ 2,1$
$NPQ = 87.\ 52.\ 24,3$
$NQP = 40\ \ 14.\ 52,7$
$QNP = 51\ \ 53.\ 4,3$
$PNH = 93.\ 25.\ 7,5$
$HNK = 27.\ 11.\ 53,3$
$CHA = 36.\ 42\ \ 3,1$
$CHn = 19.\ 38\ \ 21$
$nHK = 16.\ 26.\ 6,3$
$AHP = 94.\ 53.\ 49,7$
$PHN = 49.\ 13.\ 9,3$

Theſe are the angles as meaſured with a quadrant of two feet radius, furniſhed with a micrometer, and reduced to the horizon.

987. Let *QM* be a meridian line, and from the ſeveral ſtations draw the dotted lines perpendicular to it, and *QM* is the difference of the latitudes of Tornea and Kittis, which we want to determine in meaſure of toiſes. For this purpoſe, it was neceſſary to meaſure ſome baſe line, and connect it with the above triangles. A line *Bb* lying on the ice was therefore accurately meaſured, and found from the mean of the two meaſurements, which differed only four inches, to be 7406,86 toiſes. And the following angles were found by menſuration.

$ABb = 9^{\circ}.\ 22'.\ 0''$
$AbB = 77.\ 31\ \ 50$
$ACB = 54.\ 40.\ 28$
$BAC = 22.\ 37.\ 20$

988. Now

988. Now in any triangle, the angles and one ſide being known, the other ſides may be computed; hence, in the triangle ABb, we find $BA = 7242,76$; therefore in the triangle ABC, we find $AC = 8659,94$ toiſes. By proceeding thus with the triangles ACH, CHK, CKT, AHP, HNP, NPQ, we find $AP = 14277,43$, $PQ = 10676,9$, $CT = 24302,64$ toiſes.

989. To determine the poſition of theſe triangles in reſpect to the meridian QM, the paſſages of the ſun at Q through a vertical circle to P and N was obſerved for many days, which gave the angle $PQM = 28°.\ 51'\ 52''$, and $NQM = 11°\ 22°.\ 54'$. Hence, $PQD = 61°\ 8'.\ 8''$, $APE = 84°.\ 33'.\ 54''$, $ACF = 81°.\ 33'.\ 26''$, $CTG = 69°.\ 49'.\ 8''$; therefore by the reſolution of the right angled triangles PDQ, AEP, AFC, CGT, we get

$$\begin{array}{rl} PD = & 9350,45 \text{ toiſes.} \\ AE = & 14213,24 \\ AF = & 8566,08 \\ CG = & 22810,62 \\ \hline QM = & 54940,39 \\ \hline \end{array}$$

Which is the arc of the meridian paſſing through Kittis, and terminated by a perpendicular from Tornea.

990 The ſame may be computed from the triangles ACH, CHK, CKT, HKN, HNP, NPQ, by the reſolution of which, we find $QN = 13564,64$, $NK = 25053,25$, $KT = 16695,84$ toiſes, alſo $QNd = 11°\ 22'\ 54''$, $KNL = 86°.\ 7'.\ 12''$, $KTg = 85°\ 48'\ 7''$; hence, by the reſolution of the right angled triangles, NQd, KNL, Kgt, we get,

$$\begin{array}{rl} Nd = & 13297,88 \text{ toiſes} \\ KL = & 24995,83 \\ Kg = & 16651,05 \\ \hline QM = & 54944,76 \\ \hline \end{array}$$

The mean of theſe two values of QM gives $QM = 54942,57$ toiſes.

991. If we take the obſervations from n, we may compute the value of QM, from a great variety of triangles Accordingly, ten other values of QM were computed,

computed, the mean of which gave $QM = 54922,1$. The obſervers however (for reaſons not aſſigned in their account of the menſuration) preferred the determinations of QM from the mean of the two firſt values of it.

992 The next thing to be determined was to find the degrees in the arc QM. This was done by obſerving the difference of the zenith diſtances of the ſame ſtar at Kittis and Tornea The inſtrument they uſed for this purpoſe was made by Mr GRAHAM, and the diviſions being verified by a micrometer adapted to it, the error never amounted to above $2''$ or $3''$.

993. The ſtar δ *Draconis* was obſerved at Kittis and Tornea, and the difference of the zenith diſtances, after applying the proper corrections, was found to be $57'\,26'',9$ By α *Draconis*, it was found to be $57'\,30'',4$, the mean of theſe is $57'.\,28'',7$, but this is not the arc QM, becauſe the point at Kittis where the obſervation was made, was 3 toiſes 4 feet 8 inches more north than the point Q, and the point at Tornea where the obſervation was made, was 73 toiſes 4 feet $5\frac{1}{2}$ inches more ſouth than the point T, there muſt alſo be added 3,38 toiſes, becauſe the points T and Q are not in the ſame meridian, the ſum of theſe is 80,9, this added to 54942,57 gives 55023,47 the length of an arc of $57'\,28'',65$; hence, $57'.\,28'',65 : 1^\circ \quad 55023,47 \quad 57438$ the length of a degree of the meridian at the place of meaſurement. But we muſt ſubtract 16 toiſes from this, on account of refraction, which MAUPERTUIS neglected, hence, the length of the degree becomes 57422 toiſes. The latitude of the middle of QM was $66^\circ.\,20'$. And by comparing the length of this degree with the length 57183 toiſes of a degree meaſured by M. PICARD between Paris and Amiens, the latitude of the middle of which was $49^\circ.\,22'$, the earth was found (995) to be an *oblate* ſpheroid, and the ratio of the diameters 178 179.

994 In taking the angular diſtance of two objects upon the earth with a quadrant*, if they be at any altitude above or below the ſurface, that angular diſtance muſt be reduced to the horizon, if we want it for the purpoſe of carrying on a meaſurement upon the earth's ſurface, that reduction may be thus made. Let Z be the zenith, MN the horizon, A and B the two objects. Find their altitudes MA, NB, and alſo their diſtance AB, then in the triangle ZAB, we know all the ſides, to find the angle Z, or the arc MN, which is the angular diſtance of the two objects reduced to the horizon. We avoid this reduction however, when we obſerve with a theodolite having two teleſcopes which move vertically on an horizontal axis. FIG 223.

995. To

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* For this purpoſe, the quadrant muſt be fixed upon a center, ſo that it may be put into any poſition.

Fig. 218.

995. To find the ratio of the diameters, from the lengths of two degrees in two known latitudes, let Pp be the earth's axis, $EPDp$ any meridian, E the equator, M any place, draw the tangent Mt meeting OE in t, and DOK parallel to it, also Mv perpendicular to Mt, and Mr to OE, and let Mx be the radius of curvature to M. Now by Conics, $Mx = \frac{OK^2}{Ma}$, but $Ma = \frac{OP^2}{Mv}$, and $OK^2 = \frac{PO^2 \times OE^2}{Ma^2}$, hence, $Mx = \frac{OE^2 \times Mv^3}{OP^4}$, but a degree of latitude muſt vary as Mx, and conſequently as Mv^3. Now $Rt = \frac{OE^2}{OP^2} \times \frac{MR^2}{OR}$, and $Rt = \frac{MR^2}{vR}$, hence, $vR = \frac{OP^2}{OE^2} \times OR$. But $OR^2 = \frac{OE^2}{OP^2} \times \overline{PO^2 - MR^2}$, hence, $vR^2 = \frac{OP^4}{OE^2} - \frac{OP^2 \times MR^2}{OE^2}$, conſequently $Mv^2 = \frac{OP^4}{OE^2} - \frac{OP^2 \times MR^2}{OE^2} + MR^2$, but if $T =$ the ſine of tMR, or of latitude, $MR^2 = Mv^2 \times T^2$, by ſubſtitution therefore, we get the value of $Mv^2 = \frac{OP^4}{OE^2 \times \overline{1 - T^2} + OP^2 \times T^2}$, hence, a degree of latitude, or Mv^3, varies inverſely as $\overline{OE^2 \times \overline{1 - T^2} + OP^2 \times T^2}^{\frac{3}{2}}$, or $\overline{OE^2 - \overline{OE^2 - OP^2} \times T^2}^{\frac{3}{2}}$. But if the ſpheroid be very nearly a ſphere, and $OP : OE :: 1 : 1 + d$, then Mv^3 varies inverſely as $\overline{1 - 2dT^2}^{\frac{3}{2}}$, or inverſely as $1 - 3dT^2$ very nearly, d being very ſmall. Hence, if T, t be the ſines of any two latitudes, and m and n repreſent the lengths of a degree of each reſpectively, we have $m : n :: 1 - 3dt^2 : 1 - 3dT^2$, conſequently $d = \frac{m - n}{3mT^2 - 3nt^2}$. Hence, the ratio of the diameters is as $\frac{3mT^2 - 3nt^2}{m - n} : \frac{3mT^2 - 3nt^2}{m - n} + 1$.

996. Now the length of a degree in different latitudes is, according to

		Toiſes.
MAUPERTUIS	in latitude 66° 28′	57422
CASSINI and de la CAILLE	49. 23	57069
	45. 0	57028
BOSCOVICH	43. 0	56979
JUAN and ULLOA	at the equator	56768
BOUGUER		56753
De la CONDAMINE		56750
MASON and DIXON	39. 12	56888

Hence, we get the following ratios of the diameters of the earth,

Lat. 66° 20′ and 49° 23′ . . . . . give	130	131, or 200	201,54.
66. 20 and 45. 0 . . . . . give	150	151, or 200 .	201,33.
66. 20 and 43. 0 . . . . . give	147	148, or 200	201,36
66. 20 and 39. 12 . . . . give	140	141, or 200	201,43
66. 20 and at equat by BOUGUER	216	217, or 200	200,92
49. 23 and at equat. by BOUGUER	312	313, or 200	200,64
39. 12 and at equat by BOUGUER	370 .	371, or 200	200,54
66. 20 and mean at the equator	217	218, or 200	200,92.
45. 0 and 49°. 23′ . . give	321	322 or 200	200,62.
45. 0 and 39. 12 . . . give	112	113, or 200	201,78
45. 0 and at equat by BOUGUER	311	312, or 200	200,64.
43 0 and 39°. 12′ . . . . . give	109 .	110, or 200 .	201,83.

997 The mean of all the conſequents of the laſt ratio is 201,13; hence, the ratio of the diameters from the mean of theſe twelve comparisons is 200 : 201,13, or reduced to that ratio, the difference of whoſe terms is unity, it is 177 178, which is (993) extremely near to the ratio deduced from the meaſurement at the polar circle and in France. But the great difference of the reſults from the different comparisons, ſhow that we cannot depend upon the accuracy of the mean ratio. Indeed, other authors have deduced a mean ratio from menſuration, agreeing very nearly with Sir I. NEWTON.

998. In the year 1738, when M. BOUGUER was at Peru, meaſuring a degree of longitude, it occurred to him to put the Newtonian theory of gravity to the teſt, by examining the attraction of mountains. This he communicated to his colleague M de la CONDAMINE, and they made the trial upon the mountain Chimboraco, the attraction of which they judged would be about the 2000th part of the attraction of the whole earth, and therefore they concluded that a plumb line would be drawn out of it's vertical ſituation through an angle of 1′. 43″ towards the mountain; whereas it amounted only to 7½″. But the experiments were made under ſo many diſadvantages, that no great dependance can be placed upon the accuracy of the reſult. This ſatisfied them however that the mountain had an attraction, although it was much leſs than what was expected from it's bulk. But it appeared, that this mountain had once been a volcano, and therefore was probably hollow in many places. M. BOUGUER concludes his account thus "that as in France or in England, a hill may be found of ſufficient height for the purpoſe, and eſpecially if the obſerver would double the action, by making a ſtation on each ſide, he ſhould be happy to hear on his return to Europe, that the experiment had been repeated,

whether the reſult tended to confirm his obſervations, or to throw ſome better light upon that enquiry." Accordingly, the Royal Society requeſted Dr. MASKELYNE to undertake the buſineſs, who repeated the experiments upon Schehallien in Scotland, with an excellent zenith ſector, made by Mr. SISSON: his MAJESTY very liberally undertaking to defray the expenſes. From obſervations of ten ſtars near the zenith, he found the difference of latitudes of the two ſtations on the oppoſite ſides of the mountain, to be 54″,6, and by a meaſurement by triangles, he found the diſtance of the two parallels to be 4364,4 feet, anſwering, in that latitude, to an arc of the meridian of 42″,94, which is 11″,6 leſs than by obſervation, it's half therefore, 5″,8, is the effect of the attraction of the mountain, and from it's magnitude, compared with the bulk of the whole earth, Dr MASKELYNE diſcovered the mean denſity of the earth to be about double that of the mountain Thus, the doctrine of *Univerſal Gravitation*[*] is firmly eſtabliſhed. Dr. HUTTON in the *Phil Tranſ.* for 1778, has

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[*] Dr MASKELYNE deduced the following conſequences:

1 It appears from this experiment, that the mountain Schehallien exerts a ſenſible attraction, therefore, from the rules of philoſophiſing, we are to conclude, that every mountain, and indeed every particle of the earth, is endued with the ſame property, in proportion to it's quantity of matter

2 The law of the variation of this force, in the inverſe *ratio* of the ſquares of the diſtances, as laid down by Sir I NEWTON, is alſo confirmed by this experiment For, if the force of attraction of the hill had been only to that of the earth, as the matter in the hill to that of the earth, and had not been greatly increaſed by the near approach to it's center, the attraction thereof muſt have been wholly inſenſible But now, by only ſuppoſing the mean denſity of the earth to be double to that of the hill, which ſeems very probable from other conſiderations, the attraction of the hill will be reconciled to the general law of the variation of attraction in the inverſe duplicate *ratio* of the diſtances, as deduced by Sir I NEWTON from the compariſon of the motion of the heavenly bodies with the force of gravity at the ſurface of the earth, and the analogy of nature will be preſerved

3 We may now, therefore, be allowed to admit this law, and to acknowledge, that the mean denſity of the earth is at leaſt double of that at the ſurface, and conſequently, that the denſity of the internal parts of the earth is much greater than near the ſurface. Hence alſo, the whole quantity of matter in the earth will be at leaſt as great again as if it had been all compoſed of matter of the ſame denſity with that at the ſurface, or will be about four or five times as great as if it were all compoſed of water The idea thus afforded us, from this experiment, of the great denſity of the internal parts of the earth, is totally contrary to the hypotheſis of ſome naturaliſts, who ſuppoſe the earth to be only a great hollow ſhell of matter, ſupporting itſelf from the property of an arch, with an immenſe vacuity in the midſt of it But, were that the caſe, the attraction of mountains, and even ſmaller inequalities in the earth's ſurface, would be very great, contrary to experiment, and would affect the meaſures of the degrees of the meridian much more than we find they do, and the variation of gravity in different latitudes in going from the equator to the poles, as found by pendulums, would not be near ſo regular as it has been found by experiment to be,

4. The

has calculated the attraction of this mountain from the obſervations of Dr MASKELYNE, and found that the denſity of the mountain was to the mean denſity of the earth as 5 : 9; now the denſity of the mountain was found to be to the denſity of rain water as $2\frac{1}{2} : 1$, hence, the mean denſity of the earth is to the denſity of rain water as $4\frac{1}{2} : 1$. The internal parts of the earth are therefore much denſer than thoſe at the ſurface, but in what manner the denſe parts are diſpoſed muſt be uncertain. If we ſuppoſe the earth at firſt to have been in a fluid ſtate, and the different parts to have taken their places according to their gravity, the central parts muſt be the moſt denſe, the effects of which upon the ratio of the diameters of the earth we ſhall afterwards ſtate.

999. The vibration of pendulums upon different parts of the earth have been uſed as a means to determine the ratio of it's diameters; for this purpoſe, we muſt find the force of gravity upon different parts of it's ſurface. To inveſtigate this, let Pp be the polar, and $E\mathcal{Q}$ the equatorial diameter. By Art. 976 the attraction at M perpendicular to the ſurface varies as $\frac{1}{MO}$. But if $1=$ the mean radius of the earth, and $1+e=EO$, then (882) $MO=1+e\times$ coſ. $2MOE$, hence, $\frac{1}{MO}=\frac{1}{1+e\times \text{coſ. } 2MOE}=1-e\times$ coſ. $2MOE=1-e$ $+e\times$ ver. ſin. $2MOE$, therefore the increaſe of attraction from the equator to the poles varies as the verſed ſine of double the latitude very nearly, or as the ſquare of the ſine of the latitude; which is the ſame ratio as that by which the degrees of latitude increaſe.

1000. If the time of vibration of a pendulum be given, the length varies as the gravity, and conſequently (976) inverſely as MO. Hence, the length of a pendulum vibrating ſeconds, increaſes as it is carried towards the poles. If therefore the length of a pendulum vibrating ſeconds in two latitudes could be accurately aſcertained, we might aſcertain the ratio of the diameters of the earth, the denſity of the earth being ſuppoſed to be uniform. Now it is found by obſervations, that the length of a pendulum vibrating ſeconds increaſes from the equator towards the poles, agreeable to what ought to take place according to our theory; but if we deduce the ratio of the diameters of the earth

from

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[4] The denſity of the ſuperficial parts of the earth, being, however, ſufficient to produce ſenſible deflections in the plumb-lines of aſtronomical inſtruments, will thereby cauſe apparent inequalities in the menſurations of degrees in the meridian, and therefore it becomes a matter of great importance to chuſe thoſe places for meaſuring degrees, where the irregular attractions of the elevated parts may be ſmall, or in ſome meaſure compenſate one another, or elſe it will be neceſſary to make allowance for their effects, which cannot but be a work of great difficulty, and perhaps liable to great uncertainty.

A Table of the Lengths of a Pendulum, in French Lines, vibrating Seconds upon the Surface of the Earth, from the Observations made at Peru, Paris, and Spitsbergen.

Latitude	Length of the Pendulum	Latitude.	Length of the Pendulum.	Latitude.	Length of the Pendulum.
Degrees	Lines.	Degrees	Lines.	Degrees	Lines.
0	439,07	30	439,72	60	440,92
1	439,07	31	439,76	61	440,95
2	439,08	32	439,80	62	440,97
3	439,08	33	439,84	63	441,01
4	439,09	34	439,87	64	441,04
5	439,09	35	439,91	65	441,07
6	439,11	36	439,95	66	441,09
7	439,12	37	440,00	67	441,12
8	439,13	38	440,04	68	441,15
9	439,14	39	440,08	69	441,18
10	439,15	40	440,13	70	441,20
11	439,16	41	440,17	71	441,22
12	439,18	42	440,22	72	441,24
13	439,20	43	440,27	73	441,26
14	439,22	44	440,31	74	441,29
15	439,24	45	440,35	75	441,31
16	439,27	46	440,40	76	441,33
17	439,30	47	440,45	77	441,35
18	439,32	48	440,49	78	441,36
19	439,35	49	440,54	79	441,37
20	439,38	50	440,58	80	441,38
21	439,41	51	440,62	81	441,39
22	439,44	52	440,65	82	441,40
23	439,47	53	440,68	83	441,41
24	439,50	54	440,71	84	441,42
25	439,53	55	440,75	85	441,43
26	439,56	56	440,79	86	441,43
27	439,59	57	440,82	87	441,44
28	439,63	58	440,85	88	441,44
29	439,67	59	440,88	89	441,44
30	439,72	60	440,92	90	441,45

This Table is computed upon ſuppoſition that the increaſe of the length of the pendulum is as the ſquare of the ſine of the latitude; for when the time of vibration is the ſame, the lengths vary as the forces, therefore the variation of the lengths vary as the variation of the forces, or (999) as the ſquare of the ſine of latitude.

1007. The ratio of the diameters of *Jupiter* is leſs (982) than that given by theory, upon ſuppoſition that it is homogeneous, which M. Clairaut ſhows may happen, if Jupiter have a nucleus denſer than the other part of the planet, with a certain elliptic form (1002). It is therefore unneceſſary to ſuppoſe, with Sir I. Newton, that Jupiter is more denſe towards it's equator, and which (he thinks) may ariſe from the heat of the ſun at thoſe parts But this is much more likely to be the caſe with our earth than with Jupiter, and yet Sir Isaac thinks that the earth is denſer towards the center. He appears to have been led into his conjectures, from thinking, that an increaſe of flatneſs of the body muſt be attended with a greater increaſe of weight in going from the equator to the poles, than if the body had been homogeneous, which (1003) is not neceſſarily the caſe. The greater denſity of the internal parts of the earth, and the variation of gravity upon it's ſurface, ſeem to favour the ſuppoſition that the difference of the diameters is leſs than that which Sir I. Newton has determined. M. de la Lande aſſumes the difference to be $\frac{1}{300}$ part of the whole.

1008. The horizontal parallax of the moon is as the radius of the earth directly, and the diſtance of the moon from the center of the earth inverſely. The diſtance of the moon therefore being known, if we know alſo it's horizontal parallax, the radius of the earth will be known. If therefore two radii of the earth in two known latitudes be thus determined, the figure of the earth may be found But obſervations of ſufficient accuracy to ſettle this matter, have never been made It has alſo been propoſed to find the ratio of the diameters of the earth from ſolar eclipſes; as the computation of the parallax of the moon, and conſequently the times of the beginning and end of ſuch an eclipſe, will vary according as the ratio of the diameters of the earth vary. M. de la Lande thinks that a difference of $\frac{1}{300}$ of the diameters will make ſuch computations beſt agree with obſervations. From a conſideration of all the circumſtances, it is probable that the difference of the polar and equatorial diameters of the earth, is leſs than that which is determined by Sir I Newton.

1009. The length of a degree of latitude at the equator, taking the mean of three meaſures, is 56756$\frac{2}{3}$ toiſes, which multiplied by 6 gives 340540 French feet, and as a Paris foot : an Engliſh foot :: 4,263 : 4, a degree at the equator in Engliſh feet is 362930,5, hence, the circumference, correſponding to this arc of 1°, is 130654980, the radius of which is 3938,334 miles, which we may conſider as the radius of curvature at the equator. Aſſume 230x and 231x for the polar and equatorial radii; then, by Conics, $\frac{\overline{230x}^2}{231x} = 3938,334$, hence, $230x = \frac{3938,334 \times 231}{230} = 3955,4$ miles the polar radius, and $231x = 3972,5$ the equatorial radius, and the difference of the two radii $= 17,1$ miles, from this ratio of the diameters; alſo, the mean radius $= 3963,95$ which we may take 3964. Now the circumference correſponding to this mean radius is 24907, conſequently the length of a degree correſponding to this mean radius, or a mean degree, will be 69,2 miles.

1010. By Art. 995 the length of a degree varies inverſely as $1 - 3dT^2$, now at the equator $T = 0$, hence, the length of a degree at the equator : length in any other latitude :: $1 - 3dT^2$: 1, thus the length of a degree for any latitude may be computed. Hence alſo, the increment of a degree from the equator to the pole will increaſe as $3dT^2$, or as T^2 the ſquare of the ſine of latitude. And as the length of a degree muſt be in proportion to the radius of curvature, the variation of the radius of curvature of an ellipſe which is very nearly a circle, muſt be as the ſquare of the ſine of latitude.

Fig. 218. 1011. To find the angle OMv between a perpendicular Mv to the ſurface and MO drawn to the center. Let $PO = 1$, $OE = 1 + d$, then $OE^2 = 1 + 2d$; put $OR = x$, and draw vc perpendicular to MO. Now $Rv = x \times \frac{1}{1+2d} = x \times \overline{1-2d}$, therefore $Ov = 2dx = 2d \times$ coſ. lat. nearly, but $cv = Ov \times$ ſin. vOc $= Ov \times$ ſin. lat. nearly, therefore $cv = 2d \times$ coſ. lat. $\times$ ſin. lat. nearly $= d \times$ ſin. 2 lat. nearly, and this is the ſine of vMO to radius Mv which we may conſider here as unity. This angle OMv is the reduction of the elevation of the pole in Art. 173.

1012. The earth being ſuppoſed to be a ſphere, the length of a degree of longitude, as you go from the equator to the poles, decreaſes as the arcs of the circles parallel to the equator intercepted between any two meridians decreaſe, which arcs are as their radii, or as the coſines of latitude, therefore, radius : coſ. of latitude :: the length of a degree of longitude at the equator : the

the length of a degree of longitude at that latitude. But as the earth is a ſpheroid, this rule will want a little correction. Let POp be the axis of the earth, $EPUp$ a meridian, EOU a diameter of the equator, then the latitude of M is MvE, and the angle vMO is given in Art 1011. therefore $MvE - vMO = MOE$ is known, hence, we get QOM. But (882) if unity repreſent the mean radius of the earth, and e = the difference between the mean and the greateſt or leaſt radii, then will $1 + e$ coſ $2MOE = MO$, hence, knowing QOM and MO, we find QM, and this we muſt uſe inſtead of the coſine of the latitude, in order to find the length of a degree of longitude upon the ſurface of the earth. The length of a degree therefore being known for one latitude, the length for every other latitude may be found. Hence, the calculations for the following Table may be made by this Article and Article 1010

A Table

1013. The figure of the earth is ſo near that of a ſphere, that in eſtimating it's magnitude, we may conſider it as a ſphere whoſe radius is the mean radius of the earth. Now this mean radius is 3964 miles, hence, $3964 \times 6,28318 = 24907$ miles the circumference, alſo, $\overline{3964}^2 \times 3,14159265 \times 4 = 197459101$ the number of ſquare miles upon the earth's ſurface, laſtly, $\overline{3964}^3 \times 4,188790204785 = 260909292265$ the number of cubic miles contained in the earth. Dr. LONG eſtimated the proportion of the land and water upon the ſurface of the earth, ſo far as diſcoveries had then been made, in the following manner He took the paper off a terreſtrial globe, and then cut out the land from the ſea, and weighed the two parts; by this means he found the proportion of the water to the land as 349 : 124. The concluſion would be more accurate, if the land were cut out from the ſea *before* the paper was put upon the globe. After all the modern diſcoveries, this method would probably give the proportion of land to water, to a conſiderable degree of accuracy.

* * *

CHAP. XXXIV.

ON THE PRECESSION OF THE EQUINOXES, AND THE NUTATION OF THE EARTH'S AXIS.

Art. 1014. IT has already been obſerved (148), that the equinoctial points have a retrograde motion of about $50\frac{1}{3}''$ in a year. Sir I. NEWTON was the firſt who accounted for this motion. Having proved that, from the centrifugal force of the parts of the earth ariſing from it's rotation, the equatorial diameter muſt be greater than the polar, he proceeded to ſhow, that if we conceive a ſphere to be inſcribed in the earth, the attraction of the ſun and moon upon the exceſs of the quantity of matter in the earth above that of the ſphere will cauſe a motion in the plane of the equator, and make the points where it interſects the ecliptic go backwards upon it. But although he aſſigned the true cauſe of the preceſſion, it is acknowledged that he fell into an error in his inveſtigation of the effect. Without, however, any inquiry relative to the circumſtances in which he has erred, I ſhall ſhow how we may obtain a true ſolution from the common principles of motion.

1015. Let S be the ſun, $ABDC$ the earth, T it's center, EQ the equator, P, p the poles, draw CTB perpendicular to SAD, and join SE, which produce to meet CB in K. Call the radius TE unity, and let the force of the ſun on a particle at T be $\frac{1}{ST^2}$, then the force on a particle at $E = \frac{1}{SE^2}$; hence, if we reſolve this latter force in two others, one in the direction ET and the other in a direction parallel to TS, we have $SE : ST :: \frac{1}{SE^2}$: the force in the direction parallel to $TS = \frac{ST}{SE^3} = \frac{ST}{\overline{ST-EK}^3} = \frac{1}{ST^2} + \frac{3EK}{ST^3}$, omitting the other terms of the ſeries on account of their ſmallneſs. Hence, the force with which a particle at E is drawn *from* $CB = \frac{3EK}{ST^3}$, conſequently the effect of this force in a direction perpendicular to ET will be $\frac{3EK \times KT}{ST^3}$, hence, this force : the force of the ſun on a particle at $T :: \frac{3EK \times KT}{ST^3} : \frac{1}{ST} :: 3EK \times KT : ST$

FIG. 224.

Now

Now if P = the periodic time of the earth, p = the periodic time of a body revolving at the earth's ſurface, then (858) the force of the earth to the ſun : force of the body to the earth, or the force of gravity, : $\frac{ST}{P^2} . \frac{1}{p^2}$, hence, the force on a particle at E perpendicular to ET : force of gravity : $\frac{3EK \times KT \times p^2}{P^2} : 1$.

1016. Let v be the center of gyration, and put M = the quantity of matter in the earth, then the effect of the inertia of M placed at v to oppoſe the communication of motion is the ſame as the effect of the inertia of the earth; and hence, $TE^2 : Tv^2 (= \frac{2}{5} TE^2) . M : \frac{2}{5} M$, which is the quantity of matter to be placed at E to have the ſame effect.

FIG. 225. 1017. Let $PEpQ$ repreſent the earth, P, p it's poles, EQ the equator, $Pepq$ a ſphere whoſe diameter is the axis PTp, TA a radius directed to the ſun, and CTB a plane perpendicular to it, PXp a great circle perpendicular to $PEApQ$, and let IR repreſent a ſmall circle parallel to the equator, take the arc $XL = Xl$, and draw LM, lm, XY perpendicular to the plane CTB. Now (847) the diſturbing force of the ſun at L, l, in the directions ML, ml are as ML, ml, conſequently theſe forces are the ſame as they would be if the corpuſcles at L, l were orthographically projected upon the plane $CABD$; let us therefore conceive the whole matter in the earth to be thus projected. Draw XN, ln parallel to BC, put $p = 3{,}14159$ &c. a = the mean radius of the earth, $Ee = m$, $r = IX$ on the projection, $TX = v$, s = ſin ATE, c = coſ ATE, the arc XL, or $Xl = z$, and y = ſin XL, then (in the projection) $ln = NX = sy$, $Xn = NL = cy$, $XY = sv$ and $TY = cv$, therefore $LM = sv + cy$, $lm = sv - cy$, $TM = cv - sy$, and $Tm = cv + sy$. Now the forces at L and l in the directions ML, ml, being as ML, ml, their effects to turn the earth about T in the direction BAC are as $ML \times MT$ and $ml \times mT$, or the whole effects are as $\overline{sv+cy} \times \overline{cv-sy} + \overline{sv-cy} \times \overline{cv+sy} = 2cs \times \overline{v^2 - y^2}$, therefore the fluxion of the force of all the matter in the circumference IR is as $2cs\dot{z} \times \overline{v^2 - y^2}$, hence, the fluxion of the force to turn the earth in the direction CAB is as $2cs\dot{z} \times \overline{y^2 - v^2} = 2cs \times \frac{ry^2\dot{y}}{\sqrt{r^2 - y^2}} - 2csv\dot{z}$, whoſe fluent, when $y = r$, is $\frac{1}{2} pr \times cs \times \overline{r^2 - 2v^2}$ the force upon the ſemicircumference IR, hence, the force on that whole circumference $= pr \times cs \times \overline{r^2 - 2v^2} =$ (as $r^2 = a^2 - v^2$) $pr \times cs \times \overline{a^2 - 3v^2}$. Now $a : r . m : Ir = \frac{mr}{a}$, hence, the fluxion of the force of the annulus $IreE$ is as $pr \times \frac{mr}{a} \times cs\dot{v} \times \overline{a^2 - 3v^2} = \frac{pmcs}{a} \times \dot{v} \times \overline{a^2 - v^2} \times \overline{a^2 - 3v^2} = \frac{pmcs}{a} \times$

$a^4\dot{v}$

$\overline{a^4\dot{v}-4a^2v^2\dot{v}+3v^4\dot{v}}$, whoſe fluent, when $v=a$, is $\frac{4pmcs}{15}\times a^4$, which is as the whole force on the matter exterior to the ſphere $Pepq$, on one ſide of $E♎$, hence, $\frac{8pmcs}{15}\times a^4$ is as the whole force of the ſun upon the earth to turn it about in the direction CAB. Now $a+\frac{1}{2}m=TE$, $a-\frac{1}{2}m=Te$; hence, the ſolid content of the ſpheroid $=\frac{4}{3}p\times\overline{a-\frac{1}{2}m}\times\overline{a+\frac{1}{2}m}^2=\frac{4}{3}pa^3+\frac{2}{3}pma^2$ very nearly; and the content of the ſphere $=\frac{4}{3}p\times\overline{a-\frac{1}{2}m}^3=\frac{4}{3}pa^3-2pma^2$ very nearly, the difference of theſe is $\frac{8}{3}pma^2$ the content of the part exterior to the ſphere, place one fifth of this matter, that is, $\frac{8}{15}pma^2$, at E, then as $EK=ca$, and $KT=sa$, the effect of the ſun to turn the matter $\frac{8}{15}pma^2$ at E about $T=\frac{8}{15}pmcsa^4$, which is equal to the effect of the ſun upon the whole earth to turn it about it's center. Hence, the effect of the ſun upon the matter of the earth exterior to the ſphere to turn it about it's center, is equal to the effect which would be produced if one fifth part of that matter were placed at E.

1018. Put $q=$the quantity of matter in the earth above that of it's inſcribed ſphere, now (1017) the attraction upon the matter exterior to the ſphere would generate an angular velocity about an axis perpendicular to $CABD$, equal to the angular velocity which would be generated in a quantity of matter $=\frac{1}{5}q$ placed at E. Let us therefore ſuppoſe the ſun's attraction perpendicular to ET to be exerted upon a quantity of matter at $E=$to $\frac{1}{5}q$, and at the ſame time to have a quantity of matter to move $=\frac{2}{5}M$, and then (1016, 1017) it appears, that the effect will be the ſame as the accelerative force of the ſun to turn about the earth. Hence, that accelerative force is (1015) equal to $\frac{3EK\times KT\times p^2\times\frac{1}{5}q}{\frac{2}{5}M\times P^2}=\frac{3EK\times KT\times p^2\times q}{2M\times P^2}$. Now if $TE:TP::1:1-r$, then $M:M-q::1:1-2r$, therefore $M:q::1:2r$, hence, $\frac{q}{2M}=r$, conſequently the accelerative force$=\frac{3EK\times KT\times p^2\times r}{P^2}$, the force of gravity on the earth being unity.

1019. Let $z=$the arc deſcribed by a point of the equator about it's axis in an indefinitely ſmall given time, which may therefore repreſent it's velocity; and let az repreſent the arc deſcribed in the ſame time by a body revolving about the earth at it's ſurface, then $\frac{a^2\dot{z}^2}{2}=$the ſagitta of the arc deſcribed by

the

the body in the ſame time, and conſequently $a^2\dot{z}^2$ = the velocity generated by gravity whilſt a point of the equator deſcribes $\dot{z}$. Hence (1018), 1 : $\frac{3EK \times KT \times p^2 \times r}{P^2}$ :: $a^2\dot{z}^2$: $\frac{3EK \times KT \times p^2 \times r \times a^2\dot{z}^2}{P^2}$ the velocity of the point E perpendicular to ET, generated by the action of the ſun whilſt the equator deſcribes $\dot{z}$ about it's axis, conſequently the ratio of theſe velocities is as $\frac{3EK \times KT \times p^2 \times r \times a^2\dot{z}}{P^2}$: 1.

1020 Let y be an arc deſcribed by the ſun in the ecliptic to a radius equal to unity, whilſt a point of the equator deſcribes $\dot{z}$ about it's axis, then (as ap = the time of the earth's rotation, and the arcs deſcribed in equal times to equal radii are inverſely as the periodic times,) $\frac{1}{P} : \frac{1}{ap} :: y : \dot{z} = \frac{Py}{ap}$, hence, if v and w be put for the ſine and coſine of the ſun's declination, the ratio of the velocities in the laſt Article becomes $\frac{3aprvwy}{P}$: 1.

FIG. 226

1021 Hence, if SAL be the ecliptic to the radius unity, P the place of the ſun, SBL the equator, PE the ſun's declination, and we take $Ec : dc$ (dc being perpendicular to Ec) : 1 : $\frac{3aprvwy}{P}$, and through d, E, deſcribe the great circle TEM, then will ST be the preceſſion of the equinox, during the time the ſun deſcribes y in the ecliptic, hence, Ed, or Ec, or 1, : dc, or $\frac{3aprvwy}{P}$, :: ſin SE : $SV = \frac{3aprvw \times \text{ſin } SE \times y}{P}$, therefore the ſin. STV, or ESP, : 1 : SV : $ST = \frac{3aprvw \times \text{ſin } SE \times y}{P \times \text{ſin. } ESP}$.

1022 Now $\frac{v}{\text{ſin } ESP}$ = ſin. SP, and $w = \frac{\text{coſ. } SP}{\text{coſ. } ES}$, hence, $\frac{vw}{\text{ſin. } ESP} = \frac{\text{ſin } SP \times \text{coſ. } SP}{\text{coſ } ES}$, but $\frac{\text{coſ. } ESP}{\text{tan } ES \times \text{cot } SP} = 1$, hence, $\frac{vw}{\text{ſin. } ESP} = \frac{\text{ſin } SP \times \text{coſ } SP \times \text{coſ } ESP}{\text{coſ } ES \times \text{tan } ES \times \text{cot. } SP} = \frac{\overline{\text{ſin } SP}^2 \times \text{coſ } ESP}{\text{ſin } ES}$, conſequently $ST = \frac{3apr \times \overline{\text{ſin } SP}^2 \times \text{coſ. } ESP \times y}{P}$ = (if x = ſine of SP) $\frac{3apr \times \text{coſ } ESP \times x^2 x}{P \times \sqrt{1 - x^2}}$, whoſe fluent is $\frac{3apr \times \text{coſ. } ESP}{2P} \times \overline{y - x\sqrt{1 - x^2}}$, and when $x = 1$, it becomes $\frac{3apr \times \text{coſ. } ESP \times m}{2P}$ (y being now $= m$ a quadrant) the arc of preceſſion whilſt the ſun deſcribes 90° from the equinox; and to find the degrees, ſay $4m$: 360°

$360° :: \frac{3apr \times \text{coſ } ESP \times m}{2P} : 360° \times \frac{3apr \times \text{coſ } ESP}{8P}$, conſequently the preceſſion in a year $= 360° \times \frac{3apr \times \text{coſ. } ESP}{2P} = 21'' \; 6'''$. This would be the preceſſion of the equinox ariſing from the attraction of the ſun, if the earth were ſolid of an uniform denſity, and the ratio of the diameters as 229 230; but, from what follows, if the greateſt nutation of the earth's axis be rightly aſcertained, the preceſſion is only about $14\frac{1}{2}''$, which difference between the theory and what is deduced from obſervation, muſt ariſe, either from the fluidity of the earth's ſurface, an increaſe of denſity towards the center, or the ratio of the diameters being different from that which is here aſſumed, or probably from all the cauſes conjointly. This regreſſion of the equinoxes (cauſed by the plane of the equator moving backwards upon the ecliptic) muſt neceſſarily cauſe the poles of the earth to deſcribe circles about the poles of the ecliptic, in a direction contrary to the order of the ſigns, ſetting aſide the effect of nutation.

1023 Having aſcertained the preceſſion, the correſponding nutation may be immediately found thus Take $SB = SA = 90°$, and draw the great circle BbA, then BA is the meaſure of the angle BSA (12), and as we may conſider Tb and TA to be each equal to 90° without any ſenſible error, Ab will be the meaſure of the angle bTA, hence, Bb will meaſure the difference of the angles BSA, bTA, or the variation of the inclination of the equator to the ecliptic, or the nutation of the axis of the equator Now SV Bb . ſin. SE . ſin. BE, or coſ SE, :. tan. SE rad ; alſo, ST . SV rad. ſin. T, hence, $ST : Bb$: tan SE ſin. T, but tan. $SE = \text{coſ } T \times \text{tan. } TP$, therefore $Bb = \frac{ST \times \text{ſin. } T}{\text{coſ } T \times \text{tan. } TP}$. But $ST = \frac{3apr \times \text{coſ } ESP \times x^2 \dot{x}}{P \times \sqrt{1-x^2}}$, and x being the ſine of TP, it's tangent $= \frac{x}{\sqrt{1-x^2}}$, hence, (conſidering the angles ESP and T as equal) $Bb = \frac{3apr \times \text{ſin } ESP \times x\dot{x}}{P}$, whoſe fluent is $\frac{3apr \times \text{ſin } ESP \times x^2}{2P}$ the arc of nutation whilſt the ſun deſcribes SP, and if m = an arc of 90° of the ecliptic from Aries, we have, $4m$ $360°$ $\frac{3apr \times \text{ſin } ESP \times x^2}{2P}$ $360° \times \frac{3apr \times \text{ſin } ESP \times x^2}{8Pm}$ the angle of nutation; and when $x = 1$, we have, $360° \times \frac{3apr \times \text{ſin } ESP}{8Pm}$ for the whole nutation whilſt the ſun moves from the equinox to the tropic. Whilſt the ſun moves from the equinox to the tropic, BA is greater than bA, and therefore the inclination of the equator to the ecliptic decreaſes, but from the tropic to the equinox, BA is leſs than bA, and therefore

therefore the inclination increaſes. Now the nutation varies as the ſquare of the ſine of the ſun's longitude, and as x increaſes till the ſun comes to the tropic, and then decreaſes again until it comes to the equinox, when it is $=0$, the inclination, from this cauſe, is leaſt when the ſun is at the tropic, and greateſt when the ſun comes to the equinox, and is then the ſame as at the preceding equinox.

1024. Hence, the preceſſion during the ſun's motion from the equinox to the tropic : the nutation at any time :: coſ ESP . $\frac{\text{ſin. } ESP \times x^2}{m}$ :: m : tan $ESP \times x^2$, and at the tropic, this ratio becomes m . tan ESP, or as 1,5708 : ,4341. Hence, if we take the whole preceſſion for the time the ſun is moving from the equinox to the tropic to be one fourth of 14½″, the nutation at the tropic $= 1''$, which is the greateſt nutation ariſing from the force of the ſun, x at the tropic being the greateſt. Hence, the nutation from the time the ſun leaves the equinox $= 1'' \times x^2 = \frac{1}{2}'' - \frac{1}{2}'' \times$ coſ $2y$, and at 45° from the equinoxes, the inclination is the mean, it varying half a ſecond each way from thence. Hence, at the equinoxes, the nutation $= \frac{1}{2}''$; and at the tropic, the nutation $= -\frac{1}{2}''$ from the mean inclination, and the nutation at any time from the mean inclination $= \frac{1}{2}''$ coſ. $2y$.

1025. To find the equation of the preceſſion, we have $m : y ::$ ¼ of 14½″ : $14\frac{1}{2}'' \times \frac{y}{4m}$ = the *mean* preceſſion correſponding to the longitude y, upon ſuppoſition that the annual preceſſion ariſing from the force of the ſun is 14½″. Alſo (1022), the *mean* preceſſion correſponding to the longitude m is $\frac{3apr \times \text{coſ } ESP \times m}{2P}$, hence, $m : y :: \frac{3apr \times \text{coſ } ESP \times m}{2P} : \frac{3apr \times \text{coſ } ESP \times y}{2P}$ = the *mean* preceſſion correſponding to the longitude y, but the *true* preceſſion in the ſame time is (1022) $\frac{3apr \times \text{coſ } ESP}{2P} \times \overline{y - x\sqrt{1-x^2}}$, from which take the mean preceſſion, and we have $\frac{3apr \times \text{coſ } ESP}{2P} \times -x\sqrt{1-x^2}$ for the equation of preceſſion, which therefore is to the mean preceſſion as $-x\sqrt{1-x^2} : y :: -2x\sqrt{1-x^2} : 2y :: -\text{ſin. } 2y : 2y$. But the mean preceſſion at the ſame time is $14\frac{1}{2}'' \times \frac{y}{4m}$, hence, $2y : -\text{ſin. } 2y :: 14\frac{1}{2}'' \times \frac{y}{4m} : \frac{-14\frac{1}{2}'' \times \text{ſin. } 2y}{8m} = -1''.\ 9''' \times \text{ſin. } 2y$ the equation of the preceſſion. Hence, this equation is greateſt in the middle point between the equinox and tropic, and is there $= 1''.\ 9'''$, and it is to be ſubtracted in the firſt and third quadrants of the ecliptic, and added in the ſecond and fourth.

1026. Let

1026. Let E be the pole of the ecliptic ♈ L, P the mean pole of the equator, about which as a center deſcribe the circle $abcd$ with a radius $=\frac{1}{2}''$, draw $EaPaC$, and make the angle $aPp=$ twice the motion of the ſun in longitude from the equinox, and p will repreſent the true place of the pole, very nearly. Fig. 227.

$$\text{For } Pp=\tfrac{1}{2}'' : Pr :: \text{rad.} : \text{ſin } 2y$$
$$Pr : Cm :: \text{ſin } 23^\circ\ 28' : \text{rad.}$$
$$\therefore Cm=\frac{\frac{1}{2}''\times\text{ſin } 2y}{\text{ſin } 23^\circ\ 28'}=1'.\ 15''\times\text{ſin } 2y$$

Which is (1025) very nearly the equation of preceſſion, if we therefore ſuppoſe the true pole at p, it throws the tropic from C to m, and the equinox as much from ♈ to ♈′, and this is the caſe till $y=90^\circ$; and as y increaſes from 90° to 180°, p lies on the other ſide of EC, and throws the true from the mean equinox on the other ſide of ♈ to ♈″. This agrees with Art. 1025, where the equation was ſhown to be negative in the *former* caſe, denoting thereby the true to lie at ♈′ in reſpect to ♈ the mean equinox, and poſitive in the *latter*, denoting the true to lie at ♈″ in reſpect to ♈ the mean equinox. Alſo, $Pp=\frac{1}{2}'' : pr :: \text{rad}=1 : \text{coſ } 2y$, therefore $pr=\frac{1}{2}''\times\text{coſ. } 2y$ the nutation, which diminiſhes the mean inclination whilſt the ſun paſſes from 45° before the tropic to 45° after, and increaſes it for the other part, agreeable to Art. 1024. Hence, the inequality of the preceſſion, and the nutation, may be repreſented, by ſuppoſing the pole of the equator to deſcribe a circle of 1″ diameter about the mean pole every half year, making the true pole p to ſet off from a at the equinoxes, and to move *in conſequentia* with an angular motion about P which is equal to double the ſun's motion in longitude. The apparent diſtance therefore of every ſtar from the pole of the equator, will be ſubject to a variation of 1″ twice in a year from this cauſe. This motion of the pole of the equator, Dr. MASKELYNE has mentioned in the Preface to his Tables, publiſhed in the firſt Volume of his excellent Obſervations.

On the Preceſſion and Nutation ariſing from the Action of the Moon.

1027. The inequality of the preceſſion of the equinoxes, and the nutation of the earth's axis, ariſing from the attraction of the moon in different ſituations of it's nodes, was diſcovered by Dr. BRADLEY. With his zenith ſector fixed at Wanſtead, he informs us, that as ſoon as he diſcovered the cauſe, and ſettled the laws of the aberration of the fixed ſtars ariſing from the progreſſive motion of light, his attention was excited by another *new phænomenon*, that is, an apparent change of declination in ſome of the fixed ſtars, which ſeemed to be ſenſibly greater about that time, than a preceſſion of 50″ in a year would have occaſioned. In conſequence of this he continued his obſervations, and from 1727 to 1732 he found that ſome of the ſtars near the ſolſtitial colure had changed their declinations 9″ or 10″ *leſs* than a preceſſion of 50″ would have produced, and at the ſame time, that others near the equinoctial colure had altered their's about the ſame quantity *more* than ſuch a preceſſion would have occaſioned, the north pole of the equator ſeeming to have approached the ſtars which come to the meridian with the ſun about the vernal equinox and the winter ſolſtice, and to have receded from thoſe which come to the meridian with the ſun about the autumnal equinox and ſummer ſolſtice. Conſidering theſe circumſtances, and the ſituation of the aſcending node of the moon's orbit at the time when he firſt began his obſervations, he ſuſpected that the moon's action upon the equatorial parts of the earth might produce theſe effects for the plane of the moon's orbit being at one time, above 10° more inclined to the plane of the equator, than at another, it was reaſonable to conclude, that the part of the whole annual preceſſion, which ariſes from the moon, would in different years be varied in it's quantity, whereas, the plane of the ecliptic, wherein the ſun appears, keeping always very nearly the ſame inclination to the equator, *that* part of the preceſſion which is owing to the ſun, muſt be the ſame every year. Hence it would follow, that although the *mean* annual preceſſion, proceeding from the joint actions of the ſun and moon, was 50″, yet the *true* annual preceſſion might ſometimes exceed, and ſometimes fall ſhort, of that mean quantity, according to the various ſituations of the nodes of the moon's orbit. In the year 1727, the moon's aſcending node was near the beginning of *Aries*, and conſequently it's orbit was as much inclined to the equator as it can at any time be, and then the *true* annual preceſſion was found, by his firſt year's obſervations, to be *greater* than the *mean*, and the ſtars near the equinoctial colure, whoſe declinations are moſt affected by preceſſion, had changed *their's* above a tenth part more than a preceſſion of 50″ would have cauſed. The ſucceeding years' obſervations proved the ſame thing, and in three or four years' time the difference became ſo conſiderable, as to leave no room to ſuſpect, that it was owing to any imperfections either in the inſtrument or obſervations.

But ſome of the ſtars which he obſerved, that were near the ſolſtitial colure, having appeared to move, during the ſame time, in a manner contrary to what they ought to have done, by an increaſe in the preceſſion; and the deviations in them being as remarkable as in the others, he perceived that ſomething more than a mere change in the quantity of the preceſſion, would be requiſite

to

to ſolve the *phænomenon*. Upon comparing his obſervations of ſtars near the ſolſtitial colure, that were nearly oppoſite in right aſcenſion, he found that they were equally affected by this cauſe, for whilſt γ *Draconis* appeared to have moved northward, the ſmall ſtar, which is the thirty-fifth *Camelopardali Hevel* in the *Britiſh Catalogue*, ſeemed to have gone as much towards the ſouth, which ſhowed, that this apparent motion, in both theſe ſtars, might proceed from a nutation of the earth's axis. Upon making the like compariſon between the obſervations of other ſtars that lie nearly oppoſite in right aſcenſion, whatever their ſituations were with reſpect to the cardinal points of the equator, it appeared that their change of declination was nearly equal, but contrary, and ſuch as a nutation of the earth's axis would effect.

The moon's aſcending node being got back towards the beginning of *Capricorn* in the year 1732, the ſtars near the equinoctial colure appeared, about that time, to change their declinations no more than a preceſſion of 50″ required, whilſt ſome of thoſe near the ſolſtitial colure altered their's above 2″ in a year leſs than they ought. Soon after, he perceived the annual change of declination of the former to be diminiſhed, ſo as to become *leſs* than 50″ of preceſſion would cauſe, and it continued to diminiſh till the year 1736, when the moon's aſcending node was about the beginning of *Libra*, and it's orbit had the *leaſt* inclination to the equator. By this time, ſome of the ſtars near the ſolſtitial colure had altered their declinations 18″ *leſs*, ſince the year 1727, than they ought to have done from a preceſſion of 50″. For γ *Draconis*, which in thoſe nine years ſhould have gone about 8″ more *ſoutherly*, was obſerved in 1736 to appear 10″ more *northerly* than it did in 1727.

As this appearance of γ *Draconis* indicated a diminution of the inclination of the earth's axis to the ecliptic; and as it had been obſerved that that inclination was regularly diminiſhed, if this phænomenon depended upon ſuch a cauſe, and amounted to 18″ in nine years, the obliquity of the ecliptic would, at that rate, alter a minute in 30 years, which was faſter than had been found from obſervations. He therefore thought that ſome part of this motion, if not the whole, might ariſe from the action of the moon upon the equatorial parts of the earth; which, he conceived, might cauſe a libratory motion of the earth's axis. But as he was unable to judge, from only nine years obſervations, whether the axis would entirely recover the ſame poſition that it had in 1727, he found it neceſſary to continue his obſervations through a whole period of the moon's nodes, at the end of which he found that the ſtars returned into the ſame poſitions again, as if there had been no alteration in the inclination of the earth's axis, which convinced him that he had rightly aſſigned the cauſe of the *phænomenon*, and the very near agreement of his obſervations upon different ſtars with this theory, through a revolution of the moon's node, indiſputably confirmed it. The following Table contains his obſervations upon *γ Draconis* for 20 years. The firſt column contains the times of the obſervations, the ſecond ſhows the number of ſeconds the ſtar was ſouth of 38° 25′, that being the point of the limb of the ſector with which this ſtar was compared, the third contains the alteration of the polar diſtance, which the *mean* preceſſion, at the rate of one degree in 71½ years, would cauſe in this ſtar, from March 27, 1727, to the day on which the obſervation was taken, the fourth ſhows the aberration of light, the fifth, the equations ariſing from the aforementioned hypotheſis; and the ſixth gives the *mean* diſtance of the ſtar from the point with which it was compared, found, by collecting the ſeveral numbers, according to their ſigns, in the third, fourth and fifth columns, and applying them to the *obſerved diſtances* contained in the ſecond

If the obſervations had been perfectly exact, and the ſeveral equations of their *due* quantity, then all the numbers in the laſt column would have been equal, but ſince they differ a little from one another, if the *mean* of all be taken, and the extremes are compared with it, we ſhall find no greater difference, than what may be ſuppoſed to ariſe from the uncertainty of the obſervations themſelves, it no where amounting to more than 1½″ The hypotheſis therefore ſeems, in this ſtar, to agree extremely well with the obſervations here ſet down, but as I had made 300 of it, I took the trouble of comparing each of them with the hypotheſis: and although it might have been expected that, in ſo large a number, ſome great errors would have occurred, yet there are very few, viz. only eleven, that differ from the mean of theſe ſo much as 2″, and not one that differs ſo much as 3″. This ſurprizing agreement, therefore, in ſo long a ſeries of obſervations, taken in all the various ſeaſons of the year, as well as in the different poſitions of the moon's nodes, ſeems to be a ſufficient proof of the truth, both of *this* hypotheſis, and alſo of *that* which I formerly advanced, relating to the aberration of light, ſince the polar diſtance of this ſtar may differ, in certain circumſtances, almoſt a minute, viz. 56½″, if the corrections reſulting from both theſe hypotheſes are neglected, whereas, when thoſe equations are rightly applied, the mean place of the ſtar comes out the ſame, as nearly as can be reaſonably expected.

γ DRACONIS.

γ DRACONIS.	South of 38° 25′	Preceſſion	Aberration	Nutation	Mean Diſtance
1727 September 3	70″,5	− 0″,4	+19″,2	− 8″,9	80″,4
1728 March 18	108, 7	− 0, 8	−19, 0	− 8, 6	80, 3
—— September 6	70, 2	− 1, 2	+19, 3	− 8, 1	80, 2
1729 March 6	108, 3	− 1, 6	−19, 3	− 7, 4	80, 0
1729 September 8	69, 4	− 2, 1	+19, 3	− 6, 9	80, 2
1730 September 8	68, 0	− 2, 9	+19, 3	− 3, 4	80, 5
1731 September 8	66, 0	− 3, 8	+19, 3	− 1, 0	80, 5
1732 September 6	64, 3	− 4, 6	+19, 3	+ 2, 0	81, 0
1733 Auguſt 29	60, 8	− 5, 4	+19, 0	+ 4, 8	79, 2
1734 Auguſt 11	62, 3	− 6, 2	+16, 9	+ 6, 9	79, 9
1735 September 10	60, 0	− 7, 1	+19, 3	+ 7, 9	80, 1
1736 September 9	59, 3	− 8, 0	+19, 3	+ 9, 0	79, 6
1737 September 6	60, 8	− 8, 8	+19, 3	+ 8, 5	79, 8
1738 September 13	62, 0	− 9, 6	+19, 3	+ 7, 0	78, 7
1739 September 2	66, 6	−10, 5	+19, 2	+ 4, 7	80, 0
1740 September 5	70, 8	−11, 3	+19, 3	+ 1, 9	80, 7
1741 September 2	75, 4	−12, 1	+19, 2	− 1, 1	81, 4
1742 September 5	76, 7	−12, 9	+19, 3	− 4, 0	79, 1
1743 September 2	81, 6	−13, 7	+19, 1	− 6, 4	80, 6
1745 September 3	86, 3	−15, 4	+19, 2	− 8, 9	81, 2
1746 September 17	86, 5	−16, 2	+19, 2	− 8, 7	80, 8
1747 September 2	86, 1	−17, 0	+19, 2	− 7, 6	80, 7

The concluſion derived from theſe obſervations is, that the gradual diminution of the obliquity of the ecliptic to the equator does not ariſe from an alteration in the poſition of the earth's axis, but from ſome alteration in the ecliptic itſelf, becauſe the ſtars at the end of the period of the moon's node, appeared in the ſame places with reſpect to the equator, as they ought to have done, if the earth's axis had retained the ſame inclination to an invariable plane

Dr BRADLEY, in his obſervation upon *η Urſæ Majoris*, in the years 1740 and 1741, found that they gave the polar diſtance 3″ greater than the *mean* of the other years, and obſerves that, had there been only a ſingle obſervation in each of theſe years, part of this difference might have been ſuppoſed to have ariſen from their uncertainty, but as there were eight obſervations taken within a week in 1740, which agree well with each other, and three were made within twenty days in 1741 which likewiſe correſpond with each other, he was inclined to think that the aforementioned difference muſt be owing to ſome other cauſe, and he ſuſpected that the poſition of the moon's apogee, as well as of it's nodes, had ſome relation to this apparent motion.

1028. Dr. **Bradley** communicated his obſervations to Mr **Machin**, who ſoon after ſent him a Table of the annual preceſſion and the correſponding nutation, in the various ſituations of the moon's nodes. Theſe were calculated upon ſuppoſition that the pole of the equator, during a period of the moon's nodes, moved round in the periphery of a circle of 18″ diameter, having the center 23° 29′ from the pole of the ecliptic, that circle having an angular motion of 50″ about the ſame pole. The north pole of the equator was conceived to be in that part of the ſmall circle which is furtheſt from the north pole of the ecliptic, when the moon's aſcending node is in the beginning of Aries, and the oppoſite point of it, when the ſame node is in the beginning of Libra. But Dr **Bradley** afterwards obſerved, that the calculations would agree better with obſervations, if the true pole of the equator deſcribed an ellipſe inſtead of a circle, whoſe minor axis is about 16″, and lying parallel to the ecliptic This is confirmed by theory From all the obſervations of Dr **Bradley**, Dr **Maskelyne** fixed the whole nutation at 19″,1. In the Table we have computed, we have aſſumed it 19″.

1029 Sir I **Newton** had taken notice of the nutation of the earth's axis, and Mr **Flamstead** in his *Hiſt Cœl* Vol. 3 p 113, informs us, that he attempted to diſcover it's quantity, but found his inſtruments not ſufficiently accurate for the purpoſe M de la **Lande** alſo obſerves, that the following paſſage was found in the manuſcripts of **Romer** *Sed de altitudinibus non perinde certus reddebar, tam ob refractionum varietatem quàm ob aliam non dum liquido perſpectam cauſam; ſcilicet per hos duos annos, quemadmodum et alias, expertus ſum eſſe quandam in declinationibus varietatem quæ nec refractionibus nec parallaxibus tribui poteſt, ſine dubio ad vacillationem aliquam poli terreſtris referendam, cujus me veriſimilem dare poſſe theoriam, obſervationibus munitam, ſpero* Notwithſtanding however the nutation of the earth's axis had been ſo long ſuſpected, the diſcovery of the cauſe and quantity thereof was reſerved for Dr **Bradley**. We proceed now to conſider, how all this agrees with the concluſions deduced from the theory of gravity.

Fig 224.

1030. The effect (856) of the body at S upon a particle at E varies as the cube of the apparent diameter of S ſeen from T and the denſity of S conjointly, therefore as the apparent diameters of the ſun and moon ſeen from the earth may be conſidered as equal, the effects of the ſun and moon upon a particle E of the earth, given in poſition, will be as their denſities. Hence, if $m : 1 ::$ denſity of the moon : denſity of the ſun, and A = the whole preceſſion generated

FIG. 228.

iated by the ſun in one year, mA would repreſent the mean preceſſion by the moon in the ſame time, if it's orbit had the ſame inclination to the equator as the ecliptic has. But this is not the caſe. To find therefore the effect produced by the moon, let $fFNeE$ repreſent the orbit of the moon interſecting the ecliptic $♈CL$ in N, and let $♈ELF$ be the poſition of the equator when the moon paſſed it at F, and $aebf$ when it paſſed it again at e; biſect $♈L$ in C, and draw the great circle CrR perpendicular to $♈L$. Now (1022) the preceſſion, *cæteris paribus*, varies as the coſine of the angle which the orbit of the body makes with the equator, hence, $\cos ♈ : \cos E :: mA : mA \times \frac{\cos E}{\cos ♈}$ = the mean preceſſion ariſing from the moon in a year, hence, if t = the time the moon is moving in it's orbit from F to e, 1 year : t :: $mA \times \frac{\cos E}{\cos ♈} : tmA \times \frac{\cos E}{\cos ♈}$ = the mean preceſſion Ee cauſed by the moon in reſpect to it's own orbit. Now to reduce this to the ecliptic, and find the preceſſion during a revolution of the moon's node, we ſhall follow and explain the method given by Mr T. SIMPSON in his *Miſcellaneous Tracts*, it appearing to be as ſimple as the nature of the ſubject will admit of.

1031 As the inclination of the earth's axis at the end of every half revolution, on the return of the ſun or moon again to the equator, is (1023) reſtored to it's former quantity on the reſpective orbits, the angles E, e, F, f are equal, and the triangles DEe, DFf are ſimilar and equal in all reſpects, therefore $DE + De$ being $= DE + DF$ = a ſemicircle, both DE and De may be taken as quadrantal arcs. Now ſin ED (= rad.) : ſin e, or E, :: ſin Ee, or Ee, : ſin eDE, or $eDE = \sin E \times Ee$, alſo ſin e, or ♈ : ſin $♈D$, or $\cos ♈E$, :: ſin eLE, or eDE, : ſin $ae♈$, or $e♈ = \frac{\cos ♈E}{\sin ♈} \times eDE = \frac{\cos ♈E}{\sin ♈} \times \sin E \times Ee = tmA \times \frac{\sin E \times \cos E \times \cos ♈E}{\sin ♈ \times \cos ♈}$ = the preceſſion upon the ecliptic in the time t. Alſo, ſin r, or rad : ſin DR, or $♈E$, :: ſin eDE, or eDE, : ſin. Rr, or $Rr = \sin ♈E \times eDE = \sin ♈E \times \sin E \times Ee = tmA \times \frac{\sin ♈E \times \sin E \times \cos E}{\cos ♈}$ = the correſponding nutation. But ſin E : ſin $♈N$ :: ſin. N : ſin. $♈E$, hence, $\sin ♈E \times \sin E = \sin ♈N \times \sin N$, conſequently the nutation $Rr = tmA \times \frac{\sin ♈N \times \sin N \times \cos E}{\cos ♈}$. Hence (from the former expreſſion for the nutation), the nutation : the correſponding preceſſion :: ſin. ♈ : $\frac{\cos ♈E}{\sin ♈E}$ = cotan $♈E$. Having determined the preceſſion and nuta-

tion

tion for any given poſition of the moon's node, we proceed next to determine the ſame during a revolution of the node.

1032 Let $\Upsilon N=z$, it's ſine $=x$, coſine $=y$, the ſine of $N\Upsilon E=a$, coſine $=b$, ſine of $N=c$, it's coſine $=d$, $\Upsilon CL=e$, and the time of half a revolution of the node $=R$, and let $\Upsilon\mathcal{Q}$ be perpendicular to NE. Then, coſ $N\Upsilon$, or y, : rad. $=1$. cotan N, or $\frac{d}{c}$, . tan. $N\Upsilon\mathcal{Q}=\frac{d}{cy}$, let this be denoted by h, then the ſecant of the ſame angle $=\sqrt{1+h^2}$, it's ſine $=\frac{h}{\sqrt{1+h^2}}$, and coſine $=\frac{1}{\sqrt{1+h^2}}$, hence, the ſine of the difference of the two angles $N\Upsilon\mathcal{Q}$ and $N\Upsilon E$ will be $\frac{hb-a}{\sqrt{1+h^2}}$, and the coſine $=\frac{ha+b}{\sqrt{1+h^2}}$. Alſo, ſin. $N\Upsilon\mathcal{Q}$ ſin. $E\Upsilon\mathcal{Q}$ coſ. N : coſ $E=\frac{\overline{hb-a}\times d}{h}=bd-\frac{ad}{h}=bd-acy$, and coſ $N\Upsilon\mathcal{Q}$ coſ $E\Upsilon\mathcal{Q}$: cot. $N\Upsilon$, or $\frac{y}{x}$, . cotan $\Upsilon E=\overline{b+ha}\times\frac{y}{x}=\frac{ad+bcy}{cx}$, becauſe $h=\frac{d}{cy}$. But (1031) the ſine of nutation for the time $t=tmA\times\frac{\text{ſin }\Upsilon N\times\text{ſin }N\times\text{coſ }E}{\text{coſ }N\Upsilon E}$ $=tmA\times\frac{cx\times\overline{bd-acy}}{b}$, and the time t (whilſt the node deſcribes $\dot z$) $\cdot\ R\ \cdot.\ \dot z\quad e$, therefore $t=R\times\frac{\dot z}{e}=\frac{R\dot x}{e\sqrt{1-x^2}}$; hence, the nutation whilſt the node deſcribes $\dot z$ is (writing $\sqrt{1-x^2}$ for y) $=mAR\times\frac{c}{eb}\times\overline{\frac{bdx\dot x}{\sqrt{1-x^2}}-acx\dot x}$, whoſe correct fluent $mAR\times\frac{c}{eb}\times\overline{bd-bd\sqrt{1-x^2}-\frac{1}{2}acx^2}$ is the nutation, or decreaſe of the inclination of the equator to the ecliptic, cauſed by the moon, for the time the node has moved from Υ to N Next, with regard to the correſponding preceſſion, the increaſe thereof being (1031) in proportion to the decrement of nutation as the cotangent of ΥE to the ſine of $N\Upsilon E$, or as $\frac{ad+bcy}{cx}=\frac{ad+bc\sqrt{1-x^2}}{cx}$ to a, therefore the fluxion of the preceſſion $=mAR\times\frac{1}{abe}\times\overline{\frac{abd^2\dot x}{\sqrt{1-x^2}}+\overline{b^2-a^2}\times cd\dot x-abc^2\dot x\sqrt{1-x^2}}$, whoſe fluent $mAR\times\frac{1}{abe}\times\overline{abd^2z+\overline{b^2-a^2}\times cdx-\frac{1}{2}abc^2z-\frac{1}{2}abc^2x\sqrt{1-x^2}}$ is the true preceſſion.

1033. Hence,

1033. Hence, at the end of half a revolution of the node, or when it arrives at *Libra* L, $x=0$, and $z=e$, therefore the nutation $= mAR \times \frac{2cd}{e}$, and the preceſſion $= mAR \times \overline{1-\frac{3}{2}c^2}$, becauſe $c^2+d^2=1$ Hence, the whole quantity of nutation during half a revolution of the node from ♈ : the correſponding quantity of preceſſion $\frac{2cd}{e}$ $1-\frac{3}{2}c^2$: 10 : 174 very nearly.

1034 Hence, the mean preceſſion of the equinox from the action of the moon is to what it would be if the moon's orbit coincided with the ecliptic as $1-\frac{3}{2}c^2$ 1 But if the moon's orbit coincided with the ecliptic, the effect of the moon would (1030) be to that of the ſun in the ratio of their denſities, therefore the mean preceſſion of the moon : that from the ſun, in a compound ratio of $1-\frac{3}{2}c^2=0{,}988$ 1, and of the denſity of the moon the denſity of the ſun, consequently the denſity of the moon the denſity of the ſun preceſſion from the moon : preceſſion from the ſun $\times$ 0,988

1035 As the preceſſion in half a revolution of the node $= mAR \times \overline{d^2-\frac{1}{2}c^2}$, we have, e z $mAR \times \overline{d^2-\frac{1}{2}c^2}$ $mAR \times \frac{z}{e} \times \overline{d^2-\frac{1}{2}c^2}$ the *mean* preceſſion whilſt the node moves over the arc z, which ſubtracted from the *true* preceſſion which we found in Article 1032, we get $mAR \times \frac{1}{abe} \times \overline{b^2-a^2} \times cdx$ for the *equation of the equinoxes*, neglecting $-\frac{1}{2}abc^2x\sqrt{1-x^2}$ as never amounting to $\frac{1}{4}''$. Hence, the equation, when the node has made one fourth of a revolution, will be $mAR \times \frac{1}{abe} \times \overline{b^2-a^2} \times cd$, which is to the greateſt nutation $mAR \times \frac{2cd}{e}$ during half a revolution of the node, as b^2-a^2 : $2ab$, or as 1 : $\frac{2ab}{b^2-a^2}$, that is, as radius the tangent of double the inclination of the equator to the ecliptic

1036 As c is but very ſmall, we may neglect the laſt term in the expreſſion for the nutation, as well as in the equation of preceſſion, without any conſiderable error; hence, the nutation becomes $mAR \times \frac{cd}{e} \times \overline{1-\sqrt{1-x^2}}$, which varies as $1-\sqrt{1-x^2}$ the verſed ſine of the nodes' true longitude, and the equation of preceſſion $mAR \times \frac{1}{abe} \times \overline{b^2-a^2} \times cdx$ varies as x the ſine of the node's true longitude.

1037. If

1037 If the annual preceſſion ariſing from the ſun be taken $=21''.\ 6'''$ as in Art 1022. and the whole preceſſion $=50''$, then the part ariſing from the action of the moon will be $28''\ 54'''$, hence (1034), the denſity of the moon : denſity of the ſun : $28''\ 54''' : 21''\ 6''' \times 0{,}988 = 20''.\ 8'''$, which ratio does not agree, either with the proportion deduced from the tides, or with the accurate obſervations of Dr. BRADLEY. The beſt method of ſettling this point, is from the greateſt nutation

1038 The nutation, during half a revolution of the moon's node from Aries, Dr. MASKELYNE fixed at $19'',1$, and which we ſhall here aſſume $19''$, hence (1033), $10 : 174 :: 19'' : \frac{174 \times 19''}{10}$ the preceſſion from the moon during that time, which we may take equal 9,31 years, hence, the mean preceſſion from the moon in one year $= \frac{174 \times 19''}{10 \times 9{,}31}$, therefore if we take the whole preceſſion in a year to be $50\frac{1}{4}''$, we have $50\frac{1}{4}'' - \frac{174 \times 10''}{10 \times 9{,}31} = \frac{10 \times 9{,}31 \times 50\frac{1}{4}'' - 174 \times 19''}{10 \times 9{,}31}$ for the part of the preceſſion ariſing from the ſun Hence (1034), the denſity of the moon : the denſity of the ſun :: $174 \times 19'' : \overline{10 \times 9{,}31 \times 50\frac{1}{4}'' - 174 \times 19''} \times 0{,}988 : 2{,}44 : 1$ The part therefore of the preceſſion ariſing from the action of the moon $=35''.\ 39'''$, and that of the ſun $=14''.\ 36'''$, and the greateſt equation (1035) of the preceſſion ariſing from the moon $=17'',7$

1039. The equation of the preceſſion (1036) varies as x the ſine of the node's diſtance from ♈ meaſured contrary to the order of the ſigns, but if $L =$ the longitude of the node, ſin. $x = -$ ſin L; therefore *rad.* : *−ſin.* L :: $17'',7$: *the equation of preceſſion* $=$ *−ſin.* $L \times 17'',7$; hence, the equation is to be *ſubtracted* from the mean preceſſion when L is *leſs* than ſix ſigns, and *added*, when *greater*.

1040. As (1036) the decreaſe of the inclination of the equator to the ecliptic, from the time the node coincides with ♈, is as the verſed ſine of the node's diſtance from that point, the inclination muſt be at it's mean value when the node is in the ſolſtice, hence, the difference between the mean and true values will be as the difference between the verſed ſine and radius, or as the coſine of the node's diſtance from ♈; therefore to find the nutation at any time, ſay, *rad.* : *coſ.* z : $9'',5$: *the nutation* $= 9'',5 \times$ *coſ.* z, which muſt be *added*, when the node is in the *aſcending* ſigns ♑, ♒, ♓, ♈, ♉, ♊, but *ſubtracted*, when in the *deſcending* ſigns, ♋, ♌, ♍, ♎, ♏, ♐, to get the true obliquity of the equator to the ecliptic; hence, *the equation of the obliquity of the ecliptic* $= -9'',5 \times$ coſ. z.

The

The following Table shows the Equation of the Preceſſion, and the Equation of the Obliquity of the Ecliptic, ariſing from the Action of the Moon, both computed by the Rules here given.

The Equation of the Preceſſion of the Equinox					The Equation of the Obliquity of the Ecliptic				
☽'s ☊ from ♈	Sig. O – Sig. VI. +	Sig I – Sig. VII. +	Sig II. – Sig. VIII. +		☽'s ☊ from ♈	Sig O + Sig VI. –	Sig. I. + Sig. VII. –	Sig II + Sig. VIII. –	
0°	0″,0	8″,8	15″,3	30°	0	9″,5	8″,2	4″,7	30°
5	1, 5	10, 1	16, 1	25	5	9, 4	7, 8	4, 0	25
10	3, 0	11, 4	16, 7	20	10	9, 3	7, 3	3, 3	20
15	4, 5	12, 5	17, 2	15	15	9, 2	6, 7	2, 4	15
20	6, 0	13, 6	17, 5	10	20	9, 0	6, 1	1, 7	10
25	7, 4	14, 5	17, 7	5	25	8, 7	5, 5	0, 9	5
30	8, 8	15, 3	17, 7	0	30	8, 2	4, 7	0, 0	0
	Sig. V. – Sig XI +	Sig. IV – Sig X +	Sig III – Sig. IX +	☽'s ☋ from ♈		Sig. V – Sig XI +	Sig. IV. – Sig. X. +	Sig. III – Sig. IX. +	☽'s ☋ from ♈

1037. If the annual preceſſion ariſing from the ſun be taken $= 21''\ 6'''$ as in Art. 1022 and the whole preceſſion $= 50''$, then the part ariſing from the action of the moon will be $28''\ 54'''$, hence (1034), the denſity of the moon : denſity of the ſun : $28''.\ 54''' : 21''.\ 6''' \times 0{,}988 = 20''.\ 8'''$, which ratio does not agree, either with the proportion deduced from the tides, or with the accurate obſervations of Dr. BRADLEY. The beſt method of ſettling this point, is from the greateſt nutation.

1038. The nutation, during half a revolution of the moon's node from Aries, Dr. MASKELYNE fixed at $19'',1$, and which we ſhall here aſſume $19''$, hence (1033), $10 : 174 :: 19'' : \frac{174 \times 19''}{10}$ the preceſſion from the moon during that time, which we may take equal 9,31 years, hence, the mean preceſſion from the moon in one year $= \frac{174 \times 19''}{10 \times 9{,}31}$, therefore if we take the whole preceſſion in a year to be $50\frac{1}{4}''$, we have $50\frac{1}{4}'' - \frac{174 \times 19''}{10 \times 9{,}31} = \frac{10 \times 9{,}31 \times 50\frac{1}{4}'' - 174 \times 19''}{10 \times 9{,}31}$ for the part of the preceſſion ariſing from the ſun. Hence (1034), the denſity of the moon : the denſity of the ſun :: $174 \times 19'' : \overline{10 \times 9{,}31 \times 50\frac{1}{4}'' - 174 \times 19''} \times 0{,}988 :: 2{,}44 : 1$. The part therefore of the preceſſion ariſing from the action of the moon $= 35''\ 39'''$, and that of the ſun $= 14''.\ 36'''$, and the greateſt equation (1035) of the preceſſion ariſing from the moon $= 17'',7$.

1039. The equation of the preceſſion (1036) varies as x the ſine of the node's diſtance from ♈ meaſured contrary to the order of the ſigns, but if $L =$ the longitude of the node, ſin. $x = -$ ſin L; therefore *rad.* : *−ſin.* L :: $17'',7$: *the equation of preceſſion* $= -$*ſin* $L \times 17'',7$, hence, the equation is to be *ſubtracted* from the mean preceſſion when L is *leſs* than ſix ſigns, and *added*, when *greater*.

1040. As (1036) the decreaſe of the inclination of the equator to the ecliptic, from the time the node coincides with ♈, is as the verſed ſine of the node's diſtance from that point, the inclination muſt be at it's mean value when the node is in the ſolſtice, hence, the difference between the mean and true values will be as the difference between the verſed ſine and radius, or as the coſine of the node's diſtance from ♈, therefore to find the nutation at any time, ſay, *rad.* : *coſ* z :: $9'',5$: *the nutation* $= 9'',5 \times$ *coſ.* z, which muſt be *added*, when the node is in the *aſcending* ſigns ♑, ♒, ♓, ♈, ♉, ♊, but *ſubtracted*, when in the *deſcending* ſigns, ♋, ♌, ♍, ♎, ♏, ♐, to get the true obliquity of the equator to the ecliptic, hence, *the equation of the obliquity of the ecliptic* $= -9'',5 \times$ coſ. z.

The

The following Table shows the Equation of the Precession, and the Equation of the Obliquity of the Ecliptic, arising from the Action of the Moon, both computed by the Rules here given.

The Equation of the Preceſſion of the Equinox					The Equation of the Obliquity of the Ecliptic				
☽'s ☊ from ♈	Sig. O. – Sig. VI. +	Sig. I – Sig. VII. +	Sig. II. – Sig. VIII. +		☽'s ☊ from ♈	Sig O + Sig. VI. –	Sig. I. + Sig. VII. –	Sig II. + Sig VIII. –	
0°	0″,0	8″,8	15″,3	30°	0	9″,5	8″,2	4″,7	30°
5	1, 5	10, 1	16, 1	25	5	9, 4	7, 8	4, 0	25
10	3, 0	11, 4	16, 7	20	10	9, 3	7, 3	3, 3	20
15	4, 5	12, 5	17, 2	15	15	9, 2	6, 7	2, 4	15
20	6, 0	13, 6	17, 5	10	20	9, 0	6, 1	1, 7	10
25	7, 4	14, 5	17, 7	5	25	8, 7	5, 5	0, 9	5
30	8, 8	15, 3	17, 7	0	30	8, 2	4, 7	0, 0	0
	Sig. V. – Sig XI +	Sig. IV. – Sig. X +	Sig III – Sig. IX +	☽'s ☋ from ♈		Sig. V. – Sig XI +	Sig IV. – Sig. X. +	Sig III – Sig. IX +	☽'s ☋ from ♈

Ex. Let the diſtance of the aſcending node of the moon from the firſt point of Aries be $4^s. 18°. 40'$, to find the equations of the preceſſion and obliquity.

The equation of the preceſſion for $4^s\ 15°$ is $-12'',5$, and for $4^s\ 20°$ it is $-11'',4$, hence, $5°$ $3°\ 40'$ $1'',1$ $0'',8$, which taken from $-12'',5$ leaves $11'',7$ the equation of preceſſion. Alſo, the equation of the obliquity for $4^s\ 15°$ is $-6'',7$, and for $4^s\ 20°$ it is $-7'',3$, hence, $5°$ $3°. 40' : 0'',6$ $0'',4$, which added to $-6'',7$ gives $-7'',1$ the equation of the obliquity. Theſe equations, with thoſe ariſing from the ſun (1024, 1025), applied to the *mean* preceſſion and obliquity, give the *true*, ſo far as regards the diſplacement of the equator.

1041. The *nutation* ariſing from the moon, and the *equation of the preceſſion*, may be both together repreſented thus. Let P be the mean place of the pole of the equator $♈ V$, E the pole of the ecliptic $♈ Y$, in $EP♋$ take $PA = PB = 9'',5$ half the greateſt nutation, and deſcribe the circle $BDAG$, and draw DPG perpendicular to AB, and take PC PD $\text{coſ } 2EP \cdot \text{coſ } EP$ 6828 9173, deſcribe the ellipſe $BCAF$, and the true place of the pole of the equator will always be found in the circumference of this ellipſe, and make $APS =$ the diſtance of the moon's aſcending node from $♈$, draw SpR perpendicular to AB, and p is the true place of the pole. For draw the great circles EC, EpT, then it is manifeſt, that EA and EB will be the greateſt and leaſt diſtances of the two poles, AB being $= 19''$ the greateſt nutation, and rad cos APS, or cos v, $PS = 9'',5$ $PR = 9'',5 \times \text{coſ } z =$ (1040) the nutation, therefore ER, or Ep very nearly, is the true diſtance of the poles. Alſo, by *Conſtruction*, CP AB . $\frac{1}{2}$ coſ. $2EP$ coſ. EP $\frac{b^2-a^2}{2}$ b, and by *Spherics*, ſin PEC ſin PC, or (on account of their ſmallneſs) PEC PC, : rad. : ſin. EC, or EP, or a, hence, by compounding theſe two proportions, PEC AB : $\frac{b^2-a^2}{2}$ ab b^2-a^2 $2ab$, which proportion to find PEC is the ſame as that in Art. 1035 to determine the greateſt difference of the true and mean longitude, conſequently PEC repreſents that difference. Hence it follows, that the angle REp will expreſs the difference of the mean and true longitudes, at the given poſition of the node, for rad ſin APS : PD RS .: PC Rp .: the angle PEC REp, as it ought to be by Art. 1036. As therefore Ep is the true diſtance of the poles, and REp expreſſes the difference between the mean and true longitudes, p muſt be the true place of the pole. Now as the inclination of the equator to the ecliptic decreaſes from the time the aſcending node of the moon's orbit leaves *Aries* till it gets back to *Libra*, therefore the pole of the equator during that time has moved from A to B, and as the equation of the preceſſion for that time is to be added to the mean preceſſion by

FIG. 229

Art.

Art. 1039. it makes the true preceſſion greater than the mean, and therefore the true place of the pole muſt have been behind the mean place, conſequently the pole has moved from A in the direction ACB. This matter therefore may be ſimply explained thus. If the preceſſion were uniform, and there was no nutation, then the true place of the pole P would (1021) deſcribe a circle about E contrary to the order of the ſigns, with an angular velocity equal to that of the preceſſion; but the preceſſion is not uniform, and there is alſo a nutation, in conſequence of which, the true motion of the pole of the equator is in the ellipſe $ACBF$, whilſt the center P is carried about E as above-mentioned. The motion of the pole in the ellipſe, therefore, takes into conſideration the effect of the equation of preceſſion and the nutation.

1042. Let s be the place of a ſtar, $♈V$ the equator to the mean pole P, $♈'W$ the equator to the true pole p, draw the great circles $Psma$, $psnbd$, and let $♈c$ be perpendicular to $♈'W$, and pr to Ps. Put $v = APp$, $z = APS$, $r = APs$, $d =$ the declination of the ſtar, $a =$ it's right aſcenſion, then $sPp = r - v$, alſo, by the property of the ellipſe, $PD = 9'',5 : PC = 7'',07 :: RS : Rp :: \tan.\ z : \tan\ v$, therefore $\tan.\ v = \frac{7'',07}{9'',5} \times \tan.\ z$, and $PD = 9'',5 : PC = 7'',07 :: RS : Rp = \frac{7'',07}{9'',5} \times RS$, but $\text{rad.} = 1 : \text{ſin.}\ z :: PS = 9'',5 : RS = 9'',5 \times \text{ſin}\ z$, hence, $Rp = 7'',07 \times \text{ſin.}\ z$, alſo $\text{coſ.}\ v : \text{rad} = 1 :: PR = 9'',5 \times \text{coſ}\ z$ (1040) $: Pp = 9'',5 \times \frac{\text{coſ}\ z}{\text{coſ}\ v}$.

1043. The *mean* right aſcenſion is $♈a = ♈b + ba$*, and the *true* right aſcenſion is $♈'d = ♈'c + cd = ♈'c + ♈b$, hence, the variation of the right aſcenſion $= ♈'c - ba = ♈♈' \times \text{coſ}\ 23^\circ\ 28' - ba = T♋ \times \text{coſ}\ 23^\circ\ 28' - ba$. Now $T♋ = \frac{Rp}{\text{ſin.}\ 23^\circ\ 28'} = \frac{7'',07 \times \text{ſin}\ z}{\text{ſin.}\ 23^\circ\ 28'}$, hence, $T♋ \times \text{coſ}\ 23^\circ.\ 28' = 7'',07 \times \text{ſin}\ z \times \text{cot}\ 23^\circ.\ 28' = 16'',29 \times \text{ſin}\ z$. Alſo, $\text{ſin}\ sPp : \text{ſin}\ sp :: \text{ſin}\ Psp : \text{ſin.}\ Pp$, and as Psp and Pp are very ſmall, we may put the quantities themſelves for their ſines, hence, $Pp \times \frac{\text{ſin}\ \overline{r - v}}{\text{ſin}\ sp} = Psp = bsa$, therefore $ab = Pp \times \frac{\text{ſin}\ \overline{r - v} \times \text{coſ}\ sp}{\text{ſin}\ sp} = Pp \times \text{ſin.}\ \overline{r - v} \times \tan.\ d$. Hence, the variation of right aſcenſion

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* The arc ab is called the *deviation* in right aſcenſion, mn the *deviation* in longitude, and Pr the *deviation* in north polar diſtance, $♈♈'$ is called the *equation of the equinoxes* in longitude, and $♈c$ the *equation of the equinoxes* in right aſcenſion.

aſcenſion $= 16'',29 \times \text{ſin.}\ z - Pp \times \text{ſin}\ \overline{r-v} \times \text{tan.}\ d = 16'',29 \times \text{ſin.}\ z - 9'',5 \times \frac{\text{coſ}\ z}{\text{coſ.}\ v} \times \text{ſin}\ \overline{r-v} \times \text{tan.}\ d = 16'',29 \times \text{ſin}\ z - 9'',5 \times \frac{\text{coſ}\ z}{\text{coſ}\ v} \times \text{ſin.}\ \overline{r-v} \times \text{tan}\ d = 16'',29 \times \text{ſin.}\ z - 9'',5 \times \frac{\text{coſ}\ z}{\text{coſ}\ v} \times \text{tan.}\ d \times \overline{\text{ſin.}\ r \times \text{coſ.}\ v - \text{ſin.}\ v \times \text{coſ.}\ r} =$ (as the tangent $v = \frac{7'',07}{9'',5} \times$ the tangent z) $16'',29 \times \text{ſin.}\ z -$ the tangent $d \times \overline{9'',5 \times \text{ſin.}\ r \times \text{coſ.}\ z - 7'',07 \times \text{coſ}\ r \times \text{ſin}\ z}$

$$= -\left\{\begin{matrix} 4'',75 \times \text{ſin.}\ \overline{r-z} + 4'',75 \times \text{ſin.}\ \overline{r+z} \\ 3'',53 \times \text{ſin.}\ \overline{r-z} - 3'',53 \times \text{ſin.}\ \overline{r+z} \end{matrix}\right\} \times \text{tan.}\ d$$
$$+\ 16'',29 \times \text{ſin.}\ z.$$

But (in this figure) $r = 90° - a$, therefore $r \pm z = 90° - a \pm z =$ (ſo far as regards the angle) $a \pm z - 90°$, and the ſame is true if a be greater than 90°, and $r = a + 90°$. Hence, the variation of *Right Aſcenſion*

$$= -\left\{\begin{matrix} 4'',75 \times \text{ſin.}\ \overline{a-z-90°} + 4'',75 \times \text{ſin.}\ \overline{a+z-90°} \\ 3'',53 \times \text{ſin.}\ \overline{a-z-90°} - 3'',53 \times \text{ſin.}\ \overline{a+z-90°} \end{matrix}\right\} \times \text{tan.}\ d$$
$$+\ 16'',29 \times \text{ſin.}\ z.$$

$$= \text{tan}\ d \times (-8'',28 \times \text{ſin.}\ \overline{a-z-90°} - 1'',22 \times \text{ſin.}\ \overline{a+z-90°}) + 16'',29 \times \text{ſin.}\ z.$$

For ſouth declination d is negative.

1044. The variation in *Declination* is $sP - sp = Pr$ very nearly, $= Pp \times \text{coſ.}\ \overline{r-v} = 9'',5 \times \frac{\text{coſ.}\ z}{\text{coſ.}\ v} \times \text{coſ.}\ \overline{r-v} = 9'',5 \times \frac{\text{coſ.}\ z}{\text{coſ.}\ v} \times \overline{\text{coſ.}\ r \times \text{coſ.}\ v + \text{ſin.}\ r \times \text{ſin.}\ v}$ $= 9'',5 \times \text{coſ.}\ r \times \text{coſ.}\ z + 9'',5 \times \text{ſin.}\ r \times \text{coſ.}\ z \times \text{tan.}\ v = 9'',5 \times \text{coſ.}\ r \times \text{coſ.}\ z + 7'',07 \times \text{ſin.}\ r \times \text{ſin.}\ z$

$$= \left\{\begin{matrix} 4'',75 \times \text{coſ.}\ \overline{r-z} + 4'',75 \times \text{coſ.}\ \overline{r+z} \\ 3'',53 \times \text{coſ.}\ \overline{r-z} - 3'',53 \times \text{coſ.}\ \overline{r+z} \end{matrix}\right\}$$

$$= 8'',28 \times \text{coſ.}\ \overline{r-z} - 1'',22 \times \text{coſ.}\ \overline{r+z} = 8'',28 \times \text{ſin.}\ \overline{a-z} - 1'',22 \times \text{ſin.}\ \overline{a+z}.$$

1045. From

1045. From theſe expreſſions for the variation in right aſcenſion and declination, I have calculated the following Tables for more readily finding thoſe quantities. They have been computed by M. LAMBERT, upon ſuppoſition that the nutation was 18″, but a more correct value being 19″ (1028), I have here calculated the Tables for that quantity.

TABLE

Degrees	TABLE I			TABLE II			TABLE III			Degrees
	Signs.			Signs			Signs			
	O VI	I VII	II VIII.	O VI	I VII	II VIII	O VI	I VII	II. VIII	
	+ −	+ −	+ −	+ −	+ −	+ −	− +	− +	− +	
0	0″,00	4″,14	7″,17	0″,00	0″,61	1″,06	0″,00	8″,14	14″,11	30
1	0, 14	4, 26	7, 24	0, 02	0, 63	1, 07	0, 28	8, 39	14, 25	29
2	0, 29	4, 39	7, 31	0, 04	0, 65	1, 08	0, 57	8, 63	14, 38	28
3	0, 43	4, 51	7, 38	0, 06	0, 66	1, 09	0, 85	8, 87	14, 51	27
4	0, 58	4, 63	7, 44	0, 09	0, 68	1, 10	1, 14	9, 11	14, 64	26
5	0, 72	4, 75	7, 50	0, 11	0, 70	1, 11	1, 42	9, 34	14, 76	25
6	0, 86	4, 87	7, 56	0, 13	0, 72	1, 11	1, 70	9, 58	14, 88	24
7	1, 01	4, 98	7, 62	0, 15	0, 74	1, 12	1, 99	9, 80	15, 00	23
8	1, 15	5, 10	7, 68	0, 17	0, 75	1, 13	2, 27	10, 03	15, 10	22
9	1, 30	5, 21	7, 73	0, 19	0, 77	1, 14	2, 55	10, 25	15, 21	21
10	1, 44	5, 32	7, 78	0, 21	0, 78	1, 15	2, 83	10, 47	15, 31	20
11	1, 58	5, 43	7, 83	0, 23	0, 80	1, 15	3, 11	10, 69	15, 40	19
12	1, 72	5, 54	7, 87	0, 25	0, 82	1, 16	3, 39	10, 90	15, 49	18
13	1, 86	5, 65	7, 92	0, 27	0, 83	1, 17	3, 66	11, 11	15, 58	17
14	2, 00	5, 75	7, 96	0, 30	0, 85	1, 17	3, 94	11, 32	15, 66	16
15	2, 14	5, 85	8, 00	0, 32	0, 86	1, 18	4, 22	11, 52	15, 73	15
16	2, 28	5, 96	8, 03	0, 34	0, 88	1, 18	4, 49	11, 72	15, 81	14
17	2, 42	6, 06	8, 07	0, 36	0, 89	1, 19	4, 76	11, 91	15, 87	13
18	2, 56	6, 15	8, 10	0, 38	0, 91	1, 19	5, 03	12, 11	15, 93	12
19	2, 70	6, 24	8, 13	0, 40	0, 92	1, 20	5, 30	12, 29	15, 99	11
20	2, 83	6, 34	8, 15	0, 42	0, 93	1, 20	5, 57	12, 48	16, 04	10
21	2, 97	6, 43	8, 18	0, 44	0, 95	1, 20	5, 84	12, 66	16, 09	9
22	3, 10	6, 52	8, 20	0, 46	0, 96	1, 21	6, 10	12, 84	16, 13	8
23	3, 23	6, 61	8, 22	0, 48	0, 97	1, 21	6, 36	13, 01	16, 17	7
24	3, 37	6, 70	8, 23	0, 50	0, 99	1, 21	6, 63	13, 18	16, 20	6
25	3, 50	6, 78	8, 25	0, 52	1, 00	1, 22	6, 88	13, 34	16, 23	5
26	3, 63	6, 86	8, 26	0, 53	1, 01	1, 22	7, 14	13, 50	16, 25	4
27	3, 76	6, 94	8, 27	0, 55	1, 02	1, 22	7, 40	13, 66	16, 27	3
28	3, 89	7, 02	8, 27	0, 57	1, 03	1, 22	7, 65	13, 81	16, 28	2
29	4, 01	7, 10	8, 28	0, 59	1, 05	1, 22	7, 90	13, 96	16, 29	1
30	4, 14	7, 17	8, 28	0, 61	1, 06	1, 22	8, 14	14, 11	16, 29	0
	V XI	IV X	III IX	V. XI	IV. X	III IX	V XI	IV. X	III IX	
	+ −	+ −	+ −	+ −	+ −	+ −	− +	− +	− +	

Ex. On January 1, 1790, the right aſcenſion of *α Lyræ* was $9^s. 7^\circ. 27'$, and it's declination was $38^\circ. 35'. 44''$, to find it's nutation

The longitude of the moon's aſcending node was $7^s. 16^\circ\ 36'$, hence,

$a = 9^s\ 7^\circ\ 27'$

$z = 7\ 16.\ 36$ ..... $+11'',64$ Tab III.

$a - z = 1\ 20.\ 51$ ...... $+\ 6,42$ Tab I.

$a + z = 4.\ 24\ \ 3.$ .. $+\ 0,72$ Tab II.

Nutation in *Declination* $= +\ 7,14$

$a - z - 90^\circ = 10^s\ 20^\circ\ 51'$ $-5'',23$ Tab I

$a + z - 90^\circ = 1^s\ 24^\circ\ 3'$.. $+0,99$ Tab. II.

$-4,24$

Tan. dec. $= ,798$

$-\ 3,38$ Product.

$+11,64$

Nutation in *Right Aſcen* $= +\ 8,26$

When the declination is ſouth, it's tangent becomes negative

To find the Variation in Right Aſcenſion and Declination of a Star, from the Preceſſion of the Equinoxes

1046 Let ♈*L* be the ecliptic, ♈𝒬 the equator, *r*𝒬 it's next poſition at the end of any given time, *s* a ſtar, *sx*, *svw* two circles of declination to the two poſitions of the equator, and ♈*a* perpendicular to *r*𝒬. As we here conſider the effect ariſing only from the regreſſion of the equinoctial points, the inclination of the equator to the ecliptic remaining the ſame, we have (1031) 𝒬♈ = 90°, hence (13), rad = 1 : coſ ♈*w* : *a*♈ (= ♈*r* × ſin. ♈) : *vw* = ♈*r* × ſin ♈ × coſ. ♈*w*, the *variation of declination* FIG. 230.

1047. Hence, we can find the variation of right aſcenſion. From the variation of the right angled ſpherical triangle 𝒬*sw*, whoſe ſide 𝒬*s* is conſtant, we have[*] $vw \times \text{tan. } ws = vx \times \text{tan. } Qx$, therefore $vx = \dfrac{vw \times \text{tan. } ws}{\text{cot. ♈}w} =$ (1046) $\dfrac{\text{♈}r \times \text{ſin. ♈} \times \text{coſ. ♈}w \times \text{tan. } ws}{\text{cot. ♈}w} = \text{♈}r \times \text{ſin. ♈} \times \text{ſin. right aſcenſion} \times \text{tan. dec.}$ This is one part of the variation of right aſcenſion. But as we now reckon from r and not from ♈, it is manifeſt that there is another part ra which is common to all the ſtars, but $ra = \text{♈}r \times \text{coſ. ♈}$, hence, the *variation of right aſcenſion* $= \text{♈}r \times \text{coſ. ♈} - \text{♈}r \times \text{ſin. ♈} \times \text{ſin. right aſcen.} \times \text{tan. dec.} = \text{prec. in long.} \times \overline{\text{coſ. } 23°.\ 28' - \text{ſin. } 23°\ 28' \times \text{ſin. right aſcen.} \times \text{tan. dec.}}$ In the laſt ſix ſigns, the ſin. right aſcen. becomes negative, alſo, if the declination becomes ſouth, the tang. of declination becomes negative, the ſecond term therefore is ſometimes additive, and ſometimes ſubtractive.

On the preceding theory, Mr. **Simpson** has made the following remarks, in order to explain certain difficulties and objections that may thence ariſe.

1048 It may be obſerved, in the firſt place, that we have, all along, conſidered the effects of the ſun and moon ſeparately, and, conſequently, have ſuppoſed them to be no-ways influenced or diſturbed by each other. This may ſeem too bold an aſſumption, eſpecially, as it is known that the tides, which are produced by the very ſame forces, depend upon, and are greatly varied by, the different poſitions of the two luminaries.

FIG. 231.

1049 To remove this objection, let ♈SM♎ repreſent the plane of the earth's equator, ♈O♎ it's interſection with the plane of the ecliptic, ♈S the right aſcenſion of the ſun, ♈M the right aſcenſion of the moon, and let the forces of the two bodies to turn the earth about it's center, in thoſe poſitions, be repreſented by f and F reſpectively.

1050 Theſe forces may be conſidered as acting perpendicular to the plane of the equator in the points S and M, and will be equivalent to, and have the ſame effect with, one ſingle force, equal to them both, acting in their center of gravity N. But, by Mechanics, the force $f+F$, acting at N, will (if the radius OP be drawn through N) be equivalent to another force, acting at P, expreſſed by $\overline{f+F} \times \dfrac{ON}{OP}$, or $\overline{f+F} \times \dfrac{NQ}{PR}$ (ſuppoſing NQ, PR, as alſo SB, MC, to be perpendicular to ♈O♎).

1051. But

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[*] If S and s repreſent the ſines of the two legs A, a of a right angled triangle, C and c their coſines, T and t their tangents, and the hypothenuſe be conſtant, then $C \times c =$ rad $\times$ coſ. hyp. a conſtant quantity; hence, $C \times c + c \times C = 0$; but $c = s \times a$, and $C = S \times A$, therefore $C \times s \times a + c \times S \times A = 0$, and $\frac{s}{c} \times a = -\frac{S}{C} \times A$, or $t \times a = -T \times A$, the ſigns of the quantities thus deduced we have omitted above, as we only wanted the values of the quantities.

1051. But the quantity of preceſſion, during a given moment of time, is known (1018 to 1021) to be as the force, and as the ſine of the right aſcenſion, conjointly; from whence the two quantities ariſing from the ſun and moon, conſidered ſeparately, are expounded by $f \times SB$, and $F \times MC$, reſpectively. But, ſuppoſing both bodies to act together, or, which is the ſame, ſuppoſing one ſingle force, expreſſed by $\overline{f+F} \times \frac{NQ}{PR}$, to act at P, the quantity of the preceſſion will then (by the very ſame rule) be truly defined by $\overline{f+F} \times \frac{NQ}{PR} \times PR$, or it's equal $\overline{f+F} \times NQ$; which quantity, by the property of the center of gravity, is known to be equal to $f \times SB + F \times MC$. Hence, it is manifeſt that, whether the forces of the luminaries be joined together, or treated apart, the reſult will be the ſame.

1052. The next difficulty relates to the excentricity of the lunar orbit, and the inequality of the motion in that orbit; which may be thought ſufficient to occaſion a ſenſible deviation from rules founded on a ſuppoſition that pays no regard to them.

1053. In order to clear up this point alſo, imagine $ADBE$ to be an ellipſe, in which the moon is ſuppoſed to revolve, about the center of the earth placed in the lower focus F of the ellipſe, let AB be the tranſverſe axis of the ellipſe, perpendicular to which, through F, draw the ordinate IH, moreover, let there be drawn any two other lines DE, de, through the focus F, to make a very ſmall (given) angle DFd with each other. FIG. 232.

1054. The perturbating force of the moon, at the diſtance DF will (856) be *inverſely* as the cube of that diſtance; and the time of deſcribing the given angle DFd will * be *directly* as the ſquare of the ſame diſtance. Therefore, by compoſition, the quantity of the moon's action, during the time of deſcribing this angle, will be in the ſimple ratio of the ſaid diſtance, *inverſely*. Hence, it appears, that the ſum of the forces employed, during the times of deſcribing the oppoſite angles DFd, EFe, will be truly defined by $\frac{1}{FD} + \frac{1}{FE}$, or it's equal $\frac{FE+FD}{FE \times FD}$.

1055. Upon AB let fall the perpendiculars DN and EM, ſo ſhall $FE - FH : FI\ (FH) - FD : FM : FN$ (by the property of the ellipſe) :: $FE : FD$ by ſimilar triangles: conſequently $FE \times FD - FH \times FD = FH \times FE \quad FD \times FE$, or

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* For the angle DFd being given, the area DFd varies as $DF \times dF$, or as DF^2; therefore (805) the time of deſcribing the angle varies as DF^2.

or $2FE \times FD = FH \times \overline{FE+FD}$ therefore, as it appears from hence that $\frac{FE+FD}{FE \times FD}$, the meaſure of the ſaid forces, is, every where, equal to the conſtant quantity $\frac{2}{FH}$, it is evident that the excentricity of the orbit and the poſition of the apogee have no effect on the motion of the earth's axis.

1056. An objection may, perhaps, ariſe, with regard to the addition of the forces employed by the moon in oppoſite parts of it's orbit, which ſtep may be looked upon as arbitrary, but the reaſon upon which it is founded will be clear, by conſidering that the moon's inclination to the plane of the equator, in oppoſite points of it's orbit, is always the ſame, and that, therefore, the very ſame effect in the alteration of the poſition of the equator will be produced, whether the whole force employed during the deſcription of the correſponding oppoſite angles, be equally, or unequally, divided, with reſpect to the ſaid angles; ſince the ſaid force acts with the ſame advantage, or under the ſame circumſtance of declination, in both caſes.

1057. Another difficulty, that may ariſe, is in relation to our having made the effect of the ſun's force to be about one third part leſs than the quantity reſulting from calculations founded on hydroſtatical principles and the hypotheſis of an uniform denſity of all the parts of the earth. But, that the *phænomenon* cannot be truly accounted for, upon this hypotheſis, appears from the concurrence of all experiments in general: for, whether we regard the menſuration of the degrees of the earth, the accurate obſervations of Dr BRADLEY, or the proportions and times of the tides, the caſe is the ſame, and requires a much leſs effect from the action of the ſun than reſults from, or can conſiſt with, the ſaid hypotheſis.

1058. But if the denſity of the earth, inſtead of being uniform, is ſuppoſed to increaſe from the ſurface to the center (as there is the greateſt reaſon to imagine it does), then the *phænomenon* may be eaſily made to quadrate with *the principles of gravitation*, and that according to innumerable ſuppoſitions, reſpecting the law whereby the denſity may be conceived to increaſe.

CHAP. XXXV.

ON THE DENSITIES, QUANTITIES OF MATTER, LIGHT AND HEAT OF THE PLANETS.

Art. 1059. TO meaſure the quantity of matter in diſtant bodies, appears, at firſt ſight, to be a problem of inſuperable difficulty, and ſuch it was before the diſcovery of the laws of gravitation, but thoſe principles led Sir I. NEWTON to a very eaſy ſolution of this important problem, in all thoſe planets which have ſatellites revolving about them, and in the other planets, they alſo furniſh a method by which their quantities of matter may be aſſigned, to a conſiderable degree of accuracy, by the effects which ſuch planets produce upon the others. To underſtand the principle upon which this determination reſts, we may obſerve, that the effect of attraction at equal diſtances will be in proportion to the quantity of matter in the attracting body, and at different diſtances, as the quantity of matter and the inverſe ſquare of the diſtance conjointly. The quantity of matter is alſo in proportion to the magnitude of the body and it's denſity conjointly. If therefore we know the effects of the attraction of different bodies, together with their magnitudes, we can find their denſities, and thence their quantities of matter.

1060. To find their denſities, put

d = the denſity of the central body.
m = it's diameter.
a = it's quantity of matter.
P = the periodic time of the revolving body.
D = the mean diſtance of the revolving body from it's central body.
s = the ſine of the angle under which m appears at the diſtance D, to radius unity.

Then a varies as dm^3, but (818) P^2 varies as $\frac{D^3}{a}$ which varies as $\frac{D^3}{dm^3}$, hence, d varies as $\frac{D^3}{m^3P^2}$. But $s=\frac{m}{D}$, hence, d varies as $\frac{1}{s^3P^2}$, we will therefore aſſume $d=\frac{1}{s^3P^2}$.

For the *Sun* If we take the earth as the revolving body, $P = 365{,}25639$ days, according to M. de la CAILLE, $s = 0{,}0093155 =$ ſin. 32′ 1″,5 the mean apparent diameter of the ſun, hence, $d = \frac{1}{0{,}0093155 \times 365{,}25639} = 9{,}2722$.

For the *Earth*. Here we muſt take the moon for the revolving body, therefore $P = 27{,}32167$ days, according to MAYER, $s = 0{,}033155 =$ ſin. 1° 54′, the mean angle under which the earth's mean diameter appears at the moon, hence, $d = \frac{1}{0{,}033155^3 \times 27{,}32167^2} = 36{,}7569$.

For *Jupiter*. Mr. POUND obſerved the greateſt elongation of it's fourth ſatellite to be 8′. 16″, and the correſponding diameter of Jupiter to be 39″*, hence, the ſine s of the angle under which the diameter of Jupiter appeared at that ſatellite at that time was 0,078629, alſo, $P = 16{,}68898$ days, according to M. WARGENTIN, hence, $d = \frac{1}{0{,}078629^3 \times 16{,}68898^2} = 7{,}3857$.

For *Saturn*. According to Mr POUND, the greateſt elongation of it's fourth ſatellite is 2′. 58″, and the correſponding diameter of Saturn $= 18''$, hence, $s = 0{,}10112$, alſo, $P = 15{,}9454$ days, according to Dr HALLEY, hence, $d = \frac{1}{0{,}10112^3 \times 15{,}9454} = 3{,}8038$

For the *Georgian*. If we take the ſecond ſatellite, we have, according to Dr. HERSCHEL, it's greateſt elongation $= 44'',23$, and the correſponding diameter of the planet $= 3'',90554$, hence, $s = 0{,}0883$, alſo, $P = 13{,}462$ days, hence, $d = \frac{1}{0{,}0883^3 \times 13{,}462^2} = 8{,}0149$.

1061 Theſe denſities of the Sun, Earth, Jupiter, Saturn and the Georgian, are as 0,25226, 1, 0,20093, 0,10349 and 0,21805. The other planets not having any ſatellites revolving about them, their denſities cannot be thus determined, but they may be found, by obſerving the effects which thoſe planets

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* From the times in which the firſt and third ſatellites are in paſſing over the body of Jupiter, Sir I NEWTON computes the diameter of that planet at it's mean diſtance to be 37″,25, which he uſes But M de la GRANGE thinks it is ſafer to truſt to the diameter directly meaſured by a teleſcope, he accordingly ſuppoſes the diameter to be 39″ Sir I. NEWTON reduces the obſerved diameter 18″ of Saturn to 16″, on account of the irradiation of light (1063). From all the obſervations, we have judged 18″ to be the moſt correct value.

planets produce upon the other planets in diſturbing their motion. EULER, however, in his *Recherches ſur les Perturbations des Planets*, obſerving that the denſities of the *Earth*, *Jupiter*, and *Saturn*, were very nearly as the ſquare roots of their mean motions, or inverſely, as $d^{\frac{3}{4}}$, d being their mean diſtance from the ſun, ſuppoſed that the ſame law might hold for all the planets, from whence he eſtimated the denſities of *Mercury*, *Venus* and *Mars*. But the denſity of the *Georgian*, thus determined, does by no means agree with that found from Dr. HERSCHEL's obſervations. M de la GRANGE, in his *Théorie des Var. Sec. des Planets*, in the *Hiſt de l'Acad. Roy. des Scien* 1782, aſſumes the denſities to be inverſely as their diſtances from the ſun, as being the moſt ſimple law, and which, by his calculations, anſwers very nearly for the *Earth*, *Jupiter* and *Saturn*. He computes the diſturbing forces of all the planets upon that ſuppoſition, and from the agreement of the reſults with obſervation, he ſees no reaſon for changing his hypotheſis This gives the denſities of Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian as 2,5833, 1,3825, 1, 0,6563, 0,20155, 0,11215, and 0,052077 As however the denſity of the Georgian, deduced from it's ſecond ſatellite, by no means agrees with the above law, we may conclude that that law for the denſities is not generally true, as it cannot be ſuppoſed that Dr HERSCHEL ſhould have erred ſo much in his obſervations. M. de la LANDE makes the denſity of Venus 1,0379, as beſt anſwering to the motion of the ſun's apogee, of the aphelion of the orbit of Mercury, and of the nodes of Mercury From a diminution of the inclination of the equator to the ecliptic of 50″ in 100 years, Dr MASKELYNE has determined the denſity of Venus to be 1,024, that of the earth being unity. Dr HERSCHEL makes the time of the rotation of Mars to be 24,656 hours, and the ratio of the diameters as 16 : 15 Hence (983), the denſity of Mars is about 0,07, that of the earth being unity But from ſome obſervations of Dr. MASKELYNE upon this planet, he has reaſon to think that the ratio of it's diameters is much nearer to a ratio of equality than that given above, which renders the denſity, thus deduced, ſubject to great uncertainty. We will therefore aſſume the denſities of the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian, as 0,25226, 2,5833, 1,024, 1, 0,6563, 0,20093, 0,10349 and 0,21805

1062. By Art 1038 the denſity of the moon : denſity of the ſun as 2,44 : 1, and the denſity of the ſun being to that of the earth as 0,252 : 1, it follows, that the denſity of the moon : denſity of the earth : 0,6149 : 1. Hence, in proportion to the above denſities of the planets, the moon's denſity would be 0,6149.

To find the Ratio of the Diameters of the Sun and Planets.

1063. Dr. Herschel found the diameter of the *Georgian*, at it's mean diſtance, to be 3″,9554; if therefore it were ſeen at the mean diſtance of the earth from the ſun, it would be 74″,52. Mr. Pound found the diameter of *Saturn*, at it's mean diſtance from the earth, to be 18″, and therefore if it were ſeen at the mean diſtance of the earth from the ſun, the diameter would be 171″,71 He alſo found the diameter of *Jupiter*, at it's mean diſtance, to be 39″, hence, at the mean diſtance of the earth from the ſun, it would be 202″,82. M. Picard and Flamstead found the diameter of *Mars* to be 30″ when it's diſtance from the earth was 0,3815, the mean diſtance of the earth from the ſun being unity, hence, if Mars were ſeen at the earth's mean diſtance from the ſun, it's diameter would be 11″,4. Dr. Herschel makes it 8″,94, which is probably the moſt accurate. The apparent diameter of the *Earth* ſeen from the ſun is twice the ſun's horizontal parallax, or 17″,5 (629). In the tranſit of *Venus* over the ſun in 1761, M. de la Lande, from his own obſervations, and thoſe of Mr. Short, found it's diameter to be 57″,8, hence, the diameter of Venus, ſeen at the mean diſtance of the earth from the ſun, would be 16″,7 The diameter of *Mercury*, meaſured by Dr. Bradley in 1723, in it's tranſit over the ſun's diſc, with a micrometer to Huygen's teleſcope of 120 feet long, was found to be 10,″75, hence, it's diameter, at the mean diſtance of the earth, will be 7″,27. M de la Lande, from the tranſit in 1753, found it to be 6″,5, we will therefore take it 7″. The diameter of the ſun in it's apogee ſeen from the earth is 31′. 29″,2, according to Dr. Maskelyne, according to Mr. Short, 31′. 28″, according to M. de la Lande, 31′. 30″,5, Dr. Maskelyne's is a mean between the other two, we will therefore ſtate it at 31′ 29″, and as the difference of the diameters in it's apogee and perigee is 1′. 5″, if we add it's half, 32″,5, to 31′ 29″ we have 32′ 1″,5 for it's mean diameter. Sir I Newton ſuppoſes that there is a ſenſible aberration in all teleſcopes, which makes the image of the object in the focus of a teleſcope greater than it ought to be. He obſerves, that this aberration has a leſs ratio to the diameter of *Jupiter* in long than in ſhort teleſcopes; the latter therefore will give the greater diameter. What reduction muſt be made from the meaſured diameter by any teleſcope in order to get the real diameter, it is not eaſy to ſay. M de la Lande thinks you may allow 5″ for a ſmall teleſcope not very perfect. This will make the diameter nearly the ſame as that uſed by M. Cassini in his Tables, who probably had not a very good teleſcope for that purpoſe. On the contrary, when a planet appears *on* the ſun, it's diameter, meaſured by a teleſcope, appears leſs than it is, owing to the irradiation of the ſun. The diameter of the moon in 1748, meaſured upon the diſc of the ſun, appeared 6″ leſs than when meaſured off the ſun. Hence, the diameters of the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian are as 109,8, 0,4, 0,9543, 1, 0,5109, 11,593, 9,812 and 4,258.

1064. We have found the denſities (1060), and (1063) apparent diameters of all the planets ſeen at the mean diſtance of the earth from the ſun, which muſt repreſent the ratio of their real diameters, and as the quantities of matter in ſpherical bodies are as the cubes of their diameters and denſities conjointly, we find the ratio of the quantities of matter in the Sun, Mercury, Venus the Earth, Mars, Jupiter, Saturn and the Georgian as 333928, 0,16[illegible] 0,88993, 1, 0,08752, 312,101, 97,762 and 16,837

1065 The diameter of the earth is to that of the moon as 11 [illegible] 3 [illegible] 1 as 1 0,2727, therefore the magnitude of the earth [illegible] that of the [illegible]on [illegible] 1 ,02028, or very nearly as 49 [illegible] 1, and their denſities (1062) a[illegible] as 1 0,6149, therefore the quantity of matter in the earth [illegible] that of the [illegible]on : 1 0,1245 If we aſſume, with ſome Authors, the denſity of the m[illegible]n to that of the ſun as 2,5 · 1, the quantity of matter in the earth [illegible] that [illegible] the moon : 78 [illegible] 1, or 1 . ,0128 Alſo, the gravity of a body upon the [illegible]th is to that on the moon as 1 : 0,1677.

To find the relative Weights of Bodies upon the Surfaces of the Planets.

1066. The weight of a given body on a planet muſt be in proportion to the force with which the planet attracts it, the weight being the effect ariſing from that cauſe. Now when the force varies inverſely as the ſquare of the diſtance, if a body be at the ſurface of the ſphere, the attraction (837) varies as the diameter and denſity conjointly, conſequently the weights of equal bodies on the ſurfaces of different planets, vary as the radii and denſities conjointly, or as the diameters and denſities Hence, the weights of equal bodies on the ſurfaces of the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian are as 27,7, 1,0333, 0,9771, 1, 0,3355, 2,3287, 1,0154 and 0,9285, theſe numbers therefore repreſent the forces of gravity upon the ſurfaces of theſe reſpective bodies.

1067. The

1067. The following Table exhibits the relation of the denſities, diameters, quantities of matter, and gravity on the ſurface of the reſpective bodies.

Planets	Denſities.	Diameters.	Quantities of Matter	Grav. on Surf
Sun	0,25226	109,8	333928	27,7
Mercury	2,5833	0,4	0,16536	1,0333
Venus	1,024	0,9543	0,88993	0,9771
Earth	1	1	1	1
Mars	0,6563	0,5109	0,08752	0,3355
Jupiter	0,20093	11,59	312,101	2,3287
Saturn	0,1[illegible]49	9,812	97,762	1,0154
Georgian	0,21805	4,258	16,837	0,9285
Moon	0,6149	0,2727	0,01245	0,1677

1068. The intenſities of light and heat which the planets receive from the ſun, vary inverſely as the ſquares of their diſtances from the ſun. For let L be a point from which light or heat diverges, abc a triangle upon which they fall, produce La, Lb, Lc to A, B, C, and let AB, BC, AC be reſpectively parallel to ab, bc, ac; then the triangle abc is ſimilar to ABC, and the ſame quantity of light or heat (ſuppoſed to proceed in ſtraight lines) which fall on abc, would, if that plane were removed, fall on ABC, and there occupying a greater ſpace, the intenſity muſt be ſo much leſs in proportion as the ſpace is greater, hence, the intenſity on abc : the intenſity on ABC :: $ABC : abc :: AB^2 : ab^2 :: LB^2 : Lb^2$. To apply this to the ſun and planets, we have (217) the diſtances of Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgian from the ſun as 4, 7, 10, 15, 52, 95 and 190, the inverſe ſquares of which are as $\frac{10^2}{4^2}$, $\frac{10^2}{7^2}$, 1, $\frac{10^2}{15^2}$, $\frac{10^2}{52^2}$, $\frac{10^2}{95^2}$ and $\frac{10^2}{190^2}$, or as 6,25, 2,04, 1, 0,44375, 0,036875, 0,01106 and 0,00276 the relative intenſities of light and heat which the reſpective planets receive from the ſun.

FIG. 233.

1069. The

1069. The apparent diameter of a body is inverſely as it's diſtance. Aſſuming therefore the mean diameter of the ſun $= 32'$, we have the apparent diameter of the Sun at Mercury $= \frac{5}{2} \times 32' = 80'$, at Venus $= \frac{10}{7} \times 32' = 45',7$, at Mars $= \frac{10}{15} \times 32' = 21',33$, at Jupiter $= \frac{10}{52} \times 32' = 6',15$, at Saturn $= \frac{10}{95} \times 32' = 3',37$, and at the Georgian $= \frac{10}{195} \times 32' = 1',64$. Hence, the apparent diameter of the ſun at the Georgian is only about 2½ times greater than the apparent diameter of Jupiter ſeen from the earth at it's mean diſtance.

The following Table exhibits the relative intenſities of light and heat at the different planets, and the apparent diameter of the ſun ſeen from them.

Planets.	Intenſities of Light and Heat	Apparent Diameter of the Sun
Mercury	6,25	80′
Venus	2,04	45,7
Earth	1	32
Mars	0,44375	21,33
Jupiter	0,036875	6,15
Saturn	0.01106	3,37
Georgian	0,00276	1,64

CHAP. XXXVI.

ON THE MOTION OF THE PLANES OF THE ORBITS OF THE PLANETS, FROM THEIR MUTUAL ATTRACTIONS.

Art 1070. BY comparing the modern with the ancient obſervations, it appears that the latitudes of the fixed ſtars vary, and that the mean inclination of the ecliptic to the equator gradually diminiſhes; the former can ariſe only from an alteration in the poſition of the ecliptic. M. GODIN, in a Treatiſe on the obliquity of the ecliptic, judged that it's diminution was owing to a change in the ecliptic. He compared the poſition of the nodes of *Jupiter's* orbit, obſerved 241 years before CHRIST, with that obſerved by M. de la HIRE; and ſuppoſing the plane of Jupiter's orbit not to be changed, he concluded the ecliptic muſt. His conjecture, partly true, led him to aſſign the true cauſe of the diminution. KEPLER and TYCHO obſerved that the latitude of the ſtars was ſubject to a change, the former concluded that it was owing to a change in the poſition of the ecliptic, and that it aroſe from ſome phyſical cauſe, he ſuſpected that it might ariſe from the rotation of the ſun. But after Sir I. NEWTON had eſtabliſhed the doctrine of univerſal gravitation, it was evident that the planets muſt diſturb each other's motions, the conſequence of which muſt be, that as their orbits are inclined to the ecliptic, they muſt tend to diſturb the motion of the earth in the plane of it's orbit, and therefore ſubject the ecliptic to a change in it's poſition. EULER firſt computed theſe effects on the earth, and found that they would ſolve the above phænomena, and alſo give the variation of the inclination of their orbits, and retrograde motion of their nodes. The method here given of inveſtigating theſe matters, is ſimilar to that by which we determined the motion of the moon's nodes.

FIG 234. 1071. Let $N\mathcal{Q}n$ be the orbit of the body $\mathcal{Q}$ inclined to NAn the plane of the orbit of the body P, NSn the line of the nodes, draw PAS, and Aa perpendicular to the plane $N\mathcal{Q}n$, and AK, aK perpendicular to Nn, then AKa is the inclination of the orbit of $\mathcal{Q}$ to that of P; produce Sa to v, and draw Pv parallel to Aa, and conſequently perpendicular to the plane $N\mathcal{Q}n$, join $P\mathcal{Q}$, $v\mathcal{Q}$, which latter will lie in the plane $N\mathcal{Q}n$, becauſe v is in that plane, and draw $\mathcal{Q}u$ perpendicular to PS, and $\mathcal{Q}L$ to Nn. Put $SP=a$, $S\mathcal{Q}=b$, $Su=x$, and $s=$ ſin. AKa, alſo, let $1 : m ::$ quantity of matter in P : that in the ſun S.

Now

Now if SA repreſent the force of S towards P, then Aa is that part of the force which acts perpendicularly to the plane NQn, and $Aa = s \times AK$; but the force of S to P varies as $\frac{1}{a^2}$, and not as SA; hence, $SA : \frac{1}{a^2} :: s \times AK :$ that part of the force of P upon S which acts perpendicularly to the plane of the orbit of $Q = \frac{s}{a^2} \times \frac{AK}{SA} = \frac{s}{a^2} \times$ ſin NSA to radius unity. Now any lines mn, zx parallel to Pv will be perpendicular to the plane NQn. Hence, if Sm, Qz repreſent the forces of S to P and Q to P, then mn, zx will repreſent that part of each force which acts perpendicularly to the plane NQn. Let therefore Sm repreſent $\frac{1}{a^2}$, then mn will repreſent $\frac{s}{a^2} \times$ ſin. NSA, and let alſo Qz repreſent $\frac{1}{PQ^2}$, hence, and by ſimilar triangles,

$$Pv : SP :: mn : Sm :: \frac{s}{a^2} \times \text{ſin } NSA : \frac{1}{a^2} :: s \times \text{ſin. } NSA : 1$$

$$PQ : Pv :: Qz = \frac{1}{PQ^2} : zx$$

$$\therefore PQ : SP :: \frac{s}{PQ^2} \times \text{ſin. } NSA : zx = \frac{s \times SP}{PQ^3} \times \text{ſin } NSA = \frac{sa}{\left(a^2+b^2-2ax\right)^{\frac{3}{2}}} \times \text{ſin. } NSA,$$

which is that part of the force of P on Q which acts perpendicularly to the plane NQn. Hence, $s \times$ ſin. $NSA \times \overline{\frac{a}{\left(a^2+b^2-2ax\right)^{\frac{3}{2}}} - \frac{1}{a^2}} =$ the difference of the forces by which S and Q are drawn from the orbit NQn, or the whole force with which Q is drawn from the plane of it's orbit about S. Now $\frac{m}{b^2}$ is the force of Q towards S, hence, if Qw be an indefinitely ſmall arc, we have (as in Art. 927) $\frac{m}{b^2} : \overline{s \times \text{ſin } NSA \times \overline{\frac{a}{\left(a^2+b^2-2ax\right)^{\frac{3}{2}}} - \frac{1}{a^2}}} :: \frac{Qw^2}{b} :$ $\frac{s \times b \times Qw^2 \times \text{ſin. } NSA}{m} \times \overline{\frac{a}{\left(a^2+b^2-2ax\right)^{\frac{3}{2}}} - \frac{1}{a^2}}$ the velocity generated by the force drawing the body Q from the plane of it's orbit, the velocity in the orbit being Qw; hence, this latter velocity : the former :: 1 : $\frac{s \times b \times Qw \times \text{ſin. } NSA}{m}$ $\times \overline{\frac{a}{\left(a^2+b^2-2ax\right)^{\frac{3}{2}}} - \frac{1}{a^2}}$.

1072. Draw

1072 Draw wt perpendicular to the plane NQn, and take it to Qw as $\frac{s \times b \times Qw \times \text{ſin } NSA}{m} \times \overline{\frac{a}{\overline{a^2+b^2-2ax}|^{\frac{3}{2}}} - \frac{1}{a^2}}$ to 1, and draw the orbit $N'Qtn'$, and nn' will be the cotemporary motion of the nodes Draw $n's$ perpendicular to nQ, then Qw wt ſin Qs, or Qn, which is QL, $n's = \frac{wt \times QL}{Qw}$, and ſin snn' (s) $ns = \frac{wt \times QL}{Qw}$. rad. $= 1 : nn' = \frac{wt \times QL}{s \times Qw} = \frac{b \times Qw \times QL \times \text{ſin } NSA}{m} \times \overline{\frac{a}{\overline{a^2+b^2-2ax}|^{\frac{3}{2}}} - \frac{1}{a^2}}$.

1073 Draw Dd perpendicular to PAS, then QL is the ſin. QSN $=$ ſin. $\overline{QSD \pm NSD} =$ ſin. $QSD \times$ coſ $NSD \pm$ ſin. $NSD \times$ coſ. $QSD =$ ſin $QSD \times$ ſin $NSA \pm$ coſ. $NSA \times$ coſ QSD Now in a whole revolution of Q, the laſt term will be deſtroyed by the oppoſition of ſigns in the oppoſite quadrants, and therefore to get the *mean* motion of the nodes in a revolution, we may neglect that term, hence, if we ſubſtitute ſin $QSD \times$ ſin NSA for QL, put $b = 1$, and x for ſin QSD, we have $nn' = \frac{b \times Qw \times x \times \overline{\text{ſin } NSA}^2}{m} \times \overline{\frac{a}{\overline{a^2+1-2ax}|^{\frac{3}{2}}} - \frac{1}{a^2}}$ But to determine the value of this quantity for a whole revolution of Q, we muſt expand $\frac{1}{\overline{a^2+1-2ax}|^{\frac{3}{2}}}$ and multiply it by x, and then take thoſe terms only which contain the even powers of x, for in oppoſite quadrants x having a different ſign, the odd powers will deſtroy each other in a revolution. Now by Sir I NEWTON's Binomial Theorem, $\overline{P+PQ}|^{\frac{r}{n}} = P^{\frac{r}{n}} + \frac{r}{n} AQ + \frac{r-n}{2n} BQ + \frac{r-2n}{3n} CQ +$ &c. where A, B, C, &c repreſent the preceding term, and to make the expanſion of the above quantity converge, the powers of a muſt ſtand in the denominators, hence, $P = a^2$, $Q = \frac{1}{a^2} - \frac{2x}{a}$, $r = -3$, $n = 2$, hence, by expanding the above quantity and multiplying each term by x, and taking only the even powers of x, we have $nn' = \frac{Qw \times \overline{\text{ſin } NSA}^2}{m \times a^3} \times \left(3x^2 - \frac{15x^2}{2a^2} + \frac{35x^4}{2a^2} + \frac{105x^2}{8a^4} - \frac{315x^4}{4a^4} + \frac{693x^6}{8a^4} - \frac{315x^2}{16a^6} + \frac{3465x^4}{16a^6} - \frac{9009x^6}{16a^6} + \frac{6435x^8}{16a^6}\right)$. But by the principles of plane Trigonometry

Trigonometry (ſee CRAKELT's *Trig.*), $x^2=\frac{1}{2}-\frac{1}{2}$ coſ. $2QSD$, $x^4=\frac{3}{8}-\frac{1}{2}$ coſ. $2QSD+\frac{1}{8}$ coſ. $4QSD$, $x^6=\frac{5}{16}-\frac{15}{32}$ coſ. $2QSD+\frac{3}{16}$ coſ $4QSD-\frac{1}{32}$ coſ $6QSD$, $x^8=\frac{35}{128}-\frac{7}{16}$ coſ. $2QSD+\frac{7}{32}$ coſ. $4QSD-\frac{1}{16}$ coſ. $6QSD+\frac{1}{128}$ coſ. $8QSD$, and in a whole revolution of Q, the coſines of $2QSD$, $4QSD$, $6QSD$, $8QSD$, &c. &c. will deſtroy each other by the oppoſition of ſigns in the oppoſite quadrants, and therefore to get the *mean* motion of Q in one revolution, we may neglect thoſe quantities, and ſubſtitute only $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$, $\frac{35}{128}$, for x^2, x^4, x^6, x^8, hence, by collecting the terms, and ſubſtituting 360° for Qw, we have $nn'=\frac{3\times360^\circ\times\overline{\text{ſin } NSA}^2}{2m\times a^3}\times\overline{1+\frac{15}{8a^2}+\frac{175}{64a^4}+\frac{3675}{1024a^6}}$ for the mean motion of the nodes in one revolution of Q, the place of P being given. Now according to the increaſe of the terms 1, $\frac{15}{8}$, $\frac{175}{64}$, $\frac{3675}{1024}$, we may aſſume the next term $\frac{9}{2}$, hence, we get $nn'=\frac{3\times360^\circ\times\overline{\text{ſin. } NSA}^2}{2ma^3}\times\overline{1+\frac{15}{8a^2}+\frac{175}{64a^4}+\frac{3675}{1024a^6}+\frac{9}{2a^8}}$. If any further terms ſhould be required, the law to which they are obſerved to approach, will enable the computer to ſupply them to a very conſiderable degree of accuracy. This is neceſſary, when the difference of the diſtances SP, SQ is ſmall in reſpect to thoſe diſtances. As the $\overline{\text{ſin } NSA}^2=\frac{1}{2}-\frac{1}{2}$ coſ. $2NSA$, if we ſubſtitute this quantity for $\overline{\text{ſin. } NSA}^2$, we may, for the ſame reaſon as above, neglect $\frac{1}{2}$ coſ $2NSA$ for one revolution of P, conſequently the mean motion nn' of the nodes will be $\frac{3\times360^\circ}{4ma^3}\times\overline{1+\frac{15}{8a^2}+\frac{175}{64a^4}+\frac{3675}{1024a^6}+\frac{9}{2a^8}}$. This is the *mean* motion of the node of an inferior planet upon the orbit of a ſuperior, from the action of that ſuperior.

1074. To get the motion of the node of a ſuperior planet from the action of an inferior, a becomes leſs than b, or leſs than unity, therefore the powers of a muſt go in the numerators, hence, $P=1$, $Q=a^2-2ax$, and, by a ſimilar proceſs, we get $nn'=\frac{3\times360^\circ\times a^3}{4m}\times\overline{1+\frac{15a^2}{8}+\frac{175a^4}{64}+\frac{3675a^6}{1024}+\frac{9a^8}{2}}$. This is the

the *mean* motion of the node of a ſuperior planet upon the orbit of an inferior, from the action of that inferior.

To find how much the nodes of the orbit of *Jupiter* go back upon the orbit of *Saturn*, from the diſturbing force of Saturn. Here we have $\frac{1}{m} = \frac{97,762}{333928}$, $a = \frac{95}{52}$, hence, in one revolution of Jupiter, $nn' = 113''$, or at the rate of about 9″,9 in a year. Euler makes it 10″

To find how much the nodes of the orbit of Saturn go back upon the orbit of Jupiter, from the diſturbing force of Jupiter. Here we have $\frac{1}{m} = \frac{312,101}{333928}$, $a = \frac{52}{95}$, hence, in one revolution of Saturn, $nn' = 659''$, or at the rate of about 22″ in a year. Euler makes it about 18″.

The Motion of the Nodes of the Ecliptic upon the Orbits of all the Planets, from the Attractions of the reſpective Planets.

To find how much the nodes of the ecliptic go back upon the orbit of the *Georgian*, from the diſturbing force of the Georgian. Here we have $\frac{1}{m} = \frac{16,837}{333928}$, $a = 19$; hence, in a year $nn' = 26''''$; therefore the ſecular motion is 43‴.

To find how much the nodes of the ecliptic go back upon the orbit of *Saturn*, from the diſturbing force of Saturn. Here we have $\frac{1}{m} = \frac{97,762}{333928}$, $a = 9,5$; hence, in a year $nn' = 20''',34$, therefore the ſecular motion is 34″. Euler makes it 37″.

To find how much the nodes of the ecliptic go back upon the orbit of *Jupiter*, by the diſturbing force of Jupiter. Here we have $\frac{1}{m} = \frac{312,101}{333928}$, $a = 5,2$; hence, in a year $nn' = 6'',93$; therefore the ſecular motion is 693″. Euler makes it 695″.

To

To find how much the nodes of the ecliptic go back upon the orbit of *Mars*, from the attraction of Mars. Here we have $\frac{1}{m} = \frac{0,08752}{333928}$, $a=1,5$; hence, in a year $nn'=13''',4$; therefore the ſecular motion is $22''$

To find how much the nodes of the ecliptic go back upon the orbit of *Venus*, from the diſturbing force of Venus. Here we have $\frac{1}{m'} = \frac{0,88993}{333928}$, $a=0,72$; hence, in a year $nn'=5'',58$; therefore the ſecular motion is $558''$. EULER makes it $533''$.

To find how much the nodes of the ecliptic go back, by the action of *Mercury*, upon the orbit of Mercury. Here we have $\frac{1}{m} = \frac{0,16536}{333928}$, $a=,38$, hence, in a year $nn'=6''$, therefore the ſecular motion is $10''$.

1075. The inclination of the orbit of *Q* to that of *P* will be ſubject to a like variation with the inclination of the moon's orbit to the ecliptic, and by the very ſame reaſoning it appears, that the *mean* inclination of the two orbits will never vary. It is therefore unneceſſary to give the inveſtigation, as it would only be a repetition of what we have already very fully explained.

1076. Let *AN*, *BN* be the orbits of the two planets, *CE* the ecliptic, and let us call the planets *A* and *B* reſpectively, then (1073, 1074) the attraction of *A* upon *B* will make the nodes of the orbit of *B* go back upon the orbit of *A*, and the attraction of *B* upon *A* will make the nodes of the orbit of *A* go back upon the orbit of *B*, but (1075) the mean inclination of the two orbits will remain the ſame. Hence, if *NA*, *NB* be the orbits of two planets *A* and *B*, moving towards *N*, the attraction of *B* upon *A* will bring the orbit *AN* into the poſition *Av*, and the attraction of *A* upon *B* will bring the orbit *BN* into the poſition *Bw*, ſo that from their mutual attractions, the node will be brought to *n*, and the angle at *n* will be equal to the angle at *N*. But in reſpect to the ecliptic, the nodes of the orbit of *A* in FIG. 235. go backwards, and the nodes of the orbit of *B* go forwards; but the contrary in FIG. 236. Now it is manifeſt from the figures, that, as the points *A* and *B*, about which each orbit revolves, are 90° from the node *N*, the node of the orbit of *A* will be direct or retrograde upon the ecliptic, according as *zN* is leſs or greater than 90°, and the node of the orbit of *B* will be direct or retrograde, as *xN* is greater or leſs than 90°. Now, whether *zN*, *xN* be greater or leſs than 90°, FIG 235. 236.

may

may be known from the triangle Nxz*, where we know the angles x and z, the inclination of the two orbits, and xz the diſtance of their nodes. Now to determine the motion of the node in any given time, we have, by the variation of the triangle zNz (ſee CRAKELT's *Trig.*) ſin. z : cot. zN :: var. z : $rz = \frac{\text{var. } z \times \cot. zN}{\sin. z}$ = (as var. $z = Nv \times$ ſin. $x \times$ ſin. xz) $\frac{Nv \times \sin. x \times \sin. xz \times \cot. zN}{\sin.^2 z}$ the motion of the node of the orbit AN upon the ecliptic, in the time the node moves through Nv upon the orbit of B.

Ex. Let A repreſent *Mars*, and B *Jupiter*; then $xz = 50°.\ 22'$, the angle $Nxz = 178°.\ 41'$, and the angle $Nzx = 1°\ 51'$, alſo $Nv = 14'',2$ according to M. de la LANDE (found by Art. 1073), hence, $Nz = 135°.\ 5'$, which being greater than 90°, ſhows that the node of Mars is retrograde upon the ecliptic from the action of Jupiter, hence, $rz = \frac{14'',2 \times \sin.\ 178°\ 41' \times \sin.\ 50°\ 22' \times \cot.\ 135°.\ 5'}{\sin.\ 1°\ 51'}$ $= 7'',83$ motion of the node in a year, therefore 13° 3′ is the ſecular motion. As $Nx = 44°.\ 55'$, the node of Jupiter is regreſſive.

1077. The angles at N and v being equal (1075), the interſection A about which the orbit moves muſt be 90° from N or v. Hence, as the two orbits AN, Av, and BN, Bw, diverge from each other in each direction for 90° from A and B, it is manifeſt, that in each caſe the angle at r is leſs than that at z, and therefore the inclination upon the ecliptic is diminiſhed; but the angle at s is greater than that at x, and therefore the inclination upon the ecliptic is increaſed. Hence, by placing the nodes in all their different poſitions, we deduce this rule, given by M. de la LANDE. *Whenever the node of the orbit of the planet which attracts, is forwarder than the node of that which is attracted, the inclination of the orbit of the attracted body to the ecliptic is diminiſhed, if the diſtance of the nodes be leſs than 180°, otherwiſe, it is increaſed.* We here mean the ſame node, that is, the aſcending or deſcending one. Now if we arrange the planets according to the ſituation of their nodes, beginning with that whoſe node is forwardeſt, they will ſtand thus. Saturn, Georgian, Jupiter, Venus, Mars, and Mercury. Hence,

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* By ſpherical Trigonometry (CAGNOLI, *Traité de Trig.* p. 272.) tan. $\frac{1}{2}.\overline{Nz \sim Nx}$ = tan. $\frac{1}{2}.zx \times \frac{\sin.\ \frac{1}{2}.\overline{Nxz \sim Nzx}}{\sin.\ \frac{1}{2}.\overline{Nxz + Nzx}}$, and tan. $\frac{1}{2}.\overline{Nz + Nx}$ = tan. $\frac{1}{2}.zx \times \frac{\cos.\ \frac{1}{2}.\overline{Nxz \sim Nzx}}{\cos.\ \frac{1}{2}.\overline{Nxz + Nzx}}$; hence, finding half the ſum and difference of Nz, Nx, we get Nz and Nx. Thus we at once determine, whether the node of each is progreſſive or regreſſive.

Hence, *Saturn* diminiſhes the inclination of all the other orbits, *Jupiter* increaſes that of Saturn, and diminiſhes all the reſt, and ſo on. This is upon ſuppoſition that the ecliptic is immoveable.

1078. Now to find how much the inclination of each varies in a given time, we may conſider the triangle Nzx to vary and become vrx, where the two angles x and N remain conſtant, hence, by ſpherical Trigonometry (CRAKELT's *Trig.*), variat. $\angle z = Nv \times$ ſin $x \times$ ſin xz.

Ex. The node of *Mars* goes back upon the orbit of *Jupiter* 14″,2 in a year, alſo the angle NxE for Jupiter is 1°. 19′, and the diſtance zx of their nodes = 50°. 22′, hence, Jupiter diminiſhes the inclination of the orbit of Mars by a quantity = 14″,2 × ſin. 1° 19′ × ſin. 50°. 22′ = 0″,248 in a year, or 24″,8 in 100 years.

1079. The following Tables contain the annual movement of the nodes, and the ſecular change of the inclinations of the orbit of each planet from the attraction of the reſt, according to the theory of M. de la GRANGE.

ANNUAL MOTION OF THE NODES.

By the Action of	Mercury	Venus	Mars	Jupiter	Saturn
Mercury	− 0″,10	+ 0″,16	− 0″,32	− 0″,31	− 0″,11
Venus	− 5, 57	− 7, 46	−11, 80	−17, 56	− 8, 06
Earth	− 0, 87	− 6, 69	− 1, 77	− 0, 01	− 0, 00
Mars	− 0, 14	− 0, 29	− 0, 43	− 0, 39	− 0, 14
Jupiter	− 2, 18	− 5, 13	−11, 0	− 6, 95	−12, 28
Saturn	− 0, 12	− 0, 09	− 0, 47	+ 5, 88	− 0, 34
Total	− 8, 98	−19, 50	−25, 79	−19, 34	−20, 93
Preceſſion	50, 25	50, 25	50, 25	50, 25	50, 25
Mot. in long.	41, 27	30, 75	24, 46	30, 91	29, 32

The firſt perpendicular line ſhows the body which attracts, and the firſt horizontal line, the body which is attracted. When we ſee, for inſtance, that Mars attracts Mars − 0″,43, it is not that Mars attracts itſelf, but it diſplaces

the

the ecliptic, and makes the nodes of Mars move −0″,43 upon it. And by ſubtracting the regreſſion of the nodes from the preceſſion of the equinoxes, we get the motion of the nodes in longitude. In order to adapt the effect of Venus in the ſecond horizontal line to what it would be upon *our* ſuppoſition of it's denſity, we muſt diminiſh all the numbers in the ratio of 1,3825 : 1,024.

SECULAR CHANGE OF THE INCLINATIONS.

By the Action of	Mercury	Venus	Mars	Jupiter	Saturn
Mercury	0″,00	+1″,94	− 0″,05	− 0″,95	− 1″,10
Venus	+ 9, 46	0, 00	+17, 95	−17, 67	−26, 65
Mars	+ 0, 06	−0, 42	0, 00	− 1, 06	− 1, 25
Jupiter	+ 9, 87	+2, 60	−13, 20	0, 00	+ 5, 89
Saturn	+ 1, 04	+0, 35	− 1, 25	− 7, 51	0, 00
Total	+20, 43	+4, 47	+ 3, 45	−27, 19	−23, 11

We muſt diminiſh the ſecond horizontal line as before, in order to adapt it to the denſity which we have aſſumed. This ſecular change of inclination takes into conſideration the diſplacement of the ecliptic.

On the Change of the Plane of the Ecliptic

FIG. 237. 1080 Let ♈ *L* be the poſition of the ecliptic at any given time, *BVA* the orbit of *Venus*, ♈ *B* the equator. Then as the attraction of Venus upon the earth cauſes the orbit of the earth to go back upon the orbit of Venus, let *rcV* be the next poſition of the ecliptic. Now as the longitude ♈ *A* of the node of Venus's orbit is about 2ˢ. 14½°, and the angle $rAV = rVB$, becauſe (1075) the inclination of the two orbits is not altered, we have $Ar = 90°$, therefore *r* lies backwards beyond ♈, conſequently the ecliptic has gone forwards upon the equator from ♈ to *c*; and from the motion of the ecliptic, the latitude and longitude of the ſtars will be affected. Hence, ſuppoſing the triangle ♈*BA* to become *cBV*, and the angles *B* and *V* to remain conſtant, we have (as in Art.

Art. 1076.) $\text{♈}c = \frac{AV \times \text{ſin. } B \times \text{ſin } \text{♈}B \times \text{cot } \text{♈}A}{\text{ſin } \text{♈}}$ = (becauſe ſin. $B \times$ ſin. $\text{♈}B$ = ſin. $A \times$ ſin. $\text{♈}A$, and ſin. $\text{♈}A \times$ cot. $\text{♈}A$ = coſ. $\text{♈}A$) $\frac{AV \times \text{ſin } A \times \text{coſ. } \text{♈}A}{\text{ſin } \text{♈}}$ the progreſſion of the equinoctial points upon the equator, from the action of Venus. Now to find the value of this upon the ecliptic, draw co perpendicular to $\text{♈}L$, and then we ſhall have $\text{♈}o = \frac{AV \times \text{ſin } A \times \text{coſ } \text{♈} \times \text{coſ. } \text{♈}A}{\text{ſin } \text{♈}} = AV \times \text{ſin. } A \times \text{cot. } \text{♈} \times \text{coſ. } \text{♈}A$ the progreſſion of the equinoctial points upon the ecliptic. For *Jupiter*, *Saturn*, and the *Georgian*, A lies above three ſigns from ♈ and therefore r lies on the other ſide of ♈ as in FIG. 238. therefore c and o lie on the other ſide of ♈; conſequently they cauſe the equinoctial points to go *backwards*, but $\text{♈}A$ being leſs than three ſigns for all the other planets, they cauſe the equinoctial points to go *forwards*. FIG. 238.

1081. Now to find the variation of the angle ♈, we have, (as in Art. 1076) variat. $\angle \text{♈} = AV \times \text{ſin. } B \times \text{ſin. } \text{♈}B$ = (becauſe ſin $B \times$ ſin $\text{♈}B$ = ſin $A \times$ ſin. $\text{♈}A$) $AV \times \text{ſin. } A \times \text{ſin } \text{♈}A$. Now as Vr is 90°, Vr is converging to Ar, and therefore the angle BcV is leſs than $B\text{♈}A$, conſequently Venus diminiſhes the obliquity of the ecliptic And as $\text{♈}r$ is leſs than 90° for all the planets, they *all* tend to diminiſh the obliquity of the ecliptic When, by the motion of the equinoxes and node A, $\text{♈}A$ becomes greater than 180°, it's ſign becomes negative, and the obliquity will be increaſed Hence, when the longitude of the aſcending node of a planet's orbit is *leſs* than 180°, it *diminiſhes* the obliquity of the ecliptic, but when *greater* than 180°, it *increaſes* it

1082. To find the variation in latitude of any ſtar s Draw Svw perpendicular to Vr, and sx to rA. Then as $\text{♈}c = \frac{AV \times \text{ſin } A \times \text{coſ } \text{♈}A}{\text{ſin } \text{♈}}$, therefore $co = AV \times \text{ſin. } A \times \text{coſ } \text{♈}A$, and $co \,.\, vw \cdot \text{ſin. } r\text{♈} \quad \text{ſin } rw \quad \text{coſ. } A\text{♈} \,.\, \text{coſ } Av$, hence, $vw = AV \times \text{ſin } A \times \text{coſ } Av$, *the variation of latitude* FIG. 237.

1083. To find the variation of longitude From the variation of the right angled triangle srx, whoſe hypothenuſe sr is conſtant, we find (1047) $vx = \frac{vw \times \text{tan } vs}{\text{tan. } xr}$, hence, $vx = AV \times \text{ſin. } A \times \text{ſin. } Ax \times \text{tan lat.} = AV \times \text{ſin. } A \times$ (ſin of longitude of the ſtar – longitude of the node of the planet's orbit) × tan. of the ſtar's latitude. This is one part of the variation, but as the ecliptic has moved upon the equator from ♈ to c, the longitude of all the ſtars will be alſo altered by $\text{♈}o = AV \times \text{ſin. } A \times \text{cot. } \text{♈} \times \text{coſ } \text{♈}A$. Hence, the whole *variation in longitude* $= AV \times \text{ſin. } A \times \text{ſin } \overline{x - \text{♈}A} \times \text{tan lat.} - AV \times \text{ſin } A \times \text{cot. } \text{♈} \times \text{coſ. } \text{♈}A$, where regard muſt be had to the ſigns of ſin. $\overline{\text{♈}x - \text{♈}A}$, tan. lat.

lat. and coſ ♈A, the laſt of which changes for *Jupiter*, *Saturn* and the *Georgian*, becauſe for theſe planets, ♈A is greater than 90°. If the ſtar be very near the pole of the ecliptic, the expreſſions fail. Hence, if

m = the inclination of the planet's orbit to the ecliptic,
w = the obliquity of the ecliptic,
L = the longitude of the aſcending node of the planet's orbit,
l = the longitude of the ſtar,
t = it's latitude,
$a = AV$, then,

I. $a \times$ ſin. $m \times$ ſin. $L =$ the *diminution* of the obliquity, whilſt the ſin. L is *poſitive*, which is the caſe at preſent for all the planets. When L becomes greater than 180°, the planet will *increaſe* the obliquity (1081).

II $a \times$ ſin. $m \times$ coſ. $\overline{l-L} =$ the *increaſe* of the latitude of a ſtar on the *north* ſide of the ecliptic, within 90° longitude of A, but the *decreaſe* for one on the *ſouth* ſide. And the contrary, if more than 90° longitude from A (1082).

III. $a \times$ ſin. $m \times$ cot. $w \times$ coſ. $L =$ the *progreſſion* of the equinoctial points upon the ecliptic, when coſ. L is *poſitive*, and the *regreſſion*, when *negative* (1080).

IV. $a \times$ ſin. $m \times$ ſin. $\overline{l-L} \times$ tan. $t =$ that part of the variation of the ſtar's longitude which belongs to each particular ſtar (1083), where t muſt be written negative when the latitude is ſouth. This expreſſion is ſufficient, when we only want to compare the variation of the longitudes of two ſtars in reſpect to each other. The part (1083) common to all the ſtars is $a \times$ ſin. $m \times$ cot. $w \times$ coſ. L, which *diminiſhes* the longitude when coſ L is *poſitive*, and *increaſes* it when *negative*, and therefore it muſt be written negative to give it it's proper effect. Hence, the whole variation in longitude $= a \times$ ſin $m \times$ ſin. $\overline{l-L} \times$ tan. t $- a \times$ ſin. $m \times$ cot. $w \times$ coſ. L, regard being had to the ſigns of the quantities, ſin $\overline{l-L}$, tan. t, coſ. L, where the longitude will be *increaſed* or *diminiſhed*, according as this quantity is *poſitive* or *negative*.

1084. Theſe expreſſions agree with thoſe in *L'Hiſtoire de l'Acad. des Sciences*, 1754, given by M. EULER, who firſt diſcovered and explained the cauſe of the diminution of the obliquity of the ecliptic.

1085. The following Table contains the values of *a* for the reſpective planets, according to our determination; together with the place of the nodes of each, their ſecular motions in longitude, and the inclination of their orbits, according to M. de la LANDE.

Planets.	Node in 1750.	Secular Mot.	Inclination.	*a* =
Mercury	1ˢ. 15°. 20′. 43″	1°. 12′. 10″	7°. 0′. 0″	10″
Venus	2. 14 26. 18	0 51. 40	3. 23. 35	558
Mars	1. 17. 38 38	0. 46 40	1 51 0	22
Jupiter	3. 7. 55. 32	0. 59. 30	1 18. 56	693
Saturn	3. 21. 32. 22	0 55. 30	2. 29. 50	34
Georgian	3. 12. 33 31		0. 46. 20	0,7

The ſecular motion of the nodes of the *Georgian* cannot yet be determined from obſervation. By theory, M. de la GRANGE makes it 12″. 30‴; but M. de la LANDE makes it 20″. 40‴ from taking a different quantity of matter for *Venus*.

On the Motion of the Equinoxes from the diſturbing forces of the Planets upon the Earth

1086 By Article 1083 III. the motion of the equinoctial points upon the ecliptic in an hundred years at this time is, $\cot 23^\circ\ 28' \times (10'' \times \text{ſin. } 7^\circ \times \text{coſ } 45^\circ.\ 21' + 558'' \times \text{ſin. } 3^\circ\ 23',5 \times \text{coſ. } 74^\circ.\ 26' + 22'' \times \text{ſin. } 1^\circ.\ 51' \times \text{coſ. } 47^\circ.\ 38',5 + 693'' \times \text{ſin. } 1^\circ.\ 19' \times \text{coſ. } 97^\circ\ 55',5 + 34'' \times \text{ſin } 2^\circ.\ 30' \times \text{coſ. } 111^\circ.\ 32' + 0'',7 \times \text{ſin } 46' \times \text{coſ. } 102^\circ.\ 33') = 17'',4$, which, being poſitive, ſhows that the motion of the equinoxes upon the ecliptic, from this cauſe, is according to the order of the ſigns, and conſequently by this quantity the longitude of all the ſtars will be diminiſhed in 100 years at *this* time, independent of other cauſes. Hence, if we take the whole ſecular preceſſion to be 1°. 23′. 45″, as the motion of the equinoxes would be 17″,4 progreſſive from the planets, the whole regreſſion from the action of the ſun and moon upon the earth muſt be 1°. 24′. 2″,4.

1087. To

1087 To find the motion of the equinoxes from the ſame cauſe, for the firſt century of this æra, we have, from the ſecular motion of the nodes, the longitude of the node of *Mercury* at that time 0ˢ 24°. 53′. 53″, of *Venus* 1ˢ. 29° 47′ 58″, of *Mars* 1ˢ. 4° 27′ 18″, of *Jupiter* 2ˢ 21° 4′ 2″, of *Saturn* 3ˢ. 5° 48′. 52″, and taking the node of the *Georgian* the ſame, we have the preceſſion in that century $= \cot 23^\circ.\,36' \times (10'' \times \text{ſin. } 7^\circ \times \text{coſ. } 24^\circ\ 54' + 558'' \times \text{ſin. } 3^\circ.\,23',5 \times \text{coſ } 59^\circ\ 48' + 22'' \times \text{ſin. } 1^\circ.\,51' \times \text{coſ. } 34^\circ.\,27' + 693'' \times \text{ſin. } 1^\circ\ 19' \times \text{coſ } 81^\circ.\,4' + 34'' \times \text{ſin. } 2^\circ\ 30' \times \text{coſ } 95^\circ.\,49' + 0'',7 \times \text{ſin. } 46^\circ \times \text{coſ. } 102^\circ\ 33') = 47''$, which, being poſitive, ſhows that the motion of the equinoxes, from the attraction of the planets, was then progreſſive, by that quantity.

1088. The preceſſion (1022, 1035) of the equinoxes from the ſun and moon, by diſplacing the equator, varies as the coſine of the obliquity of the ecliptic, and therefore as the obliquity diminiſhes, the preceſſion increaſes, this will increaſe the preceſſion about 9″ in 1700 years, from this cauſe Therefore in the firſt 100 years of our æra, the preceſſion of the equinoxes, from the action of the ſun and moon, muſt have been 1°. 24′. 2″,4 − 9″ = 1° 23′. 53″,4; therefore 1° 23′ 53″,4 − 47″ = 1°. 23′. 6″,4 the whole regreſſion for that time. Hence, 1° 23′ 45″ − 1° 23′ 6″,4 = 38″,6 the quantity by which the regreſſion is faſter now in 100 years than it was in the firſt 100 years of our æra In conſequence of this, the tropical year keeps decreaſing, and this will continue till the place of the nodes of *Jupiter* and *Venus*, from which the principal cauſe ariſes, be got into ſuch a ſituation, that the diſplacements of the ecliptic and the equator together produce a *retarded* preceſſion of the equinoxes. The regreſſion of the equinoctial points is (at the above rate) faſter now by 0,386″ in a year than it was at the beginning of our æra, now the ſun takes 9″ to move over that ſpace, hence, the tropical year is 9″ ſhorter now than it was about 1700 years ago. The tropical year has therefore decreaſed at the mean rate of about *half a ſecond* in 100 years. M de la PLACE makes the year ſhorter now by 10″,33 than it was at the time of HIPPARCHUS, who lived about 1950 years ago. Theſe concluſions therefore agree very well.

1089 This increaſe of regreſſion 38″,6 in 17 ages, gives 2″,27 for every 100 years, ſuppoſing it to increaſe uniformly, and as it was 1°. 23′ 6″,4 in the firſt century, by the addition of 2″,27 we get the preceſſion for every century after

On

On the Variation of the Obliquity of the Ecliptic

1090. By Art. 1083. I. the whole diminution of the obliquity of the ecliptic, at this time, in 100 years is 10″ × ſin 45°. 21′ × ſin. 7° + 558″ × ſin. 74°. 26′ × ſin. 3°. 23′,5 + 22″ × ſin 47°. 38′,5 × ſin. 1°. 51′ + 693″ × ſin. 97°. 55′,5 × ſin. 1°. 19′ + 34″ × ſin. 111° 32′ × ſin 2°. 30′ + 0,7″ × ſin. 102°. 33′ × ſin. 46′ = 49″,35; this concluſion agrees very well with obſervations, from which it appears that at *this time* the obliquity diminiſhes at the rate of about 50″ in 100 years.

1091. Now allowing for the motion of the nodes of the planets, as in Art. 1086. we have the ſecular diminution for the *firſt* year of our æra = 10″ × ſin. 24°. 54′ × ſin. 7° + 558″ × ſin 59°. 48′ × ſin 3°. 23′,5 + 22″ × ſin. 34° 27′ × ſin. 1° 51′ + 693″ × ſin. 81° 4′ × ſin 1°. 19′ + 34″ × ſin. 95° 49′ × ſin 2°. 30′ + 0,7″ × ſin 102° 33′ × ſin. 46′ = 45″,43, which is 3″,92 leſs than in our age. We have here ſuppoſed the inclination of the orbits to remain the ſame.

1092 The motion of the node of the ecliptic upon the orbit of any planet the cotemporary variation of the inclination of the ecliptic, from the attraction of that planet (1081) · AV : AV × ſin A × ſin. ♈A :: (becauſe $\overline{\text{♈}A}$: $\overline{\text{verſ. ſin.}}$ ♈A :: rad. = 1 : ſin. ♈A) $\overline{\text{♈}A}$: ſin. A × $\overline{\text{verſ. ſin. ♈}A}$; hence, by taking the fluents, the motion of the node of the ecliptic upon the orbit of the planet : the cotemporary variation of the inclination of the ecliptic from the time of coincidence of A and ♈ :: ♈A · ſin. A × verſ. ſin. ♈A. Hence, when ♈A = 180°, the ratio becomes c : ſin. A, c being one fourth of the circumference of a circle whoſe radius is unity. Now the ſecular motion of the longitude of the node of each planet being known, the time in which ♈A from nothing becomes 180° will be known, hence, the motion AV in that time will be known, put that quantity $= d$, and we have the whole diminution of the obliquity of the ecliptic in the time the aſcending node of a planet's orbit moves from *Aries* to *Libra* $= \frac{d \times \text{ſin. } A}{c}$. Whilſt the node moves from Libra to Aries the obliquity will be increaſed by the ſame quantity. Now the value of $\frac{d \times \text{ſin. } A}{c}$ for *Mercury* = 1′. 56″; for *Venus* = 1°. 13′. 12″, for *Mars* = 1′. 44″; for *Jupiter* = 30′. 41″, and for *Saturn* = 3′. 4″, the effect of the *Georgian* we here omit, as it would be extremely ſmall. Hence, if all theſe could conſpire they would diminiſh the obliquity of the ecliptic 1° 50′ 37″, but as this is not the caſe, the diminution muſt be leſs. The principal effects are produced

produced by Venus and Jupiter, now the aſcending node of Venus would have coincided with Aries about 8700 years before 1750, and that of Jupiter, 99 years before, their effects therefore to diminiſh the obliquity do not begin at the ſame time, and for about the firſt 1200 years after the coincidence of Jupiter's node with Aries, the effect of Venus tended to increaſe the inclination, and as the ſecular motion of the nodes of Jupiter is not much greater than that of Venus, their effects will conſpire for a very conſiderable part of the time in which their nodes are moving from Aries to Libra. Now the whole diminution from Venus and Jupiter together is 1° 43'. 52"; but we cannot fix this as the limit of the variation of the equator to the ecliptic, becauſe this calculation has been upon ſuppoſition that the inclinations of the orbits of the planets to the ecliptic remain the ſame, whereas they are ſubject to a variation. The limit of the variation of the obliquity will be greater than that which we have here given, becauſe the inclinations of the orbits may be greater than they are at preſent But a calculation (1078) of the variations of all the inclinations from their mutual attractions, and their *maxima* and *minima*, and thence the limit of the variation of the obliquity of the ecliptic to the equator, would be too long to be here introduced. As the rate of variation of the inclination appears now to be nearly uniform, it muſt have arrived at it's maximum, conſequently the inclination itſelf is now about it's mean value, and according to M. de la GRANGE, the inclination of the ecliptic to the equator will never vary more than 5° 23' from the year 1700. *(Mem. de l'Acad. Roy. des Sc. à.* 1782). It is not true therefore, what ſome Authors have aſſerted, that the ecliptic was formerly perpendicular to the equator; nor what others have aſſerted, that the obliquity will continue to be diminiſhed till it coincides with the equator; nor will the ſmall alteration which has taken place from the time of the Creation, account for the changes which ſome have ſuppoſed the earth has undergone.

On the Variation of a Star's Latitude and Longitude

1093. Put A, B, C, D, E, F for the longitudes of the aſcending nodes of *Mercury*, *Venus*, *Mars*, *Jupiter*, *Saturn* and the *Georgian*; m, n, p, r, v, w for the inclinations of their orbits to the ecliptic, and a, b, c, d, e, f for the reſpective values of AV, then (1083, II.) the variation of latitude from the joint effect of all the planets $= a\times$ ſin. $m\times$ coſ. $\overline{l-A}+b\times$ ſin. $n\times$ coſ. $\overline{l-B}=c\times$ ſin. $p\times$ coſ. $\overline{l-C}$ $+d\times$ ſin. $r\times$ coſ. $\overline{l-D}+e\times$ ſin. $v\times$ coſ. $\overline{l-E}+f\times$ ſin. $w\times$ coſ. $\overline{l-F}$. But this expreſſion may be rendered more convenient, by ſubſtituting coſ. $A\times$ coſ. $l+$ ſin.

ſin. $A \times$ ſin. l for coſ $\overline{l-A}$, and making the like ſubſtitution for the other coſines; hence, the *variation of latitude* $= (a \times \text{ſin.}\ m \times \text{coſ.}\ A + b \times \text{ſin.}\ n \times \text{coſ.}\ B + c \times \text{ſin.}\ r \times \text{coſ.}\ C + d \times \text{ſin.}\ s \times \text{coſ}\ D + e \times \text{ſin.}\ v \times \text{coſ}\ E + f \times \text{ſin.}\ w \times \text{coſ}\ F) \times \text{coſ.}\ l + (a \times \text{ſin.}\ m \times \text{ſin.}\ A + b \times \text{ſin.}\ n \times \text{ſin.}\ B + c \times \text{ſin}\ r \times \text{ſin.}\ C + d \times \text{ſin.}\ s \times \text{ſin.}\ D + e \times \text{ſin}\ v \times \text{ſin.}\ E + f \times \text{ſin}\ w \times \text{ſin.}\ F) \times \text{ſin}\ l$ Now if we take the values of theſe quantities as in the Table, we have the variation of latitude for 100 years from $1750 = 7'',558 \times \text{coſ.}\ l + 49'',349 \times \text{ſin.}\ l$.

Ex. The longitude of *Regulus* on January 1, 1760, was $4^s.\ 26^\circ.\ 29'.\ 30''$, whoſe ſine $=0,55205$, coſ $= -0,8838$; hence, $7'',558 \times -0,8828 + 49'',349 \times 0,55205 = 20'',56$, the *increaſe* of the ſtar's latitude in 100 years, the latitude of the ſtar being *north*.

1094 If we make the ſame ſubſtitution for the part of the variation in longitude which is common to every ſtar (1083), we ſhall have it's value $= \overline{7'',558 \times \text{ſin.}\ l - 49'',349 \times \text{coſ}\ l} \times \tan\ t$. Now (1086) the longitude of all the ſtars will be diminiſhed $17'',4$ from the motion of the equinoctial points, independent of the above, hence, the whole *variation in longitude* $= \overline{7'',558 \times \text{ſin.}\ l - 49'',349 \times \text{coſ.}\ l} \times \tan.\ t - 17'',4$. This expreſſion is for ſtars of *north* latitude; for *ſouth* latitude, tan. t becomes negative. As the nodes are not fixed, the values of theſe formulæ will vary, and may be computed at any time, by aſſuming the places of the nodes at that time.

Ex. The longitude of *Regulus* on January 1, 1760, was $4^s.\ 26^\circ\ 29'.\ 30''$, whoſe ſine $=0,55205$, coſ. $= -0,8838$, and latitude $0^\circ\ 27'\ 27''$ N. whoſe tangent $=0,007985$, hence, $\overline{7'',558 \times 0,55205 - 49'',349 \times -0,8838} \times 0,007985 - 17'',4 = 0'',377 - 17'',4 = -17'',023$, the ſecular variation of the longitude of Regulus.

1095 If we want only to find how much the difference of the longitudes of two ſtars have varied, we may leave out that part of the variation common to all the ſtars, and only compute the variation belonging to each particular ſtar.

1096 The variations of the latitudes and longitudes of the ſtars thus determined, are found to agree very well with obſervations, by comparing the latitudes and longitudes of the ſtars as given by PTOLEMY in his Catalogue, with the latitudes and longitudes obſerved at this time.

Ex. The latitude of the ſtar N° 27 of the *Great Bear* is 54° 27′, and that of N° 10. *Draconis* is 81°. 48′, alſo,

The longitude in 1700,			
Of the firſt ſtar . .	5^s	22°.	34′
Of the ſecond . .	11	29	23
Diff. of Long. . .	6	6	49

Long. at the time of PTOLEMY,			
Of the firſt ſtar . .	4^s	29°.	50′
Of the ſecond . . .	11	8	0
Diff. of Long. . .	6.	8.	10

Hence, the difference of the longitudes of theſe two ſtars has decreaſed 1°. 21′ ſince the time of PTOLEMY, and this agrees nearly with the above theory, by computing the variation of the longitude of each ſtar for the above interval of time.

1097. In the conſtellation *Auriga*, at the time of PTOLEMY the longitude of the ſtar N°. 1. was ♊ 25°. 36′, and latitude 30° N., and in 1700 it's latitude was 30°. 49′ N. The longitude of N° 12. was ♊ 19°. 50′, and latitudes 8° 30′ and 8° 51′ at the above times. The former latitude was therefore increaſed 19′ and the latter 21′, which agrees very well with the theory.

On the Motion of the Orbits of the Satellites of Jupiter, *from their mutual Attractions.*

1098. The motion of the nodes of the ſatellites are found in the ſame manner as thoſe of the primary planets, taking Jupiter for the central body, inſtead of the ſun.

To find the motion of the node of the ſecond ſatellite of Jupiter upon the orbit of the third, from the attraction of the third. Here $\frac{1}{m} = 0{,}0000687$ (1073), $a = \frac{8}{5}$, hence, in one revolution of the ſecond ſatellite, the motion of it's node $= 42''{,}386$, and conſidering 105 revolutions in a year, it's annual motion is 1°. 14′.

To

To find the motion of the nodes of the third ſatellite of Jupiter upon the orbit of the ſecond, from the attraction of the ſecond. Here $\frac{1}{m}=0,00002417$, $=a\frac{5}{8}$, hence, in one revolution of the third ſatellite, the motion of it's node $=23'',61$; and conſidering 51 revolutions in a year, it's annual motion is $20',4$.

In this manner the motion of the nodes of all the ſatellites upon the orbits of each other, from their mutual attractions, may be determined, granting the quantity of matter in each to be rightly aſſumed, and then that motion (1077) may be referred to the plane of Jupiter's orbit.

1099. But the nodes have a further motion, ariſing from the diſturbing force of the ſun, which may be calculated in the ſame manner as the motion of the moon's node has been. But knowing the motion of the moon's nodes, the motion of the nodes of the ſatellites may be more eaſily found by Art. 857, for if p = the periodic time of the moon, P = that of the ſun, p' = the periodic time of a ſatellite of Jupiter, and P' = the time of the revolution of Jupiter about the ſun, M = the motion of the nodes of the moon, and m the cotemporary motion of the nodes of the ſatellite, then $\frac{p}{P^2} : \frac{p'}{P'^2} :: M : m = M \times \frac{P^2 \times p'}{P'^2 \times p}$.

The mean annual motion of the moon's nodes $= 19° \ 19'$ (938), alſo $p=27,32$ days, $P=365,256$, for the ſecond ſatellite of Jupiter $p'=3,55$, and $P'=4332,6$, hence, $m=19° \ 19' \times \frac{\overline{365,256}^2 \times 3,55}{\overline{4332,6}^2 \times 27,32} = 1' \ 4''$ the mean annual motion of the node of the ſecond ſatellite of Jupiter upon the plane of Jupiter's orbit. Thus we may find the whole motion of the nodes of all the ſatellites.

1100. The quantities of matter of the ſatellites of *Saturn* not having been determined, we cannot find the motion of their nodes from their mutual attractions, but we can find their motion ariſing from the diſturbing force of the ſun, in the ſame manner as we find the motion of thoſe of Jupiter.

M. de la PLACE, in his *Théorie du Mouvement et de la Figure Elliptique des Planètes*, has given the following theorems for the ſecular inequalities of the planets.

1101. Let a planet P be acted upon by another planet p, and let m be the quantity of matter in p, that of the ſun being repreſented by unity,

a = the mean diſtance of P from the ſun,
$e \times a$ = the excentricity of it's orbit,
L = the longitude of it's aphelion at any given epoch,
Γ = the longitude of it's aſcending node on a fixed plane at the ſame epoch;
γ = the inclination of it's orbit upon the ſame plane,

And let a', $e' \times a'$, L', Γ' and γ' be the ſame in reſpect to p; and let $\frac{a'}{a} = n$;

$$b = \frac{1}{\overline{1+n^2}\Big|^{\frac{3}{2}}} \times \left(1 + \overline{1-\frac{1}{4^2}} \times \overline{\frac{2n}{1+n^2}}\Big|^{2} + \overline{1-\frac{1}{4^2}} \times \overline{1-\frac{1}{8^2}} \times \overline{\frac{2n}{1+n^2}}\Big|^{4} + \overline{1-\frac{1}{4^2}} \times \overline{1-\frac{1}{8^2}} \times \overline{1-\frac{1}{12^2}} \times \overline{\frac{2n}{1+n^2}}\Big|^{6} + \&c.\right)$$

$$c = \frac{3n}{\overline{1+n^2}\Big|^{\frac{5}{2}}} \times \left(\overline{1+\frac{3}{4\cdot 3^2-1}} \times \overline{\frac{2n}{1+n^2}}\Big|^{2} + \overline{1+\frac{3}{4\cdot 3^2-1}} \times \overline{1+\frac{3}{4\cdot 5^2-1}} \times \overline{\frac{2n}{1+n^2}}\Big|^{4} + \overline{1+\frac{3}{4\cdot 3^2-1}} \times \overline{1+\frac{3}{4\cdot 5^2-1}} \times \overline{1+\frac{3}{4\cdot 7^2-1}} \times \overline{\frac{2n}{1+n^2}}\Big|^{6} + \&c.\right)$$

1102. Let v be the number of revolutions of P from the given epoch, v being negative for any time *before* the epoch, then

The increaſe of the equation of the center is

$$m \times v \times 360^\circ \times e' \times \text{ſin.}\ \overline{L'-L} \times \overline{c \times \overline{1+n^2} - 3bn}.$$

The mean motion of the aphelion according to the order of the ſigns is

$$m \times v \times 360^\circ \times \left(\tfrac{1}{4}\,nc - \tfrac{1}{2}\,.\,\frac{e'}{e} \times \text{coſ.}\ \overline{L'-L} \times \overline{c \times \overline{1+n^2} - 3bn}\right).$$

The

The diminution of the inclination of the orbit upon the fixed plane is

$$\tfrac{1}{4}\, m \times v \times 360^\circ \times nc\gamma' \times \text{ſin}\ \overline{\Gamma' - \Gamma}.$$

The retrograde motion of the node upon the fixed plane is

$$\tfrac{1}{4}\, m \times v \times 360^\circ \times nc \times \overline{1 - \frac{\gamma'}{\gamma} \times \text{coſ.}\,\overline{\Gamma' - \Gamma}}.$$

If we thus collect the effects of all the planets upon P, the ſum will give the whole variation of the elements of the orbit of P. But theſe formulæ only ſerve for a limited time from the given epoch.

1103. If we fix upon the year 1700 for the epoch, we ſhall have,

$$\tfrac{1}{4}\, m \times v \times 360^\circ \times nc\gamma' \times \text{ſin.}\ \Gamma'$$

for the diminution of the obliquity of the ecliptic, produced by the action of p upon the earth, v being the number of revolutions of the earth from 1700, γ' the inclination of the orbit of the planet p upon the ecliptic, and Γ' the longitude of the aſcending node. The ſum of all the diminutions from all the planets will give the whole diminution. When Γ' is greater than 180°, which it is not at preſent for any of the planets, the ſin. Γ' becomes negative, and the obliquity will then be increaſed by that planet

1104. Hence, by collecting the effects of all the planets, if v be the number of years *after* 1700, the whole diminution will be the difference between $932'',56$ and $932'',56 \times \text{coſ}\ 17'',7686v - 3140'',34 \times \text{ſin}\ 32'',8412v$, ſuppoſing the variation at this time to be $50''$ in 100 years. For any number of years *before* 1700, the ſign of the ſecond term muſt be changed

1105. In like manner, all the planets together will produce a preceſſion of the equinoxes equal to $50'',53353v - 3292'',28\ \text{ſin}\ 17'',7686v - 9315'',65\ \text{coſ.}\ 32'',8412v + 9315'',65$ in v years *after* 1700, for any time *before*, the ſign of v becomes negative. The inequality of the preceſſion of the equinoxes changes the ſecular motion of the ſun in reſpect to the equinoxes, this motion is 46′ in this age, but it was 45′ 23″ in the beginning of our æra Hence, the place of the ſun, calculated with the uniform ſecular motion of 46′, as in our Tables, will require a ſecular equation. And this ſecular motion of the ſun gives, for the increaſe of the year by going back from 1700, $36'',114 \times \text{ſin}\ 32'',8412v + 6'',9039 \times \text{coſ}\ 17'',7686v - 6'',9039$. Hence, at the time of HIPPARCHUS the year was about 10″,33 longer than at this time (1088).

CHAP.

CHAP. XXXVII.

ON THE EFFECTS PRODUCED ON THE MOTIONS OF THE PLANETS IN THE PLANES OF THEIR ORBITS, FROM THEIR MUTUAL ATTRACTIONS.

Fig. 239.

Art. 1106. LET S be the center of force, PM the orbit of a body deſcribed by virtue of two forces, one (f) tending from the body at P in the direction PS, and the other (F) acting in a direction Pw perpendicular to PS. Let SQ be a line given in poſition, draw PA perpendicular to SQ, and complete the parallelogram $PASV$, and draw Vr, At perpendicular to PS Of the whole force acting on P, let M be that part which acts in the direction PA, and m the part acting in the direction PV; alſo, let $r=PS$, $p=PA$, $q=PV$, $t=$the fluxion of the time, $v=$the angle PSQ, $x=$it's ſine, $y=$it's coſine, then $Mx=$that part of the force M which acts in the direction PS, and $my=$that part of the force y which acts in the ſame direction; hence, $Mx+my=f$; alſo, $My=$that part of the force M which acts in a direction parallel to tA, and $mx=$that part of the force m which acts in a direction parallel to rV, hence, $mx-My=F$. Now by the principles of motion *, $p=-Mt^2$, and $q=-mt^2$, the fluxion of the time being conſtant. But $p=rx$, and $q=ry$, and as $x=yv$, and $y=-xv$, we have,

$$\dot{p}=x\dot{r}+ry\dot{v},$$

$$(A)\quad p=xr+2y\dot{r}v+ry\ddot{v}-rx\dot{v}^2=-Mt^2,$$

$$q=yr-rx\dot{v},$$

$$(B)\quad q=yr-2x\dot{r}v-rx\ddot{v}-ryv^2=-mt^2;$$

Multiply

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* If F repreſent any accelerative force, v the velocity which the body would acquire in 1″ by that force, s the fluxion of the ſpace deſcribed in a given time t with that velocity, then if we meaſure the velocity v by the ſpace deſcribed uniformly in 1″, we have 1″ t v $s=vt$, therefore $s=\pm\dot{v}\dot{t}$; but v varies as $F\times t$, let therefore $v=F\times t$, and $s=\pm Ft^2$.

Multiply (A) by y and (B) by x, and ſubtract the latter from the former, and we get

$$(C)\ 2\dot{r}\dot{v}+r\ddot{v}=-\dot{t}^2\times\overline{My-mx}=F\dot{t}^2.$$

Multiply (A) by x and (B) by y and add them together, and we get

$$\ddot{r}-r\dot{v}^2=-\dot{t}^2\times\overline{Mx+my}=-f\dot{t}^2,\text{ or}$$
$$(D)\ r\dot{v}^2-\ddot{r}=F\dot{t}^2.$$

From the equations (C) and (D), the curve PM deſcribed by the body P may be found. Theſe fluxional equations are the ſame as thoſe determined by CLAIRAUT, EULER and MAYER, in their Treatiſes upon the theory of the moon, the integration of which is a problem of great difficulty; and as the firſt of theſe Authors has proceeded in a manner the moſt eaſy to be underſtood by the generality of readers, I ſhall here enter into a very full explanation of all his principles of inveſtigation.

1107. Let the force f conſiſt of two parts, one of which $=\frac{C}{r^2}$, C being a conſtant quantity, and the other $=D$; or let $f=\frac{C}{r^2}+D$; then $2\dot{r}\dot{v}+r\ddot{v}=F\dot{t}^2$, and $r\dot{v}^2-\ddot{r}=\overline{\frac{C}{r^2}+D}\times\dot{t}^2$. Multiply the firſt of theſe equations by $\frac{r}{\dot{t}}$, and we get $\frac{2r\dot{r}\dot{v}+r^2\ddot{v}}{\dot{t}}=Fr\dot{t}$, whoſe fluent ($\dot{t}$ being conſtant) is $\frac{r^2\dot{v}}{\dot{t}}=a+\int Fr\dot{t}$, a being a conſtant quantity. Multiply this equation by $Fr\dot{t}$, and we have $Fr^3\dot{v}=aFr\dot{t}+Fr\dot{t}\times\int Fr\dot{t}$, whoſe fluent is $\int Fr^3\dot{v}=a\int Fr\dot{t}+\frac{1}{2}\int\overline{Fr\dot{t}}^2$ *, multiply this by 2, and add a^2 to both ſides, and it becomes $a^2+2\int Fr^3\dot{v}=a^2+2a\int Fr\dot{t}+\int\overline{Fr\dot{t}}^2$, whoſe ſquare root is $\sqrt{a^2+2\int Fr^3\dot{v}}=a+\int Fr\dot{t}$. Hence, $\frac{r^2\dot{v}}{\dot{t}}=\sqrt{a^2+2\int Fr^3\dot{v}}$, conſequently $\dot{t}=\frac{r^2\dot{v}}{\sqrt{a^2+2\int Fr^3\dot{v}}}=$ (if we put $e=\frac{\int Fr^3\dot{v}}{a^2}$) $\frac{r^2\dot{v}}{a\sqrt{1+2e}}$.

1108. Let

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* For if $y=\int Fr\dot{t}$, then $\dot{y}=Fr\dot{t}$, therefore $y\dot{y}=Fr\dot{t}\times\int Fr\dot{t}$, and the fluent $=\frac{1}{2}y^2=\frac{1}{2}\int\overline{Fr\dot{t}}^2$.

1108. Let us next take the other equation $r\dot{v}^2 - r = \overline{\frac{C}{r^2} + D} \times t^2$. Now in this equation, t is conſtant and r variable, but it is well known, that in a fluxional equation of the ſecond order, where r is variable and t conſtant, that if we ſubſtitute $-\frac{rt}{t}$ for r, the equation will be changed into one in which r is conſtant and t variable, if therefore for r we ſubſtitute $r - \frac{\dot{r}t}{t}$, the equation will be changed into one in which r and $\dot{t}$ are both variable. If therefore we ſubſtitute $r - \frac{rt}{t}$ for r in the above equation, it becomes $r\dot{v}^2 - r + \frac{r\dot{t}}{t} = \overline{\frac{C}{r^2} + D} \times t^2$, in which equation we may aſſume $\dot{r}$ or $\dot{t}$ conſtant, as it may be found convenient. But (1107) $\dot{t} = \frac{r^2\dot{v}}{a\sqrt{1+2e}}$, therefore $t = \frac{2rr\dot{v} \times a \times \overline{1+2e} - ar^2\dot{v}e}{a^2 \times \overline{1+2e} \times \sqrt{1+2e}}$; hence, $\frac{r\dot{t}}{\dot{t}} = \frac{2\dot{r}^2}{r} - \frac{re}{1+2e}$; by ſubſtitution therefore we get $r\dot{v}^2 - \dot{r} + \frac{2r^2}{r} - \frac{\dot{r}e}{1+2e} = \frac{\overline{\frac{C}{r^2} + D} \times r^4\dot{v}^2}{a^2 \times \overline{1+2e}}$, divide both ſides by $r^2\dot{v}^2$, and tranſpoſe $-\frac{\dot{r}\dot{e}}{1+2e}$, and we get $\frac{1}{r} - \frac{\frac{r}{r^2} - \frac{2r^2}{r^3}}{v^2} = \frac{C + Dr^2 + \frac{a^2r\dot{e}}{r^2v^2}}{a^2 \times \overline{1+2e}}$. Now $\frac{\ddot{r}}{r^2} - \frac{2r^2}{r^3} =$ flux. $\frac{r}{r^2}$; put alſo $1 + P = \frac{1 + \frac{Dr^2}{C} + \frac{a^2\dot{r}\dot{e}}{Cr^2v^2}}{1+2e}$ = (as $e = \frac{\int Fr^3\dot{v}}{a^2}$, and therefore $\dot{e} = \frac{Fr^3\dot{v}}{a^2}$) $\frac{1 + \frac{Dr^2}{C} + \frac{Fr\dot{r}}{Cv}}{1+2e}$, and then $P = \frac{\frac{Dr^2}{C} + \frac{Fr\dot{r}}{Cv} - 2e}{1+2e}$, hence, by ſubſtitution we get $\frac{1}{r} - \frac{\text{flux.}\ \frac{r}{r^2}}{v^2} = \frac{C}{a^2} \times \overline{1+P}$, or $1 - \frac{a^2}{rC} + \frac{a^2}{Cv^2} \times \text{flux.}\ \frac{r}{r^2} + P = 0$, put $1 - \frac{a^2}{Cr} = s$, then $\dot{s} = \frac{a^2\dot{r}}{Cr^2}$, and $\ddot{s} = \frac{a^2}{C} \times \text{flux.}\ \frac{r}{r^2}$, or $\frac{C}{a^2} \times \ddot{s} = \text{flux.}\ \frac{r}{r^2}$, hence, the equation becomes $s + \frac{\ddot{s}}{\dot{v}^2} + P = 0$.

1109. To find the fluent of this fluxional equation $s + \frac{\ddot{s}}{\dot{v}^2} + P = 0$, ſuppoſing $P = \text{coſ.}\ mv$, or of $s + \frac{\ddot{s}}{\dot{v}^2} + \text{coſ}\ mv = 0$. Multiply it by coſ. $v \times \dot{v}$,

and

and it becomes $\frac{\text{coſ. } v\times s}{v}+s\times\text{coſ. } v\times v+\text{coſ. } mv\times\text{coſ. } v\times v=0$; now the fluent of the two firſt terms is coſ. $v\times\frac{\dot s}{v}+s\times\text{ſin. } v$*; alſo, coſ. $mv\times\text{coſ. } v\times\dot v=\frac{1}{2}.\text{coſ. }\overline{m+1}.v\times v+\frac{1}{2}.\text{coſ }\overline{m-1}.v\times\dot v$, whoſe fluent is $\frac{1}{2.\overline{m+1}}\times\text{ſin. }\overline{m+1}.v+\frac{1}{2.\overline{m-1}}\times\text{ſin. }\overline{m-1}.v$, but $\text{ſin. }\overline{m+1}.v=\text{ſin. } mv\times\text{coſ. } v+\text{ſin. } v\times\text{coſ } mv$, and $\text{ſin. }\overline{m-1}.v=\text{ſin. } mv\times\text{coſ. } v-\text{ſin. } v\times\text{coſ. } mv$, hence, the laſt quantity becomes $\frac{1}{2.\overline{m+1}}\times\text{ſin. } mv\times\text{coſ. } v+\frac{1}{2.\overline{m+1}}\times\text{ſin. } v\times\text{coſ. } mv+\frac{1}{2.\overline{m-1}}\times\text{ſin. } mv\times\text{coſ. } v-\frac{1}{2.\overline{m-1}}\times\text{ſin } v\times\text{coſ } mv=\frac{m}{m^2-1}\times\text{ſin. } mv\times\text{coſ. } v-\frac{1}{m^2-1}\times\text{ſin. } v\times\text{coſ. } mv$, the whole fluent therefore becomes, coſ. $v\times\frac{s}{v}+s\times\text{ſin. } v+\frac{m}{m^2-1}\times\text{ſin. } mv\times\text{coſ. } v-\frac{1}{m^2-1}\times\text{ſin. } v\times\text{coſ } mv=g$, a correction. Multiply the laſt equation by $\frac{\dot v}{\text{coſ. } v^2}$, and it becomes $\frac{\dot s}{\text{coſ. } v}+\frac{s\times\text{ſin } v\times\dot v}{\text{coſ. } v^2}+\frac{m}{m^2-1}\times\frac{\text{ſin. } mv\times\dot v}{\text{coſ. } v}-\frac{1}{m^2-}\times\frac{\text{ſin } v\times\text{coſ } mv\times v}{\text{coſ. } v^2}=g\times\frac{\dot v}{\text{coſ. } v^2}$, now the fluent of the two firſt quantities is $\frac{s}{\text{coſ } v}$, for if this quantity be put into fluxions it will be found to give thoſe terms, alſo, the fluent of the two next terms is $-\frac{1}{m^2-1}\times\frac{\text{coſ } mv}{\text{coſ } v}$, for if this quantity be put into fluxions it will produce thoſe two terms, but this laſt will want a correction, becauſe, when $v=0$, it becomes $-\frac{1}{m^2-1}$, therefore that part of the fluent corrected is $\frac{1}{m^2-1}-\frac{1}{m^2-1}\times\frac{\text{coſ. } mv}{\text{coſ. } v}$, laſtly, the fluent of

$g\times$

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* For the fluxion of coſ $v\times\frac{s}{v}+s\times\text{ſin } v$ is $\frac{\text{coſ } v\times s}{v}+\overline{\text{coſ } v}\times\frac{s}{v}+\text{ſin } v\times s+s\times\overline{\text{ſin } v}=\frac{\text{coſ } v\times s}{v}-\text{ſin. } v\times s+\text{ſin } v\times s+s\times\text{coſ } v\times v=\frac{\text{coſ } v\times s}{v}+s\times\text{coſ } v\times v$, becauſe $\overline{\text{coſ } v}=-\text{ſin } v\times v$, and $\overline{\text{ſin. } v}=\text{coſ } v\times v$ Here v is ſuppoſed conſtant, as in the laſt Art.

$g \times \frac{\dot{v}}{\text{coſ. } v^2} = g \times \dot{\overline{\text{tan } v}}$*; hence, the fluent becomes $\frac{s}{\text{coſ. } v} + \frac{1}{m^2-1} - \frac{1}{m^2-1} \times \frac{\text{coſ } mv}{\text{coſ. } v} - g \times \text{tan. } v = c$, a correction, conſequently $s - g \times \text{ſin. } v - c \times \text{coſ. } v + \frac{1}{m^2-1} \times \text{coſ. } v - \frac{1}{m^2-1} \times \text{coſ. } mv = 0$, the two laſt terms of which aroſe from the value of P, that is, from introducing the diſturbing forces D and F, without which, the whole force would have varied inverſely as the ſquare of the diſtance of the body from the center of force, and the orbit deſcribed would have been an ellipſe about the center of force in it's focus. Hence, if inſtead of aſſuming $P = \text{coſ. } mv$, we aſſume $P = a' \times \text{coſ } mv + b' \times \text{coſ. } nv +$ &c. and for s we ſubſtitute $1 - \frac{a^2}{Cr}$, we obtain $\frac{a^2}{Cr} = 1 - g \times \text{ſin } v - \left(c - \frac{a'}{m^2-1} - \frac{b'}{n^2-1} - \text{\&c}\right) \times \text{coſ. } v - \frac{a'}{m^2-1} \times \text{coſ } mv - \frac{b'}{n^2-1} \times \text{coſ. } nv - \text{\&c.} = 0$, which is the equation of the curve deſcribed by the two forces $\frac{C}{r^2} + D$ and F, the former tending to the center of force, and the latter acting in a direction perpendicular to the radius vector, upon the above ſuppoſition for the value of P, which ſuppoſition is always applicable in the caſe of the planets.

FIG. 198.

1110. It has been proved (868) that if r be equal to the radius vector from the focus of an ellipſe whoſe ſemi-parameter is equal to p, c equal the diſtance of the focus from the center divided by the ſemi-axis major, and v equal to the true anomaly, then $r = \frac{p}{1 - c \times \text{coſ. } v}$, therefore $\frac{p}{r} = 1 - c \times \text{coſ. } v$. But if we eſtimate the motion of the body from ſome other point B inſtead of A, and put the angle $BEM = v$, and $BEA = m$, then $AEM = v - m$, and we muſt put $v - m$ inſtead of v, hence, $\frac{p}{r} = 1 - c \times \text{coſ. } \overline{v-m} = 1 - c \times \overline{\text{coſ. } v \times \text{coſ. } m + \text{ſin. } v \times \text{ſin } m} =$ (if $h = c \times \text{coſ } m$, $k = c \times \text{ſin. } m$) $1 - h \times \text{coſ. } v - k \times \text{ſin. } v$. It appears therefore, that the part $\frac{a^2}{Cr} = 1 - g \times \text{ſin. } v - c \times \text{coſ. } v$ of the equation of the curve in the laſt Article, expreſſes an ellipſe, whoſe ſemi-parameter is $\frac{a^2}{C}$, deſcribed by the force $\frac{C}{r^2}$. The ſecond part therefore expreſſes the alteration ariſing from the diſturbing forces D and F. And if

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* For $\dot{v} = \frac{\dot{\overline{\text{tan } v}}}{\overline{\text{ſec. } v}^2} = \overline{\text{coſ } v}^2 \times \dot{\overline{\text{tan. } v}}$, therefore $\frac{\dot{v}}{\text{coſ. } v^2} = \dot{\overline{\text{tan. } v}}$.

we eſtimate the motion from A, $g=0$, or that correction (1109) will be unneceſſary, and the equation (putting $p=\frac{a^2}{C}$ the ſemi-parameter) becomes $\frac{p}{r}=1-(c-\frac{a'}{m^2-1}-\frac{b'}{n^2-1}-\&c)\times$ coſ. $v-\frac{a'}{m^2-1}\times$ coſ. $mv-\frac{b'}{n^2-1}\times$ coſ. $nv-\&c.$

1111. The equation of the ellipſe which would have been deſcribed without the diſturbing forces is $\frac{1}{r}=\frac{1}{p}-\frac{c}{p}\times$ coſ. v, where $c=$ the excentricity divided by the ſemi-axis major, and $p=$ the ſemi-parameter, and if we put $u=c-\frac{a'}{m^2-1}-\frac{b'}{n^2-1}-\&c$ and $\frac{1}{p}\times(-\frac{a'}{m^2-1}\times$ coſ. $mv-\frac{b'}{n^2-1}\times$ coſ. $nv-\&c.)=S$, the equation of the orbit deſcribed by P becomes $\frac{1}{r}=\frac{1}{p}-\frac{u}{p}\times$ coſ. $v+S$, where $\frac{1}{r}=\frac{1}{p}-\frac{u}{p}\times$ coſ. v is the equation of a new ellipſe. Hence, the ellipſe which would have been deſcribed without the diſturbing forces is changed into another ellipſe very nearly, the deviation from an ellipſe being only that which ariſes from the ſmall quantity S. The effect therefore of the diſturbing forces is to alter the excentricity of the ellipſe, to change the mean diſtance, and to cauſe a ſmall alteration in this new ellipſe, the dimenſions of which muſt be found from obſervations.

1112. If at the ſame time that the planet deſcribes the angle v, the apſide deſcribe the angle $v-mv$, then the motion of the planet in reſpect to the apſide $=mv$, and the true anomaly of the planet in this moveable ellipſe $=mv$, therefore the equation of the moveable ellipſe is $\frac{1}{r}=\frac{1}{d}-\frac{w}{d}\times$ coſ mv. Hence, as the apſides of the orbits of the planets are moveable, we muſt aſſume $\frac{1}{r}=\frac{1}{d}-\frac{w}{d}\times$ coſ. mv, when we determine the value of P, d being half the parameter, and $w=$ the excentricity divided by the ſemi-axis major

1113. Having determined the value of r from the general equation of the curve, we can get the time, for $t=\frac{r^2v}{a\sqrt{1+2e}}$, now $\frac{a^2}{C}=p$ the ſemi-parameter, and let us ſuppoſe $C=1$; then the orbit being ſuppoſed to have but a ſmall excentricity, and the mean diſtance being aſſumed $=1$, we may ſuppoſe $p=1$; hence, $a=1$. Therefore $t=\frac{r^2v}{\sqrt{1+2e}}=r^2v\times\overline{1+2e}|^{-\frac{1}{2}}=r^2v\times\overline{1-e}$, neglecting the other terms of the ſeries.

1114 By the property of the ellipſe (1112), $r = \frac{1}{d} - \frac{w}{d} \times \text{coſ}\ mv$, or aſſuming $d=1$, $\frac{1}{r} = 1 - w \times \text{coſ.}\ mv + S$, S being the correction of $\frac{1}{r}$ from the diſturbing forces (1111). Put $1 - w \times \text{coſ}\ mv = l$, then $r^2 = \overline{l+S}\rvert^{-2} = l^{-2} - 2l^{-3}S$, neglecting the other terms, but $l^{-2} = \overline{1 - w \times \text{coſ.}\ mv}\rvert^{-2} = 1 + 2w \times \text{coſ}\ mv$, and $l^{-3} = \overline{1 - w \times \text{coſ}\ mv}\rvert^{-3} = 1 + 3w \times \text{coſ}\ mv$, hence, $r^2 = 1 + 2w \times \text{coſ.}\ mv - 2S - 6wS \times \text{coſ.}\ mv$, ſubſtitute this value of r^2 into $t = r^2 v \times \overline{1-e}$, and we obtain $t = (1 + 2w \times \text{coſ}\ mv - 2S - 6wS \times \text{coſ.}\ mv)\ v \times \overline{1-e} = v + 2w \times \text{coſ.}\ mv \times v - 2S \times \dot{v} - 6wS \times \text{coſ}\ mv \times v - ev - 2ew \times \text{coſ}\ mv \times v$, neglecting thoſe terms where the product of the two ſmall quantities S and e enter. Now the two firſt terms are independent of e and S, and therefore we have nothing to do with them in our preſent enquiry, which is only to find the equations ariſing from the diſturbing forces; but the other terms, depending upon e and S, ariſe from the diſturbing forces, therefore the required correction of the fluxion of the time is $-\overline{2S+e} \times v - 2w \times \overline{3S+e} \times \text{coſ}\ mv \times v$. Now if the orbit had been a perfect circle, and there had been no diſturbing forces, we ſhould have had $t = v$, and $t = v$, and the motion being uniform, t would have been the mean longitude, and t being here conſtant, t muſt alſo expreſs the mean longitude of the body P. Let therefore $\alpha =$ the fluent of $2w \times \text{coſ}\ mv \times v$ ariſing from the elliptic form of the orbit, and $\beta =$ the fluent of $-\overline{2S+e} \times v - 2w \times \overline{3S+e} \times \text{coſ}\ mv \times v$ ariſing from the diſturbing forces, then $t = v + \alpha + \beta$, therefore $t - \alpha - \beta = v$; that is, to find the true place of the body, firſt correct the mean longitude by the elliptic equation (222), and then apply the fluent of $-\overline{2S+e} \times v - 2w \times \overline{3S+e} \times \text{coſ.}\ mv \times v$ with a contrary ſign. If the orbit be a circle, or if the excentricity be very ſmall, for the correction we may aſſume only the fluent of $-\overline{2S+e} \times v$ with it's ſign changed.

1115 The quantities here found are in terms of the radius ſuppoſed to be unity, and as an arc of 57°,29578 is equal to radius, the correction in the laſt Article muſt be multiplied by 57°,29578 in order to reduce it into degrees. Alſo, the forces D and F are fractions of the force of the ſun at the diſtance unity, or at the mean diſtance of the planet which is diſturbed.

FIG 240

1116 Let the ſum of the maſſes of the earth E (or the body attracted) and ſun S be unity, $M =$ the maſs of the planet P diſturbing the motion of the earth, then (861) $\frac{1}{ES^2} =$ the attraction of E to S, and $\frac{M}{EP^2} =$ the attraction

of

of E to P, complete the parallelogram $PSEW$, and put z = the angle PSE. The attraction of E to $S = \frac{1}{ES^2}$, the attraction of E to $P = \frac{M}{EP^2}$, and the attraction of S to $P = \frac{M}{SP^2}$, and by the reſolution of forces, $EP . ES :: \frac{M}{EP^2} : \frac{M \times ES}{EP^3}$ = the force of E to P in the direction ES, and $EP : EW$, or SP, $:: \frac{M}{EP^2} : \frac{M \times SP}{EP^3}$ = the part of the force of E to P which acts in a direction parallel to SP, therefore $\frac{M \times SP}{EP^3} - \frac{M}{SP^2}$ = the diſturbing force of P upon E in a direction parallel to SP, or EW, and this force, tending to draw E *from* S, muſt be written negative, and reſolving this force into two others, one in the direction SE and the other perpendicular to it, we have $F = -\left\{\frac{M \times SP}{EP^3} - \frac{M}{SP^2}\right\} \times \text{ſin } z$ = the force which acts perpendicularly to SE, this muſt be written with a contrary ſign, or +, when the body attracting is neareſt to the ſun, becauſe it then acts in an oppoſite direction, alſo, $-\left\{\frac{M \times SP}{EP^3} - \frac{M}{SP^2}\right\} \times \text{coſ. } z$ = the force which acts in the direction ES; hence, $D = \frac{M \times ES}{EP^3} - \left\{\frac{M \times SP}{EP^3} - \frac{M}{SP^2}\right\} \times \text{coſ. } z$ = the diſturbing force in the direction of the radius. The value of e is $\int F r^3 v$, a being unity (1113), alſo, $P = \frac{\frac{Dr^2}{C} + \frac{Frr}{Cv} - 2e}{1 + e}$, and as e is a very ſmall quantity, we may aſſume $P = \frac{Dr^2}{C} + \frac{Frr}{Cv} - 2e$, and as we aſſume the ſum of the maſſes of the ſun and body attracted = 1, we have $C = 1$, and therefore $P = Dr^2 + \frac{Frr}{v} - 2e$.

Hence,

Hence, we have theſe four equations,

$$D=\frac{M\times ES}{EP^3}-\left(\frac{M\times SP}{EP^3}-\frac{M}{SP^2}\right)\times \text{coſ. } z$$

$$F=\mp\left(\frac{M\times SP}{EP^3}-\frac{M}{SP^2}\right)\times \text{ſin. } z.$$

$$e=\int Fr^3\dot{v}.$$

$$P=Dr^2+\frac{Fr\dot{r}}{\dot{v}}-2e.$$

1117. Let E be at A the higher apſide in conjunction with P at B, and $z=$ the angle ESP of elongation, $v=ASE$, and let the planet P deſcribe a circle, and E an ellipſe whoſe center of force is S and the other focus c, and let $1-n$ be to 1 as the mean motion of the planet P to the mean motion of the earth E, or the body attracted, biſect SC in O, and let $AO=1$. As the motion of the earth's apogee is very ſmall, we may, for our preſent purpoſe, conſider it as fixed, and putting $u=SO$, we have, $\frac{1}{r}=\frac{1}{d}-\frac{u}{d}\times \text{coſ. } v$ Now the *mean* motion is very nearly (227) about the point c, and $2u\times \text{ſin. } v=cd$, which meaſures the angle cES, hence, $EcA=v+2u\times \text{ſin. } v$; therefore $v-z : v+2u\times \text{ſin. } v :: 1-n : 1$, and $z=nv-2u\times \overline{1-n}\times \text{ſin. } v$. If the diſturbing planet be inferior, and it's mean motion be to that of the earth as $1+n : 1$, then $z=nv+2u\times \overline{1+n}\times \text{ſin. } v$.

We proceed now to the application of theſe principles

On

On the Equations of the Earth's *Motion produced by the Action of* Jupiter.

1118. Put $EP=s$, $SP=b$, $SE=r$, and draw Eu perpendicular to SP; then $Su=r\times$ coſ. z; hence, $s=\sqrt{b^2+r^2-2br\times\text{coſ. }z}$, put $2br\times$ coſ. $z-r^2=l$, and we have $s=\sqrt{b^2-l}$, alſo,

$$D=\frac{Mr}{s^3}-\left[\frac{Mb}{s^3}-\frac{M}{b^2}\right]\times\text{coſ. }z.$$

$$F=-\left[\frac{Mb}{s^3}-\frac{M}{b^2}\right]\times\text{ſin. }z.$$

$$e=\int Fr^3\dot{v}.$$

$$P=Dr^2+\frac{Fr\dot{r}}{v}-2e.$$

Now $\frac{1}{s^3}=\frac{1}{\overline{b^2-l}|^{\frac{3}{2}}}=\overline{b^2-l}|^{-\frac{3}{2}}=\frac{1}{b^3}+\frac{3l}{2b^5}+\frac{15l^2}{8b^7}+\frac{35l^3}{16b^9}+\frac{315l^4}{128b^{11}}+$&c. ſubſtitute for l it's value $2br\times$ coſ. $z-r^2$, and for coſ. z^2, coſ. z^3, coſ. z^4, &c. put $\frac{1}{2}+\frac{1}{2}$ coſ. $2z$, $\frac{3}{4}$ coſ. $z+\frac{1}{4}$ coſ. $3z$, $\frac{3}{8}+\frac{1}{2}$ coſ. $2z+\frac{1}{8}$ coſ. $4z$, &c. and we get $\frac{1}{s^3}=\frac{1}{b^3}+\frac{9r^2}{4b^5}+\frac{225r^4}{64b^7}+\left(\frac{3r}{b^4}+\frac{45r^3}{8b^6}\right)\times$ coſ. $z+\left(\frac{15r^2}{4b^5}+\frac{105r^4}{16b^7}\right)$ coſ. $2z+\frac{35r^3}{8b^6}\times$ coſ. $3z+\frac{315r^4}{64b^7}\times$ coſ. $4z+$&c. which we might put under the general form $A+B$ coſ. $z+C$ coſ. $2z+D$ coſ. $3z+$&c. The value of $\frac{1}{s^3}$ being obtained, ſubſtitute it for it's value in the expreſſion for D, and we get the ſecond part of D $\left(\left(\frac{Mb}{s^3}-\frac{M}{b^2}\right)\times\text{coſ. }z\right)=M\times\left(\left(\frac{9r^2}{4b^4}+\frac{225r^4}{64b^6}\right)\times\text{coſ. }z+\left(\frac{3r}{b^3}+\frac{45r^3}{8b^5}\right)\times\text{coſ. }z\times\text{coſ. }z+\left(\frac{15r^2}{4b^4}+\frac{105r^4}{16b^6}\right)\times\text{coſ. }2z\times\text{coſ. }z+\frac{35r^3}{8b^5}\times\text{coſ. }3z\times\text{coſ. }z+\frac{315r^4}{64b^6}\times\text{coſ. }4z\times\text{coſ. }z\right)$; ſubſtitute for coſ. $z\times$ coſ. z, coſ. $2z\times$ coſ. z, coſ. $3z\times$ coſ. z, and coſ. $4z\times$ coſ. z their values,

values *, and we get $\left(\frac{Mb}{s^3} - \frac{M}{b^2}\right) \times \text{coſ.}\ z = M \times \left(\frac{3r}{2b^3} + \frac{45r^3}{16b^5} + \left(\frac{3r}{2b^3} + \frac{5r^3}{b^5}\right) \times \text{coſ}\ z + \left(\frac{33r^2}{8b^4} + \frac{435r^4}{64b^6}\right) \times \text{coſ}\ 2z + \left(\frac{15r^2}{8b^4} + \frac{735r^4}{128b^6}\right) \times \text{coſ.}\ 3z + \frac{35r^3}{16b^5} \times \text{coſ.}\ 4z + \frac{315r^4}{128b^6} \times \text{coſ}\ 5z\right)$, this we muſt ſubtract from the above value of $\frac{1}{s^3}$ multiplied by Mr, and we have $D = M \times \left(-\frac{r}{2b^3} - \frac{9r^3}{16b^5} + \frac{225r^5}{64b^7} - \left(\frac{9r^2}{8b^4} + \frac{75r^4}{64b^6}\right) \text{coſ.}\ z - \left(\frac{3r}{2b^3} + \frac{5r^3}{4b^5} - \frac{105r^5}{16b^7}\right) \times \text{coſ}\ 2z - \left(\frac{15r^2}{8b^4} + \frac{175r^4}{128b^6}\right) \times \text{coſ.}\ 3z - \left(\frac{35r^3}{16b^5} - \frac{315r^5}{64b^7}\right) \times \text{coſ.}\ 4z - \frac{315r^4}{128b^6} \times \text{coſ.}\ 5z\right)$.

1119. The value of F is $-\left(\frac{Mb}{s^3} - \frac{M}{b^2}\right) \times \text{ſin}\ z$, which is equal to the above value of $\left(\frac{Mb}{s^3} - \frac{M}{b^2}\right) \times \text{coſ}\ z$ multiplied by $-\text{ſin}\ z$ and divided by coſ. z, hence, $F = -M \times \left(\left(\frac{9r^2}{4b^4} + \frac{225r^4}{64b^6}\right) \times \text{ſin.}\ z + \left(\frac{3r}{b^3} + \frac{45r^3}{8b^5}\right) \times \text{coſ}\ z \times \text{ſin.}\ z + \left(\frac{15r^2}{4b^4} + \frac{105r^4}{16b^6}\right) \times \text{coſ}\ 2z \times \text{ſin.}\ z + \frac{35r^3}{8b^5} \times \text{coſ}\ 3z \times \text{ſin}\ z + \frac{315r^4}{64b^6} \times \text{coſ}\ 4z \times \text{ſin}\ z\right)$, ſubſtitute for coſ $z \times$ ſin z, coſ $2z \times$ ſin. z, coſ. $3z \times$ ſin. z and coſ. $4z \times$ ſin. z, their values †, and we get $F = -M \times \left(\left(\frac{3r^2}{8b^4} + \frac{15r^4}{64b^6}\right) \times \text{ſin}\ z + \left(\frac{3r}{2b^3} + \frac{5r^3}{8b^5}\right) \times \text{ſin.}\ 2z + \left(\frac{15r^2}{8b^4} + \frac{105r^4}{128b^6}\right) \times \text{ſin}\ 3z + \frac{35r^3}{16b^5} \times \text{ſin.}\ 4z + \frac{315r^4}{128b^6} \times \text{ſin.}\ 5z\right)$. The values of D and F being determined, we ſhall know the value of P, hence, we obtain S (1112), and thence (1114) the correction required.

1120. To find the values of the powers of r in terms of v. On account of the ſmall excentricity of the earth's orbit, we may neglect all the powers of u above the firſt, and from the ſlow motion of the apogee, we may ſuppoſe it

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* By Trigonometry, coſ $A \times$ coſ $B = \frac{1}{2}$ coſ $\overline{A+B} + \frac{1}{2}$ coſ $\overline{A-B}$, hence, coſ $z \times$ coſ. $z = \frac{1}{2}$ coſ. $2z$, coſ $2z \times$ coſ. $z = \frac{1}{2}$ coſ $3z + \frac{1}{2}$ coſ. z, coſ $3z \times$ coſ $z = \frac{1}{2}$ coſ. $4z + \frac{1}{2}$ coſ $2z$, and coſ $4z \times$ coſ. $z = \frac{1}{2}$ coſ. $5z + \frac{1}{2}$ coſ. $3z$.

† By Trigonometry, coſ $A \times$ ſin. $B = \frac{1}{2}$ ſin $\overline{A+B} - \frac{1}{2}$ ſin $\overline{A-B}$, hence, coſ $z \times$ ſin. $z = \frac{1}{2}$ ſin $2z$, coſ. $2z \times$ ſin $z = \frac{1}{2}$ ſin $3z - \frac{1}{2}$ ſin. z, coſ. $3z \times$ ſin $z = \frac{1}{2}$ ſin $4z - \frac{1}{2}$ ſin $2z$, and coſ $4z \times$ ſin. $z = \frac{1}{2}$ ſin $5z - \frac{1}{2}$ ſin. $3z$.

it fixed; hence (1111), $\frac{1}{r} = \frac{1}{p} - \frac{u}{p}$ coſ. v is the equation from which we muſt deduce r, therefore $\frac{r}{p} = 1 + u \times$ coſ v, in which the ſemi-parameter p and excentricity u are known from obſervation, therefore $\frac{r^2}{p^2} = 1 + 2u \times$ coſ v, $\frac{r^3}{p^3} = 1 + 3u \times$ coſ. v, &c. and by taking the fluxion of the firſt of theſe, we have $\frac{r\dot{r}}{p^2 v} = -u \times$ ſin. v.

1121. To find the value of ſin z, ſin. $2z$, coſ z, coſ. $2z$, in terms of v; for, in this caſe, it will not be neceſſary to go any farther in the values of D (1118) and F (1119) As $z = nv - 2u \times$ ſin $\overline{1-n} \times$ ſin v (1117), we have by Trigonometry, conſidering the coſine of the laſt term $= 1$, and the term itſelf $=$ to it's ſine on account of it's ſmallneſs,

$$\text{ſin. } z = \text{ſin. } nv - u \times \overline{1-n} \times \text{ſin } \overline{n+1} \times v + u \times \overline{1-n} \times \text{ſin. } \overline{n-1} \times v,$$
$$\text{coſ. } z = \text{coſ. } nv - u \times \overline{1-n} \times \text{coſ. } \overline{n+1} \times v + u \times \overline{1-n} \times \text{coſ } \overline{n-1} \times v,$$
$$\text{ſin. } 2z = \text{ſin } 2nv - 2u \times \overline{1-n} \times \text{ſin. } \overline{2n+1} \times v + 2u \times \overline{1-n} \times \text{ſin. } \overline{2n-1} \times v,$$
$$\text{coſ. } 2z = \text{coſ. } 2nv - 2u \times \overline{1-n} \times \text{coſ. } \overline{2n+1} \times v + 2u \times \overline{1-n} \times \text{coſ. } \overline{2n-1} \times v.$$

1122 Let us aſſume $M = \frac{1}{1067}$, according to Sir I Newton, the maſs of the ſun being unity, which we may conſider as the maſs of the ſun and earth, on account of the extreme ſmallneſs of the earth in reſpect to the ſun, and, by obſervation, the value of $\frac{1}{b} = 0{,}192451$; therefore $\frac{1}{b^3} = 0{,}0071264$, $\frac{1}{b^4} = 0{,}0013714$, $\frac{1}{b^5} = 0{,}0002639$, $\frac{1}{b^6} = 0{,}0000508$, $\frac{1}{b^7} = 0{,}00000977$, &c. alſo, $1 - n = \frac{1}{11{,}857}$, therefore $n = 0{,}915659$, and $u = 0{,}01683$, conſequently $u \times \overline{1-n} = \frac{1}{352}$, $n - 1 = -0{,}0843$, $n + 1 = 1{,}91566$, $2n - 1 = 0{,}8313$ and $2n + 1 = 2{,}8313$ Alſo, we aſſume $r = 1$, it's mean value.

1123. To find the value of $e = \int Fr^3 v$. We have (1119) $Fr^3 = -M \times \left(\left(\frac{3r^5}{8b^4} + \frac{15r^7}{64b^6}\right) \times \text{ſin. } z + \left(\frac{3r^4}{2b^3} + \frac{5r^6}{8b^5}\right) \times \text{ſin. } 2z\right) = -M \times 0{,}0005262 \times$ ſin. $z - M \times 0{,}0108545 \times$ ſin. $2z$. Now for ſin. z and ſin. $2z$, ſubſtitute their values found in Art. 1121 neglecting the terms $-n\times\overline{1-n}\times\text{ſin}\ \overline{n+1}\times v$ and $-2n\times\overline{1-n}\times\text{ſin}\ \overline{2n+1}\times v$, becauſe, when the fluent is taken, the denominators will be ſo large as to render the terms too ſmall for conſideration. This ſubſtitution being made, and ſubſtituting alſo for M it's value, we have, after taking the fluent,

$$e=0{,}0000005356\times\text{coſ.}\ nv+0{,}000005555\times\text{coſ.}\ 2nv-0{,}0000002474\times\text{coſ}\ \overline{n-1}\times v+0{,}0000004457\times\text{coſ}\ \overline{2n-1}\times v.$$

1124. To find the value of Dr^2. The firſt term $M\times\left(-\frac{r}{2b^3}-\frac{9r^3}{16b^5}+\frac{225r^5}{64b^7}\right)$ in the value of D (1118) not being multiplied into the ſine or coſine of z, may be here neglected, it not being a variable quantity depending on the place of the body, but only a conſtant part of the whole force of the earth to the ſun, and we are here conſidering only the variable quantities which produce the irregularities of the earth's motion in different parts of it's orbit. We have therefore only to multiply the next two terms of the value of D into r^2, and we obtain $-M\times\left\{\frac{9r^2}{8b^4}+\frac{75r^4}{64b^6}\right\}\times\text{coſ}\ z-M\times\left\{\frac{3r}{2b^3}+\frac{5r^3}{4b^5}-\frac{105r^5}{16b^7}\right\}\times\text{coſ}\ 2z=-M\times0{,}0016023\times\text{coſ}\ z-M\times0{,}019554\times\text{coſ}\ 2z$, ſubſtitute for M it's value, and for coſ. z and coſ $2z$ their values found in Art 1121. neglecting $-n\times\overline{1-n}\times\text{coſ.}\ \overline{n+1}\times v$ and $-2n\times\overline{1-n}\times\text{coſ.}\ \overline{2n+1}\times v$, on account of their ſmall effects, and we get (for our preſent purpoſe) the value of

$$Dr^2=-0{,}0000015017\ \text{coſ.}\ nv-0{,}0000010267\ \text{coſ.}\ 2nv-0{,}00000005265\ \text{coſ.}\ \overline{n-1}\times v-0{,}000000289\ \text{coſ.}\ \overline{2n-1}\times v.$$

1125 To find the value of $\frac{Frr}{v}$. Aſſuming $\dot{p}=1$, we have (1120) $\frac{rr}{v}=-n\times\text{ſin.}\ v$, hence, $\frac{Frr}{v}=M\times\left\{\frac{3r^2}{8b^4}+\frac{15r^4}{64b^6}\right\}\times\text{ſin.}\ z\times n\times\text{ſin.}\ v+M\times\left\{\frac{3r}{2b^3}+\frac{5r^3}{8b^5}\right\}\times\text{ſin.}\ 2z\times n\times\text{ſin.}\ v$, now for ſin. z and ſin. $2z$ ſubſtitute their values found in Art. 1121 and here all the terms which ariſe will be found ſo extremely ſmall that they might be all omitted, one of the principal of the terms, however, $+0{,}0000000848\ \text{coſ.}\ \overline{2n-1}\times v$ (ariſing from the ſubſtitution

ſubſtitution of the term $2u\times\overline{1-n}\times$ ſin. $\overline{2n-1}.v$ in the value of ſin. $2z$) of the ſame ſpecies as one of thoſe above, we ſhall retain, we therefore aſſume

$$\frac{Frr}{v}=0{,}0000000848 \text{ coſ. } \overline{2n-1}\times v.$$

1126. The value of P is (1118) $Dr^2+\frac{Frr}{v}-2e$, by ſubſtitution therefore from the three laſt Articles, we get

$$P=-0{,}0000025788 \text{ coſ. } nv-0{,}000021377 \text{ coſ. } 2nv+0{,}00000044215 \text{ coſ. } \overline{n-1}\times v-0{,}0000010956 \text{ coſ } \overline{2n-1}\times v.$$

Thus it appears, that P is expreſſed by a ſeries of coſines of the multiples of v (1109)

1127. Aſſuming $p=1$, if $P=a'\times$ coſ. $mv+b'\times$ coſ $nv+$&c then $S=\frac{-a'}{m^2-1}\times$ coſ $mv-\frac{b'}{n^2-1}\times$ coſ $nv-$&c. (1111); from the above value of P therefore we obtain the correſponding value of

$$S=-0{,}00001597 \text{ coſ } nv+0{,}0000090081 \text{ coſ. } 2nv+0{,}0000004453 \text{ coſ. } \overline{n-1}\times v-0{,}000003548 \text{ coſ } \overline{2n-1}\times v.$$

1128. The correction of the expreſſion for the time (1114) is the fluent of $-\overline{2S+e}\times\dot{v}-2w\times\overline{3S+e}\times\dot{v}$, ſubſtitute therefore for S and e their values above found, and take the fluent, retaining thoſe terms only which are of the ſame ſpecies as thoſe we have hitherto retained, and neglecting the others as being extremely ſmall, and we get the *correction of the time*

$$=0{,}00003429 \text{ ſin. } nv-0{,}00001295 \text{ ſin. } 2nv-0{,}00000194 \text{ ſin } \overline{n-1}\times v+0{,}00000735 \text{ ſin } \overline{2n-1}\times v.$$

1129. By Art. 1114. the correction of the mean longitude, ſo far as regards the diſturbing forces, is the above correction of the time with it's ſign changed, which correction is in terms of the true longitude, but here for the *true* longitude we may ſubſtitute the *mean*, without producing any ſenſible error; if therefore x repreſent the mean longitude of the earth, we have the *correction of the mean longitude*

$$= -0{,}00003429\ \text{ſin}\ nx + 0{,}00001295\ \text{ſin}\ 2nx + 0{,}00000194\ \text{ſin.}\ \overline{n-1} \times x - 0{,}00000735\ \text{ſin}\ \overline{2n-1} \times x.$$

1130. The mean motion of Jupiter : that of the earth :: $1-n$: 1, or as $\overline{1-n} \times x : x$; hence, nx = the mean longitude of the earth − that of Jupiter, which put $= y$. If therefore (1115) we multiply each of the co-efficients by $57°{,}29578$, we get *the equations of the mean motion of the earth ariſing from the action of Jupiter*

$$= -7'',1 \times \text{ſin.}\ y + 2'',7 \times \text{ſin.}\ 2y + 0'',4 \times \text{ſin}\ \overline{y-x} - 1'',5 \times \text{ſin}\ \overline{2y-x}.$$

1131. In the foregoing ſolution, we have ſuppoſed the orbit of Jupiter to be a circle, and conſequently b was conſtant; but if b repreſent the ſemi-parameter, b' the true diſtance, u' the excentricity of the orbit divided by the ſemi-axis-major, and v' the true anomaly, then $b' = \frac{b}{1 - u'\ \text{coſ}\ v'}$; the mean anomaly alſo of Jupiter $= v' + 2u'\ \text{coſ}\ v'$, proved as in Art 1117 and the mean motion of the earth being to that of Jupiter as $1+n$ to 1, the mean motion of the earth $= \overline{1+n}\ v' + \overline{1+n} \times 2u'\ \text{coſ}\ v'$, from this ſubtract v' the true motion of Jupiter, and we get $z = nv' + \overline{1+n} \times 2u'\ \text{coſ.}\ v'$. Alſo $\frac{b'}{b} = 1 + u'\ \text{coſ.}\ v'$. Proceed therefore for this value of z and $\frac{b'}{b}$, as we have done for that of z and $\frac{r}{p}$ in the foregoing operation, aſſuming thoſe terms only where u' enters, by which we ſhall obtain the equations ariſing from the excentricity of Jupiter's orbit. We have here omitted thoſe terms where the powers of the excentricity above the firſt have entered, but if we had taken any of the higher powers into conſideration, we might, in the ſame manner, have got the equations thence ariſing. If the reader wiſh to ſee an example where the excentricity of each orbit is conſidered, and alſo the ſquare of the excentricity of the orbit of the body which is diſturbed, he may conſult the *Mem. de l'Acad. Roy. des Scien.* 1761, where he will find M. de la LANDE has computed the equations of the orbit of *Mars* ariſing from the attraction of the earth, taking all theſe circumſtances into conſideration. M. de la PLACE, in computing the

the equations of *Jupiter* and *Saturn* from their mutual attractions, has taken into conſideration the third power of the excentricity. *Mem. de l'Acad.* 1785, 1786

1132. The general value $\overline{b^2+r^2-2br \text{ coſ. } z}\,\big|^{-\frac{3}{2}}$ is (aſſuming the mean radius of the earth's orbit unity) $[b^2+1+2bu' \text{ coſ. } v'+2u \text{ coſ } v-(2b+2bu \text{ coſ } v+2u' \text{ coſ } v') \text{ coſ. } z]^{-\frac{3}{2}}$. The equations may be computed, by firſt ſuppoſing the two orbits to be circular; and after that, by ſuppoſing firſt one of them to be excentric and then the other, aſſuming, in the two latter operations, the terms ariſing from the excentricities reſpectively, repeating the ſame proceſs each time Or (as we have here done) we may take into conſideration at firſt the whole effect ariſing from one of the orbits ſuppoſed to be excentric, and then compute the effect ariſing from the excentricity only of the other.

To reſolve $\overline{b^2+r^2-2br \text{ coſ. } z}\,\big|^{-\frac{3}{2}}$ *into the Series* $A+B$ *coſ.* $z+C$ *coſ.* $2z+D$ *coſ.* $3z+E$ *coſ.* $4z+$ *&c.*

1133 When the difference of the radii of the two orbits is very conſiderable in reſpect to the greater of the two radii, the ſeries expreſſing the value of $\frac{1}{s^3}$* (1118) converges ſo quick, that a few terms will ſuffice, as in the example here given, but if that difference be ſmall, the ſeries will converge but ſlow, and the labour by the direct method becoming very great, we muſt have recourſe to other means to obtain the values of the coefficients EULER, who firſt diſcovered that the ſeries might be expreſſed under the form $A+B\times$ coſ $z+C\times$ coſ. $2z+D\times$ coſ $3z+$&c. gave two methods of finding the coefficients, which we ſhall here explain.

1134. By

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* The expreſſion which we here want to expand (1118) is $\overline{b^2+r^2-2br \text{ coſ. } z}\,\big|^{-\frac{3}{2}}$, which may be reduced to $\frac{1}{\overline{b^2+r^2}|^{\frac{3}{2}}}\times\overline{1-\frac{2br}{b^2+r^2}\times \text{ coſ } z}\,\big|^{-\frac{3}{2}}=\left(\text{if } g=\frac{2br}{b^2+r^2}\right)\frac{1}{\overline{b^2+r^2}|^{\frac{3}{2}}}\times \overline{1-g \text{ coſ } z}\,\big|^{-\frac{3}{2}}$, where $\overline{1-g \text{ coſ. } z}\,\big|^{-\frac{3}{2}}$ is the variable part.

1134. By the Binomial Theorem, $\overline{1-g\ \text{coſ.}\ z}|^{-m} = 1+\frac{m}{1}g\ \text{coſ.}\ z+\frac{m.\overline{m+1}}{1.2}$ $g^2\ \overline{\text{coſ}\ z}^2+\frac{m.\overline{m+1}.\overline{m+2}}{1.2.3}g^3\ \overline{\text{coſ.}\ z}^3+$&c. Now

$$\text{Coſ.}\ z = \text{coſ.}\ z$$

$$\overline{\text{Coſ}\ z}^2 = \frac{1}{2}\left(\text{coſ.}\ 2w+\frac{1}{2}.\frac{2}{1}\right).$$

$$\overline{\text{Coſ.}\ z}^3 = \frac{1}{4}\left(\text{coſ.}\ 3w+\frac{3}{1}\ \text{coſ.}\ w\right).$$

$$\overline{\text{Coſ.}\ z}^4 = \frac{1}{8}\left(\text{coſ.}\ 4w+\frac{4}{1}\ \text{coſ.}\ 2w+\frac{1}{2}.\frac{4.3}{1.2}\right).$$

$$\overline{\text{Coſ.}\ z}^5 = \frac{1}{16}\left(\text{coſ.}\ 5w+\frac{5}{1}\ \text{coſ}\ 3w+\frac{5.4}{1.2}\ \text{coſ.}\ w\right).$$

$$\overline{\text{Coſ.}\ z}^6 = \frac{1}{32}\left(\text{coſ.}\ 6w+\frac{6}{1}\ \text{coſ.}\ 4w+\frac{6\ \ 5}{1\ \ 2}\ \text{coſ.}\ 2w+\frac{1}{2}.\frac{6.5.4}{1.2.3}\right).$$

$$\overline{\text{Coſ}\ z}^7 = \frac{1}{64}\left(\text{coſ.}\ 7w+\frac{7}{1}\ \text{coſ.}\ 5w+\frac{7\ \ 6}{1.2}\ \text{coſ.}\ 3w+\frac{7\ \ 6.5}{1.2\ \ 3}\ \text{coſ.}\ w\right).$$

&c. &c &c

1135. As therefore the values of $\overline{\text{coſ}\ z}^2$, $\overline{\text{coſ}\ z}^3$, $\overline{\text{coſ}\ z}^4$, &c. &c. are expreſſed in terms of coſ. z, coſ. $2z$, coſ. $3z$, coſ. $4z$, &c. we have, by ſubſtitution, $\overline{1-g\ \text{coſ.}\ z}|^{-m} = A+B\ \text{coſ}\ z+C\ \text{coſ}\ 2z+D\ \text{coſ}\ 3z+E\ \text{coſ.}\ 4z+$ &c. $A, B, C, D, E,$ &c being the coefficients ariſing from the ſubſtitution. Now from the ſubſtitution it appears that

$$A = \begin{cases} 1+\frac{2}{4}\times\frac{m\,.\,\overline{m+1}}{1\,.\,2}g^2+\frac{4\,.\,3}{4\,.\,8}\times\frac{m\,.\,\overline{m+1}\,.\,\overline{m+2}\,.\,\overline{m+3}}{1\,.\,2\,.\,3\,.\,4}g^4+ \\ \frac{6\,.\,5\,.\,4}{4\,.\,8\,.\,12}\times\frac{m\,.\,\overline{m+1}\,.\,\overline{m+2}\,.\,\overline{m+3}\,.\,\overline{m+4}\,.\,\overline{m+5}}{1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6}g^6+\&c. \end{cases}$$

$$B = mg\times\begin{cases} 1+\frac{3}{4}\times\frac{\overline{m+1}\,.\,\overline{m+2}}{2\,.\,3}g^2+\frac{5\,.\,4}{4\,.\,8}\times\frac{\overline{m+1}\,.\,\overline{m+2}\,.\,\overline{m+3}\,.\,\overline{m+4}}{2\,.\,3\,.\,4\,.\,5}g^4 \\ +\frac{7\,.\,6\,.\,5}{4\,.\,8\,.\,12}\times\frac{\overline{m+1}\,.\,\overline{m+2}\,.\,\overline{m+3}\,.\,\overline{m+4}\,.\,\overline{m+5}\,.\,\overline{m+6}}{2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7}g^6+\&c. \end{cases}$$

The values of the other coefficients C, D, E, &c we ſhall find in terms of A and B, and therefore having computed A and B, we ſhall very eaſily obtain C, D, E, &c

1136 Let $s=A+B\,\text{coſ.}\,z+C\,\text{coſ}\,2z+D\,\text{coſ.}\,3z+E\,\text{coſ}\,4z+F\,\text{coſ.}\,5z$ + &c $=\overline{1-g\,\text{coſ.}\,z}\big|^{-m}$, then log $s=-m\times\text{log.}\,\overline{1-g\,\text{coſ.}\,z}$, and $\frac{\dot{s}}{s}=\frac{-mg\,\text{ſin}\,z\,\dot{z}}{1-g\,\text{coſ}\,z}$, therefore $\frac{\dot{s}}{\dot{z}}\times\overline{1-g\,\text{coſ}\,z}+mgs\,\text{ſin.}\,z=0$. In this equation ſubſtitute $A+B\,\text{coſ}\,z+C\,\text{coſ}\,2z+\&c.$ for s, and $-B\,\text{ſin.}\,z.\dot{z}-2C\,\text{ſin.}\,2z\,\dot{z}-\&c$ for $\dot{s}$, and ſubſtituting alſo for ſin. $nz\times\text{coſ.}\,z$ it's value $\frac{1}{2}\,\text{ſin.}\,\overline{n+1}\,.\,z+\frac{1}{2}\,\text{ſin.}\,\overline{n-1}\,z$, and for coſ $nz\times\text{ſin.}\,z$ it's value $\frac{1}{2}\,\text{ſin.}\,\overline{n+1}.z-\frac{1}{2}\,\text{ſin.}\,\overline{n-1}.z$, according as $n=1, 2, 3, 4, 5,$ &c. and we get

$$\frac{\dot{s}}{\dot{z}}=-B$$

$$\left.\begin{array}{rlllll}
\frac{\mathring{s}}{\dot{z}}= - & B\,\text{ſin.}\,z - & 2C\,\text{ſin.}\,2z - & 3D\,\text{ſin.}\,3z - & 4E\,\text{ſin.}\,4z - & 5F\,\text{ſin.}\,5z - \&\text{c.} \\
-\frac{g\dot{s}}{z}\times\text{coſ.}\,z = & & +\frac{1}{2}\,gB\,\text{ſin.}\,2z + & \frac{2}{2}\,gC\,\text{ſin.}\,3z + & \frac{3}{2}\,gD\,\text{ſin.}\,4z + & \frac{4}{2}\,gE\,\text{ſin.}\,5z + \&\text{c} \\
 & +\frac{2}{2}\,gC\,\text{ſin.}\,z + & \frac{3}{2}\,gD\,\text{ſin.}\,2z + & \frac{4}{2}\,gE\,\text{ſin.}\,3z + & \frac{5}{2}\,gF\,\text{ſin.}\,4z + & \frac{6}{2}\,gG\,\text{ſin}\,5z + \&\text{c.} \\
mgs\,\text{ſin}\,z = & +mgA\,\text{ſin.}\,z + & \frac{1}{2}mgB\,\text{ſin.}\,2z + & \frac{1}{2}mgC\,\text{ſin}\,3z + & \frac{1}{2}mgD\,\text{ſin.}\,4z + & \frac{1}{2}mgE\,\text{ſin.}\,5z + \&\text{c.} \\
 & -\frac{1}{2}mgC\,\text{ſin.}\,z - & \frac{1}{2}mgD\,\text{ſin.}\,2z - & \frac{1}{2}mgE\,\text{ſin.}\,3z - & \frac{1}{2}mgF\,\text{ſin.}\,4z - & \frac{1}{2}mgG\,\text{ſin}\,5z - \&\text{c.}
\end{array}\right\} = 0.$$

Make the coefficients of ſin. z, ſin. $2z$, &c. reſpectively $=0$, and we get

$$C=\frac{2B-2mgA}{\overline{2-m}\times g},\quad D=\frac{4C-\overline{m+1}\,.\,gB}{\overline{3-m}\times g},\quad E=\frac{6D-\overline{m+2}\;gC}{\overline{4-m}\times g},\quad F=\frac{8E-\overline{m+3}\,.\,gD}{\overline{5-m}\times g},\quad G=\frac{10F-\overline{m+4}\;gE}{\overline{6-m}\times g},$$

where the law of continuation is manifeſt; and putting $m=\frac{3}{2}$, which is it's value in the preſent caſe, we find

$$C=\frac{4B-6gA}{g},\quad D=\frac{8C-5gB}{3g},\quad E=\frac{12D-7gC}{5g},\quad F=\frac{16E-9gD}{7g},\quad G=\frac{20F-11gE}{9g},\ \&\text{c.}$$

1137. The

1137. The principal part of this operation therefore conſiſts in finding the values of A and B. One of the methods which Euler propoſes for this purpoſe is this. In each ſeries for A and B, compute the value of a few of the firſt terms, for inſtance, of ten terms, and let the following terms be $O+OPg^2+OPQg^4+OPQRg^6+$&c and then we have only to find the ſum of this ſeries, or of the ſeries $1+Pg^2+PQg^4+PQRg^6+$&c. and as the factors P, Q, R, &c. approach to unity, we may conſider this as a recurring ſeries, and which may therefore be repreſented by a fraction. Let therefore $\frac{1+\alpha g^2+\beta g^4}{1-\gamma g^2-\delta g^4}=1+Pg^2+PQg^4+PQRg^6+$&c. and we have

$$\left.\begin{array}{llllll} 1+Pg^2+ & PQg^4+ & PQRg^6+ & PQRSg^8+ & \&c \\ -\gamma & -\gamma P & -\gamma PQ & -\gamma PQR & -\&c \\ & -\delta & -\delta P & -\delta PQ & -\&c \\ -1-\alpha & -\beta & & & \end{array}\right\}=0,$$

hence, $P-\gamma-\alpha=0$, $PQ-\gamma P-\delta-\beta=0$, $PQR-\gamma PQ-\delta P=0$, $PQRS-\gamma PQR-\delta PQ=0$, from the two laſt equations we find $\gamma=\frac{R\times\overline{Q-S}}{Q-R}$, $\delta=Q\times\overline{R+\gamma}$, therefore $\alpha=P-\gamma$ and $\beta=PQ-\gamma P-\delta$ are known. But as the factors P, Q, R, S, &c approach to unity, at an infinite diſtance we have $PQRS$ &c. $-\gamma PQR$ &c. $-\delta PQ$ &c. $=0$, and the quantities $PQRS$ &c. PQR &c. PQ &c. having each an infinite number of factors and the firſt only one more than the ſecond, and the ſecond one more than the third, and the laſt factors alſo equal to unity, we have theſe quantities $PQRS$ &c. PQR &c. PQ &c. ultimately equal, therefore $1-\gamma-\delta=0$, or $\gamma+\delta=1$; hence, $\gamma=R+\frac{R-1}{Q-1}$; $\delta=1-\gamma$, $\alpha=P-\gamma$, $\beta=PQ-\gamma P-\delta$. Thus we obtain the value of $O\times\frac{1+\alpha g^2+\beta g^4}{1-\gamma g^2-\delta g^4}$, which repreſents the value of all the terms after thoſe which were computed. To adapt this to our preſent purpoſe, we muſt aſſume $m=\frac{3}{2}$.

1138. The values of A and B may alſo be thus obtained. Let $\overline{1-g\cos. z}|^{-\frac{3}{2}}=A+B\cos. z+C\cos. 2z+D\cos. 3z+$&c and then $\overline{1-g\cos z}|^{-\frac{3}{2}}\times\dot{z}=A\dot{z}+B\cos. z.\dot{z}+C\cos. 2z\dot{z}+$&c therefore $\int\overline{1-g\cos z}|^{-\frac{3}{2}}\times\dot{z}=Az+B\times\sin z+\frac{1}{2}C\sin 2z+$&c make $z=c$ an arc

of

of 180°, and all the terms after Az will become $=0$; hence, $A=\dfrac{\int \overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}\times\dot{z}}{c}$. Now the fluent of $\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}\times\dot{z}$ is the area of a curve whoſe ordinate is $\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}$ and abſciſſa z. Let therefore FIG 241. $PZ=$ an arc of 180°, $PW=z$, $WH=\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}$, and AK the locus of all the points H, and the area $PAHW$ will be the fluent of $\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}\times\dot{z}$, but this we can find only by an approximation. Let the abſciſſa PZ be divided into a great many (α) equal parts $PQ=QR=RS=ST=$&c. and let each of theſe parts be called unity, then as the ordinates are ſuppoſed to be very near, $PQ\times QB$, or (as $PQ=1$) $QB=$ area $PAQB$ nearly, and thus $RC=$ area $QBCR$ nearly, $SD=$ area $RCDS$ nearly, &c. therefore any area $PAET=QB+RC+SD+TE$ nearly; divide therefore c into $\alpha=$ parts, and let the ſucceſſive values of z be $\frac{c}{\alpha}$, $\frac{2c}{\alpha}$, $\frac{3c}{\alpha}$, ... $\frac{\alpha c}{\alpha}$; then will $\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}$ become $\overline{1-g\ \text{coſ}\ \frac{c}{\alpha}}\Big|^{-\frac{3}{2}}=QB=$ area $PABQ$, $\overline{1-g\ \text{coſ}\ \frac{2c}{\alpha}}\Big|^{-\frac{3}{2}}=RC=$ area $QBCR$, $\overline{1-g\ \text{coſ}\ \frac{3c}{\alpha}}\Big|^{-\frac{3}{2}}=SD=$ area $RCDS$, &c. nearly, put theſe quantities $=H, I, K, L,$ &c reſpectively, and the fluent of $\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}\times\dot{z}=H+I+K+L+$&c. nearly. Now let us conſider what is the fluent of all the Az on the ſame ſuppoſition. As we have found the fluent of $\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}\times\dot{z}$ by taking the ſucceſſive increments and adding them together, we muſt find the value of the fluent of Az in the ſame manner. Now as the increment of z is called unity, the whole value of $A\dot{z}$ thus generated is $A\times$ number of theſe increments $=A\alpha$ when $z=c$. Now the fluent of $\overline{1-g\ \text{coſ}\ z}\big|^{-\frac{3}{2}}\times\dot{z}=\int A\dot{z}$ (neglecting the other terms which vaniſh when $z=c$); hence, $H+I+K+L+$&c. $=A\alpha$, and $A=\dfrac{H+I+K+L+\&c}{\alpha}$.

1139. To find the value of B, multiply $\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}=A+B\ \text{coſ}\ z+C\ \text{coſ.}\ 2z+$&c by $\text{coſ.}\ z\times\dot{z}$, and we have $\overline{1-g\ \text{coſ}\ z}\big|^{-\frac{3}{2}}\times\text{coſ}\ z\times\dot{z}=A\ \text{coſ}\ z\,.\,z+B\ \overline{\text{coſ.}\ z}^2.\,\dot{z}+C\ \text{coſ.}\ z\,.\ \text{coſ}\ 2z\,.\,z+$&c $=A\ \text{coſ.}\ z\,.\,z+B\times\overline{\tfrac{1}{2}+\tfrac{1}{2}\ \text{coſ.}\ 2z}\times z+$&c. $=A\times\text{coſ.}\ z\,.\,\dot{z}+\tfrac{1}{2}B\dot{z}+\tfrac{1}{2}B\ \text{coſ.}\ 2z\,.\,\dot{z}+$&c. Now the

the fluent of $\frac{1}{2} B\dot{z}$ is $\frac{1}{2} Bz$, and the fluent of all the other terms will be in terms of ſin z, ſin. $2z$, ſin. $3z$, &c and therefore when $z=c=180°$ they all vaniſh; hence, when z becomes c, $\int \overline{1-g \text{ coſ. } z}\big|^{-\frac{3}{2}} \times \text{coſ } z \ \dot{z} = \frac{1}{2} Bc$, therefore $B = \frac{\int \overline{1-g \text{ coſ } z}\big|^{-\frac{3}{2}} \times \text{coſ } z \ \dot{z}}{\frac{1}{2} c}$. If therefore any ordinate $WH = \int \overline{1-g \text{ coſ } z}\big|^{-\frac{3}{2}} \times \text{coſ } z$, by proceeding exactly as before, we get $B = \frac{H'+J'+K'+L'+\&c}{\frac{1}{2}\alpha}$, where H', I', K', L', &c. repreſent QB, RC, SD, TE, &c. the values of $\overline{1-g \text{ coſ. } z}\big|^{-\frac{3}{2}} \times \text{coſ. } z$, when we ſubſtitute $\frac{c}{\alpha}$, $\frac{2c}{\alpha}$, $\frac{3c}{\alpha}$, &c for z. The more parts α into which we divide c, the nearer is the approximation.

1140. But we may approximate to the area $PAKZ$ more nearly in this manner $BQ \times PR =$ area $PACR$ very nearly; $SD \times RT =$ area $RCET$ very nearly; &c in which caſe the number of ſuch quantities $= \frac{1}{2}\alpha$ Now let us, for inſtance, divide c into 20 = parts, or let $\alpha = 20$, then as we now aſſume for z the firſt, third, fifth, &c ordinates, the reſpective values of z, if we take them in parts of a right angle ϱ inſtead of two right angles c, will be $\frac{1}{10}\varrho$, $\frac{3}{10}\varrho$, $\frac{5}{10}\varrho$, $\frac{7}{10}\varrho$, $\frac{9}{10}\varrho$, $\frac{11}{10}\varrho$, $\frac{13}{10}\varrho$, $\frac{15}{10}\varrho$, $\frac{17}{10}\varrho$, $\frac{19}{10}\varrho$, the coſines of which we muſt ſubſtitute for z in $\overline{1-g \text{ coſ. } z}\big|^{-\frac{3}{2}}$ in order to get the firſt, third, fifth, &c. ordinates, and the coſ. z being negative for the laſt five values, the quantity $-g$ coſ. z becomes *poſitive*, but coſ. $\frac{19}{10}\varrho =$ ſin. $\frac{9}{10}\varrho$, coſ. $\frac{17}{10}\varrho =$ ſin. $\frac{7}{10}\varrho$, coſ. $\frac{15}{10}\varrho =$ ſin. $\frac{5}{10}\varrho$, coſ. $\frac{13}{10}\varrho =$ ſin. $\frac{3}{10}\varrho$, and coſ. $\frac{11}{10}\varrho =$ ſin. $\frac{1}{10}\varrho$, ſubſtituting therefore theſe ſines inſtead of the coſines in the value of A (1138), and $\frac{1}{2}\alpha$ inſtead of α, and we get

$$A = \frac{1}{10}\left\{\begin{array}{l} \overline{1-g \text{ coſ } \frac{1}{10}\varrho}\Big|^{-\frac{3}{2}} + \overline{1-g \text{ coſ. } \frac{3}{10}\varrho}\Big|^{-\frac{3}{2}} + \overline{1-g \text{ coſ } \frac{5}{10}\varrho}\Big|^{-\frac{3}{2}} + \&c. \\ \overline{1+g \text{ ſin. } \frac{1}{10}\varrho}\Big|^{-\frac{3}{2}} + \overline{1+g \text{ ſin. } \frac{3}{10}\varrho}\Big|^{-\frac{3}{2}} + \overline{1+g \text{ ſin. } \frac{5}{10}\varrho}\Big|^{-\frac{3}{2}} + \&c. \end{array}\right.$$

This

This is the other expreſſion for A * which EULER has given in his *Recherches des Inegalitès de Saturn et de Jupiter*, which he ſays is ſufficiently accurate. The number of the quantities is only ten, and are very eaſily computed in numbers. If z be divided into a greater number of parts, the concluſion will be more accurate.

1141. To obtain the value of B, inſtead of the quantity $\overline{1-g\ \text{coſ.}\ z}\big|^{-\frac{3}{2}}$, we have $\overline{1-g\ \text{coſ}\ z}\big|^{-\frac{3}{2}}\times \text{coſ}\ z$, ſubſtitute ſin. z for coſ z, as before, and without the vinculum coſ. z becomes negative for the laſt half of the terms; hence,

$$B=\tfrac{1}{5}\left\{\begin{array}{l}\overline{1-g\ \text{coſ.}\ \frac{1}{10}\varrho}\Big|^{-\frac{3}{2}}\times\text{coſ.}\ \frac{1}{10}\varrho+\overline{1-g\ \text{coſ.}\ \frac{3}{10}\varrho}\Big|^{-\frac{3}{2}}\times\text{coſ.}\ \frac{3}{10}\varrho+\&\text{c}\\ -\overline{1+g\ \text{ſin.}\ \frac{1}{10}\varrho}\Big|^{-\frac{3}{2}}\times\text{ſin.}\ \frac{1}{10}\varrho-\overline{1+g\ \text{ſin.}\ \frac{3}{10}\varrho}\Big|^{-\frac{3}{2}}\times\text{ſin.}\ \frac{3}{10}\varrho-\&\text{c.}\end{array}\right.$$

1142. But there is another method by which we may approximate to the area of this curve, by ſuppoſing a parabolic curve $y=a'+b'x+c'x^2+d'x^3+\&c.$ to paſs through the extremities of any number $a, b, c, d, e, \ldots l, m, n,$ of ordinates, x repreſenting the abſciſſa and y the ordinate. For inſtance, take three ordinates a, b, c, and the parabolic curve paſſing through them is $y=a'+b'x+c'x^2$, the area of which, in terms of a, b, c, is $\frac{1}{3}a+\frac{4}{3}b+\frac{1}{3}c$ † $=$ the area $PACR$, for the ſame reaſon, $\frac{1}{3}c+\frac{4}{3}d+\frac{1}{3}e=$ area $RCET$, &c. &c. hence, the whole area $=\frac{1}{3}a+\frac{4}{3}b+\frac{2}{3}c+\frac{4}{3}d+\frac{2}{3}e\ldots\ldots\frac{2}{3}l+\frac{4}{3}m+\frac{1}{3}n$. that is, the whole area $=\frac{1}{3}$ of the ſum of the firſt and laſt ordinates $+\frac{4}{3}$ of the ſum of the alternate ordinates, beginning at the ſecond, $+\frac{2}{3}$ of the ſum of the alternate ordinates, beginning at the third. Thus we may find the values of A and B from the known ordinates of the curve. Multiply theſe values of A and B by $\dfrac{1}{\overline{b^2+r^2}\big|^{\frac{3}{2}}}$, and we obtain the values of A' and B'.

1143. M. de la GRANGE

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* In the caſe of the firſt and third ſatellites of Jupiter, M. BAILLY has ſhown that this does not differ more than the 2000th part of the whole from the truth.

† This is proved in the Chapter upon Interpolation.

1143. M de la GRANGE has thus inveſtigated the values of the coefficients.

Let $V = b^2 + r^2 - 2br\ \text{coſ.}\ z$, and ſuppoſe

$$\frac{1}{V^s} = P + Q\ \text{coſ.}\ z + R\ \text{coſ.}\ 2z + \&c.$$

Take the fluxion, and we have

$$\frac{2sbr\ \text{ſin}\ z}{V^{s+1}} = Q\ \text{ſin.}\ z + 2R\ \text{ſin.}\ 2z + \&c.$$

Multiply the firſt of theſe by V, and the ſecond by it's equal b^2+r^2-2br coſ z, and

$$\frac{2sbr\ \text{ſin}\ z}{V^s} = \overline{b^2+r^2-2br\ \text{coſ}\ z} \times \overline{Q\ \text{ſin}\ z + 2R\ \text{ſin}\ 2z + \&c}\ ,$$

$$\text{Or, } 2sbr\ \text{ſin.}\ z \times \overline{P + Q\ \text{coſ.}\ z + R\ \text{coſ.}\ 2z + \&c.}$$

$$= \overline{b^2+r^2-2br\ \text{coſ}\ z} \times \overline{Q\ \text{ſin.}\ z + 2R\ \text{ſin.}\ 2z + \&c.}$$

1144. Now ſin. $z \times$ coſ $2z = \frac{1}{2}$ ſin. $3z - \frac{1}{2}$ ſin z, and coſ. $z \times$ ſin. $2z = \frac{1}{2}$ ſin $3z + \frac{1}{2}$ ſin z, in the laſt equation therefore, ſubſtitute for ſin $z \times$ coſ. $2z$, and coſ $z \times$ ſin $2z$, theſe values, and we get

$$sbr \times \overline{2P - R} \times \text{ſin}\ z + \&c = \overline{\overline{b^2+r^2} \times Q - 2brR} \times \text{ſin}\ z + \&c.$$

all the other terms containing ſin. $2z$, ſin $3z$, &c. Equating therefore the coefficients of ſin z, we get

$$sbr \times \overline{2P - R} = \overline{b^2+r^2} \times Q - 2brR,$$

hence, $R = \dfrac{\overline{b^2+r^2} \times Q - 2sbrP}{\overline{2-s} \times br}$.

1145. Let

1145. Let $\frac{1}{V^{s+1}} = P' + Q'\,\text{coſ.}\,z + R'\,\text{coſ.}\,2z + \&c$ Multiply the firſt by V, and the ſecond by $b^2+r^2-2br\,\text{coſ.}\,z$, and we have

$$\frac{1}{V^s} = \overline{b^2+r^2-2br\,\text{coſ.}\,z} \times \overline{P'+Q'\,\text{coſ.}\,z+\&c.}$$

But $\text{coſ.}\,z \times \text{coſ.}\,z = \frac{1}{2} + \frac{1}{2}\,\text{coſ}\,2z$, hence,

$$\frac{1}{V^s} = \overline{b^2+r^2} \times P' - brQ' + \&c.$$

the other terms being in terms of ſin. z, ſin. $2z$, &c. And comparing this with our firſt aſſumed value of $\frac{1}{V^s}$, we have

$$\overline{b^2+r^2} \times P' - brQ' = P.$$

1146. Next, multiply $\frac{1}{V^{s+1}} = P' + Q'\,\text{coſ.}\,z + R'\,\text{coſ.}\,2z + \&c.$ by $2sbr\,\text{ſin.}\,z$, and

$$\frac{2sbr\,\text{ſin}\,z}{V^{s+1}} = 2sbrP'\,\text{ſin.}\,z + 2sbrQ'\,\text{ſin.}\,z \times \text{coſ}\,z + 2sbrR' \times \text{ſin.}\,z \times$$

$\text{coſ.}\,2z + \&c. =$ (as $\text{ſin.}\,z \times \text{coſ.}\,2z = \frac{1}{2}\,\text{ſin.}\,3z - \frac{1}{2}\,\text{ſin.}\,z$) $\overline{2sbrP' - sbr \times R'} \times$ $\text{ſin.}\,z + \&c.$

Compare this with the value of $\frac{2sbr\,\text{ſin.}\,z}{V^{s+1}}$ in Art. 1143. and we have

$$2sbrP' - sbrR' = Q.$$

But there muſt be the ſame relation between P', Q', R', as between P, Q, R, only writing $s+1$ inſtead of s. Hence (1144), $R' = \frac{\overline{b^2+r^2} \times Q' - 2 \times \overline{s+1} \times brP'}{1 - s \times br}$, ſubſtitute this quantity for R' in $2sbrP' - sbrR' = Q$, and we get

$$\frac{s}{1-s} \times \overline{4brP' - \overline{b^2+r^2} \times Q'} = Q.$$

From

From this equation, and the above equation $\overline{b^2+r^2} \times P' - brQ' = P$, we obtain

$$P' = \frac{\overline{b^2+r^2} \times P - \frac{1-s}{s} \times brQ}{\overline{b^2-r^2}^2},$$

$$Q' = \frac{4brP - \frac{1-s}{s} \times \overline{b^2+r^2} \times Q}{\overline{b^2-r^2}^2}.$$

Knowing therefore the two firſt quantities P, Q in the value of $\frac{1}{V^s}$, we can find the two firſt quantities P', Q' in the value of $\frac{1}{V^{s+1}}$. In like manner, if $\frac{1}{V^{s+2}} = P'' + Q''$ coſ. z + &c. we have

$$P'' = \frac{\overline{b^2+r^2} \times P' + \frac{s}{1+s} \times brQ'}{\overline{b^2-r^2}^2};$$

$$Q'' = \frac{4brP' + \frac{s}{1+s} \times \overline{b^2+r^2} \times Q'}{\overline{b^2-r^2}^2}.$$

If we ſubſtitute in theſe expreſſions for P' and Q' their values in terms of P and Q, we ſhall get P'' and Q'' in terms of P and Q. Thus we get the coefficients of the terms of the ſuperior powers in terms of the coefficients of thoſe of the inferior.

1147. Let $\frac{1}{V^s} = P + Q$ coſ $z + R$ coſ. $2z + S$ coſ. $3z$ + &c. to find P, Q, R, &c. If e be a number whoſe hyperbolic log. $= 1$, then $e^{mz\sqrt{-1}} + e^{-mz\sqrt{-1}} = 2$ coſ. mz. Hence, the value of V, or b^2+r^2-2br coſ. $z =$

$b - re$

$\overline{b - re^{z\sqrt{-1}}} \times \overline{b - re^{-z\sqrt{-1}}}$, therefore $\frac{1}{V^s} = \frac{1}{\overline{b - re^{z\sqrt{-1}}}|^s} \times \frac{1}{\overline{b - re^{-z\sqrt{-1}}}|^s}$,

theſe two factors expanded give theſe two ſeries,

$$\frac{1}{b^s} + \frac{sre^{z\sqrt{-1}}}{b^{s+1}} + \frac{s\,\overline{s+1}\,r^2e^{2z\sqrt{-1}}}{2b^{s+2}} + \frac{s.\overline{s+1}\,\overline{s+2}\,r^3e^{3z\sqrt{-1}}}{2.3\,b^{s+3}} + \&c.$$

$$\frac{1}{b^s} + \frac{sre^{-z\sqrt{-1}}}{b^{s+1}} + \frac{s\,\overline{s+1}\,.\,r^2e^{-2z\sqrt{-1}}}{2b^{s+2}} + \frac{s\,\overline{s+1}\,\overline{s+2}\,r^3e^{-3z\sqrt{-1}}}{2.3\,b^{s+3}} + \&c.$$

Multiply theſe two ſeries together, and for $e^{z\sqrt{-1}} + e^{-z\sqrt{-1}}$ put 2 coſ. z, for $e^{2z\sqrt{-1}} + e^{-2z\sqrt{-1}}$ put 2 coſ. $2z$, for $e^{3z\sqrt{-1}} + e^{-3z\sqrt{-1}}$ put 2 coſ. $3z$, &c. put alſo $\alpha = s$, $\beta = s.\frac{s+1}{2}$, $\gamma = \frac{s\,\overline{s+1}\,\overline{s+2}}{2.3}$, &c. and the product, or $\frac{1}{V^s}$, becomes,

$$\frac{1}{b^{2s}} \times \left(1 + \alpha^2\frac{r^2}{b^2} + \beta^2\frac{r^4}{b^4} + \gamma^2\frac{r^6}{b^6} + \&c.\right)$$

$$+ \frac{2}{b^{2s}} \times \left(\alpha\frac{r}{b} + \alpha\beta\frac{r^3}{b^3} + \beta\gamma\frac{r^5}{b^5} + \&c.\right) \text{coſ. } z$$

$$+ \frac{2}{b^{2s}} \times \left(\beta\frac{r^2}{b^2} + \beta\gamma\frac{r^4}{b^4} + \gamma\delta\frac{r^6}{b^6} + \&c.\right) \text{coſ. } 2z$$

&c. &c &c

And comparing this with

$$\frac{1}{V^s} = P + Q \text{ coſ. } z + R \text{ coſ. } 2z, \ \&c.$$

we have

$$P = \frac{1}{b^{2s}}\left(1+\alpha^2\frac{r^2}{b^2}+\beta^2\frac{r^4}{b^4}+\gamma^2\frac{r^6}{b^6}+\&c.\right)$$

$$Q = \frac{2}{b^{2s}}\left(\alpha\frac{r}{b}+\alpha\beta\frac{r^3}{b^3}+\beta\gamma\frac{r^5}{b^5}+\&c.\right)$$

As the quantity $b^2+r^2-2br\,\text{coſ.}\,z$ is alſo $=\overline{r-be^{z\sqrt{-1}}}\times\overline{r-be^{-z\sqrt{-1}}}$, we may change r for b and b for r, ſo as to make the greater of the two enter into the denominators.

1148. In the preſent inſtance, we want to find P and Q when $s=\frac{3}{2}$, but in this caſe the coefficients α, β, γ, &c. would not be converging, we will begin therefore by taking $s=-\frac{1}{2}$, which will give the ſeries a ſufficient degree of convergency. Let us therefore compute P and Q in the equation

$$\frac{1}{V^{-\frac{1}{2}}}=P+Q\,\text{coſ.}\,z+R\,\text{coſ.}\,2z+\&c.$$

from the ſeries expreſſing their values in the laſt Article, writing $-\frac{1}{2}$ for s.

1149. Let $\frac{1}{V^{\frac{1}{2}}}=P'+Q'\,\text{coſ.}\,z+R'\,\text{coſ.}\,2z+\&c.$ then (1146)

$$P'=\frac{\overline{b^2+r^2}\times P+3brQ}{\overline{b^2-r^2}^{\,2}},\quad Q'=\frac{4brP+3\times\overline{b^2+r^2}\times Q}{\overline{b^2-r^2}^{\,2}}.$$

1150. Let $\frac{1}{V^{\frac{3}{2}}}=P''+Q''\,\text{coſ.}\,z+R''\,\text{coſ.}\,2z+\&c.$ then (1146)

$$P''=\frac{\overline{b^2+r^2}\times P'-brQ'}{\overline{b^2-r^2}^{\,2}},\quad Q''=\frac{4brP'-\overline{b^2+r^2}\times Q'}{\overline{b^2-r^2}^{\,2}}.$$

Subſtitute for P' and Q' their values found above in terms of P and Q, and we get

$$P''=\frac{P}{\overline{b^2-r^2}^{\,2}},\quad Q''=-\frac{3Q}{\overline{b^2-r^2}^{\,2}}.$$

1151. Now P and Q ariſe from the ſuppoſition $s = -\frac{1}{2}$, putting therefore $\alpha = -\frac{1}{2}$, $\beta = -\frac{1}{2} \cdot \frac{1}{4}$, $\gamma = -\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6}$, $\delta = -\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6} \cdot \frac{5}{8}$, $\epsilon = -\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6} \cdot \frac{5}{8} \cdot \frac{7}{10}$, &c. we have,

$$P'' = \frac{b}{\overline{b^2 - r^2}^{\,2}} \left(1 + \alpha^2 \frac{r^2}{b^2} + \beta^2 \frac{r^4}{b^4} + \gamma^2 \frac{r^6}{b^6} + \&c.\right)$$

$$Q'' = \frac{6b}{\overline{b^2 - r^2}^{\,2}} \left(-\alpha \frac{r}{b} - \alpha\beta \frac{r^3}{b^3} - \beta\gamma \frac{r^5}{b^5} - \&c.\right)$$

where we may change r for b and b for r. Here the coefficients α, β, γ, &c. form a ſeries decreaſing ſo faſt, that the tenth term is leſs than $\frac{1}{100}$, and theſe terms approach to a ſtate of equality; therefore, after taking the ſum of a certain number of terms, we may conſider the other terms as forming a geometric ſeries, the ratio of whoſe terms is $\frac{r^2}{b^2}$. Hence, if T be any term, the following terms will be nearly $T \frac{r^2}{b^2} + T \frac{r^4}{b^4} + \&c. = T \times \frac{r^2}{b^2 - r^2}$, but ſomething leſs, as the terms themſelves will be a little leſs than if they had proceeded accurately in geometrical progreſſion, becauſe the factors in the terms denoted by the Greek letters are leſs than unity, but approaching to unity. In the caſe of *Venus* and the *Earth*, $\frac{r}{b} = \frac{7}{10}$ nearly; therefore $\frac{r^2}{b^2}$ is leſs than $\frac{1}{2}$, and $\frac{r^2}{b^2 - r^2}$ is leſs than 1; thus, the ſum of all the terms which follow T is leſs than T, and this is the moſt unfavourable caſe that can be put. If T be the tenth term of either ſeries, it will be leſs than $\frac{1}{1000000}$, ten terms only will therefore be ſufficiently accurate for this purpoſe. In all other caſes, a ſmaller number of terms will be ſufficient.

To

To find the Equations of the Earth's *Motion ariſing from the Attraction of* Venus, *ſuppoſing the Orbit of each to be a Circle.*

1152. Let $SP=r=1$ the radius of the earth's orbit, $SE=b$ the radius of the orbit of Venus, $s=EP$; then (1118) $s=\sqrt{1+b^2-2b\,\text{coſ.}\,z}$, and $\frac{1}{s^3}=\overline{1+b^2-2b\,\text{coſ.}\,z}\big|^{-\frac{3}{2}}=A+B\,\text{coſ.}\,z+C\,\text{coſ.}\,2z+D\,\text{coſ.}\,3z+E\,\text{coſ.}\,4z$ $+\&c.$, hence, FIG. 240.

$$\frac{b}{s^3}-\frac{1}{b^2}=Ab-\frac{1}{b^2}+Bb\,\text{coſ.}\,z+Cb\,\text{coſ.}\,2z+Db\,\text{coſ.}\,3z+Eb\,\text{coſ.}\,4z+\&c$$

And

$$\left\{\frac{b}{s^3}-\frac{1}{b^2}\right\}\times\text{ſin.}\,z=Ab\,\text{ſin.}\,z-\frac{1}{b^2}\,\text{ſin.}\,z+Bb\,\text{coſ.}\,z\times\text{ſin.}\,z+Cb\,\text{coſ.}\,2z\times\text{ſin.}\,z+Db\,\text{coſ.}\,3z\times\text{ſin.}\,z+Eb\,\text{coſ.}\,4z\times\text{ſin.}\,z+\&c.$$

Now for coſ. $z\times$ ſin. z, coſ. $2z\times$ ſin. z, &c. ſubſtitute their values as in Art. 1119. and we get,

$$\left\{\frac{b}{s^3}-\frac{1}{b^2}\right\}\times\text{ſin.}\,z=(Ab-\frac{1}{b^2}-\tfrac{1}{2}Cb)\,\text{ſin.}\,z+(\tfrac{1}{2}Bb-\tfrac{1}{2}Db)\,\text{ſin.}\,2z+(\tfrac{1}{2}Cb-\tfrac{1}{2}Eb)\,\text{ſin.}\,3z+(\tfrac{1}{2}Db-\tfrac{1}{2}Fb)\,\text{ſin.}\,4z+\&c.$$

Multiply this by coſ. z and divide by ſin. z, and for ſin. $z\times$ coſ. z, ſin. $2z$ $\times$ coſ. z, &c. ſubſtitute their values (1119), and we obtain,

$$\left\{\frac{b}{s^3}-\frac{r}{b^2}\right\}\times\text{coſ.}\,z=\tfrac{1}{2}Bb+(Ab-\frac{1}{b^2}+\tfrac{1}{2}Cb)\,\text{coſ.}\,z+(\tfrac{1}{2}Bb+\tfrac{1}{2}Db)\,\text{coſ.}\,2z+(\tfrac{1}{2}Cb+\tfrac{1}{2}Eb)\,\text{coſ.}\,3z+(\tfrac{1}{2}Db+\tfrac{1}{2}Fb)\,\text{coſ.}\,4z+\&c.$$

1153. As the orbits are ſuppoſed to be circles, $z=nv$ (1117); hence,

$$F=M\left(\frac{b}{s^3}-\frac{1}{b^2}\right)\text{ſin}\ z=M\left(\left(Ab-\frac{1}{b^2}-\tfrac{1}{2}Cb\right)\text{ſin.}\ nv+\left(\tfrac{1}{2}Bb-\tfrac{1}{2}Db\right)\right.$$
$$\left.\text{ſin.}\ 2nv+\left(\tfrac{1}{2}Cb-\tfrac{1}{2}Eb\right)\text{ſin}\ 3nv+\left(\tfrac{1}{2}Db-\tfrac{1}{2}Fb\right)\text{ſin.}\ 4nv\right).$$

$$D=M\left(\frac{1}{s^3}-\left(\frac{b}{s^3}-\frac{1}{b^2}\right)\text{coſ.}\ z\right)=M\left(A-\tfrac{1}{2}Bb+\left(B-Ab+\frac{1}{b}\right.\right.$$
$$\left.-\tfrac{1}{2}Cb\right)\text{coſ}\ nv+\left(C-\tfrac{1}{2}Bb-\tfrac{1}{2}Db\right)\text{coſ.}\ 2nv+\left(D-\tfrac{1}{2}Cb-\tfrac{1}{2}Eb\right)\text{coſ.}\ 3nv.$$
$$\left.+\left(E-\tfrac{1}{2}Db-\tfrac{1}{2}Fb\right)\text{coſ.}\ 4nv\right).$$

1154. The value of $e=\int Fv$, the radius r being unity, multiply therefore the above value of F by v, and take the fluent, and we have

$$e=M\left(\left(-Ab+\frac{1}{b^2}+\tfrac{1}{2}Cb\right)\frac{1}{n}\text{coſ.}\ nv-\left(Bb-Db\right)\frac{1}{4n}\text{coſ.}\ 2nv-\left(Cb-\right.\right.$$
$$\left.\left.Eb\right)\frac{1}{6n}\text{coſ}\ 3nv-\left(Db-Fb\right)\frac{1}{8n}\text{coſ.}\ 4nv\right).$$

1155. As $r=1$, we have $r=0$, and the value of $P=D-2e$, but the firſt term $A-\frac{1}{2}Bb$ in the value of D may be omitted, it not being a variable quantity, and therefore cannot be concerned in producing any of the irregularities of the earth's motion in different parts of it's orbit, hence, we here aſſume

$$P=M\left(\left(B+\overline{\frac{2}{n}-1}\times Ab-\overline{\frac{2}{n}-1}\times\frac{1}{b^2}-\overline{\frac{1}{n}+\tfrac{1}{2}}\times Cb\right)\text{coſ}\ nv\right.$$
$$+\left(C+\overline{\frac{1}{2n}-\tfrac{1}{2}}\times Bb-\overline{\frac{1}{2n}+\tfrac{1}{2}}\times Db\right)\text{coſ.}\ 2nv$$
$$+\left(D+\overline{\frac{1}{3n}-\tfrac{1}{2}}\times Cb-\overline{\frac{1}{3n}+\tfrac{1}{2}}\times Eb\right)\text{coſ.}\ 3nv$$
$$\left.+\left(E+\overline{\frac{1}{4n}-\tfrac{1}{2}}\times Db-\overline{\frac{1}{4n}+\tfrac{1}{2}}\times Fb\right)\text{coſ.}\ 4nv\right).$$

1156. By

1156. By obſervation, we have $b=0{,}72348$, $n=0{,}625$, hence, $b^2=0{,}52342$, $1+b^2=1{,}52342$, $2b=1{,}44696$, thereſore $\overline{1+b^2-2b\ \text{coſ.}\ z}|^{-\frac{3}{2}} = \overline{1{,}52342-1{,}44696\ \text{coſ.}\ z}|^{-\frac{3}{2}} = A+B\ \text{coſ}\ z+C\ \text{coſ.}\ 2z+D\ \text{coſ.}\ 3z+E\ \text{coſ.}\ 4z+F\ \text{coſ.}\ 5z+\&\text{c.}$ and by the rules (1133, &c.) we have already given, we find $A=4{,}9996$, $B=8{,}8861$; hence (1136), $C=7{,}425$, $D=6{,}0368$, $E=4{,}8572$, $F=3{,}9281$, from whence $Ab=3{,}6171$, $Bb=6{,}4289$, $Cb=5{,}3718$, $Db=4{,}367$, $Eb=3{,}5143$, $Fb=2{,}8419$.

1157. Subſtitute theſe numbers into the above values of e and P, and we have

$$e=M\,(1{,}5699\ \text{coſ.}\ nv-0{,}8274\ \text{coſ.}\ 2nv-0{,}4954\ \text{coſ}\ 3nv-0{,}305\ \text{coſ}\ 4nv)$$

$$P=M\,(1{,}3598\ \text{coſ}\ nv+3{,}6765\ \text{coſ}\ 2nv+2{,}584\ \text{coſ.}\ 3nv+1{,}8828\ \text{coſ}\ 4nv)$$

1158. Aſſuming $d=1$, if $P=a'\ \text{coſ}\ mz+b'\ \text{coſ.}\ nz+\&\text{c.}$ then $S=-\frac{a'}{m^2-1}\times\text{coſ.}\ mz-\frac{b'}{n^2-1}\times\text{coſ}\ nz-\&\text{c.}$ (1111), in this caſe, the diviſors m^2-1, n^2-1, &c. become $n^2-1=-0{,}609375$, $4n^2-1=0{,}5625$, $9n^2-1=2{,}5156$, $16n^2-1=5{,}25$, alſo $a'=1{,}3598$, $b'=3{,}6765$, $c'=2{,}584$, $d'=1{,}8828$. Hence,

$$S=M\,(2{,}2314\ \text{coſ}\ nv-6{,}536\ \text{coſ.}\ 2nv-1{,}027\ \text{coſ}\ 3nv-0{,}3586\ \text{coſ.}\ 4nv).$$

1159. The expreſſion for the correction of the time is the fluent of $-\overline{2S+e}\times v$, ſubſtituting therefore for S and e their values here found, we have

$$-\overline{2S+e}\times\dot{v}=-M\,(6{,}0327\ \text{coſ.}\ nv\,\dot{v}-13{,}8994\ \text{coſ.}\ 2nv.\dot{v}-2{,}5494\ \text{coſ}\ 3nv\,.v-1{,}0222\ \text{coſ.}\ 4nv\,.v).$$

Hence, the *correction of the time*

$$-\int\overline{2S+e}\times v=-M\left(\frac{6,0327}{n}\,\text{ſin}\ nv-\frac{13,8994}{2n}\,\text{ſin}\ 2nv-\frac{2,5494}{3n}\,\text{ſin}\ 3nv\right.$$
$$\left.-\frac{1,0222}{4n}\,\text{ſin}\ 4nv\right)$$

$$=-M(9,6475\,\text{ſin}\ nv-11,1174\,\text{ſin}\ 2nv-1,3597\,\text{ſin}\ 3nv$$
$$-0,4089\,\text{ſin}\ 4nv)$$

1160 The correction of the longitude ſo far as regards the diſturbing forces (1114), is the above correction of the time with its ſign changed ſubſtituting therefore the mean longitude v for v and putting $t=nx=$ the mean longitude of Venus – that of the earth, we have the correction of the mean longitude

$$=M(9,6475\,\text{ſin}\ t-11,1174\,\text{ſin}\ 2t-1,3597\,\text{ſin}\ 3t-0,4089\,\text{ſin}\ 4t)$$

1161 Now (1067) the maſs of the ſun being to that of Venus as 333928 : 0,88993 we have $M=\frac{0,88993}{333928}=0,000002665$ ſubſtitute therefore this value for M, and multiply the whole by 57,29578 in order to reduce it into degrees, and we get *the correction of the longitude*

$$=5'',3\,\text{ſin}\ t-6'',1\,\text{ſin}\ 2t-0',7\,\text{ſin}\ 3t-0',2\,\text{ſin}\ 4t$$

1162 Theſe are the equations of the earth's motion from the action of *Venus*, upon ſuppoſition that the orbits are circles if we conſider the orbits as being excentric, we may compute ſeparately the effects ariſing from the excentricities, as before ſhown Thus we may obtain the equations of the motions of all the planets ariſing from their mutual attractions See *Hiſt de l'Acad à Paris*, 1754, 1760, 1761, 1785, 1786, and *Hiſt de l'Acad à Berlin*, 1781, 1782, 1783, 1784; alſo, EULER's *Recherches des Inégalites de Saturn et Jupiter*

On

On the Motion of the Moon's Apogee

1163 The action of one planet upon another to diſturb it's motion about the ſun is ſimilar to the action of the ſun upon the moon to diſturb it's motion about the earth The general equation of the curve therefore which the diſturbed planet deſcribes may repreſent that which the moon deſcribes about the earth, and thence the irregularities of the moon's motion may be inveſtigated But on account of the length of the ſubject, we cannot here give an inveſtigation of all the lunar inequalities we propoſe however to ſhow how the motion of the moon's apogee may be found and that being underſtood, together with the method of deducing the equations of the motions of the planets the reader will be prepared to enter upon any further reſearches of this kind

1164 Let E repreſent the earth, M the moon, S the ſun, and at A the higher apſide, let the moon be in conjunction with the ſun at B, put the mean diſtance of the moon from the earth $=1$, $d=$ the ſemi parameter, $r=EM$, $ES=b$, $MS=s$, $u=$ the excentricity of the moon's orbit, $v=AEM$, $x=BES$, $z=SEM$ Now (1118) Fig 242

$$D=\frac{M\times r}{b^3}-\left(\frac{M\times b}{s^3}-\frac{M}{b^2}\right)\text{coſ }z$$

$$F=-\left(\frac{M\times b}{s^3}-\frac{M}{b}\right)\text{ſin }z$$

But $MS=\overline{b^2+r^2-2br\,\text{coſ }z}\big|^{\frac{1}{2}}=$ (as r is very ſmall in reſpect to b) $b\times\overline{1-\frac{r}{b}\text{coſ }z}$, and $\frac{1}{s^3}=\frac{1}{MS^3}=\frac{1}{b^3\times\overline{1-\frac{r}{b}\text{coſ }z}\big|^3}=\frac{1}{b^3}\times\overline{1+\frac{3r}{b}\text{coſ }z}$

Hence, $D=\frac{Mr}{b^3}-\frac{3Mr}{b^3}\times\overline{\text{coſ }z}^2$, neglecting that term where b^4 enters into the denominator, as being extremely ſmall when compared with the reſt Alſo, $F=-\frac{3Mr}{b^3}\times\text{coſ }z\times\text{ſin }z$, but $\text{coſ }z^2=\frac{1}{2}-\frac{1}{2}\text{coſ }2z$, and $\text{coſ }z\times\text{ſin }z=\frac{1}{2}\text{ſin }2z$, hence,

$D=-$

$$D = -\frac{\frac{1}{2}Mr}{b^3} - \frac{\frac{3}{2}Mr}{b^3}\,\text{coſ}\ 2z$$

$$F = -\frac{\frac{3}{2}Mr}{b^3}\,\text{ſin}\ 2z$$

But $e = \int \frac{Fr^3 v}{a} = \int \frac{Fr^3 v}{p}$, C being aſſumed $= 1$ and $P = Dr^2 + \frac{Fr^2}{v} - 2e$ ſubſtituting therefore for F and D their above values, and putting $\alpha = \frac{Md^3}{b^3}$, we obtain

$$e = -\frac{3\alpha d}{2p}\int \frac{r^4}{d^4} \times \text{ſin}\ 2z \times v$$

$$P = -\frac{\alpha r^3}{2d^3} - \frac{3\alpha r^3}{2d^3}\,\text{coſ}\ 2z - \frac{3\alpha r^4}{2d^3 v} \times \text{ſin}\ 2z - 2e,$$

neglecting the denominator $1 + 2e$ of this laſt quantity

1165 To explain the principles of the method by which we propoſe to find the motion of the moon's apogee, we will take a more ſimple caſe of the problem, neglecting the force which acts perpendicularly to the radius vector, in which caſe D becomes the only diſturbing force, and $P = Dr$ Suppoſe $D = \frac{h}{r^3}$, and then the whole force to the center $= \frac{1}{r} + \frac{h}{r^3}$ Now $P = Dr^2 = \frac{h}{r}$, but (1112) $\frac{1}{r} = \frac{1}{d} - \frac{w}{d}\,\text{coſ}\ mv$ hence, $P = \frac{h}{d} - \frac{hw}{d}\,\text{coſ}\ mv$, comparing therefore this with the general value of $P = a'\,\text{coſ}\ mv + b'\,\text{coſ}\ nv$, we have $a = -\frac{hw}{d}$, $b = \frac{h}{d}$, $n = 0$, hence (1111), the equation of the curve becomes $\frac{1}{r} = \frac{1}{p} \times \overline{1 + \frac{h}{k}} - \frac{1}{p}\left(c + \frac{hw}{d \times \overline{m^2 - 1}} + \frac{h}{d}\right)\text{coſ}\ v + \frac{hw}{pd \times \overline{m^2 - 1}}\,\text{coſ}\ mv$ Now it is manifeſt that this ſuppoſition $\frac{1}{r} = \frac{1}{d} - \frac{w}{d}\,\text{coſ}\ mv$ will be juſtified, provided we can make this equation coincide with the above equation of the curve, to do which, we aſſume $\frac{1}{p} \times \overline{1 + \frac{h}{k}} = \frac{1}{d}$, $c + \frac{hw}{d \times \overline{m^2 - 1}} + \frac{h}{d} = 0$,

hw

$\frac{hw}{pd\times\overline{m^2-1}} = -\frac{w}{d}$ from which equations we find $m = \sqrt{1-\frac{h}{p}}$, $d=p-h$, $w = \frac{c\times\overline{p-h}+h}{p}$, thus we have found such values for m, d and w as will make the equation of the moveable ellipſe coincide with the equation of the curve deſcribed by the body. We ſee therefore that the effect of the force $\frac{h}{r^3}$ added to the force $\frac{1}{r}$, changes the ellipſe expreſſed by $\frac{1}{r} = \frac{1}{p} - \frac{c}{p}\times\cos v$ into a moveable ellipſe expreſſed by $\frac{1}{r} = \frac{1}{p-h} - \frac{c\times\overline{p-h}+h}{p\times\overline{p-h}}\cos mv$ the parameter and excentricity of which are different from thoſe of the ellipſe deſcribed without the diſturbing forces, and the angular motion of the planet that of the apogee $v \quad v-mv \quad 1 \quad 1-m \quad 1 \quad 1-\sqrt{1-\frac{h}{p}}$. This agrees with what we determined in Art. 833, as will appear by ſubſtituting in the general expreſſion for the force $\frac{b\times SP + c\times SP^n}{SP^3}$, $b=1$, $c=h$, $m=1$, $n=0$ and making $p=1$. Upon the ſame principle we ſhall proceed to inveſtigate the motion of the apogee, when the force F is ſuppoſed to act.

1166. As $\frac{1}{r} = \frac{1}{d} - \frac{w}{d}\cos mv$ we have $\frac{r}{d} = \frac{1}{1-w\cos mv} = 1 + w \cos mv + w^2\,\overline{\cos mv}^2 + \&c$ raiſe this quantity to the ſquare, cube, and fourth power ſubſtituting for $\overline{\cos mv}^2$ $\overline{\cos mv}^3$ $\overline{\cos mv}^4$ &c their values $\frac{1}{2}+\frac{1}{2}\cos 2mv$, $\frac{3}{4}\cos mv + \frac{1}{4}\cos 3mv$, $\frac{3}{8} + \frac{1}{2}\cos 2mv + \frac{1}{8}\cos 4mv$, and put $\alpha' = 1 + 3w$, $\beta = w + \frac{3}{4}w^3$, $\gamma = 1 + 5w^2$, $\delta = w + \frac{15w^3}{4}$, $\epsilon' = 1 + \frac{1}{2}w$, and we get

$$\frac{r}{d} = \epsilon\ (1 + 2w\cos mv + \frac{1}{2}w^2\cos 2mv)$$

$$\frac{r^4}{d^4} = \gamma' + 4\delta'\cos mv + 5w\cos 2mv$$

$$\frac{r^3}{d^3} = \alpha' + 3\beta\cos mv + 3w^2\cos 2mv$$

$$\frac{3r^2 r}{d^3 v} = -3\beta' m\sin mv - 6w^2 m\sin 2mv$$

1167. By Art. 1113, $t =$ [illegible] which is sufficiently accurate for our preſent purpoſe, that is $t = v + 2e$ coſ nv v + $^3 w$ coſ $2mv \times v$ very nearly, aſſuming $e = 1$, $d = 1$, hence, $t = v + \frac{2w}{n}$ ſin $mv + \frac{3w}{4n}$ ſin $2mv$ the mean motion (1114) of the moon correſponding to the true motion v, in which time the ſun deſcribes the angle x; let therefore $1 - \frac{1}{n}$: 1 as the mean motion of the ſun : that of the moon, and we have $v + \frac{2w}{m}$ ſin $mv + \frac{3w}{4m}$ ſin $2mv$: x, or $v - x$: 1 : 1 : $\frac{1}{n}$, therefore if we put $a = \frac{2w}{m} \times \overline{1 - \frac{1}{n}}$, $b' = \frac{3w}{4m} \times \overline{1 - \frac{1}{n}}$, we have,

$$z = \frac{v}{n} - a' \text{ ſin } mv - b \text{ ſin } 2mv$$

1168. Now to find the values of ſin $2z$, coſ $2z$, we may conſider $2z$ as made up of two parts, $\frac{2v}{n}$ and $-2a$ ſin $mv - 2b'$ ſin $2mv$, hence, by Trigonometry, we have ſin $2z =$ ſin $\frac{2v}{n} \times$ coſ $(2a$ ſin $mv + 2b$ ſin $2mv)$ $-$ coſ $\frac{2v}{n} \times$ ſin $(2a'$ ſin $mv + 2b$ ſin $2mv)$, but as a and b are ſmall, we we may conſider coſ $(2a$ ſin $mv + 2b'$ ſin $2mv)$ as equal to unity, and ſin $(2a' \times$ ſin $mv + 2b' \times$ ſin $2mv) = 2a$ ſin $mv + 2b$ ſin $2mv$, and ſubſtituting for the two products of coſine × ſine, their values in terms of half the ſine of their ſum − half the ſine of their difference we have

$$\text{ſin } 2z = \text{ſin } \frac{2v}{n} + a \text{ ſin } \overline{\tfrac{2}{n} - m}\; v - a \text{ ſin } \overline{\tfrac{2}{n} + m}\; v + b \text{ ſin } \overline{\tfrac{2}{n} - 2m}\; v - b \text{ ſin } \overline{\tfrac{2}{n} + 2m}\; v$$

1169. Alſo, coſ $2z =$ coſ $\frac{2v}{n} \times$ coſ $(2a$ ſin $mv + 2b$ ſin $2mv)$ + ſin $\frac{2v}{n} \times$ ſin $(2a$ ſin $mv + 2b$ ſin $2mv)$, aſſuming therefore the ſame coſine $= 1$ as before, and ſubſtituting for the products of the two ſines their values in terms of half the coſine of their difference − half the coſine of their ſum, we have

coſ 2z

$$\text{coſ}\,2z=\text{coſ}\,\frac{2v}{n}+a\,\text{coſ}\,\overline{\tfrac{2}{n}-m}\,v-a\,\text{coſ}\,\overline{\tfrac{2}{n}+m}\,v$$
$$+b\,\text{coſ}\,\overline{\tfrac{2}{n}-2m}\,v-b\,\text{coſ}\,\overline{\tfrac{2}{n}+2m}\,v$$

1170 The value of $-\frac{\alpha r^3}{2d^3}$ is $-\frac{\gamma\alpha}{2}-\frac{3\alpha\beta}{2}\,\text{coſ}\,mv-\frac{3\alpha w}{2}\,\text{coſ}\,2mv$, which is the firſt quantity conſtituting the value of P.

1171 Having found the values of $\frac{r^4}{d^4}$ and of ſin $2z$, we have $e=-\frac{3\alpha d}{2p}\int\frac{r^4}{d^4}\times\text{ſin}\,2z\times\dot{v}=-\frac{3\alpha d}{2p}\int(\gamma+4\delta\,\text{coſ}\,mv+5w\,\text{coſ}\,2mv)\times(\text{ſin}\,\frac{2v}{n}+a\,\text{ſin}\,\overline{\tfrac{2}{n}-m}\,v-a\,\text{ſin}\,\overline{\tfrac{2}{n}+m}\,v+b\,\text{ſin}\,\overline{\tfrac{2}{n}-2m}\,v-b\,\text{ſin}\,\overline{\tfrac{2}{n}+2m}\,v)\times\dot{v}$. Now leaving out the general multiplier $-\frac{3\alpha d\dot{v}}{2p}$ and actually multiplying the other terms, let us only retain thoſe terms which will produce quantities of this kind, ſin $\frac{2v}{n}$, ſin $\overline{\frac{2}{n}-m}\,v$, ſin $\overline{\frac{2}{n}+m}\,v$, ſin $\overline{\frac{2}{n}-2m}\,v$, as we will here neglect the other terms, ſin $\overline{\frac{2}{n}+2m}\,v$, ſin $\overline{\frac{2}{n}+3m}\,v$, &c. on account of their ſmallneſs. In our multiplication therefore we retain theſe quantities, $\gamma\,\text{ſin}\,\frac{2v}{n}+\gamma a\,\text{ſin}\,\overline{\tfrac{2}{n}-m}\,v-\gamma a'\,\text{ſin}\,\overline{\tfrac{2}{n}+m}\,v+\gamma b\,\text{ſin}\,\overline{\tfrac{2}{n}-2m}\,v+4\delta\,\text{coſ}\,mv\times\text{ſin}\,\frac{2v}{n}+4\delta a'\,\text{coſ}\,mv\times\text{ſin}\,\overline{\tfrac{2}{n}-m}\,v+5w\,\text{coſ}\,2mv\times\text{ſin}\,\frac{2v}{n}$. Now

$$4\delta\times\text{ſin}\,\frac{2v}{n}\times\text{coſ}\,mv=2\delta\times\text{ſin}\,\overline{\tfrac{2}{n}+m}\,v+2\delta\times\text{ſin}\,\overline{\tfrac{2}{n}-m}\,v$$

$$4\delta a'\times\text{ſin}\,\overline{\tfrac{2}{n}-m}\,v\times\text{coſ}\,mv=2\delta a'\times\text{ſin}\,\frac{2v}{n}+2\delta a'\times\text{ſin}\,\overline{\tfrac{2}{n}-2m}\,v$$

$$5w\times\text{ſin}\,\frac{2v}{n}\times\text{coſ}\,2mv=\frac{5}{2}w\,\text{ſin}\,\overline{\tfrac{2}{n}+2m}\,v+\frac{5}{2}w\,\text{ſin}\,\overline{\tfrac{2}{n}-2m}\,v$$

Collecting therefore the coefficients of the above mentioned terms (neglecting $2\delta a'$ ſin $\frac{2v}{n}$ on account of its ſmallneſs) we get

$$e = -\frac{3\alpha d}{2p}\int \gamma'\ \text{ſin}\ \frac{2v}{n}\ v + \overline{2\delta'+\gamma'a} \times \text{ſin}\ \overline{\frac{2}{n}-m}\ v \times v + \overline{2\delta - \gamma a'} \times \text{ſin}\ \overline{\frac{2}{n}+m}\ v \times v + \overline{\gamma b + 2\delta a \times {}^{3}w} \times \text{ſin}\ \overline{\frac{2}{n}-2m}\ v \times v$$

Putting therefore $\mathrm{a} = \frac{3d\gamma n}{4p}$, $\mathrm{b} = \frac{3d}{2p} \times \frac{2\delta'+\gamma'a'}{\frac{2}{n}-m}$, $\mathrm{c} = \frac{3d}{2p} \times \frac{2\delta-\gamma a'}{\frac{2}{n}+m}$, $\mathrm{d} = \frac{3d}{2p} \times \frac{\gamma b' + 2\delta a + {}^{3}w}{2m-\frac{2}{n}}$, $\mathrm{p}=\mathrm{a}+\mathrm{b}+\mathrm{c}-\mathrm{d}$, and taking the fluent, we have

$$-2e = -2\mathrm{a}\alpha\ \text{coſ}\ \frac{2v}{n} - 2\mathrm{b}\alpha\ \text{coſ}\ \overline{\frac{2}{n}-m}\ v - 2\mathrm{c}\alpha\ \text{coſ}\ \overline{\frac{2}{n}+m}\ v$$
$$+ 2\mathrm{d}\alpha\ \text{coſ}\ \overline{\frac{2}{n}-2m}\ v + 2\mathrm{p}\alpha,$$

where $\mathrm{p}\alpha$ is the correction, ſo that e may be $=0$ when $v=0$

1172 To determine the value of $\frac{3\alpha r^{2}r}{2d^{3}v} \times$ ſin $2z$, we have (1166) $\frac{3r'r}{d^{3}v} = -3\beta m\ \text{ſin}\ mv - 6wm\ \text{ſin}\ 2mv$ and ſin $2z = \text{ſin}\ \frac{2v}{n} + a\ \text{ſin}\ \overline{\frac{2}{n}-m}\ v - a\ \text{ſin}\ \overline{\frac{2}{n}+m}\ v + b'\ \text{ſin}\ \overline{\frac{2}{n}-2m}\ v - b'\ \text{ſin}\ \overline{\frac{2}{n}+2m}\ v$, hence, $\frac{3\alpha r^{2}r}{2d^{3}v} \times \text{ſin}\ 2z = \frac{\alpha}{2} \times \left(\left(-3\beta m\ \text{ſin}\ mv - 6wm\ \text{ſin}\ 2mv\right) \times \left(\text{ſin}\ \frac{2v}{n} + a\ \text{ſin}\ \overline{\frac{2}{n}-m}\ v - a\ \text{ſin}\ \overline{\frac{2}{n}+m}\ v + b'\ \text{ſin}\ \overline{\frac{2}{n}-2m}\ v - b'\ \text{ſin}\ \overline{\frac{2}{n}+2m}\ v\right)\right)$, now if we actually multiply theſe together, and retain thoſe terms which are of the ſame ſpecies as thoſe retained in the laſt Article, we have $\frac{3\alpha r^{2}r}{2d^{3}v} \times \text{ſin}\ 2z = \frac{\alpha}{2} \times$ $\left(-3\beta'm\ \text{ſin}\ mv \times \text{ſin}\ \frac{2v}{n} - 3\beta'ma\ \text{ſin}\ mv \times \text{ſin}\ \overline{\frac{2}{n}-m}\ v - 6w'm\ \text{ſin}\ 2mv\right.$

× ſin

$\times\ \text{ſin}\ \frac{2v}{n}$[*]$\Big) = \frac{\alpha}{2} \times \Big(-3\beta m \times \big(\frac{1}{2}\,\text{coſ}\ \overline{\tfrac{2}{n}-m}\ v - \frac{1}{2}\,\text{coſ}\ \overline{\tfrac{2}{n}+m}\ v\big) - 3\beta m a \times \big(\frac{1}{2}\,\text{coſ}\ \overline{\tfrac{2}{n}-2m}\ v - \frac{1}{2}\,\text{coſ}\ \frac{2v}{n}\big) - 6w m \times \big(\frac{1}{2}\,\text{coſ}\ \overline{\tfrac{2}{n}-2m}\ v - \frac{1}{2}\,\text{coſ}\ \overline{\tfrac{2}{n}+2m}\ v\big)\Big)$, putting therefore $a' = \frac{3}{2}\beta m a$, $b' = \frac{3}{4}\beta m$, $c' = \frac{3}{4}\beta' m a + 3 w m$, we get

$$-\frac{3\alpha r^{2} r}{2 d^{3} v} \times \text{ſin}\ 2z = -a'\alpha\ \text{coſ}\ \frac{2v}{n} + b'\alpha\ \text{coſ}\ \overline{\tfrac{2}{n}-m}\ v - b\alpha\ \text{coſ}\ \overline{\tfrac{2}{n}+m}\ v$$

$$+ c'\alpha\ \text{coſ}\ \overline{\tfrac{2}{n}-2m}\ v$$

1173 To find the value of $\frac{3\alpha r^{3}}{2d^{3}}\ \text{coſ}\ 2z$, we have (1662) $\frac{r^{3}}{d^{3}} = \alpha + 3\beta'\ \text{coſ}\ mv + 3w\ \text{coſ}\ 2mv$, and $\text{coſ}\ 2z = \text{coſ}\ \frac{2v}{n} + a\ \text{coſ}\ \overline{\tfrac{2}{n}-m}\ v - a\ \text{coſ}\ \overline{\tfrac{2}{n}+m}\ v + b'\ \text{coſ}\ \overline{\tfrac{2}{n}-2m}\ v - b'\ \text{coſ}\ \overline{\tfrac{2}{n}+2m}\ v$, hence, $\frac{3\alpha r^{3}}{d^{3}}\ \text{coſ}\ 2z = 3\alpha \times \Big(\big(\alpha + 3\beta'\ \text{coſ}\ mv + 3w\ \text{coſ}\ 2mv\big) \times \big(\text{coſ}\ \frac{2v}{n} + a\ \text{coſ}\ \overline{\tfrac{2}{n}-m}\ v - a\ \text{coſ}\ \overline{\tfrac{2}{n}+m}\ v + b'\ \text{coſ}\ \overline{\tfrac{2}{n}-2m}\ v - b'\ \text{coſ}\ \overline{\tfrac{2}{n}+2m}\ v\big)\Big)$, now if we multiply theſe together and retain only thoſe terms which are of the ſame ſpecies as thoſe which we before retained, neglecting ſome terms of the ſame ſpecies whoſe coefficients are very ſmall, we have $\frac{3\alpha r^{3}}{2d^{3}}\ \text{coſ}\ 2z = \alpha \times \text{coſ}\ \frac{2v}{n} + \alpha a\ \text{coſ}\ \overline{\tfrac{2}{n}-m}\ v - \alpha a\ \text{coſ}\ \overline{\tfrac{2}{n}+m}\ v + 3\beta'\ \text{coſ}\ mv \times \text{coſ}\ \frac{2v}{n} + 3w\ \text{coſ}\ 2mv \times \text{coſ}\ \frac{2v}{n} + 3\beta' a\ \text{coſ}\ mv \times \text{coſ}\ \overline{\tfrac{2}{n}-m}\ v + \alpha b'\ \text{coſ}\ \overline{\tfrac{2}{n}-2m}\ v$, but

$3\beta'$

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[*] Some of the other terms beſides theſe will give terms of the required ſpecies but the coefficients are ſo ſmall that they may be neglected

$$_{3}\beta\ \text{coſ}\ mv \times \text{coſ}\ \frac{2v}{n} = \beta\left(\text{coſ}\ \overline{\frac{2}{n}-m}\ v + \text{coſ}\ \overline{\frac{2}{n}+m}\ v\right)$$

$$3w\ \text{coſ}\ 2mv \times \text{coſ}\ \frac{2v}{n} = {}^{3}w\left(\text{coſ}\ \overline{\frac{2}{n}-2m}\ v + \text{coſ}\ \overline{\frac{2}{n}+2m}\ v\right)$$

$$_{3}\beta a\ \text{coſ}\ mv \times \text{coſ}\ \overline{\frac{2}{n}-m}\ v = {}^{3}\beta a\left(\text{coſ}\ \overline{\frac{2}{n}-2m}\ v + \text{coſ}\ \frac{2v}{n}\right),$$

putting therefore $\pi = {}^{3}\alpha$, $\rho = \frac{9}{4}\beta' + {}^{3}\alpha a$, $\sigma = \frac{9}{4}\beta' - {}^{3}\alpha a$, $\tau = {}^{3}\alpha b' + \frac{9}{4}\beta a + {}_{4}w$, we have (neglecting the laſt term on account of its ſmallneſs)

$$-\frac{3\alpha r^{3}}{2d^{3}}\ \text{coſ}\ 2z = -\pi\alpha\ \text{coſ}\ \frac{2v}{n} - \rho\alpha\ \text{coſ}\ \overline{\frac{2}{n}-m}\ v$$

$$-\sigma\alpha\ \text{coſ}\ \overline{\frac{2}{n}+m}\ v - \tau n\ \text{coſ}\ \overline{\frac{2}{n}-2m}\ v$$

1174 Having found the values of all the terms which compoſe the quantity P, let us add them together, and collect the coefficients of the like terms, and putting $A = 2a + a + \pi$, $B = 2b - b + \rho$, $C = 2c + b + \sigma$, $D = 2d + c' - \tau$, $E = {}^{3}\beta$, $L = 2p - \frac{\alpha}{2}$, we have

$$P = L\alpha - E\alpha\ \text{coſ}\ mv - A\alpha\ \text{coſ}\ \frac{2v}{n} - B\alpha\ \text{coſ}\ \overline{\frac{2}{n}-m}\ v$$

$$-C\alpha\ \text{coſ}\ \overline{\frac{2}{n}+m}\ v + D\alpha\ \text{coſ}\ \overline{\frac{2}{n}-2m}\ v$$

1175 Subſtitute this value of P in the general equation of the curve in Art 1110 and (dividing by p) we get

$\frac{1}{r}$ -

$$\frac{1}{r} = \frac{1 + I\alpha}{p} - \frac{1}{p}\left(c - \frac{\Gamma\alpha}{1-n} + \frac{A\alpha}{\frac{4}{n^2} - 1} - \frac{B\alpha}{\overline{1 - \frac{2}{n} - m}} + \&c\right) \text{coſ}\ v$$

$$- \frac{L\alpha}{p \times \overline{1-m}} \times \text{coſ}\ mv + \frac{A\alpha}{p \times \overline{\frac{4}{n} - 1}} \times \text{coſ}\ \frac{2v}{n}$$

$$- \frac{B\alpha}{p \times \overline{1 - \frac{2}{n} - m}} \times \text{coſ}\ \overline{\frac{2}{n} - m}\ v + \frac{C\alpha}{p \times \overline{\frac{2}{n} + m} - 1} \times \text{coſ}\ \overline{\frac{2}{n} + m}\ v$$

$$+ \frac{D\alpha}{p \times \overline{1 - \frac{2}{n} - 2m}} \times \text{coſ}\ \overline{\frac{2}{n} - 2m}\ v,$$

the Equation of the Lunar Orbit Make $1 + L\alpha = \frac{p}{d}$, $w = \frac{L\alpha d}{p \times \overline{1-m}}$, $c - \frac{E\alpha}{1-m} + \frac{A\alpha}{\frac{4}{n} - 1} - \&c = 0$, where d, p, c, w, m, have ſuch values as will ſatisfy theſe equations, put alſo $p' = \frac{A\alpha d}{p \times \overline{\frac{4}{n} - 1}}$, $q = \frac{B\alpha d}{p \times \overline{1 - \frac{2}{n} - m}}$, $r' = \frac{C\alpha d}{p \times \overline{\frac{2}{n} + m} - 1}$ $s = \frac{D\alpha d}{p \times \overline{1 - \frac{2}{n} - 2m}}$, and the equation becomes

$$\frac{d}{r} = 1 - \ v\ \text{coſ}\ mv + p'\ \text{coſ}\ \frac{2v}{n} - q\ \text{coſ}\ \overline{\frac{2}{n} - m}\ v + r'\ \text{coſ}\ \overline{\frac{2}{n} + m}\ v + s'\ \text{coſ}\ \overline{\frac{2}{n} - 2m}\ v$$

Now, $\frac{d}{r} = 1 - w\ \text{coſ}\ mv$ expreſſes (1112) the curve deſcribed by a moveable ellipſe and the other terms being ſmall compared with theſe firſt find the value of r from this equation and ſubſtituting it into the value of P a ſecond equation is found approximating more nearly to the truth, thus (were we to proceed in the theory of the moon) we might correct the equation of the curve, or the equation which gives the relation between the diſtance and

and the angle deſcribed But we here propoſe only to ſhow the method of finding the mean motion of the apogee

1176 Now $\frac{d}{r} = 1 - w \cos mv$ are the principal terms of the equation, and (1112) denote a moveable ellipſe, containing the great equation of the moon's motion, that is, the equation of the center, alſo the motion of the apogee And as this equation does not depend upon the ſituation of the ſun, the motion of the apogee which is denoted by it, may be conſidered as the mean effect of the diſturbing force This motion of the apogee is conſtantly progreſſive and (1112) is in proportion to the motion of the body as $1 - m$ to 1, if therefore 1 repreſent the mean motion of the moon, the mean motion of the apogee will be repreſented by $1 - m$ The other terms are ſmall, and depending on the poſition of the ſun in reſpect to the moon, they will produce ſome of the ſmaller equations of the moon's motion, and the equations of the motion of the apogee Hence, we may conſider $\frac{d}{r} = 1 - w \cos mv$ as an equation repreſenting the baſis of the lunar orbit

1177 By obſervation, $w = 0\ 05505$ the mean excentricity of the moon's orbit, according to Sir I NEWTON Alſo, $1 - \frac{1}{n} \quad 1 \quad 0{,}0748 \quad 1$ or $1 - \frac{1}{n} = 0{,}0748$ And if p = the periodic time of the moon, P = that of the earth, we have (818) $p \quad P \quad \frac{d^3}{1} \quad \frac{b^3}{M}$, ſetting aſide the effect of the diſturbing forces, hence, $\frac{p^2}{P^2} = \frac{Md^3}{b^3} = \alpha$, and $\alpha \left(= \frac{p}{P}\right) = 0{,}005595$ very nearly

1178 The equation $c - \frac{E\alpha}{1 - m^2} + \frac{A\alpha}{\frac{4}{n} - 1} - \frac{B\alpha}{1 - \frac{2}{n} - m} + \&c = 0$, would (as E, A, B &c are in terms of w) give the relation between c and w from which we get the ratio of the excentricity of the orbit which would have been deſcribed without any diſturbing forces to the excentricity of the true orbit but as c enters not into any future part of the proceſs, it is unneceſſary to determine that point

1179 The equation $1 + L\alpha = \frac{p}{d}$ is uſed to determine the value of d, that quantity entering into the values of p', q', r, s'

1180 The

1180 The equation $w = \frac{E\alpha d}{p \times \overline{1-m}}$ contains an element of great importance in the theory of the moon, as from it we obtain the value of m, and thence (1112) the motion of the apogee

1181 As the motion of the apogee is ſlow, we may firſt aſſume $m=1$, and afterwards correct that aſſumption From theſe values therefore of w, $1-\frac{1}{n}$, α and m, we obtain from the equations in the preceding articles, $\alpha=1{,}0091$, $\beta=0{,}0555$, $\gamma=1{,}0151$, $\delta'=0{,}0557$, $w=0{,}00303$, $b=0{,}00824$, $b'=0{,}00017$, $\pi=1{,}5136$, $\rho=0{,}1374$, $\sigma=0{,}1124$, $\tau=0{,}00718$, $'=0{,}0007$, $\mathrm{b}'=0{,}0416$, $\mathrm{c}'=0{,}00458$

1182 The coefficients a, b, c, d, as they are expreſſed in terms of $\frac{d}{p}$ cannot be known till after the reſolution of the equation $1+L\alpha=\frac{p}{d}$, in which L itſelf depends upon $\frac{d}{p}$ Now p repreſents the ſemi parameter of the orbit that would have been deſcribed without any diſturbing forces, and d the true ſemi parameter, and as the magnitude of the orbit can ſuffer but a very ſmall alteration from the diſturbing forces, we may aſſume $\frac{d}{p}=1$, and afterwards correct it Hence, we find $\mathrm{a}=0{,}8229$, $\mathrm{b}=0{,}2107$, $\mathrm{c}=0{,}0543$, $\mathrm{d}=0{,}0869$, and $\mathrm{p}=1{,}001$, conſequently $A=3{,}1595$, $B=0{,}5172$, $C=0{,}2627$, $D=0{,}1721$, $L=1{,}4975$ Subſtitute this value of L into the equation $1+L\alpha=\frac{p}{d}$, and we get $\frac{p}{d}=1{,}00838$, hence, $\frac{d}{p}=0{,}9917$ Now as the quantities expreſſing the values of a, b, c, d, have the quantity $\frac{d}{p}$ as a multiplier, the above values of theſe quantities muſt be diminiſhed in the ratio of 1 : 0,9917, therefore $\mathrm{a}=0{,}81607$, $\mathrm{b}=0{,}20895$, $\mathrm{c}=0{,}053849$, $\mathrm{d}=0{,}086179$, conſequently we get a more correct value of $A=3{,}1557$, $B=0{,}5162$, $C=0{,}2624$, $D=0{,}1708$ Hence, $p'=0{,}00722$, $q'=0{,}01035$, $r=0{,}000205$, $s'=0{,}00097$, of which values, q' and s' are thoſe where the ſubſtitution of 1 for m cauſes the greateſt error, on account of the diviſors $\overline{1-\frac{2}{n}-m}$ and $\overline{1-\frac{2}{n}-2m}$

1183 From the equation $w = \frac{E\alpha d}{p\times\overline{1-m}}$ we get $1-m^2 = \frac{E\alpha d}{wp}$, now $E=0{,}0838$, and $\frac{d}{p}\times\alpha = 0\ 9917\times 0{,}005595 = 0\ 00555$, therefore $1-m^2 =$ 0,008388, conſequently $1-m = 0\ 004186$ According to this concluſion therefore, the mean motion of the apogee that of the moon 0 004186 1, whereas the ratio ought to be that of 0,008455 1, this concluſion therefore gives the motion of the apogee only about one half of what it ought to be according to obſervation We muſt therefore ſee whether we have not omitted any terms, which, in the preſent caſe, may have been too conſiderable to be neglected

1184 In ſubſtituting the value of $\frac{d}{r}$ into e, into $\frac{3r^3}{2d^3v}\times$ ſin $2z$, and into $\frac{3r^3}{2d^3}\times$ coſ $2z$, we aſſumed it equal to $1-w$ coſ mv (1166), whereas, if we aſſume $\frac{d}{r} = 1 - w$ coſ $mv + p'$ coſ $\frac{2v}{n} - q'$ coſ $\overline{\frac{2}{n}-m}\ v + \&c$ (1175), the introduction of theſe terms p' coſ $\frac{2v}{n} - q'$ coſ $\overline{\frac{2}{n}-m}\ v + \&c$ will be found to have a very conſiderable effect in the value of $-\beta$, or of E, and conſequently of $1-m$ Now that term which will produce any conſiderable effect on E is $-q'$ coſ $\overline{\frac{2}{n}-m}\ v$ in the terms $\frac{r^3}{d^3}$, $\frac{r^4}{d^4}$ and $\frac{r\,r}{v}$, when joined with ſin $\frac{2v}{n}$, coſ $\frac{2v}{n}$ in the values of ſin $2z$, coſ $2z$ We will take each caſe ſeparately

1185 Aſſume $\frac{d}{r} = 1 - w$ coſ $mv - q$ coſ $\overline{\frac{2}{n}-m}\ v$, hence, $\frac{d^4}{r^4} = 1 - 4w$ coſ $mv - 4q'$ coſ $\overline{\frac{2}{n}-m}\ v$, therefore $\frac{r^4}{d^4} = 1 + 4w$ coſ $mv + 4q'$ coſ $\overline{\frac{2}{n}-m}\ v$, the two firſt terms we have already conſidered, we have therefore to aſſume $\frac{r^4}{d^4} = 4q'$ coſ $\overline{\frac{2}{n}-m}\ v$, and taking only the firſt term, ſin $\frac{2v}{n}$, in the value of ſin $2z$, we have $\frac{r^4}{d^4}\times$ ſin $2z\times v = 4q'$ coſ $\overline{\frac{2}{n}-m}\ v\times$ ſin $\frac{2v}{n}\times v = 2q'$ ſin $mv\ v + 2q'$ ſin $\overline{\frac{4}{n}-m}\ v\times v$ which laſt term we may omit, it not being of the ſpecies which we here want, hence, $e = -\frac{3\alpha d}{p}\int 2q'$ ſin $mv\times\dot{v} =$

$3\alpha dq$

$\frac{3\alpha dq'}{pm}$ coſ mv, the correction therefore of $-2e$ in the value of P is $-\frac{6\alpha dq}{pm}$ coſ mv.

1186. In like manner in the quantity $\frac{r^3}{d^3}$ we get the term $3q'$ coſ $\overline{\frac{2}{n}-m}\,v$ which has not been conſidered and taking only the firſt term, coſ $\frac{2v}{n}$, in the value of coſ $2z$, we have $\frac{r^3}{d^3}\times$ coſ $2z = 3q'$ coſ $\overline{\frac{2}{n}-m}\,v \times$ coſ $\frac{2v}{n} = \frac{3}{2}q'$ coſ $mv + \frac{3}{2}q'$ coſ $\overline{\frac{4}{n}-m}\,v$ which laſt term we omit, as not being of the ſpecies which we here want, hence we aſſume $\frac{3\alpha r^3}{2d^3}$ coſ $2z = \frac{9}{4}\alpha q'$ coſ mv, which is the correction of $\frac{3\alpha r^3}{2d^3}$ coſ $2z$ in the value of P.

1187. Again aſſuming $d=1$, in the quantity r^3 we introduce the term $3q'$ coſ $\overline{\frac{2}{n}-m}\,v$, therefore $\frac{r^2 r}{v} = \overline{\frac{2}{n}-m}\times q'$ ſin $\overline{\frac{2}{n}-m}\,v$ hence, we aſſume $\frac{3\alpha r^2 r}{2d^3 v} = \frac{3\alpha q'}{2}\times\overline{\frac{2}{n}-m}\times$ ſin $\overline{\frac{2}{n}-m}\,v$, and taking only the firſt term, ſin $\frac{2v}{n}$ in the value of ſin $2z$, we have $\frac{3\alpha r^2 r}{2d^3 v}\times$ ſin $2z = \frac{3\alpha q'}{2}\times\overline{\frac{2}{n}-m}\times$ ſin $\overline{\frac{2}{n}-m}\,v\times$ ſin $\frac{2v}{n} = \frac{3\alpha q'}{4}\times\overline{\frac{2}{n}-m}\times$ coſ $mv - \frac{3\alpha q'}{4}\times\overline{\frac{2}{n}-m}\times$ coſ $\overline{\frac{4}{n}-m}\,v$, which laſt term we omit, as not being of the ſpecies we here want, hence, we aſſume $\frac{3\alpha r^2 r}{2d^3 v}\times$ ſin $2z = \frac{3\alpha q'}{4}\times\overline{\frac{2}{n}-m}\times$ coſ mv, which is the correction of $\frac{3\alpha r^2 r}{2d^3 v}\times$ ſin $2z$ in the value of P.

1188. The three corrections therefore which P receives make $-\left(\frac{6d}{pm}+\frac{9}{4}-\frac{3}{4}\times\overline{\frac{2}{n}-m}\right)\alpha q'$ coſ mv, but in the value of P before this term was $-\frac{3}{4}\alpha\beta$ coſ mv, and we put $E=\frac{3}{4}\beta$; we muſt now therefore put $E=\frac{3}{4}\beta+\left(\frac{6d}{pm}+\frac{9}{4}-\frac{3}{4}\times\overline{\frac{2}{n}-m}\right)q'$. E is therefore increaſed by $\left(\frac{6d}{pm}+\frac{9}{4}-\frac{3}{4}\times\overline{\frac{2}{n}-m}\right)q' =$ 0,0784, and the former value of $\frac{3}{4}\beta$ being 0,0839, the corrected value of

$E=$

$I = 0,1622$ hence, $1 - m^2 = \frac{I a d}{v p} = 0,0162,6$, and $1 - m = 0,00836$, which is very near its true value 0,008455 Now there are, after this correction, only ſome very ſmall quantities omitted, the operation therefore ought to be a very near approximation, and accordingly we find it to be ſo, hence, we may conclude that the theory of gravity is ſufficient to give the true motion of the moon's apogee

1189 Having determined the value of m, we might correct all the quantities which depend upon it and then proceed to find the correction of the time, and thence that of the true longitude, in like manner as we obtained the ſame for the planets, taking into conſideration the excentricity of the earth's orbit, and the inclination of the moon's orbit And in the ſame manner as for the moon, we may find the motion of the apogee of the orbits of the planets But a full inveſtigation of all theſe things would exceed the bounds to which we muſt confine this work

* * *

CHAP

CHAP. XXXVIII.

ON THE TIDES.

Art. 1190. THE phænomenon of the tides is a circumſtance which muſt have been obſerved in the firſt ages of the world. According to Mr. COSTARD, in his Hiſtory of Aſtronomy, HOMER is the moſt ancient author who has mentioned it. HERODOTUS ſpeaking of the *Red Sea* or the *Arabian Gulph*, ſays, that "there is a flux and reflux of water in it every day." And DIODORUS SICULUS deſcribes it to be "a great and rapid tide." Theſe writers however, do not attempt to gueſs at the cauſe. PYTHEAS of *Marſeilles* who lived about the time of ALEXANDER *the Great* was the firſt perſon who ſuſpected that the phænomenon was owing to the moon. PLINY ſays directly, that it is cauſed by the ſun and moon, *Æſtus maris accedere et reciprocare maxime mirum eſt; pluribus quidem modis accidit, verum cauſa in ſole lunaque. Bis inter duos exortus lunæ affluunt, biſque remeant, vicenis quaterniſque ſemper horis*, and from further obſervations which he has made upon the tides, it appears, that they muſt have been very accurately obſerved in his time. GALILEO thought that the tides were owing to the rotatory motion of the earth about it's axis, and it's revolution about the ſun, but the phænomena can by no means be ſolved from theſe cauſes; as the former motion could only make the earth put on the form of an oblate ſpheroid ſubject to no change, and the latter would produce no effect on the ſurface. DES CARTES imagined the tides to be cauſed by the preſſure of the moon, but according to this hypotheſis, the tides ought to be loweſt when the moon is on the meridian; nor could the effect be the ſame when the moon is below as when above the horizon. Dr. WALLIS ſuppoſed the phænomenon might be ſolved by the earth and moon revolving about their center of gravity, but as the tides depend on the ſituation of the moon to the ſun, and are greateſt when they are in conjunction, and leaſt when in quadratures, it is manifeſt, that they muſt be partly owing to the ſun, and therefore as the ſun and moon appear to act in like manner to produce the effect, this cannot be the true principle. KEPLER was the firſt who aſſigned the true phyſical cauſe; he ſays, that the waters of the ſea gravitate towards the moon, and cauſes the tides (220). Laſtly Sir I. NEWTON has ſhown, that from the principles of gravity, the phænomena of the tides may be ſolved; but the effects from theory muſt be interrupted variouſly from local circumſtances, as the

the theory ſuppoſes the whole ſurface of the earth to be covered with water, which would, in ſuch a caſe, have a free motion

Fig 243 1191 Let $PEpD$ be the earth at reſt without any rotation about its axis, S a body at reſt attracting it, draw $SEOD$, and Pp perpendicular to it, then, by the attraction of S, the earth will put on the form of a ſpheroid, whoſe minor axis is Pp, and major ED. Let P be the attraction of the ſpheroid at P, E its attraction at E, independent of the diſturbing force of S, then (975) the attraction of M in the direction MR is $P \times \frac{QO}{PO}$, and in the direction MQ it is $E \times \frac{OR}{OE}$ Let m repreſent the additious force of S upon the point P, and n that upon the point E, then (845) as the additious force varies as the diſtance from O, $PO \quad MO \quad m \quad m \times \frac{MO}{PO}$ the additious force at M, hence, $MO \quad MR$, or QO, $\quad m \times \frac{MO}{PO} \quad m \times \frac{QO}{PO}$ the additious force at M in the direction MR, alſo $EO \quad OM \quad n \quad n \times \frac{OM}{EO}$ the additious force at M, hence, $OM \quad MQ$, or $OR \quad n \times \frac{OM}{EO} \quad n \times \frac{OR}{EO}$ the additious force at M which acts in the direction MQ therefore (847) $2n \times \frac{OR}{EO} =$ the whole diſturbing force on M by the action of S in the direction QM Hence, the whole force of M in the direction MQ is $\overline{E-2n} \times \frac{OR}{OE}$, and in the direction MR it is $\overline{P+m} \times \frac{QO}{PO}$, let theſe two forces be repreſented by Mg, Mr, and complete the parallelogram $Mrqg$, and produce Mq to G then gq, or $Mr \quad Mg \quad QG \quad QM$, that is, $\overline{P+m} \times \frac{QO}{PO} \quad \overline{E-2n} \times \frac{OR}{OE} \quad QG = \frac{OF}{OP} \times OQ$ (975) QM, or $P+m \quad E-2n \quad OE \quad OP$, therefore a ſpheroid having two axes in ſuch a ratio, the direction of the whole force at every point will be perpendicular to the ſurface, conſequently the fluid will be at reſt

1192 By Art 858 it appears, that the additious force of the ſun on a particle at the earth's ſurface the force of gravity at the earth's ſurface the ſquare of the periodic time of a body revolving at the earth's ſurface the ſquare of the periodic time of the earth 1 $38604600 = w$, and this (without any ſenſible error in the preſent caſe) we may take for the ratio on any part of the earth's ſurface, conſidered firſt as a ſphere, and then only diſturbed by the ſun and moon Let therefore $P = w$, $m = 1$, and put $OP \quad OE \quad 1 \quad 1+d$, then

then as $m : n :: OP : OE :: 1 : 1+d$, we have $n = 1+d$, alſo (977, 978) $1+\frac{4}{5}d : 1+\frac{3}{5}d :: w : L = w \times \overline{1-\frac{1}{5}d}$; hence, by ſubſtituting in the laſt article $w+1 : w \times \overline{1-\frac{1}{5}d} - 2 - 2d :: 1+d : 1$, and, by neglecting the terms where d^2 enters on account of their extreme ſmallneſs, we get $d = \frac{15}{4w-10} =$ (neglecting 10 on account of its ſmallneſs) $\frac{15}{4w} = \frac{1}{10294560}$ the part of the earth's radius which expreſſes the difference of the two ſemidiameters; hence, as the radius = 3964 miles (1009) we have $d = 2,033$ feet for the effect of the ſun in elevating the tides. Now (836) the effect of the ſun is to that of the moon as the cubes of their apparent diameters and denſities conjointly; and if we take with Sir I. NEWTON their mean ſemidiameters to be 32′ 12″ and 31′ 16½″ their cubes will be as 1 : 1,091, and (1038) the denſities are as 1 : 2,44; hence 1 : 1,091 × 2,44 :: 2,033 : 5,412 feet, the height to which the moon will raiſe the tides. When therefore the ſun and moon are in conjunction or oppoſition, in which caſe they both tend to raiſe the tides in the ſame place, the height of the tide will be 7,445 feet. Thus we have determined the figure of the earth and the height of the tides, upon the ſuppoſition of an equilibrium in all it's parts.

1193. We have here ſuppoſed that the earth was a ſphere and not a ſpheroid, but if we ſuppoſe it to be a ſpheroid and the ſun in the equator, then the mean radius of the earth which we have aſſumed will be to the radius at the equator as 3964 : 3972,5, or as 464 : 465. Now as the addititious force varies as the radius, it will be increaſed at the equator in the ratio of 464 : 465; alſo, as gravity on the earth's ſurface varies inverſely as the diſtance (984), the gravity will be diminiſhed in the ſame ratio; hence, the ratio of gravity w to the addititious force will be diminiſhed in the ratio of 465 : 464; conſequently d will be increaſed in the ratio of 464 : 465^2. But w, in this caſe, inſtead of being $= P$ will be leſs in the ratio of the radius at the mean diſtance to PO, or as 465 : 464; hence, d will be increaſed in the ratio of $464^3 : 465^3$.

1194. It has been here ſuppoſed, that the high tide was under the luminary, and that there was a general equilibrium of the waters; but the high tide is at ſome diſtance from the luminary, and the waters riſe and fall by a reciprocation; alſo, the free motion of the waters in the open ſeas is hindered by ſhallow places, rocks and iſlands; in conſequence of which, the tides in ſome of the open ſeas, at the time of the conjunction of the luminaries are found to riſe only to the height of about three feet. Thus the theory alone will afford no practical concluſions.* We ſhall now therefore proceed to explain, as briefly a we can, what M. D. BERNOULLI has written upon this ſubject, as, by correcting his

theory

theory by obſervation, he has been enabled to deduce practical rules for finding the times and height of the high tides

Fig 244 1195 Let $gbhd$ be the ſurface of the earth undiſturbed by the ſun or moon, and ſuppoſed to be a ſphere, $EPQp$ the ſpheroidical figure ariſing from the tides Let $OM = 1$ $Pg = s$, $Eb = r$, $p = 3,14159$, &c then the content of the ſphere $= \frac{4p}{3} \times 1^3$, and the content of the ſpheroid $= \frac{4p}{3} \times \overline{1+r} \times \overline{1-s} = \frac{4p}{3} \times \overline{1+r-2s}$, neglecting the terms where s^2 enters on account of their ſmallneſs, hence, $\frac{4p}{3} \times 1^3 = \frac{4p}{3} \times \overline{1+r-2s}$, therefore $2s = r$, or $2Pg = Eb$ Hence, the altitude of the high tide above the level of the water if there had been no tide is double of the depreſſion of the low tide below therefore the middle between the higheſt and loweſt tides is not the height of the ſea if it were undiſturbed

1196 Draw Ozv, and wzy perpendicular to QE, then as we may conſider the angle zvw a right one, the triangles zvw, zyO will be ſimilar, therefore zO zy Zw $Zv = zw \times \frac{zy}{zO}$ Let $Oy = s$, $Oz = b$, then $zy = \sqrt{b - s}$, and if $OE - OP = m$, then (1195) $bE = \frac{2}{3} m$, and $Pg = \frac{1}{3} m$, and by the property of the ellipſe, $yw = \frac{OP}{OE} \times \sqrt{Ey \times yQ} = \frac{b - \frac{1}{3}m}{b + \frac{2}{3}m} \times \sqrt{\overline{b + \frac{2}{3}m - s} \times \overline{b + \frac{2}{3}m + s}} =$ (by dividing $b - \frac{1}{3}m$ by $b + \frac{2}{3}m$ and extracting the root of the quantity under the radical ſign, omitting the terms where the powers of m above the firſt enter) $\overline{1 - \frac{m}{b}} \times \overline{\sqrt{b^2 - s^2} + \frac{bm}{\sqrt{b - s^2}}} = \sqrt{b^2 - s} + \frac{3s - b}{3b\sqrt{b - s}} \times m$ very nearly, from which ſubtract $\sqrt{b - s^2} = zy$, and there remains $\frac{3s - b^2}{3b\sqrt{b^2 - s}} \times m = zw$, hence, $zv = \frac{3s^2 - b^2}{3b^2} \times m$ Now $Eb = \frac{2}{3} m$, hence, $Eb - zv = \frac{2}{3} m - \frac{3s - b^2}{3b} \times m = \frac{b^2 - s^2}{b} \times m$, conſequently the falling of the water from the higheſt point is as the ſquare of the ſine of the hour angle from the time of the high tide, E being a point in the equator to which the luminary is vertical When $zv = 0$, we have $3s - b = 0$, hence, $s = b\sqrt{\frac{1}{3}} =$ (if $b = 1$) 0,57736 the coſine of 54° 44′ the angle EOM, the diſtance of the high tide from the point where the water is at the ſame height at which it would have been if there had been no tide

1197 If we ſuppoſe both the ſun and moon to be in conjunction at E, and if $m = OE - OP$ ariſing from the ſun, and $n =$ the difference cauſed by the moon, then if we take theſe quantities at the time when the ſun and moon are at at their mean diſtances, at which time we may conſider their apparent ſemi diameters as equal the effects produced will be as their denſities (856) therefore their denſities will be as m n Hence (1196), $zv = \frac{3s - b^{2}}{3b} \times m + \frac{3s^{2} - b}{3b} \times n$ the altitude from the joint effects of the ſun and moon when in conjunction or oppoſition

1198 If the ſun and moon be not in conjunction, but the ſun be vertical to E and the moon to F, then if r be the ſine of the angle FQz, the altitude zv of the tide at z will be $\frac{3s - b}{3b} \times m + \frac{3r^{2} - b}{3b} \times n$ Now to find at what point the tide will be higheſt, we muſt make the fluxion $= 0$, hence $mss + nrr = 0$ Put $A =$ the arc bz, $a =$ the arc Fz, C and c their reſpective coſines, then $s = \frac{C}{b} \times A$, and $r = \frac{c}{b} \times a$, hence, by ſubſtitution, $\frac{msC}{b} \times A + \frac{nrc}{b} \times a = 0$, but as $A + a$ is conſtant, $A + a = 0$, therefore $a = -A$, conſequently $msC = nrc$ But $sC = \frac{b}{2} \times$ ſin $2A$, and $rc = \frac{b}{2} \times$ ſin $2a$, therefore $m \times$ ſin $2A = n \times$ ſin $2a$ or m n ſin $2a$ ſin $2A$ Hence, we have only to divide an arc $2bF$ into two parts, that the ratio of the ſines may be given, and the half of each part will give bz and Fz To do that, let $bac = 2bF$, draw be parallel Fig to ac, and take ab be m n, and join ae, and it will divide it in the ratio 24. required And to compute the two parts, in the triangle abe we have the angle abe the ſupplement of the given angle cab, and the ratio of ab be as m n, hence, $n+m$ $n-m$ tan $\frac{1}{2}\,\overline{eab+aeb}$ tang $\frac{1}{2}\,\overline{eab-aeb}$, therefore we know the angles themſelves Thus we get the point where the tide is higheſt If the arcs A and a be very ſmall, ſo that the ſines may be taken for the arcs, then $m \times 2A = n \times 2a$, or $m \times A = n \times a$, and hence, m n a A M D Bernoulli has ſolved this problem analytically

1199 M D Bernoulli proceeds from hence to find the ratio of the denſities m n in the following manner Conceive on one day the ſun and moon to be in conjunction at L, and the high tide at E when they are on the meridian Now a mean lunar day being 24h 50 let us ſuppoſe the next day when the ſun is returned to F that the moon is got to F, ſo that the earth has to deſcribe an angle of 50 of time before the moon comes to the meridian Now the greateſt tide at z has been found from the mean of a great number of obſervations, to be ,5 after the ſun paſſed the meridian hence as theſe arcs bz Fz are ſo ſmall that they may be taken as their ſines we have, by the laſt Article, $m : n :: 15' : 35'$, therefore $n = \frac{35}{15} m = 2\frac{1}{3} m$. The arc Fz ſhows the time of the high tide from the tranſit of the moon over the meridian. We may compute this ratio at any poſition of the ſun and moon only by conſidering how much in time the moon recedes from the ſun in a day, and how much the high tide one day precedes that on the next, and thence find Fz and bz. When the ſun and moon are in quadratures, if $A = 90°$, then $a = 0$, for in this caſe $m \times \text{ſin. } 2A = 0 = n \times \text{ſin. } 2a$, agreeable to the conditions; hence, in this ſituation, the time of the high tide would be at the paſſage of the moon over the meridian. Afterwards the point z of high tide will lie on the other ſide of b. Hence, from ſyzygies to quadratures the high tide precedes the time of the moon's paſſage over the meridian, and from quadratures to ſyzygies it follows it. Now in quadratures, the difference of the times of the two tides was found to be 85′, therefore we have, $m : n :: 85 - 50 : 85$, conſequently $n = \frac{85}{35} \times m = 2\frac{3}{7} m$. From the mean of all the reſults, M. D. BERNOULLI concluded $n = 2\frac{1}{2} m$, which agrees very well with that deduced in Art. 1038. from the preceſſion of the equinoxes, and the nutation of the earth.

1039. The method uſed by Sir I. NEWTON to determine the ratio of $m : n$, was by obſerving the greateſt (a) and leaſt (b) aſcent of the waters from low to high tide at the vernal and autumnal equinox in conjunction or oppoſition and in quadratures, and then as the effects of the ſun and moon are in proportion to their denſities, their mean diameters being nearly equal, we have, $n + m : n - m :: a : b$; hence, $n : m :: \frac{1}{2}\overline{a+b} : \frac{1}{2}\overline{a-b}$. By this method Sir I. NEWTON found $n : m :: 3\frac{1}{2} : 1$ from obſervations made at Briſtol. From obſervations of a like kind made at St. Malo, by M. THOUROUD, it appeared that $n : m :: 1\frac{6}{7} : 1$. The ratio of m and n here found repreſents the proportional diſturbing forces of the ſun and moon at their mean diſtances, where we ſuppoſed their apparent ſemidiameters equal, and conſequently they will alſo repreſent their denſities. But at any other diſtances, we muſt add to this ratio that of the cubes of their apparent diameters. Hence, when the moon is in it's perigee and the earth in it's aphelion, the ratio of $m : n$ will be very nearly as $1 : 3$, and when the moon is in it's apogee and the earth in it's perihelion, it will be nearly as $1 : 2$, we will therefore take theſe two ratios for the limits of the ratio of $m : n$. The great difference therefore in the reſults of the ratio of $n : m$ from the obſervations at Briſtol and St. Malo, cannot be accounted for from the difference which may poſſibly take place, and therefore the method of Sir I. NEWTON gives a concluſion which is ſubject to too great a degree of inaccuracy to be depended upon. From the agreement of

conclusions found by M D BERNOULLI with that deduced in Art
we may ſuppoſe the denſities of the ſun and moon as 1 2½ as being
ly very near the truth

Hence, by computing the angle EOz, or FOz, for every day from
w or full moon, we might get the time of the high tide when compared
he paſſage of the ſun or moon over the meridian, and thus, from theory
ght conſtruct a Table, ſhowing the times of the high tides, if, as we have
o ſuppoſed, the whole effect of the ſun and moon upon the waters took
mmediately upon the operation of the cauſe, and that an equilibrium
waters was the conſequence But although the ſun and moon exert
reateſt influence when they are in the meridian yet they continue to
ne time after from which and the inertia of the water, it happens
e high tide is not at that time, a Table therefore conſtructed upon
alone muſt neceſſarily want to be corrected We ſhall therefore
the principles upon which M D BERNOULLI has applied this
on

When the ſun and moon are in ſyzygies b at Breſt, it appears, from FIG 246
an of a great number of obſervations, that the high tide happens at
, and when the ſun is at b and the moon in quadratures at g, it happens
o′, the difference is 5*h* 12′ This difference was obſerved to be the
Dunkirk, and at other ports, although the abſolute times were different
t us conſider what is the difference from theory When the ſun and
re in ſyzygies at b, the high tide is at b at that time, or at 12 o clock
the moon is in quadratures at g, it is low tide on the earth at b now
he earth is revolving about its axis ſo as to bring this point at b up to
on, the moon is got to v about 3 from g and by our computation
the point z of high tide is about [illegible] beyond v, hence, the point on the
b at low tide muſt deſcribe an arc bz of 95 before it be at the high
ich arc it deſcribes in 6*h* 20 This interval of the two tides therefore
eory does not agree with obſervation Take the point F at 20 before
uppoſe the moon to be in F and the high tide by theory to be at z',
y computation, this happens at 11*h* 2, lunar time Now let us take
before g and ſuppoſe the moon at F and the high tide at z', then,
putation, this is found to happen at 3*h* 59½ lunar time, which gives
val of 4*h* 57½ lunar time, or 5*h* 8′ ſolar time This therefore agrees
l with the interval between the times of the high tides when the moon
njunction and quadratures Hence, to get the true interval of the
the tides, we muſt compute from our theory, by ſuppoſing the moon
nd its true place, and then we ſhall get the true interval, agreeing

with

with obſervation. Hence, the following Table was computed, the firſt column of which ſhows the moon's true diſtance from the ſun when the moon paſſes the meridian, for every 10 from conjunction to oppoſition, the next three columns ſhow the times of the high ſea in reſpect to the paſſage of the moon over the meridian, for the perigee, mean diſtance and apogee of the moon, and the laſt three ſhow the abſolute hours

A TABLE

A TABLE

For finding the Time of the High Tides

Diſt ☉ a ☾	Time of High Tide before and after the paſſage of the moon over the meridian			True Time of the High Tide nearly		
	Perig of ☾	Mean Diſt ☾	Apog of ☾	Per of ☾	Me Diſ ☾	Apo of ☾
0°	18 after	22′ after	27½′ after	0h 18′	0h 22′	0h 27½′
10	9½ after	11½ after	14 after	0 49½	0 51½	0 54
20	0	0	0	1 20	1 20	1 20
30	9½ before	11½ before	14 before	1 50½	1 48½	1 46
40	18 before	22 before	27½ before	2 22	2 18	2 12½
50	26 before	31½ before	39½ before	2 54	2 48½	2 40½
60	33 before	40 before	50 before	3 27	3 20	3 10
70	37½ before	45 before	56 before	4 2½	3 55	3 44
80	38½ before	46½ before	58 before	4 41½	4 33½	4 22
90	33½ before	40½ before	50½ before	5 26½	5 19½	5 9½
100	21 before	25 before	31 before	6 19	6 15	6 9
110	0	0	0	7 20	7 20	7 20
120	21 after	25 after	31 after	8 21	8 25	8 31
130	33½ after	40½ after	50½ after	9 13½	9 20½	9 30½
140	38½ after	46½ after	58 after	9 58½	10 6½	10 18
150	37½ after	45 after	56 after	10 37½	10 45	10 56
160	33 after	40 after	50 after	11 13	11 20	11 30
170	26 after	31½ after	39½ after	11 46	11 51½	11 59½
180	18 after	22 after	27½ after	0 18	0 22	0 27½

1202 This Table gives the true interval of the tides, and alſo the time very nearly, upon ſuppoſition that the luminaries are in the equator, and that the effect could take place as ſuppoſed in the theory But from the inertia of the water, and the obſtructions it meets with in its paſſage from rocks, iſlands, ſhores, &c this Table cannot exhibit the abſolute time of the high tide at every port, which muſt vary according to the effect of theſe circumſtances although it ſhows the difference of the times To determine therefore the true time at any port, we muſt find from obſervation what is the difference between the true time and that ſhown by the Table, and then that difference added to the time ſhown by the Table will give the time of the high tide For the ſame obſtacles remaining, there muſt always be nearly the ſame retardations, the greater however the tides are, the leſs the ſame cauſes will retard, and the leſs they are the more they will retard, and accordingly it is found from obſervation, that the higheſt tides always come ſooner to their height, and the loweſt later, than the calculations give it, the above difference being determined from obſervations on the mean high tides The declination of the luminaries, as it alters the quantity of the tides, and alſo their direction, will cauſe ſome ſmall variation of the difference, and the different direction of the winds muſt have alſo ſome effect

1203 But beſides the ſmall variation of difference ariſing from the declination of the ſun and moon for the reaſons in the laſt article the time will alſo be altered from hence, that is bg is not the equator the arcs upon it will not be the meaſures of the hour angle As the moon's orbit makes but a ſmall angle with the ecliptic, we may ſuppoſe them to coincide Hence, when the moon is in the equator, the arc of the moon's orbit included between two meridians the correſponding arc of the equator 1 rad cos $23\frac{1}{2}$, or as 100 92, and when the declination is the greateſt, theſe arcs are as cos dec rad or as 90 100 Hence, the numbers in the ſecond, third, and fourth columns muſt in the former caſe be multiplied by $\frac{92}{100}$, and in the latter by $\frac{100}{92}$ Very nearly in the middle point between theſe ſituations the arcs will be equal and for any intermediate points, we may compute the multiplier by the Note, Art 128

FIG 244 1204 From the value of zv in Art 1197 we may compute the altitude of the ſea at any time, and by Art 1198 we can find the points where the tide is higheſt, and at 90° from thence it is loweſt Thus we can find the altitude of the higheſt and loweſt tides for all times, ſuppoſing (as in Art 1201) the moon to be 20 behind its true place But this can only give the relative altitudes, the true altitudes varying very much in different parts, from their ſituation. M. D. BERNOULLI therefore puts A and B for the mean height of the high tides when the luminaries are in conjunction or oppoſition, and when they are in quadratures, to be determined from obſervation, the height of the tide being the aſcent of the water from the low to the high tide. Hence, $n+m=A$, $n-m=B$, therefore $n=\frac{A+B}{2}$, $m=\frac{A-B}{2}$, and putting theſe values for n and m, he has conſtructed the following Table, the firſt column of which ſhows the diſtances of the ſun and moon, at the time the moon paſſes the meridian, and the other three ſhow the height of the high tides for the perigee, mean diſtance, and apogee of the moon.

A TABLE

A TABLE

To show the Height of the High Tide.

Dist	Alt in Perigee	Alt at Mean Dist	Alt in Apogee
0	$0{,}995A+0{,}149B$	$0{,}883A+0{,}117B$	$0{,}795A+0{,}082B$
10	$1{,}164A+0{,}038B$	$0{,}970A+0{,}030B$	$0{,}874A+0{,}021B$
20	$1{,}138A+0{,}000B$	$1{,}000A+0{,}000B$	$0{,}901A+0{,}000B$
30	$1{,}104A+0{,}038B$	$0{,}970A+0{,}030B$	$0{,}874A+0{,}021B$
40	$0{,}995A+0{,}149B$	$0{,}883A+0{,}117B$	$0{,}795A+0{,}082B$
50	$0{,}853A+0{,}319B$	$0{,}750A+0{,}250B$	$0{,}676A+0{,}176B$
60	$0{,}668A+0{,}527B$	$0{,}587A+0{,}413B$	$0{,}529A+0{,}290B$
70	$0{,}460A+0{,}749B$	$0{,}413A+0{,}587B$	$0{,}327A+0{,}412B$
80	$0{,}284A+0{,}958B$	$0{,}250A+0{,}750B$	$0{,}225A+0{,}527B$
90	$0{,}133A+1{,}127B$	$0{,}117A+0{,}883B$	$0{,}105A+0{,}621B$
100	$0{,}034A+1{,}238B$	$0{,}030A+0{,}970B$	$0{,}027A+0{,}682B$
110	$0{,}000A+1{,}277B$	$0{,}000A+1{,}000B$	$0{,}000A+0{,}703B$
120	$0{,}034A+1{,}238B$	$0{,}030A+0{,}970B$	$0{,}027A+0{,}682B$
130	$0{,}133A+1{,}127B$	$0{,}117A+0{,}883B$	$0{,}105A+0{,}621B$
140	$0{,}284A+0{,}958B$	$0{,}250A+0{,}750B$	$0{,}225A+0{,}527B$
150	$0{,}460A+0{,}749B$	$0{,}413A+0{,}507B$	$0{,}372A+0{,}412B$
160	$0{,}668A+0{,}527B$	$0{,}587A+0{,}413B$	$0{,}529A+0{,}290B$
170	$0{,}853A+0{,}319B$	$0{,}750A+0{,}250B$	$0{,}676A+0{,}176B$
180	$0{,}995A+0{,}149B$	$0{,}883A+0{,}117B$	$0{,}795A+0{,}082B$

It is manifeſt from this Table, that the higheſt tides are when the moon has paſſed conjunction 20, or about $1\frac{1}{2}$ days after, and the loweſt tides when the moon has ſo far paſſed her quadratures

1205 We come in the next place to conſider the effect ariſing from the declination of the moon It appears by Art 1196 that the fall of the water from E to $v=\frac{b-s}{b^2}\times m$, therefore at P it is $=m$, hence, $m-\frac{b-s^2}{b}\times m$ $=\frac{s^2}{b}\times m=$ the riſe of the water from P to v, put therefore $c=$ the coſine of the angle EOv to radius unity, and we have $c\,m=$ the height of the water above the loweſt point

1206 Let $GBAH$ be a meridian of the earth paſſing through the moon vertical to B, AG the axis of the earth, DMK the equator, and CEL any parallel to it, and aſſume E any place, then will the diameter BH be the axis of the ſpheroid which, as it differs but very little from a ſphere, may be regarded as ſuch, ſo far as reſpects the triangles on its ſurface Put therefore radius $=1$, $S=$ ſine of AB $C=$ its coſine, $s=$ ſine of AE, $c=$ its coſine, $y=$ coſine of the angle BAE, and $q=$ coſine of BE, then, by Spherical Trigonometry $q=Ssy+Cc$, therefore, by the laſt Article, $\overline{Ssy+Cc}\times m=$ the height of the water above the loweſt point Hence, we may conſider the following caſes FIG 247

I Let e be the point where the water is loweſt, then $Ssy+Cc=0$, hence $y=-\frac{Cc}{Ss}$ the coſine of BAe

II When $C=s$ and $c=S$ then $y=-1$ therefore the angle $CAe=180^\circ$, and conſequently e coincides with L Hence, when the latitude of the place $=$ the complement of the moon's declination the low tide happens at L, diſtant from the high tide at C twelve hours, in this caſe therefore there is only one high and one low tide in twenty four hours

III When s is leſs than C or when the diſtance of the place from the pole is leſs than the moon's declination, then $\overline{Ssy+Cc}\times m$ never can become $=0$, within the limits of y Hence the altitude diminiſhes from the paſſage of the moon over the meridian to the oppoſite meridian and conſequently from the parallel whoſe diſtance from the pole $=$ the moon's declination to the pole, there is only one high and one low tide in twenty four lunar hours And if we make $y=1$, and $y=-1$, we have $\overline{Ss+Cc}\times m-\overline{Cc-Ss}\times m=4SsCcm$ for the difference of the altitudes of the two tides

IV When the declination of the moon is equal to the latitude of the place, $S=s$, $C=c$, and make $y=1$, hence, $Ss+Cc=S^2+C^2=1$, therefore the greateſt altitude $=m$, alſo y (Caſe I) $=\frac{Cc}{Ss}=\frac{C}{S}=$ coſ BAe

V When the moon is in the equator, $S=1$, $C=0$, and the altitude of the tide $=s^2m$, which therefore varies as the ſquare of the coſine of latitude Hence, in this caſe, at the pole there is no tide

VI The height of the tide when the moon paſſes the meridian $=\overline{Ss+Cc}^2\times m$, and when the moon is at the oppoſite meridian the height is $\overline{-Ss+Cc}^2\times m$ Hence when the moon is in the equator $C=0$, and the height of both tides are equal To a place on the north of the equator, when the moon has ſouth declination, C becomes negative, and the latter tides are the greateſt, but when the moon has north declination, C is poſitive, and the former is the greateſt Hence, to us in this caſe, the high tide is greater when the moon is above the horizon than when below In all caſes, e is nearer to or further from C, according as $y\left(-\frac{Cc}{Ss}\right)$ is poſitive or negative The difference of the two tides is always $=4SsCcm$

VII The height of the two tides when the moon paſſes the meridian being $\overline{Ss+Cc}^2\times m$ and $\overline{-Ss+Cc}^2\times m$, the mean height is $\overline{S^2s^2+C^2c^2}\times m$

VIII Hence the ſame north and ſouth declination of the moon give the ſame mean altitude This is confirmed by obſervations

IX In latitude 45°, $S^2=C^2=\frac{1}{2}$, hence the mean altitude $=\frac{1}{2}\times\overline{s^2+c^2}\times m=\frac{1}{2}m$, therefore whatever be the declination of the moon, the mean altitude is in this latitude always the ſame Hence, in our latitude the mean altitude will vary but very little

X Under the equator, the mean height $=S^2m$, which therefore varies as the ſquare of the coſine of the moon's declination

1207 As the tides riſe from the collecting of the waters on the whole ſurface of the main ſea if there be any quantities of water ſeparated from it, the FIG 244 variation muſt be proportionally ſmaller For if rr be a ſmall ſurface of water detached from the reſt its ſurface will put on the figure vs ſimilar to dt, conſequently the variation xr from the mean altitude muſt be very ſmall Hence, there have never been any tides obſerved in the Caſpian ſea, for from the dimenſions of that ſea, the greateſt altitude will not be above 1½ inches at the eaſtern and weſtern extremities, according to M de la LANDE, who has corrected an error made by M D BERNOULLI in his computation, and it is manifeſt, that the middle of the ſea will never be affected Very ſmall tides have been obſerved in the Black ſea, which, from its connection with the Mediterranean

Mediterranean ſea only by a very ſmall paſſage, may be conſidered as a detached ſea. The Mediterranean ſea is connected to the main ſea only by a narrow paſſage at Gibraltar, ſo that only a ſmall quantity of tide from the open ſea can flow in, and the ſea itſelf is not large enough to produce any very ſenſible tides; accordingly we find the tides there to be but very ſmall. The beſt obſervations are thoſe which have been made by M. le Chevalier d'Angos at Toulon, from which it appears that the tides produce a variation of about one foot in the height of the water. There were frequently greater variations, but thoſe appeared to ariſe from high winds.

1208. The general phænomena of the tides from obſervation agree very well with the concluſions deduced from the theory of gravity; indeed much more accurately than could have been expected, when we conſider how many circumſtances there are which take place, and which cannot be reduced to computation. The theory ſuppoſes the whole ſurface of the earth to be covered with deep waters—that there is no inertia of the waters—that the major axis of the ſpheroid is conſtantly directed to the moon, and that there is an equilibrium of all the parts. But the inertia of the waters will make them continue to riſe after they have paſſed the moon, although the action of the moon begins to decreaſe, and they come to their greateſt altitude in the open ſeas about three hours after, at which time there is not a general equilibrium, but the waters riſe and fall by a reciprocation; hence, the longeſt axis is not directed to the moon, nor is the figure a perfect ſpheroid.—The waters have not a free motion on account of the ſhallow places, rocks, iſlands and continents, the force of currents and winds; alſo, as the waters approach the equator where the earth has a greater velocity about its axis, they muſt neceſſarily be left behind and obſtruct the regular motion of the water when it moves from weſt to eaſt, but conſpire with that from eaſt to weſt. All theſe circumſtances muſt affect the meaſures of the phænomena as deduced from theory; it may however in many caſes give the relative meaſures without any great error, ſo that by accurate obſervations once made on their abſolute quantity in ſome one particular caſe, the meaſures, in all other caſes, may be aſcertained to a conſiderable degree of accuracy.

1209. If a place communicate with two ſeas, or has two inlets to the ſame ſea, two tides may arrive at that place at different times, and produce various phænomena. An inſtance of this kind takes place at Batſha, a port in the kingdom of Tunquin in the Eaſt Indies, in 20 50 north latitude. The day in which the moon paſſes the equator the water ſtagnates; as the moon recedes from the equator towards the north, the water begins to riſe and fall once a day; and it is high water at the ſetting of the moon, and low water at its riſing. This daily tide increaſes for ſix or eight days, and then decreaſes for the ſame time by the ſame degrees and the motion ceaſes when the moon has returned to the equator. When it has paſſed the equator and approaches the ſouth pole the water riſes and falls as before but it is now high water at the riſing, and low water at the ſetting of the moon. Sir I. NEWTON thus accounts for this phænomenon. There are two inlets to this port, one from the Chineſe ocean between the continent and the Manillas, the other from the Indian ocean between the continent and Borneo, and he ſuppoſes that a tide may arrive at Batſha through one inlet, at the third hour of the moon, and through the other inlet, ſix hours after and ſuppoſing theſe tides to be equal one flowing in whilſt the other flows out the water muſt ſtagnate. Now they are equal when the moon is in the equator, but as the moon begins to decline on the ſame ſide of the equator with Batſha, the diurnal tides exceed the nocturnal (as appears by the foregoing principles) ſo that two greater and two leſſer tides muſt arrive at Batſha by turns. The difference of theſe will produce a motion of the water, which will riſe to its greateſt height at the mean time between the two greateſt tides, and fall loweſt at the mean time between the two loweſt tides, ſo that it will be high water about the ſixth hour at the ſetting of the moon, and low water at its riſing. When the moon has got to the other ſide of the equator, the nocturnal tide will exceed the diurnal and therefore the high water will be at the riſing and low water at the ſetting of the moon. Theſe principles will account for other extraordinary tides which are obſerved in thoſe places whoſe ſituation expoſes them to ſuch irregularities.

* * *

CHAP.

CHAP XXXIX

ON THE PRINCIPLES OF PROJECTION, AND THE CONSTRUCTION OF GEOGRAPHICAL MAPS

Art 1210 THE projection of an object is it's repreſentation upon a plane and is formed by drawing lines from the eye to the plane through every point of the object and according to different ſituations of the eye, the object and the plane the repreſentations will be different The projection in order to be perfect, ſhould be a perfect repreſentation of the object that is the proportion and relative ſituation of all the parts of the figure ſhould be the ſame as in the object, but in the conſtruction of geographical maps this is not practicable, it being impoſſible to give a true repreſentation of a ſpherical ſurface upon a plane, retaining the true proportion of the figures, magnitudes and poſitions of the countries with the relative degrees of latitude and longitude We will firſt ſhow the principles of the different projections, and then apply them to our preſent purpoſe

On the Orthographic *Projection*

1211 If the eye be ſuppoſed to be at an indefinitely great diſtance, ſo that all the lines drawn from it to the object may be conſidered as parallel, and alſo perpendicular to the plane of projection, the projection is called *Orthographic*

1212 The figure of a ſtraight line *AB* is a ſtraight line in the projection Fig 248
For draw *AE* *BD* perpendicular to *xy* the plane of the projection, and join *ED*, and it will repreſent the interſection of the plane paſſing through *EABD* with the plane of projection draw *mn* perpendicular to *ED*, and *n* is the projection of *m*, thus it appears that *ED* is the projection of *AB* Draw *AC* parallel to *ED*, then *ACDE* is a parallelogram, and *AC*=*ED*, *AC* may therefore repreſent the projection of *AB* Hence, if we want to make the repreſentation upon a plane at a diſtance from the body it will be all the ſame if we ſuppoſe the plane to touch the body the paralleliſm of the plane remaining the ſame

1213 The

Fig. 249 1213. The figure of the projection of a circle is an ellipſe. For let ABC be a ſemicircle conceived to be inclined to the plane of the paper, which we will make the plane of projection; draw BE perpendicular to that plane, and ED, BD perpendicular to the diameter AC, then ED is the projection of BD (1212). By the property of the circle, $AD \times DC = BD^2$; but as the angle BDE is conſtant, being the inclination of the circle to the plane of projection, BD is to DE in a conſtant ratio, namely, that of radius to the coſine of BDE, therefore DE varies as BD, and conſequently DE^2 varies as BD^2; hence, $AD \times DC$ varies as DE^2, which is the property of an ellipſe; the curve AEC is therefore an ellipſe. And by the laſt article it appears, that the projection will be the ſame, at whatever diſtance the circle is from the plane of projection. Let O be the center, and draw OP, OQ perpendicular to AC, then OQ is the minor axis; hence, to the radius of the circle, the minor axis is the coſine of the inclination of the circle to the plane of projection. If the circle be parallel to the plane of projection, the projection will be a circle equal to it.

1214. By the property of the ellipſe and circle, the area ABC : the area AEC :: BD : ED :: rad : coſ BDE; hence, the area of the circle will be diminiſhed by this projection in the ratio of radius : coſine of the inclination of the plane of the body to the plane of projection. And this muſt manifeſtly be true whatever be the form of the body ABC (conſidered as a plane), becauſe every line in the body is to its correſponding line in the projection in that ratio. Alſo, the projection is not ſimilar to the body. Hence, equal parts upon the ſurface of a ſphere will not be projected into parts either equal or ſimilar.

1215. If ABC be perpendicular to the plane of projection, E and D coincide, and D is the projection of B; thus the circle ABC is projected into its diameter; the arc AB is projected into its verſed ſine AD, and BP is projected into DO, which is equal to the ſine of BP, or the coſine of AB. If $AB = 60°$, then $BP = 30°$, and $AD = DO$; theſe two arcs therefore, one of which is double of the other, have their projections equal. This projection is uſed in the conſtruction of ſolar eclipſes, Chap. XXIII.

On the Stereographic *Projection*

1216 Let an eye be ſituated any where upon the ſurface of a ſphere, and from it draw a diameter, and perpendicular to this diameter draw a great circle, then if all the circles in the hemiſphere oppoſite to the eye be projected upon that great circle by lines drawn to the eye, the projection is called *Stereographic*, and the point oppoſite to the eye is called the *Pole*

1217 Let *EQPR* be a ſphere whoſe center is *O*, *E* the place of the eye draw the diameter *EOP*, and *QOR* perpendicular to it, repreſenting the plane of projection, draw *ECA EDB* and join *BO* Now *DO* is the projection of *PB* but *DO* is the tangent of the angle $DEO = \frac{1}{2} BOP$ Hence the projection of an arc meaſured from the pole is equal to the tangent of half that arc Fig 250

1218 Conceive the chord *AB* to be the diameter of a circle deſcribed upon the ſurface, and let *AEB* repreſent a cone having that circle for its baſe Draw *BG* parallel to *QR* then the arc $LB = EG$ therefore the angle $EAB = EBG$ = (as *BG* is parallel to *QR*) *EDC* therefore as *CD*, *AB* repreſent two ſections of the cone cutting its ſides at the ſame angle, the ſections muſt be ſimilar, but the ſection *AB* is a circle, therefore the ſection *CD* is a circle Hence, the projection of every circle is a circle

1219 Join *EG* then *BG* being the diameter of a circle parallel to the plane of projection *DH* will be the diameter of the circle of projection, whoſe center is *O* Hence the projection of all circles parallel to the plane of projection will be concentric circles, the radii of which are the tangents of half the diſtances of the circles from the poles

1220 The projection of the arc *QPR* is the ſtraight line *QOR*, and the ſame for every great circle paſſing through *P*

1221 Produce *BO* to *I*, join *EI* and produce it to meet *QR* produced in *K* and biſect *DK* in *r* then conſidering *BOI* is the diameter of a circle whoſe plane is inclined to the plane of projection in the angle *QOB*, *DK* will be the diameter of the projection of that circle (1216) Now $DK = OK + OD = \tan OEK + \tan OED = \tan \frac{1}{2} PI + \tan \frac{1}{2} PB = \cot \frac{1}{2} PB + \tan \frac{1}{2} PB = \frac{2}{\text{ſin } PB} = 2 \text{ coſec } PB = 2 \text{ ſec } QB$, therefore $\frac{1}{2} DK = \text{ſec } QB$ Hence, the radius of projection of any great circle is the ſecant of the angle between the plane of the circle and the plane of projection From theſe four articles it appears, that the projection of the parts of the ſphere will not properly repreſent in magnitude and ſituation, the parts themſelves

1222 If

1222 If E be the pole of the earth the meridians (as they paſs through P) will be projected into ſtraight lines (1220) and the parallels to the equator will be projected into circles whoſe centre is O (1219) And if EOI be the diameter of the ecliptic, DK is its projection (1221) This is called the *Polar* projection, and was uſed by PTOLEMY

1223 If $EQPR$ be the equator, then the point on the ſphere vertical to O will be the pole and BOI will be the diameter of any meridian And if Q be the point from which the longitude is reckoned, then the projection of the radius of that meridian will be the ſecant of its longitude (1221)

1224 To find the projection of the parallels of latitude, let EOP repreſent the plane of the equator, the repreſentation being a ſtraight line paſſing through the eye, then IG is the diameter of a parallel, the projection of which is $HK = OK - OH = \tan \frac{1}{2} PI - \tan \frac{1}{2} PG = \cot \frac{1}{2} EI - \tan \frac{1}{2} PG = \cot \frac{1}{2} PG - \tan \frac{1}{2} PG = 2 \cot PG$ Hence, the radius of the projection of the parallels is the cot of their latitudes This is called an *Equatorial* projection

1225 The ſtereographic projection is very convenient for practice, as all the circles are projected into circles or ſtraight lines, which are more eaſily deſcribed than any other figures

On MERCATOR's Projection

FIG 251

1226 Let P be the pole of the earth, ſuppoſed to be a ſphere EQ the equator, PL, PR two meridians, mn a ſmall circle parallel to ER PC the radius of the earth, mr, nr perpendicular to it, and join EC, RC Then mr, nr being parallel to EC, RC reſpectively the angle $mrn = ECR$, hence by ſimilar ſectors, ER mn EC mr, but when the angle is given the arc of a degree is in proportion to the radius alſo the length of a degree of the great circle ER is equal to a degree of latitude, and the length of a degree of the circle mn is a degree of longitude Hence a degree of latitude a degree of longitude EC mr radius coſine of latitude

1227 In this projection the ſphere is projected upon a plane and the meridians LP, PP are projected into ſtraight and parallel lines, conſequently P in the projection muſt be at an infinite diſtance from EQ In this caſe, the arc mn being the ſame for all latitudes the length of a degree of longitude is every where the ſame, to preſerve therefore the proper proportion between the degrees of latitude and longitude, the degrees of latitude muſt increaſe as you go from the equator, ſo that they may always be to a degree of longitude in the proportion of radius coſine of latitude

1228 Let

1228. Let P be the pole, E the equator, PCQ the axis of the earth, C the center, m a place on the ſurface; draw mr perpendicular to PQ, and join mC, mQ. Put $Cm = r$, $Em = x$, Cr (the ſine of Em the latitude of m) $= y$, and the length of En on the projection $= z$, called the *Meridional Parts*. Then (1227) $\sqrt{r^2 - y^2}$ (coſ. of lat.) $: r :: x : z = \frac{rx}{\sqrt{r^2 - y^2}}$, but $x = \frac{ry}{\sqrt{r^2 - y^2}}$; hence, $z = \frac{r^2 y}{r^2 - y^2} = \frac{r}{2} \times \frac{2ry}{r^2 - y^2}$, therefore $z = \frac{r}{2} \times \text{h. l. } \frac{r+y}{r-y} + \text{cor.} = r \times \text{h. l. } \sqrt{\frac{r+y}{r-y}} + \text{cor.}$ But by plane Trigonometry, $\sqrt{r^2 - y^2}$ $(mr) : r + y$ $(rQ) :: r$ (rad.) $: \frac{r \times \overline{r+y}}{\sqrt{r^2 - y^2}} = r \times \sqrt{\frac{r+y}{r-y}}$ the tangent of the angle rmQ = cotan. of rQm = cotan. of $\frac{1}{2} rCm$ = cotan. of half the complement of latitude. Hence $\sqrt{\frac{r+y}{r-y}} = \frac{\text{cotan. } \frac{1}{2} \text{ comp. lat.}}{r}$, conſequently $z = r \times \text{h. l. } \frac{\text{cotan. } \frac{1}{2} \text{ comp. lat.}}{r} + \text{cor.}$ but when $z = 0$, cotan. $\frac{1}{2}$ comp. lat. $= r$, therefore the laſt equation becomes $0 = r \times \text{h. l. } \frac{r}{r} + \text{cor.} = r \times \text{h. l. } 1 + \text{cor.} = 0 + \text{cor.}$ conſequently the correction $= 0$, hence, $z = r \times \text{h. l. } \frac{\text{cotan. } \frac{1}{2} \text{ comp. lat.}}{r} = r \times \text{h. l. cotan. } \frac{1}{2} \text{ comp. lat.} - r \times \text{h. l. } r$ the length of the meridian Em on the projection. If therefore we take the latitude = 1, 2, 3° ... 90, we can conſtruct a Table ſhowing the length of the meridian on the projection for every degree of latitude. In like manner it may be conſtructed for every minute. Such a Table is called a Table of *Meridional Parts*. Fig. 252.

1229. If we take the earth of its true figure, that of a ſpheroid, we may compute the meridional parts upon the ſame principles; but we ſhall not here give the inveſtigation, as we only want to explain the nature of this projection, ſo far as it may be neceſſary to ſhow how the charts are conſtructed. If we aſſume Sir I. NEWTON's ratio of the diameters of the earth, for 50° latitude the difference of the meridional parts on a ſphere and ſpheroid will not be above the 60th part of the whole. It is manifeſt that this projection cannot give the true proportion of the parts of the earth, for the figure of the part $ERnm$ on the projection would be a parallelogram. It is however very convenient for navigation, becauſe the rumbs are all projected into ſtraight lines, for the meridians being all ſtraight and parallel lines, the line which cuts them all at the ſame angle muſt be a ſtraight line, which is the property of the rumb (1251). Fig. 251.

On the Conſtruction of Maps

1230 A map is the repreſentation of the whole, or part of the ſurface of the earth upon a plane; and to be perfect it ought *firſt*, to ſhow the true latitude and longitude of every place; *ſecondly*, it ought to ſhow all countries of their proper figures and magnitude; *thirdly*, the relative ſituations of the countries ſhould be truly laid down. But all theſe circumſtances cannot take place in any conſtruction, as has been already obſerved; conſequently no map upon a plane can be a true repreſentation of the countries upon the earth's ſurface.

1231 A *Rumb* upon the globe is a line which cuts all the meridians at the ſame angle. Let EQ be the equator, P the pole, PE, PR two meridians,
FIG. 253 and $Asmv$ a rumb; then the angle $Psm=Pmv$. Draw the ſmall circle mn parallel to EQ, and conceive PE, PR to be indefinitely near each other, then we may conſider the triangle smn to be a plane one; hence, $sm : sn$ :: rad : coſ nsm; but the angle nsm is conſtant for the ſame rumb; therefore $sm : sn$ in a conſtant ratio; that is, as the rumb approaches the pole, the increment of the rumb : the correſponding increment of it's latitude in a conſtant ratio; therefore componendo the whole increaſe of the rumb : the whole correſponding increaſe of latitude in the ſame conſtant ratio. When therefore a ſhip runs upon a rumb, and varies it's latitude 5° for inſtance, if it continue on the ſame rumb and deſcribe the ſame ſpace, it will have altered it's latitude again 5°. In general, equal parts of the ſame rumb are contained between equidiſtant parallels of latitude.

The Orthographic *Projection of Maps*

FIG. 254 1232 Let FIG. 254 repreſent the orthographic projection of the earth upon the plane of the meridian, P, p repreſenting the poles, and EQ the equator. Then the meridians are all projected into ellipſes (1213), whoſe ſemi-minor axes Oa, Ob, Oc, Od are the coſines of the diſtances of the meridians from $PEpQ$; and as the coſine of the meridians near $PEpQ$ vary but very ſlowly, thoſe meridians will be crowded together, and the figures of the countries very much contracted in longitude. The parallels of latitude being

being perpendicular to the plane of projection, will (1215) be projected into right lines in their proper proportion This is called a *Meridional* projection

Fig 255 1233 If the projection be made upon the plane of the equator $EQUA$ as in Fig 255, the meridians will be projected into right lines (1215), and the parallels of latitude into circles concentric with $EQUA$ (1213) But theſe circles diminiſhing ſlowly near the equator, will be there crowded together, and the figures of the countries will there be very much contracted in latitude The contraction of the extreme parts of the map therefore, with the objections before ſtated, render this projection very unfit for the conſtruction of maps This is called an *Equatorial* projection

The Stereographic *Projection of Maps*

Fig 256 1234 Let $PEpQ$ be a meridian upon which the projection is to be made P, p, the poles, EQ the equator, and E the point from which the longitude is reckoned Now the meridians are projected into circles, the radii of which are the ſecants of their diſtances from PEp (1221) Suppoſe therefore it were required to deſcribe the meridian whoſe longitude is 60 Now conſidering the radius PO as unity, the ſecant of 60 is 2, with an extent of compaſs therefore equal to 2, ſet one foot in P, and extend the other to EQ (produced in this caſe), and with that point as a center deſcribe the circle Pdp and it will be the true projection of that meridian And thus all the required meridians may be drawn

1235 To deſcribe the circles of latitude their radii will be the cotangents of the latitude (1224) Let it therefore be required to deſcribe the parallel of 60 Take $Ea=60$, and with an extent of compaſs $=0,577$ the cotangent of 60, ſet one foot in a and extend the other to Pp (produced), and with that point as a center deſcribe the circle acb, and it will be the projection of that parallel In like manner may the other parallels be drawn, as many as may be required

1236 Having laid down the meridian and parallels of latitude for this half of the earth, draw another equal circle $BQCU$ touching the former at Q and upon that deſcribe the meridians and parallels of latitude in like manner and the projection of the earth will be divided into latitude and longitude, then draw the countries, laying down the places according to their reſpective latitudes and longitudes and the map will be completed This is called a *Meridional*

projection,

projection and is the method commonly made uſe of in the conſtruction of maps

Fig 257 1237 If the eye be at the pole *P* the projection is made upon the equator *EQUA*, and here all the meridians are projected into ſtraight lines (122[illegible]) the planes of the meridians being all perpendicular to the plane of projection, and the parallels of latitude are projected into circles (1219) the radii of which are the tangents of half the diſtances of the parallels from the pole This is called an *Equatorial* projection The rumb line *ab* in this projection is manifeſtly the logarithmic ſpiral as it is alſo in the equatorial orthographic projection

1238 To make a projection upon the horizon of any place, the eye is ſup-poſed to be oppoſite to that place In this caſe, the pole is projected within Fig 258 the circle of projection, and it's diſtance *OP* from the center *O* is equal to the tangent of half the compliment of latitude (1217) The meridians are drawn by Art 1221 This projection is called *Horizontal*, and is repreſented in Fig 258

1239 There is another meridional projection called *Globular*, which differs from the above only in its placing the meridians in the projection at equal diſtances from each other which comes nearer to the nature of the globe, and maps are frequently thus conſtructed

On MERCATOR's Projection of Maps

1240 This projection is of great uſe in navigation, on account of it being conſtructed by right lines only and no others are neceſſary in it's uſe The moſt convenient way for a ſhip in going from one place to another is always to ſail upon one point of the compaſs, or upon the ſame *rumb* (1231) and by means of this projection you can determine immediately what rumb you are to ſail upon For let Fig 259 repreſent a partial map of Fig 259 this conſtruction, *AB* repreſenting 4 of longitude, and *AC* perpendicular to it 4 of latitude, the former degrees are equal, and the latter are increaſing (1227), and are to be laid down by a Table of meridional parts (1228) Suppoſe a ſhip wants to go from *a* in longitude 7 and latitude 31, to *b* in longitude 10 and latitude [illegible], and it is required to find the rumb it muſt ſail upon Join *ab* and that is the rumb (1229) Now to determine what rumb this is, there is always in theſe maps, one or more point from which are drawn thirty two ſtraight lines repreſenting the thirty two points of the compaſs and you may eaſily diſcover to which of theſe lines, or neareſt to which, *ab* is parallel and thus you get the point of the compaſs you are to ſail upon For this purpoſe, a parallel ruler may be very uſeful laying one edge to coincide with *ab*, and bringing the other edge over the point from which the lines of the compaſs are drawn

CHAP

CHAP. XI.

ON THE USE OF INTERPOLATIONS IN ASTRONOMY.

Art. 1241. IF any quantity vary, and its value at certain intervals of time be known, it is the buſineſs of interpolation to find its value accurately, or nearly ſo, at any other point of time. In Aſtronomy the quantities between which we want to interpolate are of ſuch a nature, that if they be taken at equal intervals of time, and you take their differences, and the differences of thoſe differences, and ſo on, the laſt differences will become accurately, or nearly equal to nothing. Hence, if x repreſent the interval from the firſt time at which the value of the quantity was taken, and y be the value of the quantity correſponding, then may y be repreſented by $A+B\times x+C\times x\times\overline{x-1}+D\times x\times\overline{x-1}\times\overline{x-2}+$&c. for if we take r terms, and for x write any equidiſtant ſet of numbers, as 0, 1, 2, 3, 4, &c. the r^{th} differences of the reſults, or of the values of y, will become equal to nothing.

For take two terms $A+Bx$, and for x write 0, 1, 2, 3, 4, &c. and we have theſe reſults,

$$\begin{array}{l} A \\ A+B \\ A+2B \\ A+3B \end{array}$$

The firſt differences of which are B, and the ſecond differences $=0$.

Take three terms, $A+Bx+C\times x\times\overline{x-1}$, and for x write 0, 1, 2, 3, 4, &c. and we have theſe reſults,

$$\begin{array}{l} A \\ A+B \\ A+2B+2C \\ A+3B+6C \\ A+4B+12C \end{array}$$

The

The firſt differences of which are,

$$\begin{array}{l} B \\ B+2C \\ B+4C \\ B+6C \end{array}$$

The ſecond differences are $2C$, and the third differences $=0$

Thus it appears, that the laſt differences will always become $=0$

1242 In general therefore, let the ſucceſſive values of y be a, b, c, d, &c and let it be required to find the coefficients A, B, C, &c Firſt, take the ſucceſſive differences thus,

$$\begin{array}{ccccccccc} a, & & b & & c & & d & & \&c \\ & b-a, & & c-b, & & d-c, & & \&c & \\ & & c-2b+a, & & d-2c+b, & & \&c & & \\ & & & d-3c+3b-a, & & \&c & & & \\ & & & \&c & \&c & & & & \end{array}$$

And put $P=b-a$, $Q=c-2b+a$, $R=d-3c+3b-a$, &c Now for x write 0 1 2, 3, 4, &c and the ſucceſſive correſponding values of y being a, b, c, d &c we have the following equations, $a=A$, $b=A+B$, $C=A+2B+2C$ $d=A+3B+6C+6D$, &c Hence $A=a$, $B=b-a=P$, $C=\frac{c-A-2B}{2}=\frac{c-2b+a}{2}=\frac{Q}{2}$, $D=\frac{d-A-3B-6C}{6}=\frac{d-3c+3b-a}{2\cdot 3}=\frac{R}{2\cdot 3}$ &c therefore $y=a+Px+\frac{Q\times x\times \overline{x-1}}{2}+\frac{R\times x\times \overline{x-1}\times \overline{x-2}}{2\cdot 3}+$&c where the law of continuation is manifeſt Hence, if a b, c, d &c be the values of a variable quantity taken at *any* ſucceſſive equal intervals of time, beginning at *any* inſtant and if ſuch be their law that their laſt differences always become $=0$, we ſhall get at any intermediate time the accurate value of that quantity, becauſe then all its intermediate values follow the ſame law as the values of y from the equation, but if the differences do not at laſt become accurately $=0$ we ſhall then get only an approximate value becauſe then the intermediate values do not follow accurately the ſame law whereas the values of y found from our equation muſt always follow the ſame law, and therefore the value of y will be only an approximation to the value of the quantity at any intermediate

time

time between thoſe at which y was aſſumed accurately equal to it, the approximation however will be ſufficiently accurate for all practical purpoſes provided the differences become at laſt very ſmall which is the caſe in the application of this rule to interpolations in Aſtronomy The uſe of interpolations is therefore to determine the place of a body or the value of a quantity, at any time from knowing the place or value at three or four times near to the given time

1243 But beſides the uſe of the above equation to find the value of any term of a ſeries from its poſition being given, the converſe is often required that is, having given any term to find its poſition or diſtance from the firſt term In this caſe, we have the value of y given to find x, which will be determined from the ſolution of the equation which will riſe in its dimenſions as it may be neceſſary to increaſe the number of terms, and this depends upon how many orders of differences you muſt take before they become equal to nothing

Example I On March, 178, the ſun's declination at noon at Greenwich, by the Nautical Ephemeris was as follows on the 19th N 28 41′ $=1721'=a$ on the 20th N 5 $=300'=b$, on the 21ſt S -18 41′ $=-1121'=c$, to find the time of the equinox

The value of c is here written negative, becauſe the declination has paſſed through 0 Hence, we proceed thus,

$$1721, \quad 300, \quad -1121,$$
$$-1421, \quad -1421$$

Here $a=1721$, $P=-1421$, hence, $y=1721-1421\times x$, now when the ſun comes to the equator, y the declination becomes $=0$, therefore $1721-1421 \times x=0$, and $x=\frac{1721}{1421}=1d\ 5h\ \ '\ 53$ the time from the 19th, hence, 20d 5h 3 53 is the time required

If at any place we obſerve the ſun's declination for three or four days at the equinox by the aſtronomical quadrant, we may thus determine the time at that place when the ſun comes to the equator, without the Ephemeris

EXAMPLE II To find the time, from the Nautical Almanac, when the ſun entered the tropic in June, 1783

The ſun enters the tropic when it's longitude is three ſigns Here y repreſents the longitude, let us therefore take three longitudes the neareſt to the time, which in this caſe will be a ſufficient number Now on the 20th day the longitude is 2 28 55′ 7″, on the 21ſt it is 2 29 52′ 21″ and on the 22d it is 3 0 49′ 34″, but it will render the operation a little more ſimple, that is, the numbers will be ſmaller, if we take from each two ſigns, in which caſe it is manifeſt that y begins at the beginning of the third ſign, and conſequently when y becomes equal to one ſign the ſun enters the tropic Therefore 28 55′ 7″ $= 104107'' = a$, 29 52 21″ $= 107541'' = b$, 1 0 49′ 34″ $= 110974'' = c$, hence,

104107, 107541, 110974,
3434, 3433,
1

Here $P = 3434$, and $Q = 1$, which being ſo very ſmall compared with P, we may omit it, conſequently $y = 104107 + 3434x$, but at the tropic, $y = 1$ ſign $= 108000''$, hence, $108000 = 104107 + 3434x$, therefore $x = \frac{3893}{3434} =$ 1d 3h 12′ 28″ the time from the 20th day, and therefore the ſun enters the tropic the 21d 3h 12′ 28″

EXAMPLE III Given five places of a *Comet* as follows, on November 5, at 8h 17′ in Cancer 2 30′ $= 150' = a$ on the 6th at 8h 17 in 4° 7′ $= 247' = b$, on the 7th at 8h 17 in 6 20 $= 380' = c$, on the 8th at 8h 17′ in 9 10′ $= 550 = d$, on the 9th at 8h 17′ in 12 40′ $= 760 = e$, to find it's place on the 7th at 14h 17′

Firſt, ſubtract 5d 8h 17′ from 7d 14h 17′ and there remains 2d 6h = 2,25 for the interval of time between the firſt obſervation and the given time at which the place is required, this therefore is the value of x to which we want to find the correſponding value of y, hence,

150, 247, 380, 550, 760,
97, 133, 170, 210,
36, 37, 40,
1, 3,
2

Here $P=97$, $Q=36$, $R=1$ $S=2$ hence $y=150+97\times2{,}25+\frac{6}{2}\times2{,}25\times1{,}25+\frac{1}{2.3}\times2{,}25\times1{,}25\times{,}25+\frac{2}{2.3.4}\times2{,}25\times1{,}25\times{,}25\times-{,}75=416{,}96$ $=6°\ 58'\ 57''$ the place required

EXAMPLE IV In October 1788 the *Moon's* declination at noon at Greenwich appears from the Nautical Almanac, to have been as follows; on the 9th S 12° 42′ =762′=a on the 10th S 8° 44′=524′=b on the 11th S 4° 24′=264′=c, on the 12th N −0° 10′=$-10=d$ on the 13th N −4° 49′ $=-289=e$, to find the declination on the 10th at 6h 30′

First, 1d 6h 30′ = $\frac{61}{35}$ is the interval of time from the 9th to the given time or the value of x and the values of y are the declinations those being written negative which are north, the declination having passed through o, hence,

$$\begin{array}{ccccccccc} 762, & & 524 & & 264, & & -10, & & -289, \\ & -238, & & -260, & & -274, & & -279, & \\ & & 22, & & 14, & & 5, & & \\ & & & -8 & & -9 & & & \\ & & & & -1 & & & & \end{array}$$

Here $P=-238$ $Q=22$, $R=-8$, $S=-1$, consequently $y=762-238\times\frac{61}{48}+\frac{22}{2}\times\frac{61}{48}\times\frac{13}{48}-\frac{8}{2.3}\times\frac{61}{48}\times\frac{13}{48}\times-\frac{35}{48}-\frac{1}{2.3.4}\times\frac{61}{48}\times\frac{13}{48}\times-\frac{35}{48}\times-\frac{83}{48}$ but the terms after the three first are so small that they may be neglected, and therefore the second differences are sufficient for our purpose, hence, $y=7°\ 43\frac{1}{4}'$ the declination required

EXAMPLE V The *Moon's* longitude at noon at Greenwich in September, 1788, was as follows, on the 16th 0^s 5° 9′ 12″=18552″=a, on the 17th 0^s 17° 47′ 26″=64046″=b on the 18th 1^s 0° 36′ 12″=110172″=c, on the 19th 1^s 13° 36′ 1″=156961″=d, to find ☽'s longitude on the 17th at 6h

Here

Here $x = 1d\ 6h = 1{,}25$ the interval between the firſt given time and the time at which the longitude is required, hence,

$$\begin{array}{ccccccc} 18552, & & 64046, & & 110172, & & 156961, \\ & 45494 & & 46126 & & 46789 & \\ & & 632, & & 663 & & \\ & & & 31 & & & \end{array}$$

Here $P = 45494$ $Q = 632$, $R = 31$ but it will be ſufficient here to go only to the ſecond differences, as the value of the third would produce an effect only of one ſecond, hence $y = 18552 + 45494 \times 1{,}25 + \frac{632}{2} \times 1{,}25 \times {,}25 = 75518$ $= 20\ 58'\ 38$ the longitude at the given time

EXAMPLE VI In June 1740 the place of *Mercury* in its orbit at noon was found from obſervation to have been on the 14th in Aries $2\ 4\ 30 = 7470 = a$ on the 15th in $7\ 7\ 7' = 25627 = b$, on the 16th in $12\ 19'\ 30'' = 44370 = c$ on the 17th in $17\ 41\ 44' = 63704' = d$, to find its place on the 15th at $6h$

Here $x = 1d\ 6h = 1{,}25$ the interval between the firſt given time and that at which the place is required, hence,

$$\begin{array}{ccccccc} 7470 & & 25627, & & 44370, & & 63704, \\ & 18157 & & 18743, & & 19334 & \\ & & 586, & & 591, & & \\ & & & 5 & & & \end{array}$$

Therefore $y = 7470 + 18157 \times 1{,}25 + \frac{586}{2} \times 1{,}25 \times {,}25 = 8\ 26\ 54'$ the place required omitting the conſideration of the laſt difference as it would not affect the concluſion $\frac{1}{5}$ of a ſecond

1244 Becauſe y repreſents the velocity with which y increaſes or decreaſes, therefore to determine that velocity, take the fluxion of both ſides of the equation and you will get the relation between y and x Hence if we ſubſtitute for x any interval of time we ſhall get the quantity by which y would be increaſed in that time with the velocity continued uniform

EXAMPLE Suppoſe it were required to find, from the laſt Example, the velocity of *Mercury* on the 15th at 6*h*

Here $y=7470+18157\times x+293\times x\times\overline{x-1}=7470+17864\times x+293\times xx$, hence, $y=17864\times x+586\times xx$, now let us ſuppoſe $x=1$, which anſwers to 24 hours, and then $y=17864+586=18450=5\ \ 7'\ 30'$ the angle that would have been deſcribed by Mercury in 24 hours with the velocity at the given time Thus we muſt proceed in all other caſes to find the velocity with which y increaſes or decreaſes

1245 When y becomes a maximum or a minimum it s variation is then infinitely leſs than that of x, for it s fluxion is then equal to nothing Now as our equation only gives an approximate relation between the quantities required any ſmall variation of the law of the quantity to be interpolated from the law given by the equation will in this caſe, produce a conſiderable variation in the value of x, or of the time When therefore the quantity to be interpolated becomes a maximum or a minimum, or near to it, we cannot depend upon our equation for giving the time with ſufficient accuracy For example, if we take three declinations of the ſun near the ſolſtice, and find the value of y and make it fluxion equal to nothing, we cannot be certain that we ſhall get the time of the ſolſtice ſufficiently accurate Hence the rule given by Dr HALLEY for determining the time of the ſolſtice, from deſcribing a parabola through three points, cannot be depended upon and other authors ſince his time have fallen into the ſame miſtake If between any two values of x, y has changed it s ſign, it has paſſed through a maximum or a minimum, according as it s ſign was firſt poſitive or negative Or the ſame will happen when the firſt differences firſt increaſe and then decreaſe, or decreaſe and then increaſe

1246 It has been hitherto ſuppoſed, that the quantities to be interpolated were taken at equal intervals of time, but as it frequently happens that obſervations cannot be taken at equal intervals, we ſhall inveſtigate a rule for interpolations in this caſe Let $y=A+Bx+C\times x\times\overline{x-p}+D\times x\times\overline{x-p}\times\overline{x-q}+E\times x\times\overline{x-p}\times\overline{x-q}\times\overline{x-r}+$ &c Now when $x=o, p, q, r, s,$ &c let the correſponding values of y be $a, b, c, d, e,$ &c hence, $a=A$, $b=A+Bp$, $c=A+Bq+C\times q\times\overline{q-p}$, $d=A+Br+C\times r\times\overline{r-p}+D\times r\times\overline{r-p}\times\overline{r-q}$, $e=A+B\times s+C\times s\times\overline{s-p}+D\times s\times\overline{s-p}\times\overline{s-q}+E\times s\times\overline{s-p}\times\overline{s-q}\times\overline{s-r}$, &c hence, $A=a$, $B=\frac{b-a}{p}$, $C=\frac{c-a-Bq}{q\times\overline{q-p}}$, $D=\frac{d-a-Br-C\times r\times\overline{r-p}}{r\times\overline{r-p}\times\overline{r-q}}$,

$E=$

$E=\frac{e-a-Bs-C\times s\times\overline{s-p}-D\times s\times\overline{s-p}\times\overline{s-q}}{s\times\overline{s-p}\times\overline{s-q}\times\overline{s-r}}$, &c hence, $y=a+\frac{b-a}{p}\times x+\frac{c-a-Bq}{q\times\overline{q-p}}\times x\times\overline{x-p}+\frac{d-a-Br-C\times r\times\overline{r-p}}{r\times\overline{r-p}\times\overline{r-q}}\times x\times\overline{x-p}\times\overline{x-q}+$ $\frac{e-a-Bs-C\times s\times\overline{s-p}-D\times s\times\overline{s-p}\times\overline{s-q}}{s\times\overline{s-p}\times\overline{s-q}\times\overline{s-r}}\times x\times\overline{x-p}\times\overline{x-q}\times\overline{x-r}+$&c

EXAMPLE Let the place of a *Planet* at midnight be as follows, on November 8, in Leo 10 14′ 6″=36846″$=a$, on the 9th in 10 28′ 15″ =37695″$=b$, on the 11th in 11 4′ 10′=39850″$=c$, on the 14th in 12 2=43320′$=d$, to find its place on the 11th of November, at the 6th hour

Here $p=1$, $q=3$, $r=6$ $x=3\frac{1}{4}$, $B=\frac{b-a}{p}=849$, $C=\frac{c-a-Bq}{q\times\overline{q-p}}=\frac{3004-2547}{6}=76\frac{1}{6}$, $D=\frac{d-a-Br-C\times r\times\overline{r-p}}{r\times\overline{r-p}\times\overline{r-q}}=\frac{6474-5094-2285}{90}$ $=-10\frac{1}{18}$, hence, $y=36846+849\times3\frac{1}{4}+76\frac{1}{6}\times3\frac{1}{4}\times2\frac{1}{4}-10\frac{1}{18}\times3\frac{1}{4}\times2\frac{1}{4}\times\frac{1}{4}$ $=40144'=11$ 9′ 4″ the place required

Theſe examples and obſervations may be ſufficient to ſhow the uſe and extent of the doctrine of interpolations in Aſtronomy As the rules, under the reſtrictions here pointed out, ſerve for the interpolations of all quantities under like circumſtances in reſpect to their differences, the reader can never be at a loſs to know when they are applicable

1247 The ſeries $y=A+Bx+C\times x\times\overline{x-1}+D\times x\times\overline{x-1}\times\overline{x-2}+$&c is of the ſame kind as this, $y=P+Qx+Rx^2+Sx^3+$&c for by actually multiplying the factors, and collecting the coefficients of the like powers of x, and putting $P=A$ the abſolute term, Q, R, S &c = the ſum of all the coefficients of x, x^2, x^3 &c the former ſeries becomes $y=P+Qx+Rx^2+Sx^3$ +&c This we may conſider as an equation to a parabolic curve whoſe abſciſſa is $PW=x$, and ordinate $WH=y$ The interpolation therefore of the terms of this ſeries is the ſame as the interpolation of an ordinate of this curve, having given any number of ordinates If two ordinates be given, we aſſume $y=P+Qx$ the equation being a ſtraight line paſſing through the extremities, if three ordinates be given, we aſſume $y=P+Qx+Rx^2$ and if there be n ordinates given, we aſſume $y=P+Qx+Rx^2+$&c to n terms, becauſe we then have n given values of y and n corresponding given values of x, by ſubſtituting Fig 241

thereſore

therefore in the equation ſucceſſively the correſponding values of y and x, we get n equations and n unknown quantities P, Q, R, &c. from whence theſe quantities may be found. Thus we may deſcribe a parabolic curve paſſing through n given points, that is, we can find the equation of the curve which ſhall paſs through thoſe points.

1248. To find the area of this curve, we have $y\dot{x}=P\dot{x}+Qx\dot{x}+Rx^2\dot{x}+Sx^3\dot{x}+$&c. whoſe fluent is $Px+\frac{1}{2}Qx^2+\frac{1}{3}Rx^3+\frac{1}{4}Sx^4+$&c. = the area.

Let the intervals of the ordinates AP, BQ, CR, DS, &c. be unity, that is, let $x=0, 1, 2, 3, 4$, &c. and the correſponding ordinates be a, b, c, d, e, &c. Then from the equation $y=P+Qx+Rx^2+Sx^3+$&c. if we take $x=1$, there are two ordinates AP, BQ, and we take two terms; if we take $x=2$, there are three ordinates AP, BQ, CR, and we take three terms, &c. Hence we have the following caſes.

CASE I. For *two* ordinates.

Here $x=1$, $a=P$, $b=P+Q$, therefore $Q=b-a$, and the area $APQB$ $=\frac{1}{2}a+\frac{1}{2}b$.

CASE II. For *three* ordinates.

Here $x=2$, and
$$
\begin{aligned}
a&=P\\
b&=P+Q+R\\
c&=P+2Q+4R\\
\hline
\text{therefore}\quad b-a&=Q+R\\
c-b&=Q+3R\\
\hline
\text{hence,}\quad c-2b+a&=2R\\
\text{and}\quad \tfrac{1}{2}c-b+\tfrac{1}{2}a&=R\\
\hline
\text{conſequently}\quad Q=b-a-R&=-\tfrac{3}{2}a+2b-\tfrac{1}{2}c
\end{aligned}
$$

Hence, the area $APRC=2a+\frac{1}{2}\times\overline{-\frac{3}{2}a+2b-\frac{1}{2}c}\times 4+\frac{1}{3}\times\overline{\frac{1}{2}c-b+\frac{1}{2}a}\times 8$ $=\frac{1}{3}a+\frac{4}{3}b+\frac{1}{3}c$. This is the caſe in Art. 1142.

CASE III For *four* ordinates

Here $x=3$, and
$$\begin{aligned} a&=P\\ b&=P+Q+R+S\\ c&=P+2Q+4R+8S\\ d&=P+3Q+9R+27S \end{aligned}$$

therefore
$$\begin{aligned} b-a&=Q+R+S\\ c-b&=Q+3R+5S\\ d-c&=Q+5R+19S \end{aligned}$$

hence,
$$\begin{aligned} c-2b+a&=2R+6S\\ d-2c+b&=2R+12S \end{aligned}$$

therefore $d-3c+3b-a=6S$

and $S=-\frac{1}{6}a+\frac{1}{2}b-\frac{1}{2}c+\frac{1}{6}d$

hence, $R=\frac{1}{2}c-b+\frac{1}{2}a-3S=a-\frac{5}{2}b+2c-\frac{1}{2}d$

$Q=b-a-R-S=-\frac{11}{6}a+3b-\frac{3}{2}c+\frac{1}{3}d$

Therefore the area $APSD=3a+\frac{1}{2}\times\overline{-\frac{11}{6}a+3b-\frac{3}{2}c+\frac{1}{3}d}\times 9+\frac{1}{3}\times\overline{a-\frac{5}{2}b+2c-\frac{1}{2}d}\times 27+\frac{1}{4}\times\overline{-\frac{1}{6}a+\frac{1}{2}b-\frac{1}{2}c+\frac{1}{6}d}\times 81=\frac{3}{8}a+\frac{9}{8}b+\frac{9}{8}c+\frac{3}{8}d$

Thus we may proceed for any number of ordinates

CHAP

CHAP. XLI.

THE HISTORY OF ASTRONOMY.

Art. 1249. THE apparent motions of the heavenly bodies are the moſt obvious phænomena in nature; and as a knowledge of the return of the ſeaſons muſt always have been neceſſary for the huſbandman, the courſe of the ſun probably engaged the attention of mankind in the early ages of the world. The appearance and diſappearance of the moon alſo at ſtated times, and the utility attending a knowledge of theſe circumſtances, would neceſſarily excite an enquiry into the motions of that body. Attention to theſe things would lead to the inveſtigation of the motions of the other bodies in the heavens. Accordingly, we find it recorded, that the ſcience of Aſtronomy was cultivated by the immediate deſcendents of ADAM; for JOSEPHUS informs us, that the ſons of SETH employed themſelves in the ſtudy of Aſtronomy; and that they wrote their obſervations upon two pillars, one of brick and the other of ſtone, to preſerve them againſt the deſtruction which ADAM had foretold ſhould come upon the earth. He alſo relates, that ABRAHAM argued "the unity and power of GOD from the orderly courſe of things both at ſea and land, in their times and ſeaſons, and from his obſervations upon the motions and influences of the ſun, moon and ſtars; and that he read lectures in Aſtronomy and arithmetic to the Egyptians, which they underſtood nothing of, till ABRAHAM brought them from Chaldea to Egypt, and from thence they paſſed to the Greeks." BEROSUS alſo obſerves, "that ABRAHAM was a great and juſt man, and famous for his celeſtial obſervations;" the making of which, theſe ſages thought ſo neceſſary to the human welfare, that they aſſign it as the principal cauſe of the ALMIGHTY's prolonging the life of man. For the ſame author giving an account of the longevity of the antediluvians, ſays, "that Providence found it neceſſary for the ſtudy and advancement of virtue, and for the improvement of Geometry and Aſtronomy, which required at leaſt ſix hundred years for making and perfecting obſervations." The few accounts however which we have of theſe matters before the flood, and the little dependence to be put upon them, will juſtify our ſaying any thing further upon the progreſs which might have been made in Aſtronomy at that time.

On

On the Aſtronomy of the Egyptians *and* Chaldeans.

1250. Before the reign of **Psammeticus**, or 680 A. C. the Egyptians, according to **Herodotus**, thought themſelves to have been the firſt inhabitants of the earth, and it was only from the time of this prince that the Greeks had any regular accounts of the affairs of Egypt. It is therefore no wonder that the Egyptians endeavoured to impoſe upon them fictitious accounts of their more early ages, far before the creation according to the account of **Moses**. The ambiguity of the word *Year* very well promoted their views; which, though at that time it uſually ſignified the apparent annual revolution of the ſun, yet originally it meant a *revolution* in general, and was uſed for that of the moon: for **Plutarch** ſays, the Egyptian year was a month, he adds however, "afterwards it conſiſted of four months." And **Censorinus** ſays, "et in Ægypto, quidem, antiquiſſimum ferunt annum bimenſtrem fuiſſe, poſt deinde ab **Isone** Rege quadrimenſtrem factum, noviſſime **Arminon** ad tredecim menſes et dies quinque perduxiſſe." At what diſtinct times theſe ſeveral changes in the length of the year were made, would be an enquiry not to our preſent purpoſe; the Egyptians however, to impoſe upon the Greeks, would undoubtedly take the ſhorteſt time for the year, that of a month. And this muſt alſo render the heathen chronology ſubject to great uncertainty.

1251. The Egyptians appear to have ſtudied Aſtronomy very early; but the meaſure of their year being for a conſiderable time ſubject to very different lengths, cauſed great confuſion in their chronology. The Thebans, who paſſed into Egypt, are ſuppoſed to have been the firſt who cultivated Aſtronomy there. They eſtabliſhed a year of 360 days; but it was ſoon found neceſſary to add five days more. They ſeem to have founded their obſervations upon the heliacal riſing of *Sirius*; for obſerving the interval between two days in which Sirius thus roſe, they determined the length of the year. But it was afterwards diſcovered that by making the year to conſiſt of 365 days, Sirius roſe later every year by ſix hours. They then made the year to conſiſt of 365¼ days. The year however, ſo far as regarded their religious ceremonies, ſtill conſiſted of 365 days. Making therefore theſe two years to begin together, they would not coincide again 'till after four times 365 years, or 1460 years. This was called the *Sothiacal* period. Mr. E. **Barnard** ſays, that the Egyptians diſcovered that the ſtars had an annual motion of $50''\ 9''\ 45''''$ in a year (*Phil. Tranſ.* N. 158.) According to **Macrobius**, the Egyptians made the planets revolve about the ſun in the ſame order as we do; but it does not appear at what time the planets were diſcovered. He alſo ſays, that they divided the zodiac in the ſame manner as the Chaldeans did, and fixed the commencement at the firſt degree of Aries DIODORUS SICULUS ſays that the Egyptians diſcovered that the planets had ſometimes a direct and ſometimes a retrograde motion, and that they were ſometimes ſtationary He alſo aſſerts, that they made the ſun move in a circle inclined to the equator, and in a direction contrary to the diurnal motion The idea of dedicating the ſeven days of the week to the planets is alſo aſcribed to them DIOGENES LAERTIUS, from MANETHO ſays, that the Egyptians believed the earth to be ſpherical, and that the moon was eclipſed by falling into the earth's ſhadow They attempted to meaſure the diameter of the ſun by obſerving the motion of the ſhadow of the gnomon in the time the body of the ſun was aſcending above the horizon The diſcovery of the planets and their motions we may conſider as a proof of the early arrangement of the ſtars into conſtellations, as it muſt be by comparing the places of the planets with the fixed ſtars, that their motions could be diſcovered

1252 When ALEXANDER took Babylon, CALLISTHENES found that the moſt ancient obſervations made by the Chaldeans were not above 1903 years before that time, which carries them to about the time of the diſperſion of mankind by the confuſion of tongues Theſe obſervations are ſuppoſed to have been made in the temple of JUPITER BELUS at Babylon M GOUET however thinks this account is not to be depended upon, as it was firſt publiſhed by SIMPLICIUS in the ſixth century, who took it from PORPHYRY, and HIPPARCHUS and PTOLEMY, who lived long before, knew nothing of them, though they made a very diligent ſearch after the writings of the ancient Aſtronomers They met with no obſervations made at Babylon before the time of NABONASSAR who began to reign in the year 747 A C EPIGENES ſpeaks of Babylonian obſervations for the ſpace of 720 years BEROSUS allows them to have been made 480 years before his time, which carries them back to 746 A C and this is in ſome meaſure confirmed by the oldeſt eclipſes which are recorded by PTOLEMY one of which is mentioned to have happened 721 years A C and two, 720 A C About this time, the Babylonians ſent to HEZEKIAH to enquire about the ſhadow's going back on the dial of AHAZ The period of 223 lunar months, comprehending 6585⅓ days according to the Chaldeans, make the periodic time of the moon to be 27*d* 7*h* 43′ 13′, and it ſynodic time to be 29*d* 12*h* 44′ 7″ They moreover determined it's motion not to be uniform, but to be ſomething more than 11 in a day and leſs than 12 when it moves ſloweſt, and ſomething more than 15 and leſs than 16 when it moves quickeſt, and the mean daily motion they fixed at 13 10 35′ It does not appear by what methods they determined theſe matters, but it is probable that it was from obſerving the path of of the moon by the fixed ſtars, for there are ſtars called by the Arabs, *Manazil Al Kamar*, or *Manſions of the Moon*, by which is meant, ſuch ſtars as the moon approaches at night in the courſe of it's revolution They were twenty eight in number the latitudes and longitudes of which are given by ULUGH BEIGH It is alſo aſſerted by PTOLEMY, that they were acquainted with the motion of the nodes and apogee of the moon and they ſuppoſed the former made a revolution in 18*y* 15*d* 8*h* which period, containing 223 complete lunations is uſually called the *Chaldean Saros* The 223 lunations they reckoned to contain 6585*d* 8*h* And if we take the period of the moon as now determined, they would make 6585*d* 7*h* 43′, which ſhows how very nearly the Chaldeans had determined the ſynodic revolution of the moon This period completes the revolution in reſpect to the node and apogee The knowledge of this might probably enable them to foretel eclipſes, at leaſt thoſe of the moon ARISTOTLE informs us, that the Chaldeans made obſervations on the occultations of the fixed ſtars and planets by the moon from which they were led to conclude, that an eclipſe of the ſun was cauſed by the moon And DIODORUS SICULUS ſpeaks of their having obſerved comets which they held to be laſting bodies, having revolutions like the planets, but in more extenſive orbits The ſame author alſo ſays, that ‘ the ſouthern parts of Arabia are made up of ſandy plains of a prodigious extent, the travellers through which direct their courſe by the *Bears*, in the ſame manner as is done at ſea ’ It appears therefore, that the inhabitants were acquainted with ſome of the conſtellations The Phœnicians were probably the firſt people who ſailed by the ſtar He further obſerves, that the Chaldeans made the annual motion of the ſun oblique to the ecliptic and contrary to the daily motion Dialling was alſo firſt known amongſt them and long before any thing upon that ſubject is related by the Greeks HERODOTUS ſays, that the Greeks borrowed the uſe of the *Pole* and *Gnomon*, and the method of dividing the day into twelve parts, from the Babylonians The gnomon ſeems to have been the moſt ancient aſtronomical inſtrument The Chaldeans made thirty ſix conſtellations twelve in the zodiac, and twenty four without They alſo made an obſervation on *Saturn* in the year 228 A C which is preſerved by PTOLEMY, and it appears to be the only one which they made on the planets The diſtance was meaſured by digits, of which 24 made a degree They obſerved that the ſun, moon and planets paſſed through the twelve ſigns, the ſun in a year, the moon in a month and that the planets had their particular periods They placed the moon below all the ſtars and planets, made it the leaſt of all, placed it neareſt to the earth, and made the time of it's revolution the leaſt, according to DIODORUS SICULUS Their civil year conſiſted of 365¼ days and ALBATEGNIUS relates, that the Chaldeans made the ſidereal year

365*d*

365*d* 6*h* 11′ They formed ſome idea of the magnitude of the earth, for they diſcovered, it is ſaid that a man, walking a good rate, might follow the ſun round the earth, that is, he might make the tour of the earth in a year Now if we allow a good rate to mean three miles in an hour then there being 8760 hours in a year of 365 days the circumference of the earth would be 26280 miles, which does not differ a great deal from the truth Reſpecting the Chaldean Aſtronomers, hiſtory give us but a very little information BEROSUS is ſuppoſed to be the oldeſt from a very abſurd opinion of his reſpecting the phaſes of the moon and its eclipſes According to him the moon is a globe having one ſide luminous, and the other ſide a ſky blue At what time he lived is uncertain

On the Aſtronomy of the Chineſe *and* Indians

1253 M BAILLY ſtates, that the firſt king of the Indies lived about 3553 years before our æra this is a little more than 400 years before their aſtronomical epoch which is ſuppoſed by M le GENTIL to be 3101 A C Their zodiac had two different diviſions, one of twenty eight, and another of twelve, and they divided their zodiac into twenty ſeven conſtellations they alſo had a moveable zodiac, to explain the preceſſion of the equinoxes, the motion of which they ſtated at 54″ in a year and the entire revolution in 24000 years This diſcovery they appear to have made about the year 2250 A C They regulated their chronology by periods of 60 years, but in their aſtronomical calculations, they employed the period of 3600 years which is ſix times the luni ſolar period The Brahmins were acquainted with the obliquity of the ecliptic and they conſtructed Tables ſhowing the increaſe of the days ariſing from the change of the ſun's declination, for different latitudes M le GENTIL found that according to theſe Tables, the obliquity of the ecliptic muſt have been more than 25 They have a Table of the time which the ſun employs to move through each ſign of the ecliptic The ſign in which it moves ſloweſt they make Gemini, and that in which it moves quickeſt is Sagittarius The apogee of the ſun was therefore leſs advanced by a ſign when theſe Tables were conſtructed than it is now, this carries their conſtruction back to the year 78 of our æra, at which time died SALIVAGENA, one of their kings, who was a great encourager of Aſtronomy They applied alſo to the ſun a correction which anſwers to our equation of the center, being ſubtractive in the firſt ſix ſigns of anomaly, and additive in the laſt ſix, the greateſt ſubtractive 1 [illegible]5′, and anſwers to 20 of Gemini, and the

the greateſt additive is 11, and anſwers to 20 of Sagittarius this ſeems to have been the joint effect of two different corrections The Brahmins alſo made uſe of the gnomon, and got a meridian by deſcribing concentric circles in the manner we do By this they alſo found the latitude of the place from the length of it's ſhadow at the day of the equinox In the reign of HOANG TI 2697 years A C the Chineſe had invented and conſtructed a ſphere, with the various circles belonging to it In a book written in the reign of YAO, about 2332 years A C we collect the following circumſtances 1ſt HI and HO (two Aſtronomers who were charged with compoſing a calendar for the people to regulate their huſbandry) obſerved the places of the ſun, moon and ſtars and inſtructed the people with reſpect to the ſeaſons 2dly The equality of days and nights and the ſtar *Niao*, determined the ſpring equinox 3dly The equality of days and night, and the ſtar *Hiu* marked the autumnal equinox 4thly On the longeſt day the ſtar *Ho* marked the ſummer ſolſtice 5thly On the ſhorteſt day, the ſtar *Mao* marked the winter ſolſtice 6thly One year conſiſted of 366 days and three years of 365 days And by ſpeaking of a lunar intercalary, they muſt alſo have had a luni ſolar year The Brahmins placed the earth in the centre of the world, with the ſeven planets revolving about it, but they do not appear to have been acquainted with it's diurnal motion The earth they placed upon a mountain of gold, they thought the ſtars moved and the planets they called fiſhes, becauſe they moved in the ether as fiſhes do in water Theſe abſurdities ſhow the great antiquity of their Aſtronomy M BAILLY ſpeaks of their Tables as being probably 5 or 6000 years old M le GENTIL thinks, that the Indians perceived that their Tables wanted a correction, becauſe at the time when they calculated the mean longitude of the ſun and moon, they ſubtracted from it a conſtant quantity which was probably the correction of their epoch, and which they diſcovered by comparing their calculation with their obſervations in conjunction and oppoſition, the points where they wanted them to agree, for the purpoſe of calculating their eclipſes It is conjectured that this correction was made about the year 78 at which time their Aſtronomy underwent a great reform In the year 2952 A C reigned FOHI, the firſt emperor of China and he appears to have been the firſt who compoſed aſtronomical Tables, and gave the figure of the heavenly bodies The ſolſtices ſeem to have been known at that time, as that emperor every year ſacrificed an animal at the time of each ſolſtice His ſucceſſor added a feaſt at each equinox Under the reign of HOANG TI 2697 A C YU CHI obſerved the pole ſtar, and the conſtellations about it He alſo conſtructed a ſphere, with ſeveral fixed and moveable circles And it is ſaid that he made many experiments upon the weather and the air About the ſame time, the circle of of 60 years was eſtabliſhed. HOANG TI was the author of many inſtruments to obſerve the ſtars, and alſo of an inſtrument to find the cardinal points, without any reference to the heavens: this muſt have been the compaſs. Traces of this are found 1400 years after. The ſame prince inſtituted a ſociety of mathematicians and of hiſtorians. The emperor CHUENI, in the year 2513 A. C. compoſed an Ephemeris of the motions of five planets: and it is ſaid, that he was raiſed to the empire for his knowledge in Aſtronomy. Under the reign of CHOU KANG, 2169 years A. C. happened an eclipſe, which is the moſt ancient we have any records of. From the reign of this prince to the year 776 A. C. hiſtory makes no mention of any eclipſe having been obſerved. From the year 480 A. C. to 206 A. C. Aſtronomy was almoſt entirely neglected, the empire was divided into ſmall ſtates, and the ſociety of mathematicians was deſtroyed. TSIN CHI HOANG united again the ſtates, and formed one great empire: and in the year 246 A. C. he collected the hiſtorical and mathematical works and burned them. But the ſtudy of Aſtronomy revived again under LIEOU PANG in the year 206 A. C. About 300 years before our era the Chineſe made their year 365¼ days. The period of the moon they knew within 20 or 30, but they had a very inaccurate knowledge of its revolution in reſpect to the nodes, and in reſpect to that of the apogee, they ſeem to have been altogether unacquainted. At the ſame time they had a method of calculating eclipſes. In the year 104 A. C. SSE MA TSIEN and LOHIA HONG formed precepts for calculating the motions of the planets and eclipſes. At that time they made the obliquity of the ecliptic 23 39. Towards the year 90 J. C. the emperor TCHANG TI corrected the calendar, having found that the ſolſtice had gone back 5. About the year 164 J. C. TCHANG HENG made a catalogue of 2500 ſtars, which is now loſt. In the year 206 J. C. LIEOU HONG and TSAY YONG diſcovered that the moon's motion was not uniform, but ſubject to an inequality, the maximum of which was 4 55′ 41″, and they made the obliquity of its orbit to be about 6. They knew the ſolar year to be a little leſs than 365¼ days, and they determined the revolution of the moon in reſpect to it's apogee to be 27*d* 13*h* 16 50″. In the year 284, KIANG KI determined the true place of the ſun by means of eclipſes of the moon. About the year 460, TSOU CHONG determined the motion of the ſtars to be 1 in 46 years. LIEOU HIAO TSUN and LIEOU TCHO, about the year 584, employed the firſt equation of the inequality of the ſun. Y HANG, who lived about the year 720, made the ſun's inequality 2 21′ 30″. He alſo determined the time of *Jupiter's* revolution, and made the latitude of *Sirius* 39° 45′ 30″. In the year 822, SU GANG explained the parallax of longitude, and ſhowed its uſe in calculating ſolar eclipſes. Towards the year 1280, the Chineſe erected a gnomon a gnomon of 40 feet, from the obſervations of which by Co CHEOU KING, they found the obliquity of the ecliptic to be 23 35′ 40′ and if this be corrected by refraction it becomes 23° 34′ 36″ It is ſaid that this Aſtronomer was the firſt amongſt them who underſtood any thing of ſpherical Trigonometry He alſo invented a method of calculating ſolar eclipſes

1254 For our firſt knowledge of the modern ſtate of Aſtronomy in India, we are indebted to M la LOUBERE, who returning in 1687 from an embaſſy to Siam, brought with him a Siameſe manuſcript, containing Tables and rules for calculating the places of the ſun and moon, theſe were put into the hands of CASSINI, to be explained After that, two other ſets of Aſtronomical Tables were ſent to Paris from Hindoſtan But for the beſt information of the ſtate of the Indian Aſtronomy, we are obliged to M le GENTIL, who went to India to obſerve the tranſit of *Venus* in 1769 and who, during his ſtay there acquired a knowledge thereof They appear to have no theory but content themſelves with calculations, particularly of the eclipſes of the ſun and moon A Brahmin of Tirvalore inſtructed M le GENTIL in the method of calculating eclipſes, and communicated to him the Tables and rules for that purpoſe, which he publiſhed in the *Mem de l Acad Roy des Scien* 1772 They divide the zodiac into 27 conſtellations, and the ecliptic into twelve ſigns, of 30 each, and make the annual preceſſion of the equinoxes to be 54′ Their year begins at the beginning of their moveable zodiac The epoch of the Tables of Siam is for the year 638 of our æra, as determined by CASSINI The length of their ſidereal year is 365*d* 6*h* 12′ 36″, and that of the tropical is 365*d* 5*h* 50′ 41 Theſe Tables have a correction of the ſun s mean place, anſwering to our equation of the center, the maximum of which is 2 12, and it is made to vary as the ſine of the mean diſtance from the apogee, which they make 80 from the beginning of the zodiac The motion of the moon is deduced, by certain intercalations, from a period of 19 years The moon s apogee is ſuppoſed to have been in the beginning of the moveable zodiac 621 days after the epoch of March 21, 638, and to make a revolution in 3232 days The firſt of theſe ſuppoſitions agrees with MAYER's Tables to leſs than 1, and the ſecond differs from them only 11*h* 14 51″ The Siameſe rules, which calculate only for conjunctions and oppoſitions, give but one inequality to the moon, the maximum of which is 4 56′ and is applied when the moon is 90° from the apogee in other ſituations, the equation is as the ſine of the moon's diſtance from the apogee Theſe Tables go no further

1255 Another ſet of aſtronomical Tables were ſent from Chriſnabouram, a town in the Carnatic, about the year 1750 They are fifteen in number, and contain, beſides the mean motions of the ſun, moon and planets, the equations of the center of the ſun and moon, and two corrections for each of the planets

planets. They are accompanied with rules and examples. The epoch of theſe Tables anſwers to March 10, 1491, when the ſun was entering the moveable zodiac. The equation of the ſun's center is 2 10 30, and of the moon 5 2 47, and the inclination of the orbit is 4 30; alſo the motions of the apogee and node are very nearly true. Another ſet of Tables were alſo ſent about the ſame time, probably from Narſipour, as that place anſwers to the length of the day there given; theſe do not materially differ from the laſt mentioned.

1256. The Tables and methods of the Brahmins of Tirvalore differ in many reſpects from thoſe above deſcribed; they ſuppoſe however the ſame length of the year, the ſame mean motions, the ſame inequalities of the ſun and moon, and are adapted nearly to the ſame meridian. The epoch however goes back to 3102 A. C. The ſolar year is divided into twelve unequal months, each being the time the ſun is moving through a ſign; and in their calculations for a day, they employ the time the ſun moves 1 in the ecliptic. The ſidereal year conſiſts of 365*d* 6*h* 12′ 30″, and the tropical of 365*d* 5*h* 50 35. The Brahmins make the obliquity of the ecliptic 24, which at the time of their epoch differed very little from the truth. They aſſign two inequalities to the motions of the planets, anſwering very well to the annual parallax and the equation of the center. Theſe Tables have their radical places for the year 1491, of our æra. The equation of *Saturn's* center is 7 39 44′, which agrees very well with what it ought to have been at their epoch 3102 A. C. M. de la PLACE ſays, "I find by my theory that at the Indian epoch of 3102 A. C. the apparent and annual mean motion of *Saturn* was 12° 13 14, and the Indian Tables make it 12° 13′ 13″. And that the annual and apparent mean motion of *Jupiter* at that epoch was 30 20′ 42″, preciſely as in the Indian Aſtronomy." From various agreements of this kind, there is the higheſt degree of probability that the Indian Aſtronomy is as ancient as it is ſtated to be by the Brahmins*. In the *Phil. Tranſ.* 1777, the reader will find an account of the Brahmins obſervatory at Benares, by Sir ROBERT BARKER. See alſo an account of the chronology of the Hindoos, by W. MARSDEN, Eſq. in the *Phil. Tranſ.* 1790, and Mr. CAVENDISH on the civil year of the Hindoos, in the ſame work for 1792.

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* For the proofs in ſupport of this, ſee Profeſſor PLAYFAIR's Remarks on the Aſtronomy of the Brahmins, *Edinburgh Tranſ.* Vol. 2.

On

On the Aſtronomy of the Greeks *to the Time of* PTOLEMY

1257 It is agreed that the Greeks borrowed their knowledge of Aſtronomy from the Egyptians and Chaldeans PLUTARCH relates that about the time of HESIOD, the ſciences began to unfold themſelves, but the progreſs which they made was very ſlow until the time of THALES, about 600 years before the Chriſtian æra That philoſopher rendered himſelf famous by foretelling an eclipſe of the ſun, he however only predicted the year in which it would happen, and this he was probably enabled to do by the *Chaldean Saros*, a period of 223 lunations, after which, the eclipſes return again nearly in the ſame order He alſo explained the nature of eclipſes, and was the firſt Greek who went into Egypt for improvement He is ſaid, by HYERONYMUS in DIOGENES LAERTIUS, to have diſcovered the year to conſiſt of 365 days, this he might have got from the Egyptians but LAERTIUS ſeems to give a reaſon to the contrary, for he ſays that "he firſt found out the tranſit from the tropic to the tropic, and was the firſt who called the laſt day of the month the thirtieth He is ſaid by PLINY to have determined the coſmical riſing of the *Pleiades* to have been 25 days after the autumnal equinox He ſtudied the courſe of the ſun, and made it's diameter to be the 720th part of the whole heaven, or half a degree, he was alſo acquainted with the zodiac, and it's obliquity to the equator The ſame author informs us that he computed a calendar, containing the times of the riſing and ſetting of the ſtars He is ſaid, by CALLIMACHUS, to have formed the conſtellation of the *Leſſer Bear* WEIDLER attributes to THALES two works, one on marine Aſtronomy, and the other upon the ſolſtices and tropics When THALES was in Egypt he meaſured the height of the pyramids by the length of their ſhadow, when the ſun was 45 high He died 549 A C

1258 The ſcholar and ſucceſſor of THALES was ANAXIMANDER According to LAERTIUS, he taught that the earth was a ſphere, and in the center of the world that the moon borrowed it's light from the ſun, and that the ſun was not leſs than the earth He is ſaid to have introduced the uſe of the gnomon PLINY mentions that he firſt diſcovered the obliquity of the ecliptic He formed a geographical chart, but it is uncertain whether he was the inventor The invention of the ſun dial is given to him, this however is doubtful He died in the year 547 A C

1259 ANAXAGORAS who lived about 530 years A C is ſaid to have predicted an eclipſe of the ſun, which, according to THUCYDIDES, happened in the firſt year of the *Peloponneſian* war. He taught that the moon was inhabited, and had plains, hills and water, as our earth has.

1260. At the ſame time lived PYTHAGORAS. LAERTIUS aſſerts, that the ſyſtem he taught was, that the earth was in the center, with a diurnal motion; in the next place the moon, then the ſun, and then the orbits of the planets. PLUTARCH however informs us, that in his old age he repented that he had not aſſigned to the earth its proper place. From this we may conclude, that he was acquainted with, and approved of, the true ſyſtem taught by PHILOLAUS, his cotemporary. He aſcribed the brightneſs of the *Milky Way* to the effect of a great number of very ſmall ſtars. And it is ſaid to have been this philoſopher, who firſt taught that the morning and evening ſtar (Venus) were one and the ſame planet. He is alſo mentioned to have taught the obliquity of the ecliptic, and the poſition of the tropical circles, by the ſphere. He thought the earth was a globe, and admitted that there might be inhabitants at the antipodes.

1261. PHILOLAUS, a diſciple of PYTHAGORAS, lived about 450 years A. C. He taught the true ſyſtem, placing the ſun in the center, and making the earth and all the planets revolve about it. He was perſecuted for propagating this opinion, and obliged to fly for it; and it is remarkable, that GALILEO loſt his liberty for maintaining the ſame. Soon after this, we find HICETAS, a Syracuſan, aſſerting the diurnal motion of the earth.

1262. METON formed a cycle of 19 years, containing 19 lunar years, and ſeven intercalary months. This cycle commenced July 16, in the year 432 A. C. 19 days after the ſummer ſolſtice; and the new moon which happened on the ſame day at *7h* 43′ in the evening, was the beginning of it. This is called the *Metonic* cycle.

1263. EUDOXUS, a ſcholar of PLATO, flouriſhed about 360 years A. C. When young he travelled into Egypt, and converſing with the prieſts, he learned many things from them relating to Aſtronomy. He made the ſolar year to conſiſt of 365¼ days, and the ſynodic revolution of the moon to be *29d* *12h* 43′ 38″. ARCHIMEDES ſays, that he eſtimated the diameter of the ſun to be nine times greater than that of the moon. SENECA ſays, that he carried into Greece the elements of the motions of the planets. He wrote a treatiſe on the conſtellations, which is loſt. VITRUVIUS ſays, that EUDOXUS made a ſun dial upon a plane. He alſo conſtructed a ſphere. The orbit of the moon he diſcovered to have been inclined to the ecliptic; and that her greateſt latitudes did not always anſwer to the ſame place, but that they went backwards; thus he diſcovered the motion of the moon's nodes. He compoſed two works, one called the *Mirror*, the other, the *Phænomena*. In the firſt,

firſt, he deſcribed the conſtellations, and in the ſecond, he explained the times of their riſings and ſettings HIPPARCHUS was in poſſeſſion of theſe two works, but they are now loſt He adviſed mankind not to put any faith in the predictions of aſtrologers

1264 ARISTOTLE relates ſeveral obſervations which he made He ſaw an eclipſe of *Mars* by the moon, and an occultation of a ſtar in *Gemini* by *Jupiter* He obſerved alſo a comet, the tail of which extended over one third of the heavens

1265 CALIPPUS lived about 330 years A C He corrected the cycle of METON, and formed that of 76 years He alſo made a collection of obſervations of the riſing of the ſtars, and joined to them meteorological remarks for the ſake of agriculture

1266 PYTHEAS, who lived at Marſeilles in the time of ALEXANDER the Great, by means of a gnomon, found the length of the ſhadow at the ſummer ſolſtice to it s height as 600 to 209, from which it follows, that the obliquity of the ecliptic at that time was 23 50 He ſays alſo, that the ſame proportion of the height of the gnomon to the length of the ſhadow was true at Byſance, but this could not be, the truth of the above concluſion therefore is thus rendered doubtful

1267 ARISTILLUS and TIMOCHARIS lived about 300 A C and were the firſt Aſtronomers of the Alexandrine ſchool Their obſervations were principally confined to the ſtars, in order to fix their poſitions in the heavens, and it appears that HIPPARCHUS made great uſe of them, diſcovering from them that the ſtars had a motion in longitude of about 1 in 100 years which, though not very exact, ſerved to diſcover that the equinoctial points had a retrograde motion

1268 About 276 A C lived ARISTARCHUS of Samos He obſerved the elongation of the moon from the ſun at the time of the moon s dichotomy and found it to be not leſs than 87 from whence he concluded that the diſtance of the ſun was not leſs than 19 times the diſtance of the moon He made the diameter of the ſun to that of the earth greater than 19 to 3 and leſs than 43 to 6, and the diameter of the moon to that of the earth, greater than 43 to 108 and leſs than 19 to 60, this laſt determination is not far from the truth He agreed with PHILOLAUS in reſpect to the motion of the earth about the ſun He ſays in his treatiſe entitled *De Magnitudinibus et Diſtantiis Solis et Lunæ*, that the diameter of the moon is about [illegible] this was probably only what he firſt eſtimated it at, for ARCHIMEDES informs us that he made it about 30' He thought there was no proportion between the diſtance of the ſun and that of the fixed ſtars VITRUVIUS relates, that he made a clock, which is ſuppoſed to mean a ſun dial

1269 The next celebrated Aſtronomer was Eratosthenes, who was born 276 A C He was invited to come from Athens to Alexandria by Ptolemy Euergetes, and made by him keeper of the royal library. At his requeſt, Ptolemy fixed up *Armillas* or circles of braſs, in the portico at Alexandria, for the purpoſe of making aſtronomical obſervations With theſe, Eratosthenes meaſured the obliquity of the ecliptic and determined the diſtance of the tropics to be to an entire circle as 11 to 83, this gives the obliquity 23 51 20, and it is a very important obſervation, as, from its accuracy, it proves very ſatisfactorily the obliquity is decreaſing He alſo meaſured a degree upon the earth's ſurface by taking the difference of the zenith diſtances of a ſtar at Alexandria and Syena, which he found to be 7 12, and the diſtance upon the earth was 5000 ſtades hence, a degree is $694\frac{4}{9}$ ſtades How near this is to the truth we cannot ſay, as we do not know in what ſtades the diſtance was meaſured, alſo Alexandria and Syena are not under the ſame meridian We owe however to him the merit of diſcovering the method M Weidler, from Plutarch, ſays, that Eratosthenes made the diſtance of the moon 780000 ſtades and the diſtance of the ſun 804000000 ſtades, but it is not mentioned by what method He attempted to number the ſtars and went as far as 675 He lived to the age of eighty, when he loſt his ſight, which ſo afflicted him, that, it is ſaid, he ſtarved himſelf

1270 Archimedes, beſides being the firſt of the ancient mathematicians, was alſo a good Aſtronomer Hipparchus informs us, that he obſerved the ſolſtice, and, according to Macrobius, he determined the diſtance of the moon and planets, but to what accuracy it does not appear Cicero relates that he made a ſphere, on which were repreſented the motions of the ſun, moon and five planets, each with their proper relative velocities He was the firſt who aſſigned the true area of a curvilinear ſpace When Syracuſe was taken by Marcellus in the year 211 A C he gave orders to ſave Archimedes but he was killed by a ſoldier whilſt he was drawing geometrical figures upon the ground, for not anſwering the queſtions that were put to him Marcellus however rendered him that honour which was due to his memory

1271 Apollonius of Perga lived about the ſame time He was the firſt who explained the cauſes of the ſtations and retrogade motions of the planets by epicycles (210) He is alſo celebrated for his Treatiſe on the Conic Sections, and the method of projections is alſo attributed to him

1272 Hipparchus the father of Aſtronomy lived between 160 and 125 years before our æra, and was born at Nice in Bithynia He was the firſt perſon who cultivated every part of Aſtronomy He began, by verifying the obliquity

obliquity of the ecliptic, obſerved by ERATOSTHENES and found it very correct and this was afterwards confirmed by PTOLEMY He fixed the latitude of Alexandria at 30 58′ The length of the tropical year he determined from the interval of the return of the ſun to the ſame tropic, or the ſame equinox and he conceived, that if he could get two correſponding obſervations after a great number of revolutions, the error would be proportionally diminiſhed, and this is the method by which future Aſtronomers determined the mean motions of all the planets, a diſcovery of the firſt importance in Aſtronomy He compared the obſervation of a ſolſtice by ARISTARCHUS with one made by himſelf at the interval of 145 years and found that the ſolſtice happened half a day ſooner than it ought to have done, if the year conſiſted of 365¼ days, as the Greeks believed before him Thus he determined the tropical year to be 365*d* 5*h* 55′ 12′ From his obſervations of the equinoxes and ſolſtices he found that they did not divide the year into four equal parts and he diſcovered that the interval from the vernal to the autumnal equinox was 186 days, about ſeven days longer than the interval from the autumnal to the vernal equinox Thus he found that the motion of the ſun was not uniform But inſtead of explaining this by an epicycle, which he did at firſt, he conceived the idea of placing the earth out of the center of the circle in which the ſun was ſuppoſed to move, which would account for this irregularity The invention therefore of the excentricity of the orbit is due to this Aſtronomer Upon this principle he computed two Tables of the ſun's motion one of its mean motion and the other of its inequalities, which is the principle of all our aſtronomical Tables at this time The diſcovery of the inequality of the ſun's motion led him to another of great importance, the inequality of days He knew the two cauſes which produced this inequality and ſaid their effects were not ſenſible in one day but became ſo by accumulation He was miſtaken however in the quantity making the maximum 33 20″ We are indebted to him therefore for the diſcovery of the principle of the equation of time TIMOCHARIS found that the ſtar called *Spica Virginis* preceded the autumnal equinox 8 , HIPPARCHUS diſcovered that at his time it did not precede more than 6 , and he found the ſame of other ſtars But not conceiving it probable that all the ſtars ſhould have a motion without changing their relative ſituations, he conjectured that the equinoctial point had a retrograde motion, which he made about 1 in 100 years He found that the ſtars changed their declination , but not their latitude , on this account he referred the ſtars, not to the equator but to the ecliptic Having diſcovered the preceſſion of the equinoxes he found that the ſidereal year was more than 365*d* 6*h* By obſervations on the moon, he diſcovered that it departed from the ecliptic 5 on each ſide, and thus he fixed the inclination of

its

it's orbit at that quantity He diſcovered alſo that it's nodes were moveable By obſerving the diſtance of the moon from the ſtars during the night, after allowing for it's motion, he found that it's ſituation in reſpect to the ſtars varied alſo as it's altitude varied He alſo knew that an eclipſe of the ſun was not the ſame on different parts of the earth, and thus he diſcovered the moon to have a parallax This ſuggeſted to him the method of finding the diſtance of the ſun (156) He compared the ancient obſervations with his own, and found, that in the interval of 126007*d* 1*h* the moon made 4573 revolutions in reſpect to it's apogee, and 4712 revolution, wanting 7 30′, in reſpect to the ſtars But that the latitude, and conſequently the eclipſes, did not return the ſame till about 5458 months, during which time the moon made 5923 revolutions in reſpect to it's nodes Hence, he found it's revolutions in reſpect to the

Sun	29ᵈ	12ʰ	44′	3¼″
Apogee	27	13	18	34
Node	27	5	5	35¼
Stars	27	7	43	13

He diſcovered an inequality of about 5 of the moon, ſimilar to that which he had diſcovered of the ſun, and repreſented it in the ſame manner M **Bailly** thinks that **Hipparchus** had determined the mean motions of the planets **Ptolemy** does not mention upon what obſervations the revolutions, ſtated by him, were made, and his ſilence on that point, renders it probable that the determinations are not his, but thoſe of **Hipparchus** He made an inſtrument to meaſure the diameters of the ſun and moon but **Ptolemy** does not ſay that he made uſe of it it is probable however that he did, as he makes the diameter of the moon 33¼′ at the mean diſtance A new ſtar having appeared in his time he determined to make a catalogue of the fixed ſtars, in order that poſterity might know whether any changes had taken place in the heavens This catalogue contains the latitude and longitude of 102 ſtars, with their apparent magnitudes **Ptolemy** publiſhed theſe in his Almageſt, with their longitudes for that time **Hipparchus** divided the heavens into 49 conſtellations that is, 12 in the ecliptic, 21 to the north, and 16 to the ſouth He carried into Geography the plan which he had followed in Aſtronomy, and laid down the places upon the earth's ſurface by their latitudes and longitudes The determination of the circumference of the earth alſo occupied his attention and he added 25000 ſtades to the meaſure given by **Eratosthenes** He appears to have collected all the eclipſes of the

the ſun and moon he could meet with obſerved by the Babylonians The works of this great Aſtronomer are found in PTOLEMY's Almageſt

1273 Aſtronomy made little or no progreſs from the time of HIPPARCHUS to that of PTOLEMY, who was born in the year 69 of our æra His great work on Aſtronomy is entitled Μεγαλη Συνταξις or *Great Conſtruction*, and the Arabs give it the name of *Almageſt* by which it is now uſually called This work is invaluable, both as containing his own diſcoveries, and as preſerving thoſe of HIPPARCHUS It was known that the moon was ſubject to a very conſiderable equation, which was fixed at 5 1 at its maximum But PTOLEMY diſcovered that this was true only when the apſides were in quadratures, and that when they were in ſyzygies it amounted to 7 40 the difference 2 39 he called a ſecond inequality which differs but a very little from what is obſerved at this time He conſtructed an inſtrument to find the parallax of the moon which he made 1 7 at the zenith diſtance 50 , a quantity much too great He alſo attempted to find the parallax of the ſun which he made 2 51 He ſaid that when the latitude of the moon was greater than the ſum of the ſemi diameters of the moon and earth's ſhadow, that there could be no eclipſe of the moon And if the latitude of the moon were leſs than the ſum of the ſemi diameters of the ſun and moon, there would be an eclipſe of the ſun But to tell whether it would happen at any particular place, he found it neceſſary to apply the moon's parallax When he therefore diſcovered that the earth would be ſomewhere eclipſed, to determine whether it would happen at any particular place, he computed the place of the center of the moon for ſeveral ſucceſſive inſtants, and applied to them the reſpective effects of parallax, and thus he got the apparent diſtance of the centers of the ſun and moon, from which he deduced the times of the beginning and end The element called the *reduction to the ecliptic* he perceived and explained It appears alſo, that before the time of PTOLEMY they reckoned the digits by the twelfth parts of the ſurface and not of the diameter The ſyſtem of the planets which he embraced is well known He explained their motions by means of an epicycle revolving upon an excentric circle, but he was not able to find their relative diſtances He confirmed the diſcovery of HIPPARCHUS, of the preceſſion of the equinoxes Both PTOLEMY and HIPPARCHUS determined the longitudes of the ſtars by comparing them with the ſun And as they are not viſible in the day they uſed the moon as an intermediate obſervation He reduced the 49 conſtellations to 48 He mentions *Berenices Hair*, but did not admit it a conſtellation He was acquainted with the effect of refraction, having written a Treatiſe on Optic where it is mentioned ALHAZEN quoted this Treatiſe, which exiſted in his time The Almageſt is divided into thirteen books

In

In the firſt, he endeavours to ſhow that the earth is at reſt in the centre of the univerſe; that it is ſpherical, and but a point in compariſon of the diſtance of the fixed ſtars: he alſo treats of the ſeveral circles of the earth, and their poſitions in a right ſphere.

In the ſecond, he treats of the habitable parts of the earth, and of the nature and poſitions of its circles in a right ſphere.

The third book treats of the true length of the year, of the unequal motion of the ſun in the ecliptic, and alſo of the unequal length of days and nights; it likewiſe contains Tables of the ſun's mean motion, and precepts for uſing them.

In the fourth, he treats of the lunar motions, gives Tables for computing them, and exhibits the principles, and obſervations on which they are founded.

In the fifth, he treats of the excentricity of the lunar orbit, and the inequalities of the moon's motion; aſſigns the magnitudes of the ſun, moon and earth, and their diſtances from one another.

In the ſixth, he treats of the conjunctions and oppoſitions of the ſun and moon, the limits of ſolar and lunar eclipſes, and gives Tables for computing the times when they will happen.

In the ſeventh, he treats of the fixed ſtars; deſcribes the various conſtellations by means of an artificial ſphere, rectifies their places to his own time, and ſhows how different they then were from what they had been in the times of TIMOCHARIS, CALIPPUS, HIPPARCHUS and others; and concludes with a catalogue of the ſtars in the northern hemiſphere.

The eighth book contains a catalogue of the ſtars in the ſouthern hemiſphere, as alſo a catalogue of the ſtars in the twelve zodiacal conſtellations; this catalogue of the ſtars is the oldeſt extant, and therefore conſtitutes a very valuable part of the work. The book concludes with a diſcourſe on the *Galaxy*, or *Milky way*, and an account of the riſing and ſetting of the ſun and fixed ſtars.

The ninth book treats of the order of the planets, and of their periodical revolutions; contains Tables of their mean motions, and concludes with the theory of Mercury, and accounting for its various phænomena as ſeen from the earth.

Books the tenth and eleventh treat in like manner, of the various phænomena of the planets Venus, Mars, Jupiter, and Saturn, and ſhows how the Tables have been corrected from the obſervations of preceding Aſtronomers.

The twelfth book treats of the ſtationary and retrograde appearance of the planets; and the thirteenth of their latitudes, the inclination of their orbits, riſings, ſettings, &c.

Of

On the Aſtronomy of the Arabs, Perſians, *and* Tartars

1274 ALMAMON, an Arabian Aſtronomer, lived about the year 818 He began his obſervations upon the obliquity of the ecliptic, which he determined to be 23 , 35′, according to GOLIUS, but in another edition of this author, it is 23° 35′ This is related in the Elements of Aſtronomy, by ALFERGEN About the ſame time it was determined by CALID, ABULFIBUS SENED, and ALIS, to be 23 33′ 52′ We may therefore fix the obliquity of the ecliptic at that time at 23 34′, with the probability of it s being near the truth

1275 THEBITH, who was born about the year 836 determined the length of the ſidereal year to be 365*d* 6*h* 9′ 11 , and made the equinoctial points to have a motion ſometimes direct, and ſometimes retrograde He found the obliquity of the ecliptic to be 23 33 30′, and concluded it to be variable

1276 ALBATEGNIUS, who flouriſhed amongſt the Arabs about the middle of the 9th century, was the greateſt Aſtronomer ſince PTOLEMY Dr HALLEY calls him, *Auctor pro ſuo ſæculo admirandi acuminis, ac in adminiſtrandis obſervationibus exercitatiſſimus* Finding that PTOLEMY s Tables of the moon and planets were defective he conſtructed new and more correct Tables He made the motion of the equinoxes to be 1 in 66 years, inſtead of 100 years, as it had been before ſuppoſed and examining the obliquity of the ecliptic, he found it 23 35, but if his obſervations be corrected for parallax and refraction, it becomes 23 35 47 The theory of the ſun alſo engaged his attention, he found the excentricity of the earth s orbit to be 3465, the radius being 100000 The place of the apogee he fixed at 22 17′ of Gemini and PTOLEMY having placed it at 5 30, he diſcovered that the apogee had an annual progreſſive motion of 59″ 4 in reſpect to the equinoxes, and (according to him) the motion of the equinoxes being 54′ 32‴ the real motion from hence is 4′ 32‴ He gave two equations to the moon, the ſame as thoſe which PTOLEMY diſcovered He alſo obſerved two eclipſes of the moon, and two of the ſun Theſe diſcoveries appear in a work of his, entitled, *De Numeris et Motibus Stellarum*, which contains alſo ſeveral problems upon the doctrine of the ſphere

1277 ABU MAHMUD AL CHOCANDI, who lived about the year 992, with a ſextant of 40 cubits radius, found the obliquity of the ecliptic 23 32 21′, the limb of this quadrant was divided into ſeconds

1278 ALBATURNIUS ABUL RIAN, about the year 995 according to Mr BERNARD, or about 1070 according to ABULFARAGIUS, with a quadrant of 15 cubits radius, made the obliquity of the ecliptic 23 35.

1279 ARSACHEL, who lived about the year 1076, corrected the theory of the ſun, by making a great number of obſervations in various parts of the orbit. He made the obliquity of the ecliptic 23 34.

1280 ALHAZEN wrote upon the twilight, the beginning of which he made when the ſun was about 19 below the horizon, and he computed the height of the atmoſphere to be 51.8 miles, ſuppoſing the circumference of the earth to be 24000 miles. He wrote a Treatiſe on Optics in ſeven books, in which he explained the true principles of the refraction of a ray of light through the air, and gave a method of finding the quantity of it. He lived in the eleventh century.

1281 The uſe of the pendulum was known to the Arabs, but no account is given of the inventor, or at what time he lived. This very important diſcovery ought to have immortaliſed its author.

1282 In the year 1072, the Sultan MELICSHAH employed Aſtronomers to correct the length of the year, and OMAR CHEYAM determined the length of the tropical year to be 365*d* 5*h* 48' 48", the very ſame quantity at which it is now fixed by M. de la LANDE. He alſo corrected the calendar.

1283 In the twelfth century CHIONIADES, a great mathematician of Conſtantinople, obtained permiſſion to import many books from Trebizond in Perſia, from which we find that the Perſians had cultivated Aſtronomy with great ſucceſs, as their Tables of the motions of the planets, thoſe of Mercury excepted, were very exact. M. de l'ISLE deduced from theſe Tables the tropical year of the Perſians to be 365*d* 5*h* 49' 5" 30‴, the annual motion of the apogee 5" 25‴ 12⁗, and the ſidereal year 365*d* 6*h* 9' 55" 30‴. The obliquity of the ecliptic in theſe Tables is 23 35, the equation of the center of the ſun 2 0' 30", and the place of the apogee, in the firſt year of IESDEGIRD, is 2 17 50' 7".

1284 In the thirteenth century lived NASSIREDDIN, who conſtructed Tables from obſervations made at Maragh. He made the obliquity of the ecliptic 23 30.

1285 AL NODDAM, who lived ſoon after NASSIREDDIN, made the obliquity of the ecliptic 23 33, and diſcovered that the obliquity was decreaſing.

1286 In the fifteenth century lived ULUGH BEIGH, a prince of Tartary, and grandſon of the famous TAMERLANE. He had a gnomon 180 feet high, with which he made his obſervations of the ſun, and the Tables which were conſtructed from them were ſo exact, that they differed but a very little from thoſe

thoſe of TYCHO The Tables are divided into four parts, the firſt treats of the epochs the ſecond, of the times of revolution, the third, of the motions of the planets, the fourth of the fixed ſtars He determined the preceſſion of the equinoxes to be 1 in 70 years We find alſo in his Tables the equation of the ſun's center 1 55′ 33′ 12′, which differs only 21′ 30″ from the Tables of M de la CAILLE Alſo the epoch of the ſun for July 3, 1437, 19*h* 40 40″ at Greenwich, is found to be 4 29 16′ 24′, which is only 1 48′ leſs than in the Tables of HALLEY It is remarkable, that the Orientaliſts diſpoſed the Tables ſo as to have all their equations additive which we have ſince followed The ſidereal year he made 365*d* 6*h* 10 8′ 9′ 23″″ He alſo determined the obliquity of the ecliptic to be 23 30 17, but if his obſervations be properly corrected for parallax and refraction, it comes out 23 31′ 58 He alſo made a new catalogue of the fixed ſtars He was put to death by his own ſon in the year 1449

1287 It is to the Arabians that we are indebted for the preſent form of Trigonometry They firſt made uſe of the ſines inſtead of the chords of double the arcs ARZAKEL left a Table of ſines, ſuppoſing the diameter to be divided into 300 parts GEBER invented two theorems which are the foundation of modern Trigonometry

1288 In the ninth century, the Arabians made their way into Spain, and thus by their intercourſe with all the weſtern parts of Europe, they communicated to them the knowledge of Aſtronomy, which for ſome time had been loſt in that part of the world

On the Progreſs of Aſtronomy, from its Reſtoration in Europe

1289 The emperor FREDERIC II of Germany, who lived about the year 1230 was a great encourager of the arts and ſciences, and eſtabliſhed univerſities for that purpoſe He ordered a tranſlation of the Almageſt of PTOLEMY

1290 SACROBOSCO, born at Halifax, acquired great reputation by his knowledge in Aſtronomy He ſtudied at Oxford, and afterwards at Paris He wrote a Treatiſe on the Sphere, containing an account of the celeſtial circles, the motions of the ſtars and planets, the riſing and ſetting of the ſigns, the diverſity of days, nights and climates and the cauſes of eclipſes This was compiled from the Almageſt, and from ſome of the Arabian Aſtronomers He alſo wrote upon the calendar, and the eccleſiaſtical computations which was at that time a ſubject that engaged the attention of Aſtronomers He died in the year 1246

1291 About

1291 About the year 1240 lived ALPHONSUS X. king of Caſtile. The Tables of PTOLEMY becoming defective, he, in the life of his father, called together the learned in Aſtronomy, who met at Toledo, and made new Tables, which were called the *Alphonſine* Tables in honour of that prince. They were publiſhed in the year 1252, the firſt of his reign. The Tables were founded upon the ſame principles as thoſe of PTOLEMY, and on the ſame ſyſtem, and the correction conſiſted in that of the mean motions. The Jew R. ISAAC ABENSID ſurnamed HAZAN, was the principal author of theſe Tables. This king was depoſed by his ſon, and MARIANA ſaid, that he loſt the earth by contemplating the heavens.

1292 About this time lived ROGER BACON, a learned Franciſcan, who wrote ſeveral works relating to Aſtronomy. He obſerved, that from the time of the reformation of the calendar by JULIUS CÆSAR, the equinoxes and tropics had anticipated nine days upon the time when PTOLEMY had obſerved them, and thence concluded that the anticipation was at the rate of one day in 125 years. He had the honour therefore of diſcovering the neceſſity of correcting the calendar. He underſtood the effect of ſpherical lenſes, but it does not appear that he combined them to form a teleſcope.

1293 The cardinal NICOLAS CUSA diſcovered ſome errors in the Tables of ALPHONSUS, and he wrote alſo upon the reformation of the calendar. He remarked, that the motion of the fixed ſtars in the Alphonſine Tables did not agree with the obſervations of PTOLEMY. He appears to have been the firſt among the moderns who renewed the idea of the earth's motion. He died in the year 1464.

1294 About the ſame time lived PURBACH, a conſiderable Aſtronomer, who ſtudied in the univerſity at Vienna. He wrote upon the theory of the planets, and attempted to correct the Tables of PTOLEMY and ALPHONSUS. He calculated a Table of ſines for every ten minutes, to a radius of 60, as PTOLEMY had done, but he augmented it by five cyphers, on which account he is ſuppoſed to have been the inventor of decimal arithmetic. He conſtructed a celeſtial globe, and added to it a catalogue of ſtars. By combining the hypotheſes and elements of PTOLEMY and ALPHONSUS, he gave a more convenient form to the equations. In the year 1460, he printed a work, entitled, *Theorie des Planetes*. In his ſolar Tables he placed the ſun's apogee in the beginning of Cancer, and he uſed 23 33 30″ for the obliquity of the ecliptic. M. WEIDLER gives a catalogue of twenty works of this Aſtronomer.

1295 BLANCHINUS of Bologna, an Aſtronomer cotemporary with PURBACH, was profeſſor of that ſcience at Ferrara about 1458. He compoſed new Tables of the celeſtial motions, which he dedicated to the emperor FREDERIC III. They were firſt printed at Venice in 1485.

1296 The

1296 The next celebrated Aſtronomer was JOHN MULLER, of Koniſberg, a town in Franconia, better known by the name of REGIOMONTANUS that word being a Latin tranſlation of the German word *Koniſberg* Encouraged by the reputation of PURBACH, he went to Venice at the age of fifteen, and became his pupil Upon the death of PURBACH he went to Rome, and made there ſome aſtronomical obſervations, but in 1471, he retired to Nuremberg, where he met with BERNARD WALTHER, a zealous friend to Aſtronomy, who was at the expence of conſtructing ſome valuable aſtronomical inſtruments, with which REGIOMONTANUS made obſervations PURBACH and REGIOMONTANUS diſcovered the imperfections of the ancient obſervations, by an obſervation of Mars compared with two ſtars near it whereby this planet was found 2 diſtant from the place given by the Tables The inſtrument which WALTHER made was an armilla, but much more complete than any which had been before conſtructed It ſerved to obſerve in the plane of the ecliptic the equator and in the circles which are perpendicular to it Thus they obſerved the latitudes and longitudes of the heavenly bodies to a conſiderable degree of accuracy PURBACH and REGIOMONTANUS conſidered the heavens as a great dial, and that the ſtars would paſs in ſucceſſion over any meridian at the rate of 15 in an hour This is the principle of the modern method of finding the right aſcenſions of the ſtars REGIOMONTANUS computed an Ephemeris for 30 years forward In the month of February, 1472, a comet appeared, on which he made obſervations, and it was the firſt that had been obſerved in Europe Pope SIXTUS IV wiſhed to reform the calendar and ſent for REGIOMONTANUS to aſſiſt in that work, in conſequence of which he went to Rome in the year 1475, and died of the plague in the year following Other accounts however ſtate that he was put to death by the ſon of TRAPEZUNTIUS in revenge for his having detected errors in the tranſlation of the Almageſt by their father SCHONER does him the honour of aſſerting, that he was a favourer of the ſyſtem of the earth's motion

1297 BERNARD WALTHER was born at Nuremberg in the year 1430 After the death of REGIOMONTANUS he continued to make obſervations, and in the year 1484, he made uſe of clocks in order to meaſure time The firſt obſervation he made with a clock was, to find how long Mercury roſe before the ſun In an eclipſe of the moon February 8 1487 we find he marked the time by the clock HIPPARCHUS and PTOLEMY found the longitude of the ſtars by comparing them with the ſun making uſe of the moon as an intermediate obſervation, but WALTHER made uſe of Venus inſtead of the moon which was much more exact, becauſe its motion is ſlower and alſo on account of its parallax being ſo ſmall He made the longitude of Aldebaran 2 35′ of Gemini, in the year 1491 His obſervations were of eclipſes,

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of the longitudes of the planets and fixed ſtars, of the conjunctions of the planets with the planets and with the fixed ſtars, of their diſtances and occultations. He obſerved the effect of refraction upon the ſun in the horizon.

1298 DOMINIC MARIA was born at Ferrara in 1464, and was mathematical profeſſor at Bologna. He applied himſelf with great diligence to the making of celeſtial obſervations, and it was by his example and encouragement, that COPERNICUS was excited to cultivate practical Aſtronomy. He made the obliquity of the ecliptic 23° 29′.

1299 JOHN WERNER was born at Nuremberg in year 1468. He went to Rome in 1493, where he devoted himſelf to mathematics, aſtronomy, and aſtronomical obſervations. In 1498, he returned into his own country, and deſcribed the motion of a comet, which appeared in April in the year 1500. He determined the motion of the fixed ſtars to be 1° 10′ in 100 years, and made the obliquity of the ecliptic to be 23° 28′. He conſtructed a machine to repreſent the motions of the planets according to the Ptolemaic ſyſtem.

1300 The next Aſtronomer of eminence was NICHOLAS COPERNICUS, who was born at Thorn in Pruſſia, January 19, 1472. From his earlieſt years he was very fond of mathematical ſtudies, and when he had learned the uſe of the aſtrolabe, and began to underſtand the principles of Aſtronomy, he was ſo ſtruck with the reputation of REGIOMONTANUS, that he reſolved to give up all his attention to that ſtudy. He went to Bologna to viſit DOMINIC MARIA, a profeſſor of Aſtronomy at that place, from thence he went to Rome, where he was made profeſſor of mathematics, and where he made ſome obſervations about the year 1500. Returning to his own country, he applied himſelf to the ſtudy of Aſtronomy. He meditated upon the various ſyſtems, and examined all the hypotheſes; a circular motion of the planets about the earth he found would not ſolve the phænomena, and he could not admit the doctrine of the epicycles of HIPPARCHUS and PTOLEMY, which ſuppoſe the bodies to revolve about an imaginary center, and which, from its complexity, he thought altogether unworthy of its great Author. He diſcovered that PHILOLAUS had placed the ſun in the center, and that NICETAS had given the earth a rotation about its axis, and this led him to form the ſyſtem which now goes under his name. He was pleaſed with the idea of placing the ſun, as being the moſt glorious body in the heavens, in the center, ruling as it were and directing all the reſt, and was delighted with the ſimplicity, and harmony of the whole. He ſaid that 'by long obſervations he diſcovered, that if the motions of the planets be compared with that of the earth, and be eſtimated according to the times in which they perform their revolutions, not only their ſeveral appearances will follow from this hypotheſis, but it will ſo connect the order of the planets, their orbits, magnitudes and diſtances,

diſtances, and even the apparent motion of the fixed ſtars, that it will be impoſſible to remove one of theſe bodies out of its place, without diſordering the reſt and even the whole univerſe alſo He alſo reaſoned upon gravity and defines it to be "a certain natural deſire, given by the Supreme Being to all the parts of matter, by means of which they tend to unite under the form of a globe" Having eſtabliſhed his ſyſtem, he made obſervations and compared them with the ancient ones, in order to correct the Tables For this purpoſe he made himſelf a quadrant and parallactic rulers, and other inſtruments deſcribed by PTOLEMY He made the preceſſion of the equinoxes to be 1 in 72 years, the obliquity of the ecliptic 23 28′ 14″, the excentricity of the earth's orbit 323, the radius being 10000 and the place of the earth's apogee 3 6° 40 In treating of the retrogradation of the equinoxes he obſerved that it had not a libratory motion, as THEBITH imagined He remarked, that the obliquity of the ecliptic decreaſed, and alſo the excentricity of the earth's orbit, and thence concluded, that theſe circumſtances depended upon the ſame cauſe He made the length of the tropical year 365*d* 5*h* 49 24, which differing from the determinations of PTOLEMY and ALBATEGNIUS, he concluded that it was ſubject to change In order to explain the irregularities of the motions of the planets, he retained the epicicles of PTOLEMY He adopted PTOLEMY's two equations of the moon, and having obſerved its parallaxes he found the greateſt to be 65′ 48, and the leaſt 50 19 and the correſponding diſtances 52⅓ and 68⅓ ſemidiameters of the earth, the mean diſtance therefore was 60⅓ He attempted to get the parallax of the ſun by the method uſed by PTOLEMY and found it to be 3′, and thence the ſun's diſtance 1179 ſemidiameters of the earth The diameter of the ſun in its apogee he made 31′ 48 and in its perigee 33 54′ When the moon was in its apogee and in conjunction or oppoſition, he made its diameter 30, when in perigee 35 when the apogee was in quadratures, the diameters he found to be 28 45 and 36 44 His great work on Aſtronomy is intitled *Aſtronomia Inſtaurata*, and is divided into ſix books The *firſt* contains an account of his ſyſtem, and his reaſons for aſſuming it together with ſome geometrical theorems, and the doctrine of plane and ſpherical Trigonometry The *ſecond* contains the doctrine of the ſphere The *third* treats of the equinoxes, ſolſtices, obliquity of the ecliptic, the theory of the earth's motion, and the inequality of ſolar days The *fourth* treats of the motion of the moon The *fifth* and *ſixth* are upon the theory of the planets This work was completed about the year 1530, but it was with the utmoſt difficulty that his friends, even in the latter part of his life, could perſuade him to publiſh it at length however their entreaties prevailed and he delivered it into their hands to be publiſhed, and received a copy of it, only a few hours before

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he died, which happened May 23, 1543, in the ſeventy firſt year of his age.

1301 Erasmus Reinold, born at Thuring in the year 1511, publiſhed ſeveral pieces on Aſtronomy. The Pruſſian Tables which he publiſhed for the meridian of Koniſberg are ſaid, by M. de la Lande, to be more correct than thoſe of Copernicus.

1302 Reinerus Gemma, ſurnamed Frisius, was born in 1508. He invented a new projection placing the eye in the vernal equinox, and projecting the circles upon the ſolſtitial colure. He alſo wrote two tracts on Aſtronomy, and propoſed to find the longitude by the moon.

1303 Schoner was born at Caroldſtead, and ſtudied Aſtronomy and mathematics at Nuremberg. He made two obſervations on Mercury, which were of great uſe to Copernicus. He improved the methods of making obſervations, explained the calendar, and publiſhed a deſcription of the earth by means of the terreſtrial globe. He died in the year 1547.

1304 Jean Homelius was born in 1518. He made many aſtronomical obſervations, and amongſt others, he found the height of the pole at Leipſic, which Tycho approved of. Sulterus, a pupil of his, was the maſter of Tycho.

1305 Peter Nonius was born at Alcazar in Portugal, in the year 149. He frequently made aſtronomical obſervations, and being diſſatisfied with the inſtruments then in uſe, he invented a quadrant, and graduated it in the following manner. He deſcribed ſeveral concentric circles, the outermoſt he divided into 90 equal parts, or degrees, the next was divided into 89 equal parts, the next into 88, and ſo on; hence the index of the quadrant muſt reſt upon, or very near ſome one of the diviſions, from which he eaſily computed the degrees in the arc. He wrote upon the twilight, upon navigation, upon the properties of the rhumb lines on the globe, on aſtronomical inſtruments, upon various aſtronomical problems, gave a deſcription of a nautical plane ſphere, reſolved a problem of Aristotle concerning a ſhip driven by oars, and made ſome remarks on the theorems of Purbach. He publiſhed a Treatiſe of Algebra in the Spaniſh language, at Antwerp, and proved that Orontius was deceived in ſuppoſing that he had ſquared the circle, and doubled the cube by geometry. He alſo found the time of the year when the twilight is ſhorteſt. He died in 1577.

1306 Peter Appian was born at Leiſnig, in Poland, in the year 1495. He publiſhed a work in 1540, called the *Cæſarean Aſtronomy*, dedicated to the emperor Charles V. and Ferdinand his brother, to ſhow how aſtronomical problems may be reſolved by inſtruments. He ſhowed how to obſerve the places of the ſtars by the aſtrolabe, and taught how to predict eclipſes, and delineate

delineate them on a plane He obſerved five comets, one in the year 1531, one in 1532 one in 1533 one in 1538 and one in 1539 and he firſt remarked that their tails are always turned from the ſun He died in 1552, leaving behind him an Ephemeris from 1534 to 1570 unfiniſhed

1307 GEORGE JOACHIM RHETICUS was born in Rhetia He was a pupil of COPERNICUS, and in order to facilitate aſtronomical calculations, he began to conſtruct a Table of ſines, tangents and ſecants, for every ten ſeconds of the quadrant, but he did not live to finiſh the work, He died at Caſſobia, in Hungary, in the year 1576

1308 WILLIAM IV Landgrave of Heſſe, diſtinguiſhed himſelf greatly by promoting the ſtudy of Aſtronomy He applied cloſely to this ſcience, and attached himſelf to CHRISTOPHER ROTHMAN an aſtronomer, and JUSTUS BURGIUS, an excellent inſtrument maker With this aſſiſtance he erected an obſervatory on the top of his palace at Caſſel and furniſhed it with quadrants, ſextants, and various other inſtruments, and with theſe he made a great number of obſervations which HEVELIUS preferred to thoſe of TYCHO From theſe obſervations he determined the latitude and longitude of 400 ſtars, which he inſerted in a catalogue rectifying their places to the year 1593 He died in the year 1592

1309 GERARD MERCATOR, born in Flanders in the year 1512, made globes, and conſtructed a great many geographical maps

1310 The next Aſtronomer of any conſequence was TYCHO BRAHE, born of noble parents at Knudſtrop in Scania in the year 1546 When he was only fourteen years old he was ſtruck with aſtoniſhment at obſerving an eclipſe of the ſun to happen ſo very near the time it was predicted, and it ſeems as if this led him to the ſtudy of Aſtronomy In 1563, he obſerved the great conjunction of the ſuperior planets and in tracing the courſes of the planets, and comparing them with the Tables of ALPHONSUS and COPERNICUS, he ſaw that the Tables were ſubject to great errors In announcing the great conjunction, the Tables of ALPHONSUS erred a month November 11, 1572, he diſcovered a new ſtar in *Caſſiopea's Chair* this ſtar was greater and more brilliant than *Lyra* and *Procyon*, and was ſeen in the middle of the day, but at length its brightneſs declined, and it died away gradually, and diſappeared in 1574 It was obſerved by all the Aſtronomers in Europe This phænomenon excited TYCHO to make a new catalogue of the fixed ſtars which contained the places of 777 rectified to the beginning of the year 1600 Inſtead of the moon which was uſed to connect the ſun and the ſtars he made uſe of *Venus*, as WALTHER had done before him TYCHO being recommended by the Landgrave of Heſſe, to FREDERIC II king of Denmark he gave him the iſland of Huenna, and ſupplied him with money to build an obſervatory, to furniſh it with inſtruments, and to ſupport himſelf This TYCHO very gladly accepted, and called the name of the building *Uraniburg* It was furniſhed with the beſt inſtruments, conſiſting of quadrants, ſextants, circles, armillæ, parallactic rulers, rings, aſtrolabes, globes, clocks and ſun-dials Theſe inſtruments were of excellent workmanſhip and far more accurate than any which had been before made Moſt of the diviſions were diagonal but he had one quadrant divided according to the method of PETER NONIUS The whole expence is ſaid to have amounted to 200000 crowns Here TYCHO made all his obſervations of the ſtars comets and planets, even to *Mercury*, which COPERNICUS had never been able to ſee He firſt determined the place of a ſtar by obſerving its azimuth and the time of paſſing over it, but his clocks did not give the time with ſufficient accuracy, he therefore determined the place, by obſerving its diſtance from two known fixed ſtars In the courſe of the obſervations, TYCHO made a very important diſcovery, that of the refraction of the air, and this he found from comparing the height of the equator as determined from obſervations of the ſolſtices, and of the circumpolar ſtars, for he found that they conſtantly differed by 4, this he imputed to the refraction of the air He made the horizontal refraction 34′, and at 45 altitude he made it nothing, and calculated a Table ſhowing the refraction at all altitudes up to 45 He conſtructed new Tables of the ſun, and determined the preceſſion of the equinoxes to be 1 in 71 years he alſo found that the latitude of the ſtars ſince the time of TIMOCHARIS and HIPPARCHUS, had varied thus he diſcovered that the ecliptic was ſubject to a variation The theory of the moon alſo engaged his attention, and he diſcovered a third equation, called the *Variation* He alſo found that the motion of the nodes was not uniform, and that the inclination of the orbit was variable, the leaſt inclination he made 4 58′ 30′, and the greateſt 5 17′ 30′, which is a great proof of the goodneſs of his obſervations He very happily repreſented the variation of the motion of the nodes and of the inclination, by the motion of the pole of the lunar orbit in a ſmall circle Theſe diſcoveries relating to the moon do him great honour He obſerved a comet in the year 1577, and diſcovered that it had a parallax of 20′, and thence concluded that it was about three times as far from us as the moon He conjectured that they revolved about the ſun The ſyſtem which he invented we have already explained He made the obliquity of the ecliptic 23 31′ 30′, and found the length of the ſidereal year 365*d* 6*h* 9 26′ 45′, and the tropical 365*d* 5*h* 48′ 45′, which is within 2 or 3′ of the preſent determination He found the diameter of the ſun in apogee to be 30′, and in perigee 32′, and its mean diſtance 1150 ſemidiameters of the earth Upon the death of FREDERIC II it was repreſented to the young king that the treaſury

treaſury was exhauſted and that it was neceſſary to retrench the penſions, in conſequence of this, Tycho was deprived of his, upon which he removed to Copenhagen with ſuch of his inſtruments as he could carry, but he there received an order to diſcontinue his obſervations Upon this he went to Holſtein, and was introduced to the emperor Rodolphus, who ſettled a penſion of 3000 crowns upon him, and gave him a magnificent houſe, where he renewed his ſtudies, and the famous Kepler, who called him the Hipparchus of his age became his ſcholar and aſſiſtant He died October 24, 1601, in the fifty fifth year of his age, ſolacing himſelf that he had not lived in vain, and that his labours would redound to the glory of God

1311 Longomontanus was a pupil and aſſiſtant of Tycho at Uraniburg He aſſiſted him in his catalogue of the ſtars, and in his theory of the moon He afterwards went to Copenhagen, and was made profeſſor, and acquired great reputation by his aſtronomical knowledge He died in the year 1647

1312 Kepler was born at Wiel in Wirtemberg, the 27th of December, in the year 1570 He began to ſtudy Aſtronomy very early, and had a turn for ſeeking analogies and harmonies in nature, and having, as he thought, diſcovered a curious one, he publiſhed it in 1596, and ſent it to Tycho, who, although he diſapproved of the work ſaw ſo much ingenuity in it, that he ſent for Kepler to reſide with him at Prague, and from the obſervations of Tycho, he made his important diſcoveries He made the refraction the ſame for all bodies at the ſame height, and did not agree with Tycho, that there was no refraction above 45 he alſo obſerved that it was different on different parts of the earth He publiſhed a Treatiſe on Optical Aſtronomy, in which he treats of parallax, and the calculation of ſolar eclipſes, and applies them to find the longitude on the earth's ſurface He ſpeaks of gravitation, and applies it properly to the caſe of the earth and moon, and to the cauſe of the tides (220) He directed his attention to the motion of *Mars*, and publiſhed a work entitled *De Motibus Stellæ Martis* He firſt employed a circular orbit, and determined its excentricity, but by comparing ſome diſtances of Mars from the ſun from obſervation, with the computed diſtances, he found ſo great a diſagreement, that he concluded the orbit was not a circle, he then ſuppoſed it to be an ellipſe, and found the calculated diſtances agreed with thoſe deduced from obſervation, hence he concluded that the planets revolved about the ſun in ellipſes, having the ſun in the focus (217) Having determined the periodic times and mean diſtances of the planets, he diſcovered, by trial, the famous law, that the ſquares of their periodic times are as the cubes of their mean diſtances He alſo found, that in the apſides the areas deſcribed by the planets in equal times were equal and he ſuppoſed that the ſame was true at every other point, and thence he concluded that the planets deſcribe about

about the ſun equal areas in equal times These three important diſcoveries are the foundation of all plane and phyſical Aſtronomy He ſolved the problem, now called KEPLER's *Problem*, of cutting off from an ellipſe by a line drawn to the focus, an area equal to a given area He alſo announced the paſſage of *Mercury* over the ſun in the year 1631, and the tranſits of *Venus* in 1631 and 1761 The works which this celebrated Aſtronomer publiſhed are,

1 Coſmographical Myſtery, in 1596
2 Optical Aſtronomy, in 604
3 An Account of a new Star in Sagittarius, in 1605
4 On the Motions of Mars in 1609
5 Diſſertations with his Nuncius Sidereus of GALILEO, in 1612
6 New Ephemeris, from 1617 to 1620
7 Three Books on the Copernican Aſtronomy in 1618
8 The Harmony of the World, and three Books on Comets in 1619
9 Three more Books on the Copernican Aſtronomy, in 1622
10 Rodolphine Tables, in 1627

Beſides theſe he wrote ſeveral things in *Chronology*, the *Geometry of Solids*, *Trigonometry*, *Logarithms*, and *Dioptrics*

1313 NAPIER, in the year 1614, publiſhed his invention of logarithms, a diſcovery of ineſtimable uſe in the practice of Aſtronomy

1314 About this time lived BAYER, an Aſtronomer at Auſbourg in Germany, who rendered himſelf memorable in Aſtronomy by his work, entitled, *Uranometria* which is a very complete celeſtial atlas containing all the conſtellations viſible in Europe, in this the ſtars are marked with the Greek letters

1315 LANSBERGIUS, born at Gand in 1560 publiſhed a ſet of aſtronomical Tables in the year 1632, and in 166[illegible] ſeveral other works of his were publiſhed

1316 SCHIKARD was born at Wirtemberg he made many obſervations, and compoſed ſeveral works upon Aſtronomy He died in 16[illegible]

1317 HORROX was born at Hoole near Liverpool He and his friend CRABTREE were the firſt perſons who obſerved a tranſit of *Venus* over the ſun's diſc, this tranſit happened November 24 1639, according to his own prediction An account of this he wrote and entitled it *Venus in Sole viſa*, which was publiſhed by HEVELIUS He gave a new theory of the moon, making it move in an ellipſe about the earth in its focus From obſervations on the diameter of the moon, he found that its apogee was ſubject to an annual equation of 12 ,5 This extraordinary young man died in 1641, about the

the age of 22 years His poſthumous works, publiſhed by WALLIS in 1673, are, Aſtronomia KEPLERIANA defenſa et promota Excerpta ex Epiſtolis ad CRABTRÆUM ſuum Obſervationum cæleſtium catalogus Lunæ theoria nova

1318 WILLIAM CRABTREE, the friend of HORROX, lived at Broughton near Mancheſter he obſerved the tranſit of *Venus* in 1639, and made many aſtronomical obſervations, ſome of which were publiſhed in the works of HORROX He died in 1641

1319 GALILEO was born at Florence in the year 1564 He diſcovered the laws by which bodies falling freely are accelerated and the uſe of the pendulum for meaſuring time Having heard of the invention of the teleſcope, he, by conſidering the principles of refraction conſtructed one which magnified more than 30 times With this he immediately diſcovered the ſpots in the ſun from which he ſaw that it had a rotation about its axis he ſaw alſo various new appearances upon the face of the moon, from which he concluded that it was very rough with hills and vallies He alſo found that *Venus* put on the ſame phaſes as the moon And making his obſervations upon *Jupiter*, he ſoon diſcovered that it had four moons Afterwards he thought of making uſe of them for finding the longitudes of places upon the ſurface of the earth He alſo diſcovered as he thought, that *Saturn* was either compoſed of three bodies, or that it was in the ſhape of an olive, as he expreſſed it Upon directing his teleſcope to the fixed ſtars he was ſurpriſed to find that, inſtead of being magnified they were diminiſhed, appearing only as points He was a zealous defender of the Copernican ſyſtem, for which he was perſecuted, and caſt into priſon He died in the year 1642 The principal works which this great man publiſhed, are

1 The Operations of the Compaſs, geometrical and military
2 A Diſcourſe on the Floating of Bodies upon, and their Submerſion in Water
3 Mechanics, or the Benefits derived from that Science
4 His Balance for finding the Proportion of Alloy, or mixed Metals
5 His Nuncius Sidereus
6 A Continuation of the laſt Work, containing his laſt Obſervations on Saturn, Mars, Venus and the Sun
7 A Letter concerning the Libration of the Moon
8 On the Solar Spots, with an Ephemeris of the Motions of Jupiter's Satellites
9 Problems in Mathematics
10 Mathematical Diſcourſes
11 A Treatiſe on the Mundane Syſtem

1320 Le

1320 Le P SCHEINER, the aſſociate of GALILEO, made a great many obſervations upon the ſolar ſpots, which he publiſhed in a work entitled, *Roſa Urſina* He was the firſt who paid attention to the elliptical figure of the ſun when near the horizon He died in 1650

1321 GODEFROI VENDELINUS an Aſtronomer in Holland, publiſhed in 1626, a Diſſertation on the obliquity of the ecliptic, and eſtabliſhed its variation He was the firſt who reduced the parallax of the ſun to 15″, by the method of ARISTARCHUS

1322 GASSENDUS obſerved the tranſit of *Mercury* over the ſun on Nov 7, 1631, and took ſeveral meaſures of its diſtance from the center of the ſun

1323 RICCIOLUS, a Jeſuit, was a man indefatigable in his aſtronomical purſuits He publiſhed a work entitled *Almageſtum Novum*, containing a collection of all the known obſervations the methods, the determinations the opinions and phyſical explanations of the phænomena He publiſhed alſo his *Aſtronomia reformata*, and *Geographia reformata*, containing very valuable collections He attempted to meaſure the earth His death happened in 1671

1324 PEYRESC, the protector and friend of GASSENDUS, was born in 1580 He diſcovered the times of the revolutions of Jupiter's ſatellites, but announced that they were not very accurate He alſo conſidered their configurations, and the method of finding the longitude from them

1325 JOHN BAPTIST MORIN, born at Villefranch in 1583, diſtinguiſhed himſelf by his attempts to diſcover the longitude by means of the moon His method was good in theory, but not practicable, on this account he could not obtain the rewards which had been offered for the diſcovery

1326 SETH WARD, biſhop of Saliſbury, was born April 15, 1617 He was for ſome time of Sidney College in this Univerſity and afterwards went to Oxford, where he was made Savilian Profeſſor of Aſtronomy In inveſtigating the place of a planet, he ſuppoſed that the motion of a body in an ellipſe was uniform about the other focus (that focus in which the ſun is not), it appears however that BULLIALDUS had advanced the ſame eight years before, and it is ſaid that ALBERT CURTIUS firſt ſuggeſted it It is called however WARD's *Hypotheſis* He publiſhed a Diſcourſe concerning Comets, an Enquiry into the Principles of BULLIALDUS's philolaic Aſtronomy on Trigonometry and on Geometrical Aſtronomy He died in 1689

1327 ANDREW TACQUET was born at Anvers, and wrote ſome good elementary things in Aſtronomy He died in 1660

1328 THOMAS STREET an Engliſhman, wrote a Treatiſe entitled *Aſtronomia Carolina*, which were in uſe for a long time They firſt appeared in 1661, and an edition of them was publiſhed by Dr HALLEY in 1710 He conſtructed the *Logiſtic* logarithms

1329 AZOUT

1329 AZOUT was the inventor of the micrometer with a moveable wire, and he and PICARD applied teleſcopes to quadrants He alſo made many obſervations which are recorded by MONNIER in his *Hiſtoire Celeſte* He died in 1691

1330 ALPHONSUS BORELLI was born at Naples in 1608, and in the year 1666 he publiſhed a Theory of the Satellites of Jupiter

1331 VINCENT WING born in Rutlandſhire in 1619, publiſhed a work entitled, *Aſtronomia Britannica*, containing many uſeful Tables and obſervations

1332 NICOLAS MERCATOR was born at Holſtein He publiſhed his Coſmography in 1651, his Aſtronomical Inſtitutes in 1676, and his Logarithmotechnica in 1678

1333 BULLIALDUS was born at London in 1605 He made many aſtronomical obſervations and publiſhed a valuable work, entitled, *Aſtronomia Philolaica* He attempted to explain the three inequalities of the moon, according to the idea of HORROX

1334 MICHEL LANGRENUS of Anvers, mathematician to PHILIP IV king of Spain diſtinguiſhed himſelf by his obſervations on the ſpots of the moon, theſe he made ſubſervient to finding the longitude of places on the earth's ſurface, by obſerving, in a lunar eclipſe, the times at which the ſpots entered the ſhadow

1335 JOHN HEVELIUS was born at Dantzic, on January 28, 1611 In 1641, he founded an obſervatory, and furniſhed it with the beſt inſtruments that could be procured Some of them were divided into every five ſeconds by a diviſion ſimilar to that of the vernier Of theſe he gave a deſcription in his *Machina Cæleſtis* The obſervatory with all the inſtruments and books which were in it were deſtroyed by fire, on September 26, 1679 The damage was eſtimated at 30000 crowns The ſecond part of his *Machina Cæleſtis* is very ſcarce nearly the whole impreſſion having been burnt He publiſhed a great work, entitled *Selenographia* or a deſcription of the face of the moon and its ſpots, with very fine engravings He completed the explanation of the libration of the moon which was begun by GALILEO adding that of the longitude Dr HALLEY went to ſee him in 1679, and was charmed with the accuracy of his obſervations He publiſhed his *Cometographia* in 1668, containing a catalogue of all the comets which had been obſerved, with many new obſervations and curious reſearches reſpecting their nature He publiſhed a work called *Prodromus Aſtronomiæ et novæ Tabulæ ſolares una cum integro fixarum Catalogo*, from his own obſervations, his catalogue contains 1888 ſtars He died January 28, 1687

1336 HUYGENS

1336 **HUYGENS** was born in 1629. His firſt work was his *Syſtema Saturnium*, in which he has explained the various appearances of Saturn's ring, and in the ſame work he announced the diſcovery of a ſatellite of the ſame planet. In a work entitled *Horologium oſcillatorium*, he explained the method of applying pendulums to clocks, ſo that the time of all the vibrations ſhould be equal. He was the firſt who inveſtigated the center of oſcillation.

1337 **ROBERT HOOK** was born in 1635. He invented the zenith ſector in order to diſcover whether the earth had any ſenſible annual parallax, and this led to the diſcovery of the aberration of light in the fixed ſtars. He alſo diſcovered a ſpot upon *Jupiter* in 1664, and made many other obſervations. He was the firſt who formed the idea of making a quadrant to take angles by reflexion.

1338 **DAVID GREGORY** was profeſſor of Aſtronomy at Oxford. He publiſhed a Syſtem of Aſtronomy, in which he explained ſome parts of Sir I. **NEWTON**'s *Principia*. He died in 1708.

1339 **WILLIAM WHISTON** was Lucaſian profeſſor of mathematics at Cambridge. He publiſhed his Theory of the Earth in 1696, and in 1707, he publiſhed his Lectures on Aſtronomy.

1340 **JOHN DOMINIC CASSINI** was born at Perinaldo on June 8, 1625. In the ſetting out of his ſtudies, a book of Aſtronomy fell into his hands, which greatly attracted his attention, and led him to the purſuit of that ſcience; and he was very early appointed profeſſor of Aſtronomy at Bologna. Here he found a meridian line in the church of St. Petronia, but it not being correct, he obtained leave to rectify it. With this he determined the obliquity of the ecliptic, and the quantity of the refraction of the air. He alſo diſcovered with it, the unequal motion of the ſun, and conſtructed new ſolar Tables. He reſolved the problem, to find the elements of the orbit of a planet from three obſervations. He taught the method of calculating eclipſes, and conceived a projection which ſerved to find the longitude of places upon the earth's ſurface. He compoſed a work upon comets, from thoſe which he obſerved in 1664, and 1665. In 1665 he diſcovered the rotation of *Jupiter* and *Mars* about their axes, by means of their ſpots, and afterwards he diſcovered the rotation of *Venus*. He employed himſelf upon the theory of Jupiter's ſatellites, and publiſhed Tables of their motions. And from comparing the obſervations of their eclipſes with the times as calculated from theſe Tables, they were found to agree much better than was expected. At the requeſt of **LEWIS** XIV he went and ſettled at Paris, and the Royal Obſervatory was entruſted to his care, and in 1671 he began to make regular obſervations. In the *Elemens d'Aſtronomie*, by his ſon, we find that theſe obſervations were upon

upon the equinoxes, ſolſtices, oppoſitions and conjunctions of the planets. In 1672, he determined the parallax of the ſun to be 9″ and in the years 1671, 1672, and 1684, he diſcovered four of the ſatellites of Saturn, and further obſervations ſhowed him that the fifth ſatellite diſappeared regularly for about half a revolution, when it was to the eaſt of Saturn, from this he concluded that it revolved about it's axis and Sir I. NEWTON further concluded, that the time of it's rotation was equal to the time of it's revolution about Saturn. He obſerved a belt upon Saturn and a black mark upon the ring parallel to it's edge, dividing it into two equal parts. He alſo gave new Tables of the ſatellites of Jupiter and diſcovered that the duration of the eclipſes of theſe ſatellites was variable increaſing in length for three years, and then decreaſing in length for the ſame time, this proved to him that their orbits were inclined to the orbit of Jupiter. Upon examining the diſc of Jupiter he diſcovered that it was not a circle and by meaſuring the diameters he made them differ the fifteenth part of the whole. He gave a new theory of the libration of the moon, which he explained by two movements about two different poles. In the year 1683, he diſcovered the zodiacal light. Theſe great and important diſcoveries form an epoch in the ſcience of Aſtronomy. He died September 14, 1712, leaving for his ſucceſſor his ſon JOHN JAMES CASSINI.

1341 JOHN PICARD was born at Anjou. He and AUZOUT were the firſt who put teleſcope to quadrants. In the year 1669, he gave a meaſurement of the earth and in 1673, he eſtabliſhed the Royal Obſervatory, which had been committed to his care. In the *Hiſtoire Celeſte*, the reader may ſee his ideas for improving Aſtronomy. He publiſhed the *Connoiſſance des Temps* for 1679, and died in the year 1682.

1342 ROMER, or ROEMER, was born in Denmark in the year 1644, and came to France in 1672. He diſcovered that light was progreſſive, from the eclipſes of Jupiter's ſatellites. In 1681, he returned to Copenhagen, and died in 1710.

1343 PHILIP DE LA HIRE was born at Paris in the year 1640 and in 1687, he publiſhed his aſtronomical Tables, he likewiſe made a great number of aſtronomical obſervations.

1344 JOHN FLAMSTEAD, the celebrated Engliſh Aſtronomer was born at Derby Auguſt 19, 1646. In 1669, he calculated ſome eclipſes of the fixed ſtars by the moon, and ſent them to the Royal Society, and received the thanks of that body. In 1673 he wrote a ſmall tract on the diameters of the planets, when at their greateſt and leaſt diſtances from the earth "which I lent, ſays he, to Mr NEWTON in 1685 who has made uſe of it in his fourth book of his *Principia Phil Nat Mathemat*." By the time he was

26 years old he explained the true principles of the equation of time, a thing of the firſt importance in Aſtronomy. CHARLES II. having built an obſervatory at Greenwich, Mr FLAMSTEAD was appointed Aſtronomer Royal in the year 1676, at the recommendation of Sir I. MOORE. A deſcription of the inſtruments with which the obſervatory was furniſhed is given in the *Prolegomena* to the third volume of the *Hiſtoria Cæleſtis* which was publiſhed in 1725. A volume of his obſervations were publiſhed in 1712 by Dr HALLEY, by the order of queen ANN: this diſpleaſed FLAMSTEAD, and he ſet about preparing his obſervations for the preſs; but he did not live to finiſh the work, and it was printed after his death. The firſt volume of this great work contains the obſervations which he made firſt at Derby and afterwards at Greenwich upon the fixed ſtars, planets, comets, ſpots of the ſun, and the ſatellites of Jupiter, during 33 years. The ſecond volume contains the paſſage of the fixed ſtars and planets over the meridian with the places of the planets deduced from them. The third volume contains a prolegomena on the hiſtory of Aſtronomy, giving a deſcription of the inſtruments uſed by TYCHO and himſelf, a catalogue of the fixed ſtars of PTOLEMY, of ULUGH BEIGH, of TYCHO, of the Landgrave of Heſſe, of HEVELIUS, of the ſouthern ſtars which had not been obſerved above our horizon; and laſtly, the Britiſh catalogue of 2884 ſtars, with their right aſcenſions, north polar diſtances, latitudes, and longitudes, and the annual variations of the right aſcenſions and north polar diſtances. To theſe are added ſome aſtronomical Tables conſtructed by ABRAHAM SHARP. This great work is an invaluable treaſure to Aſtronomers. His *Atlas Celeſtis* was publiſhed in 1753. He gave new ſolar Tables, and a theory of the moon according to HORROX. He alſo publiſhed a Treatiſe on the doctrine of the ſphere, in which he ſhowed how to conſtruct eclipſes of the ſun and moon, and occultations of the fixed ſtars by the moon. Dr HALLEY's Tables and Sir I. NEWTON's theory of the moon were founded on Mr FLAMSTEAD's obſervations. This great Aſtronomer died October 31, 1719.

1345. Sir I. NEWTON, the founder of phyſical Aſtronomy, was born December 25, 1642. In the year 1660, he was admitted at Trinity College, Cambridge, and in 1667, was choſen fellow of that ſociety, and in 1669, he was elected Lucaſian profeſſor of mathematics, upon the reſignation of Dr BARROW. His great work, entitled *Philoſophiæ Naturalis Principia Mathematica* was firſt publiſhed in 1686, and another enlarged edition was publiſhed in 1713, with a preface by COTES. In this work he unfolds the law of attraction, and ſhows how it will ſolve the motions and the principal phænomena of the different bodies in the ſyſtem. And the ſame principles have been ſince further applied, and ſhown to be competent to account for all the ſmall inequalities of the motions of the heavenly bodies. His philoſophy is founded upon experiment

ment and demonſtration, and therefore its truth cannot be controverted. His Treatiſe on Optics alone would have immortalized him. To enumerate his various diſcoveries, and the extent to which his principles lead, is here unneceſſary, as they have already appeared in the courſe of this work. His inventions alſo in pure mathematics are well known to have been no leſs important than thoſe in philoſophy; and by a union of theſe, the progreſs of ſcience ſeems to be unbounded. His deep inſight into nature led him but the more to adore its AUTHOR. He ſpent a conſiderable part of his life in examining the ſacred records, and that examination confirmed him in his belief of the relations contained in them. He died March 10, 1727, and was buried in Weſtminſter Abbey.

1346. ROGER COTES was born at Burbage in Leiceſterſhire, July 10, 1682. He was educated at Trinity College, Cambridge, and in the year 1706, was elected *Plumian* Profeſſor of Aſtronomy and Experimental Philoſophy, being the firſt who was appointed to that office. In the year 1713, he publiſhed a ſecond edition of Sir I. NEWTON's *Principia*, and inſerted all the author's improvements. To this edition he prefixed a preface, in which he explained the true method of philoſophizing, ſhowed the foundation on which the Newtonian philoſophy was built, and refuted the objections which had been made to it. This extraordinary man died in 1716.

1347. JAMES PHILIP MARALDI was born Auguſt 21, 1665. He determined the retrograde motion of the nodes of Jupiter, and the progreſſive motion of its aphelion. He alſo corrected the theory of Mars. In 1704 he perceived that the motion of Saturn was diminiſhing, and in 1714 he gave a full explanation of the phænomena of its ring. From obſervations on the eclipſes of Jupiter's ſatellites, he concluded that the inclinations of their orbits were ſubject to a variation. This enquiry was purſued till his death, which happened in 1729. He left a nephew JOHN DOMINIC MARALDI, who obſerved a variation in the inclination of the orbit of the third ſatellite, and an excentricity of the orbit of the fourth.

1348. At this time lived Mr. POUND. He meaſured the diameters of Jupiter, and found them to be as 12 : 13, and publiſhed new Tables of the firſt ſatellite, for the computation of its eclipſes, making the equations all additive. He rectified the motions of the ſatellites of Saturn, and made ſome accurate obſervations on them. He alſo made further aſtronomical obſervations, as may be ſeen in the *Phil. Tranſ.*

1349. About the ſame time lived GABRIEL PHILIP de la HIRE, who examined the motions of Jupiter, and found that the progreſſive motion of its aphelion, as given in the Rodolphine Tables, was too ſlow. He alſo conſtructed

ſtructed a ſyſtem of croſs wires in the focus of the object glaſs of a teleſcope, for the obſervation of eclipſes.

1350. FRANCIS BIANCHINI, born at Verona on December 13, 1662, publiſhed a work on the rotation of *Venus*. He alſo made a great many obſervations.

1351. De LOUVILLE was born July 14, 1671, and made many aſtronomical obſervations at Paris. He was the firſt who applied the micrometer to the quadrant.

1352. The celebrated Aſtronomer Dr. EDMUND HALLEY was born November 8, 1656, and entered at Oxford at the age of 17 years. Two years after, he publiſhed a direct and geometrical method of finding the aphelia and excentricities of the orbits of the planets. At the age of 20 years he went to St. Helena to make a catalogue of the ſouthern ſtars, which he publiſhed in 1679. During his ſtay there he obſerved the tranſit of *Mercury* over the ſun's diſc, and that ſuggeſted to him the idea of finding the parallax of the ſun by the tranſit of *Venus* over its diſc; this important problem he ſolved, and recommended to future Aſtronomers to put it in practice. Had he done nothing elſe in Aſtronomy, this would have immortalized him. In 1679, he made a viſit to HEVELIUS, with whom he ſtayed and obſerved for ſome time, and returning home, he ſoon after ſet out to make a tour upon the continent with Mr. R. NELSON, his ſchool fellow. In his way from Calais to Paris, he obſerved the remarkable comet in its aſcent from the ſun, which he had before obſerved in its deſcent. Upon his return, he married and ſettled at Iſlington, where he ſet up his aſtronomical inſtruments. In 1683, he publiſhed his *Theory of the Variation of the magnetical Compaſs*, in which he ſuppoſes that the earth has four magnetic poles. In 1684, he turned his thoughts to the ſubject of the relation between the periodic times and diſtances of the planets, and concluded from it that the centripetal force muſt vary inverſely as the ſquares of the diſtances; but not being able to prove it, he applied to Mr. HOOK and Sir CHRISTOPHER WREN; they however not being able to give him ſatisfaction, he went to Cambridge to Mr. (afterwards Sir ISAAC) NEWTON, who ſoon gave him a proof of his poſition. Dr. HALLEY becoming acquainted with Mr. NEWTON, he perſuaded him to publiſh his *Philoſophiæ Naturalis Principia Mathematica*, and undertook the care of the publication. In 1685, he publiſhed the method of finding altitudes by the barometer; and in the next year came out his account of the trade winds and monſoons. He alſo publiſhed a map, repreſenting their directions. In the year 1687, he undertook to explain the reaſons why the Mediterranean Sea is not obſerved to ſwell, notwithſtanding there is no viſible diſcharge of the pro digious quantity of water which runs into it from ſo many large rivers, and the conſtant ſetting in of the current from the ſtreight. He conſtructed equations of three and four dimenſions, and gave a rule for approximating to the roots of equations. He next undertook to publiſh a correct Ephemeris for 1688. In the beginning of 1691 he publiſhed Tables of the conjunctions of *Mercury* and *Venus* with the ſun. The next year he produced his Tables for ſhowing the value of annuities for life, founded on the bills of mortality, and ſoon after, he publiſhed his univerſal theorem for the foci of lenſes. Wiſhing to make obſervations in order to determine the variation of the needle, he applied to king WILLIAM III. who appointed him captain of a veſſel with proper aſſiſtants. He croſſed the line and proceeded as far as 52 ſouth latitude, and in his way back he touched at St. Helena, the coaſt of Brazil, Cape Verd, Barbadoes, Madeira, the Canaries, the coaſt of Barbary, &c. And on his return home, he publiſhed a chart with curve lines denoting the variation of the compaſs. Soon after this he went out to obſerve the courſe of the tides in the Britiſh channel, with the ſituations of the principal head lands. Upon the death of Dr. WALLIS, he was appointed Savilian Profeſſor of Geometry at Oxford, and, by requeſt, he tranſlated Apollonius from the Arabic into Latin. In 1705, he announced the return of a comet in the year 1759, which happened accordingly within about a month of the time he predicted. He had the glory of being the firſt who foretold an event of this kind, and it is the only one which has been predicted and the prediction fulfilled. He publiſhed a *Synopſis of the Aſtronomy of Comets*. In 1713, he was appointed ſecretary to the Royal Society. As perfecting the theory of the moon was his great object, he was now determined to complete it, and in 1715, he finiſhed it, ſo far as regarded the ſyzygies; ſo that his calculation of eclipſe anſwered to a degree of accuracy which had never been before experienced. His reputation was now ſo great, that upon the death of Mr. FLAMSTEAD in 1719, he was appointed the Aſtronomer Royal at Greenwich, in which year he publiſhed new Tables of the ſun, moon and planets. This gave him an opportunity of completing the theory of the moon's motion. He therefore immediately fixed up a tranſit-inſtrument, and began his obſervations; and though he was then in the 64th year of his age, yet he attended to obſerve the moon's tranſit for 18 years afterward; in the firſt nine years of which he made 1500 obſervations, which he announced to the public, and ſhowed how they tended to correct the theory of the moon. In the year 1725, he procured a mural quadrant, with which he alſo obſerved. Upon the acceſſion of GEORGE II. to the throne, his conſort, Queen CAROLINE, made a viſit to the Royal Obſervatory at Greenwich, and was much pleaſed with the reception ſhe there met with; and Dr. HALLEY having formerly ſerved as a captain in the navy, ſhe obtained for him his half pay for that commiſſion, which he enjoyed during life An offer was made him of being mathematical preceptor to the Duke of Cumberland but he declined that honour by reaſon of his age and alſo as it would interfere with his duty at the obſervatory In 1729 he was admitted a foreign member of the academy of ſciences at Paris in the room of Signior BIANCHINI After his death (which happened the 14th of January 1742) M MAIRAN read an elogy on him before the academy, in which he ſpeaks of the univerſality of his genius as comprehending a knowledge of almoſt all the ſciences, aſtronomy, geometry, algebra optics artillery ſpeculative and experimental philoſophy, natural hiſtory, antiquities, philology and criticiſm, abounding with ideas new, ſingular and uſeful And concludes with obſerving, that he had all the qualifications neceſſary to recommend him to the attention of princes, and the applauſe of the learned He was buried at Lee near Greenwich

1353 JOHN JAMES CASSINI (ſon of JOHN DOMINIC CASSINI, before mentioned) was born at Paris February 18, 1677 and died April 15, 1756 He publiſhed a Syſtem of Aſtronomy, with aſtronomical Tables, a very valuable work A great part of this was founded upon the obſervations of his father He alſo publiſhed many other things in the different Memoirs CÆSAR FRANCIS CASSINI de THURY his ſon died in 1784, after having made a great many uſeful obſervations in Aſtronomy JOHN DOMINIC CASSINI (ſon of M de THURY) is now at the obſervatory of Paris

1354 BOUGUER was born at Croiſic February 10, 1698 His Treatiſe on the figure of the earth is a valuable work He, GODIN, and De la CONDAMINE went to South America to meaſure a degree, and in order to put the doctrine of univerſal attraction to the teſt, they found that the Cordilleras actually attracted the plumb line and drew it ſenſibly from its perpendicular poſition

1355 MAUPERTIUS was born at St Malo September 28, 1698, and is celebrated for his journey to Lapland in order to meaſure a degree of latitude His colleagues were CLAIRAUT, CAMUS, Le MONNIER, the Abbe OUTHIER and CELSUS MAUPERTIUS publiſhed alſo the Elements of Geography, and Nautical Aſtronomy He died in 1759

1356 De la CAILLE was born in 1713, and was one of the firſt Aſtronomers of his time He publiſhed an Ephemeris, Tables of the Sun, a Catalogue of the fixed ſtars on Parallax, Refraction, and the Figure of the Earth, on Comets and Eclipſes His obſervations may be found in the *Memoires de l Academie* He went to the Cape to make obſervations in order to determine, in conjunction with thoſe made in Europe, the parallax of the moon And from theſe obſervations he alſo determined the quantity of re ſtruction His Aſtronomical Leſſons were publiſhed in 1746 This celebrated Aſtronomer died in 1762

1357 PETER HORREBOW made a great many aſtronomical obſervations and publiſhed his *Clavis Aſtronomiæ*, and *Baſis Aſtronomiæ* He died at Copenhagen in the year 1764

1358 JOSEPH NICHOLAS de l'ISLE was born at Paris in 1688 By his reſearches, calculations and obſervations he contributed much to the progreſs of Aſtronomy He died in 1768

1359 The celebrated Engliſh Aſtronomer JAMES BRADLEY was born in 1692 He immortalized himſelf by two of the moſt delicate and important diſcoveries that ever were made in Aſtronomy, the aberration of light in the heavenly bodies and the nutation of the earth's axis the former of which he ſhow'd to ariſe from the progreſſive motion of light and of the earth in it's orbit and the latter from the attraction of the moon upon the protuberant parts of the earth above that of it's inſcribed ſphere He obſerved the comets which appeared in 1723, 1736 1743 and 1757 and computed the elements of their orbits He conſtructed new Tables of *Jupiter's* ſatellites from his own obſervations and thoſe of Mr POUND On the death of Dr HALLEY in 1742 he ſucceeded him at the obſervatory at Greenwich, from which time to that of his death, he was indefatigable in obſerving the ſun, moon, planets and fixed ſtars He ſettled the quantity and laws of refraction to a great degree of accuracy and gave a very elegant rule for correcting the mean refraction from the variation of the weight and temperature of the air In the year 1750, he procured a very fine tranſit inſtrument to be made for the obſervatory, by Mr BIRD and alſo a mural quadrant of braſs of eight feet radius With theſe inſtruments he continued his obſervations till the time of his death, which happened in 1762 The firſt volume of theſe obſervations is juſt now publiſhed by Dr HORNSBY, profeſſor of Aſtronomy at Oxford who, on account of his health, has conſigned the publication of the remaining part to Mr ROBERTSON Savilian profeſſor of geometry

1360 TOBIAS MAYER was born at Marbach in Wirtemberg, February 17, 1723 His firſt obſervations were made at Nuremberg afterwards he went to Gottingen, where he continued to obſerve with very excellent inſtruments His great object was to conſtruct correct Tables of the moon for which purpoſe he compoſed a very elegant theory, with which, and his obſervations he formed new and very correct Tables of the motion of the moon A copy of theſe in 1755 were ſent here to the Right Honourable the Lords Commiſſioners of the Admiralty, putting in a claim for the reward offered for the diſcovery of the longitude Dr BRADLEY compared them with his own accurate obſervations, and was convinced of the excellency of the Tables But

MAYER

MAYER continued correcting them till the time of his death (which happened in 1762) and left behind him a more complete ſet of lunar Tables, and alſo a very correct ſet of ſolar Tables For theſe his widow received £3000 He alſo left a catalogue of the fixed ſtars A volume of his poſthumous work was publiſhed in 1771 and it is to be hoped that the world will be favoured with thoſe which remain unpubliſhed

1361 M **WARGENTIN**, an excellent Aſtronomer of Sweden, publiſhed a ſet of Tables for computing the times of the eclipſes of *Jupiter's* ſatellites, in the *Upſal Acts* for 1741, and ſince that time he has applied various corrections to them

1362 Of thoſe who have written upon phyſical Aſtronomy M **CLAIRAUT** **D'ALEMBERT**, **EULER**, **MAYER**, **FRISI**, **SIMPSON** **LA PLACE** and M de la **GRANGE** are the moſt eminent By the labours of theſe celebrated mathematicians, the Tables of the motions of the bodies in our ſyſtem have been corrected to an extreme degree of accuracy, and their names will go down to poſterity, as completing that ſuperſtructure of which the **GREAT NEWTON** laid the foundation

1363 Upon the death of Dr **BRADLEY**, Mr **BLISS**, Savilian profeſſor of Aſtronomy at Oxford, ſucceeded him at the obſervatory at Greenwich, who lived there but a very little time dying in the year 1765

1364 M **BAILLY** publiſhed at Paris the Hiſtory of ancient and modern Aſtronomy We are alſo indebted to him for a valuable Treatiſe on the theory of Jupiter's ſatellites, which was printed in the year 1766

1365 Dr **LONG**, maſter of Pembroke Hall, Cambridge, and Lowndian Profeſſor of Aſtronomy, publiſhed a Treatiſe on Aſtronomy in five books He alſo conſtructed a ſphere of 17½ feet diameter, in which there is a floor ſo ſuſpended, that the ſphere has a free motion about it's axis On the concave ſurface the conſtellations are painted The mechaniſm is very ſimple and ingenious He died in 1770, in the 90th year of his age

1366 In the year 1739, Mr **DUNTHORNE** publiſhed his *Aſtronomy of the Moon*, the Tables were conſtructed from Sir I **NEWTON'S** theory, to which he added precepts for calculating eclipſes

1367 In 1741, M le **MONNIER** publiſhed his *Hiſtoire Celeſte*, containing a collection of obſervations from the year 1666 to 1685, made by order of the king, with a preliminary diſcourſe

1368 M **PINGRE** publiſhed a very valuable work, entitled *Cometographie*, containing the hiſtory and theory of comets This was printed in 1783, and 1784

1369 To that celebrated Aſtronomer M de la **LANDE** the world is indebted for the moſt important improvements in the ſcience of Aſtronomy

Though

Through ſo extenſive a field he has left no track unbeaten almoſt every part has received improvements from him but we cannot here enter into a detail of them His Syſtem of Aſtronomy is invaluable and has tended far more to the general promotion of that ſcience than all other works which ever appeared upon the ſubject The labours of this great Aſtronomer will perpetuate his name

1370 For the diſcovery of a ſeventh primary planet we are indebted to Mr (now Dr) HERSCHEL By his great ſkill and induſtry in the conſtruction of very large ſpecula, he has made teleſcopes which have opened new views of the heavens, and penetrated into the depths of the univerſe unfolding ſcenes which excite no leſs our wonder than our admiration To this new planet he has diſcovered ſix ſatellites, and alſo two more belonging to *Saturn*, thus he has added nine bodies to our ſyſtem ! The various and intereſting diſcoveries of this illuſtrious Aſtronomer the reader may ſee in the *Philoſophical Tranſactions*, they are ſuch as muſt tranſmit his name to the lateſt poſterity

1371 Dr MASKELYNE ſucceeded Dr BLISS at the obſervatory at Greenwich To the abilities and indefatigable attention of this celebrated Aſtronomer, nautical Aſtronomy is altogether indebted for its preſent ſtate of perfection Of our *Nautical Almanac*, that great Aſtronomer M de la LANDE, thus writes "On a fait a Bologne, a Vienne, a Berlin, a Milan, mais le Nautical Almanac de Londres eſt l ephemeride la plus parfaite qu il y ait jamais eu" He has eſtabliſhed the Newtonian doctrine of univerſal attraction upon the firmeſt foundation, by his experiments upon Schehallien His regular obſervations of the ſun moon planets, and fixed ſtars, which are every year publiſhed, are allowed to poſſeſs an unrivalled degree of accuracy, and we may conſider them as the baſis of future improvements of the Tables of the planetary motions M de la LANDE in his Aſtronomy (Vol ii p 121 laſt edit) ſpeaking of aſtronomical obſervations, ſays, ' Le recueil le plus moderne et le plus precieux de tous eſt celui de M MASKELYNE Aſtronome Royal d'Angleterre, qui commence a 1765, et qui forme deja deux volumes in folio juſqu a 1786 La preciſion de ces obſervations eſt ſi grande, qu on trouve ſouvent la meme ſeconde pour l aſcenſion droite d une planete deduite de differentes étoiles quoiqu on y emploie la meſure du temps His catalogue of fundamental ſtars is an invaluable treaſure Theſe and his other various important improvements in this ſcience entitle him to the moſt diſtinguiſhed rank amongſt Aſtronomers, and will render his name illuſtrious, as long as the ſcience of Aſtronomy ſhall continue to be cultivated

We muſt leave it to poſterity to do ample juſtice to thoſe whoſe labours are not yet at an end

CONCLUSION.

Art. 1372. IT has been juſtly obſerved, that the knowledge of nature is the firmeſt bulwark againſt Atheiſm. At a time therefore when infidelity ſo much prevails, and when even philoſophy has been charged with promoting it, it may be proper, in order to ſtop the prevalence of the former, and refute the unjuſt charge againſt the latter, after having treated on the ſyſtem of the univerſe, to take ſome notice of thoſe extraordinary marks of deſign in it's conſtruction, which prove ſo clearly that it could not have owed it's formation to chance, but to the contrivance of infinite WISDOM. The DEITY can only be known by his works, and the works of the creation afford a very convincing proof of a ſupreme BEING, who formed ſuch vaſt bodies, and 'gave them laws that ſhould not be broken.' If we trace not the cauſe from the effect, we neglect to direct our knowledge to that end to which all our enquiries into nature ought to tend. From the works of GOD, we muſt ſeek to know him. Let us not deny the being of a ſupreme INTELLIGENCE, who is the cauſe of all things, becauſe he is not the object of our corporeal ſenſes. "he has not left himſelf without witneſs." his exiſtence and attributes are manifeſt from the conſtruction of the univerſe, and the ends for which it was formed, but the nature of his eſſence ſurpaſſes the conception of our limited faculties. "We ſee but in part."

The obvious argument for the exiſtence of a DEITY, who formed and governs the univerſe, is founded upon the uniformity of the laws which takes place in the production of ſimilar effects, and from the ſimplicity of the cauſes which produce the various phænomena. The moſt common views of nature, however imperfect and of ſmall extent, ſuggeſt the idea of the government of a GOD, and every further diſcovery tends to confirm that perſuaſion. The ancient philoſophers, who ſcarce knew a ſingle law by which the bodies of the ſyſtem are governed, ſtill ſaw the DEITY in his works, how viſible therefore ought he to be to us, who are acquainted with the laws by which the whole is directed. The *ſame* law takes place in our ſyſtem between the periodic times and diſtances of every body revolving about the ſame center. Every body deſcribes about it's reſpective center equal areas in equal times. Every body is ſpherical. Every planet, as far as our obſervations reach, is found to revolve about an axis, and the axis of each is obſerved to continue parallel to itſelf. Now as the

circum

circumſtances which might have attended theſe bodies are indefinite in variety, the *uniform ſimilarity* which is found to exiſt amongſt them, is an irrefragable argument of deſign. To produce a ſucceſſion of day and night either the ſun muſt revolve every day about the earth, or the earth muſt revolve about it's axis; the latter is the more ſimple cauſe, and accordingly we find that the regular return of day and night is ſo produced. As far alſo as obſervations have enabled us to diſcover, the return of day and night in the planets is produced by the operation of a ſimilar cauſe. It is alſo found that the axis of each planet is inclined to the plane of it's orbit, by which a proviſion is made for a variety of ſeaſons, and by preſerving the axis always parallel to itſelf, ſummer and winter return at their ſtated periods. Where there are ſuch inconteſtable marks of deſign, there muſt be a DESIGNER, and the unity of deſign through the whole ſyſtem proves it to be the work of ONE. The general laws of nature ſhow the exiſtence of a divine INTELLIGENCE, in a much ſtronger point of view, than any work of man can prove him to have acted from intention, inaſmuch as the operations of the former are uniform and ſubject to no variation, whereas in the latter caſe, we ſee continual alterations of plan, and deviations from eſtabliſhed rules. And without this permanent order of things, experience could not have directed man in reſpect to his future operations. Theſe fixed laws of nature, ſo neceſſary for us, is an irreſiſtible argument that the world is the work of a wiſe and benevolent BEING. The laws of nature are the laws of GOD; and how far ſoever we may be able to trace up cauſes, they muſt terminate in his will. We ſee nothing in the heavens which argues imperfection; the whole creation is ſtamped with the marks of DIVINITY.

We can form no idea of that power called *Attraction*, by which diſtant bodies are made to act upon each other without any apparent connection; and yet we know that all the bodies in our ſyſtem are retained in their courſes by ſuch a power. And it is a very ſingular inſtance of the unerring wiſdom of the CREATOR, that the law which this power obſerves is ſuch, that notwithſtanding the mutual attractions of the bodies, the ſyſtem will never fall into ruin, but is capable of preſerving itſelf to all eternity. Moreover, the mutual attraction which takes place between diſtant bodies could not of itſelf, either produce their motion about the ſun, nor the rotation about their axes; it required an external impulſe to operate in conjunction with it, to produce theſe effects, an act which nothing but the arm of OMNIPOTENCE could accompliſh. And the power which thus connects the diſtant bodies, operates alſo on the conſtituent particles of the ſame body, and preſerves it's figure; for without attraction, the particles muſt have been diſſipated by their rotation. An inviſible power pervades the whole ſyſtem, and preſerves it

In the effects produced by man, we ſee the operation of the cauſe, but "the ways of the ALMIGHTY are paſt finding out."

The ſun, that great and only fountain of light and heat, is placed in the center of the ſyſtem; and whilſt by it's influence it retains the planets in their orbits, it pours forth it's rays and gives life to the creation. The formation of ſuch a glorious body, and it's arrangement, are circumſtances which afford the cleareſt evidence of deſign.

Hence, in whatever point of view we take a ſurvey of our ſyſtem, we trace the power, wiſdom, and goodneſs of the CREATOR. His power, in it's formation, his wiſdom in the ſimplicity of the means to produce the ends, and his goodneſs in making thoſe ends ſubſervient to our uſe and enjoyment. Thus we are led by our enquiries into the ſtructure of the univerſe, to the proofs of the exiſtence and attributes of a ſupreme BEING who formed and directs the whole. Arguments of this kind produce conviction which no ſophiſtry can confound. 'Every man may ſee it; man may behold it afar off.' Let not therefore the ignorant declaim againſt thoſe purſuits which direct us to a knowledge of our CREATOR, and furniſh us with unanſwerable arguments againſt the Infidel and the Atheiſt.

But if we carry our views up to the firmament of the fixed ſtars, the power of the DEITY will be ſtill more aſtoniſhing. Let a man contemplate the ſtarry heavens, and conſider thoſe glorious bodies only in reſpect to number, magnitude and diſtance, and it can ſcarcely fail to convince him of the exiſtence of an omnipotent BEING. By the late improvements of teleſcopes, the ſtarry ſyſtem appears to be without bounds, and the greater part of theſe bodies not being viſible to the naked eye, we may conclude that they were not made for our uſe, nor for the uſe of any part of our ſyſtem. They are undoubtedly bodies ſimilar to our ſun, appearing ſo ſmall from their immenſe diſtances, for opaque bodies at that diſtance could not be ſeen by reflected light. From the uniformity of nature, in all thoſe parts which we have been able to examine and inveſtigate, we may conclude, that bodies ſimilar to our ſun were created for the ſame cauſe, that of giving light and heat to the inhabitants of ſyſtems of planets ſurrounding them. We may therefore conceive the whole univerſe to be filled with created beings, enjoying the bounty of their CREATOR, and admiring his works. This benevolence of the DEITY in giving life to an almoſt infinite number of beings, muſt raiſe our admiration, till we are loſt in contemplating his goodneſs. That every individual ſhould exiſt under his protection, and be regularly ſupplied by his bountiful hand with every thing which is neceſſary for enjoyment, ought to make us very humble before him. And that every being in the univerſe ſhould be under his care, and training up here for the further enjoyment of him hereafter, is a thought which, if duly impreſſed, would penetrate us with the deepeſt ſenſe of gratitude to our CREATOR, and excite us to love and obedience. The diſappearance of ſome ſtars may be the deſtruction of that ſyſtem at the time appointed by the DEITY for the probation of its inhabitants, and the appearance of new ſtars may be the formation of new ſyſtems, for new races of beings then called into exiſtence to adore the works of their CREATOR. Thus we may conceive the DEITY to have been employed from all eternity, and thus continue to be employed for endleſs ages, forming new ſyſtems of beings to adore him, and tranſplanting thoſe beings already formed into happier regions, where they may have better opportunities of meditating on his works, and ſtill riſing in their enjoyments, go on to contemplate ſyſtem after ſyſtem through the boundleſs univerſe.

TABLES

FOR

FACILITATING

ASTRONOMICAL CALCULATIONS

A GREAT many of theſe Tables are taken from Dr MASKELYNE's firſt volume of his excellent obſervations, but Table VI contains the right aſcenſion of his fundamental ſtars as more accurately ſettled by him from obſervations which he afterwards made. From the improvement of inſtruments and the modern theories of Aſtronomy a great many irregularities of the motions of the bodies in our ſyſtem have been diſcovered, the labour therefore of making computations in Aſtronomy has of late been ſo much increaſed, that it is neceſſary to give every poſſible aſſiſtance to the practical Aſtronomer. The Tables here ſubjoined are intended for that purpoſe, and will be found very uſeful in facilitating aſtronomical calculations.

TABLE I

For converting Degrees Minutes and Seconds into Time at the Rate of 360 Degrees for 24 Hours

Deg Min	Hou Min	Min. Sec	Deg Min	Hou Min	Min Sec	Sec	Dec of Sec
1	0	4	30	2	0	1	067
2	0	8	40	2	40	2	133
3	0	12	50	3	20	3	2
4	0	16	60	4	0	4	,266
5	0	20	70	4	40	5	333
6	0	24	80	5	20	6	4
7	0	28	90	6	0	7	466
8	0	32	100	6	40	8	,533
9	0	36	200	13	20	9	6
10	0	40	300	20	0	10	666
20		20					

TABLE II

For converting Time into Degrees Minutes and Seconds at the Rate of 24 Hours for 360 Degrees

Hou	Deg	Min Sec	Deg Min	Min Sec	Dec of Sec	Sec
1	15	1	0	15	1	15
2	30	2	0	30	2	30
3	45	3	0	45	3	45
4	60	4	1	0	4	60
5	75	5	1	15	5	75
6	90	6	1	30	6	90
7	105	7	1	45	7	105
8	120	8	2	0	8	120
9	135	9	2	15	9	135
10	150	10	2	30		
11	165	20	5	0		
12	180	30	7	30		
16	240	40	10	0		
20	300	50	12	30		

TABLE III

Decimal Parts of an Hour

1	01666		00028
2	03333	2	,00056
3	05		00083
4	06666	4	00111
5	08333	5	00139
6	,1	6	,00167
7	11666	7	00194
8	13333	8	00222
9	15	9	00250
10	16666	10	00277
20	33333	20	00556
30	5	30	,00833
40	66666	40	01111
50	83333	50	,01388

TABLE IV

For finding the Length of Circular Arcs to Radius Unity

Deg	Length	Deg	Length	Min	Length	Sec	Length
1	0 0174533	60	1,0471976	1	0 0002909	1	0 0000048
2	0 0349066	70	1 2217305	2	0 0005818	2	0 0000097
3	0 0523599	80	1 3962634	3	0 0008727	3	0 0000145
4	0 0698132	90	1 5707963	4	0 0011636	4	0 0000194
5	0 0872665	100	1 7453293	5	0 0014544	5	0 0000242
6	0,1047198	120	2 0943951	6	0 0017453	6	0 0000291
7	0 1221730	150	2 6179939	7	0 0020362	7	0 0000339
8	0 1396263	180	3 1415927	8	0 0023271	8	0 0000388
9	0 1570796	210	3 6651914	9	0 0026180	9	0 0000436
10	0 1745329	240	4 1887902	10	0 0029089	10	0 0000485
20	0,3490659	270	4 7123890	20	0 0058178	20	0 0000970
30	0 5235988	300	5 2359878	30	0 0087266	30	0 0001454
40	0 6981317	330	5 7595865	40	0 0116355	40	0 0001939
50	0 8726646	360	6 2831853	50	0,0145444	50	0 0002424

TABLE V

Sun's Parallax in Altitude

Sun's Alt	Sun's Pa
0	8 75
10	8 62
20	8 22
30	7 58
40	6,70
50	5 62
55	5 02
60	4,37
65	[illegible]
70	2 99
75	2 [illegible]9
80	1 52
85	0,76
90	0 00

TABLE VI.

The Mean Right Aſcenſions and North Polar Diſtances of Thirty ſix principal Stars for the Beginning of the Year 1790, with their Annual Preceſſions.

STARS	Mean Right Aſcenſion in Sidereal Time 1790	Annual Preceſſion in Time	Mean Right Aſcenſion in Deg 1790	Annual Preceſſion in Deg	Annual Prop Mot in Time	Annual Prop Mot in Deg	Diſtance from the North Pole	Annual Preceſſion
γ Pegaſi	0ʰ [illegible] [illegible]6 0	+3 071	0 36 32 1	46 06	−0,008	−0 1[illegible]0	7[illegible] 59 4,9	−[illegible]0,0[illegible]
α Arietis	1 55 21 99	3 334	[illegible]8 50 [illegible]9 8	50 01	+0,006	+0 090	67 3[illegible] 15 3	−17 6
α Ceti	[illegible] 51 18 89	3 119	4[illegible] 49 43 3	46 [illegible]8	−0 011	−0 163	86 44 26 7	−1[illegible] 76
Aldebaran	4 [illegible]3 53 08	3 420	63 58 16 [illegible]	51 30	+0 001	+0 015	73 53 33 5	−8 [illegible]6
Capella	5 [illegible] [illegible] 0[illegible]	4 93	73 18 0 3	63 9[illegible]	+0 019	+0 [illegible]85	44 13 59 4	−5 [illegible]1
Rigel	5 4 [illegible]0 87	2 873	76 6 43 0	43 12	−0 008	−0 1[illegible]0	98 [illegible] [illegible]2 6	−4 88
β Tauri	5 13 1 60	3 7[illegible]6	[illegible]8 13 [illegible]4 0	56 64	+0 00[illegible]	+0 030	61 35 9 9	−4 19
α Orionis	5 43 48 18	3 [illegible]40	83 57 [illegible] 7	48 60	−0 001	−0 015	8[illegible] 38 46 3	−1 51
Sirius	6 33 53 44	[illegible] 6[illegible]9	98 58 [illegible]1 6	40 18	−0 03[illegible]	−0 480	106 [illegible]6 21 1	+4 [illegible]5
Caſtor	7 [illegible]1 0 [illegible]6	3 86[illegible]	110 17 9	57 93	−0 007	−0 103	5[illegible] 40 8 3	+6 85
Procyon	7 28 1 [illegible]3	3 193	1[illegible] 4 [illegible]3 9	4[illegible] 89	−0 036	−0 840	8[illegible] 14 33 8	+ [illegible] 45
Pollux	7 32 [illegible]6 31	3 [illegible]3[illegible]	113 6 [illegible] 6	56 05	−0 030	−0 [illegible]30	61 28 53 4	+7 7[illegible]
α Hydræ	9 1[illegible] 15 6[illegible]	[illegible] 930	139 18 54 3	[illegible]4 [illegible]3	−0 013	−0 [illegible]3	97 43 20 6	+15 14
Regulus	9 3[illegible] 9 98	3 [illegible]6	1[illegible]9 1[illegible] [illegible]9 7	48 39	−0 0[illegible]	−0 330	7[illegible] 0 4[illegible] 0	+1[illegible] 10
β Leonis	1[illegible] 38 19 87	104	[illegible] 1 36 0	46 36	−0 01[illegible]	−0 630	[illegible]4 3 1 3	+19 93
β Virginis	11 39 [illegible]3 6	3 [illegible]	[illegible]4 36 1[illegible] 4	46 11	+0 048	+0 [illegible]20	[illegible] 3 2 4	+19 90
Spica Virginis	[illegible] 14 [illegible] 7[illegible]	3 1[illegible]3	98 3[illegible] 11 1	[illegible] 11	−0 006	−0 090	100 3 34 7	+19 0[illegible]
Arcturus	1[illegible] 5 4 09	[illegible] 9 0	[illegible]1 31 [illegible] 8	4[illegible] 13	−0 008	−1 3 0	69 [illegible] [illegible] 9	+19 13

TABLE VI *Continued*

STARS	Mean Right Afcenfion in Sidereal Time 1690	Annual Preceffion in Time	Mean Right Afcenfion in Deg 1690	Annual Preceffion in Deg	Annual Prop Mot in Tim	Annual Prop Mot in Deg	Diftance from the North Pole	Annual Preceffion
α 1 Libræ	14ʰ 39 5 [illegible]	+3,299	219 46 26 2	49 48				
α 2 Libræ	14 39 17 08	3 [illegible]98	219 49 16 [illegible]	49 4[illegible]	−0 009	−0,135	105 9 [illegible]6 8	+1[illegible] 40
α Coronæ Borealis	15 [illegible] 47 86	[illegible]	[illegible]31 [illegible]6 57,9	37 87	+0 018	+0,270	6[illegible] 34 7 1	+1[illegible] 56
α Serpentis	15 33 5[illegible],84	[illegible] 934	23[illegible] 28 57 6	44,01	+0 00[illegible]	+0 030	8[illegible] 54 11 0	+1[illegible] 0
Antares	16 16 [illegible] [illegible]	3,653	244 8 18 6	54 [illegible]9	−0,008	−0,1[illegible]0	115 56 56 7	+ 8 84
α Herculis	17 5 [illegible] [illegible]0	[illegible]	[illegible]56 16 7 5	40 9[illegible]	−0 00[illegible]	−0 045		
α Ophiuchi	17 [illegible]5 11 [illegible]9	2 770	261 17 49 3	41 5[illegible]	−0 00[illegible]	−0 030	7[illegible] 16 23 2	+ 3 12
α Lyræ	18 29 49 61	[illegible] 011	[illegible]7 27 24,1	30 16	+0 017	+0 255	51 24 16 8	− 2 54
γ Aquilæ	19 [illegible]6 16 13	[illegible] 850	294 4 1 9	4[illegible] 75	−0 01[illegible]	−0 195	79 5[illegible] 1[illegible] 8	− 8 10
α Aquilæ	19 40 [illegible] 84	2,891	[illegible]95 7 57,6	43 [illegible]6	+0 0[illegible]7	+0 405	81 40 [illegible]8 [illegible]	− 8 [illegible]
β Aquilæ	19 44 [illegible]9 [illegible]8	2,944	[illegible]96 14 53,7	44,16	−0,005	−0,075	84 6 9 9	− 8 79
α 1 Capricorni	[illegible]0 5 59 49	3 334	301 29 5[illegible] 3	50 0	−0,004	−0,060	10[illegible] 8 45 9	−10 [illegible]9
α [illegible] Capricorni	[illegible]0 6 [illegible],17	3,3[illegible]	[illegible]01 [illegible] 47 5	49 99	−0,00[illegible]	−0 030	103 11 [illegible] 4	−10,42
α Cygni	[illegible]0 34 16 34	2 040	308 34 5 1	0 60	−0,006	−0 090	45 [illegible] 47 1	−12,46
α Aquarii	[illegible]1 54 59 08	3 084	3[illegible]8 44 46 2	46,26	−0 017	−0,[illegible]55	91 19 59,4	−17 09
Fomalhaut	[illegible] 46 0,[illegible]0	[illegible]	341 30 4,5	49 80	+0 010	+0 150	120 [illegible] 48 4	−18 98
α Pegafi	22 54 18 3[illegible]	[illegible],973	343 34 35,2	44,59	−0,009	−0 1[illegible]	75 55 15 9	−19 [illegible]
α Andromedæ	2[illegible] 5[illegible] 33 67	[illegible],060	[illegible]59 2[illegible] [illegible]5 0	45 90	+0 005	+0 075	6[illegible] 4 9 [illegible]	−[illegible]0,04

TABLE VIII

Corrections in Degrees *of the Right Aſcenſions of Thirty one Principal Fixed Stars, to every Tenth Degree of Longitude of the Moon's Aſcending Node.*

Argument Longitude of the ☽'s ☊			Pegaſi	α Arietis	α Ceti	Aldebaran	Capella	Rigel	β Tauri	α Orionis	Sirius	Caſtor	Procyon
S	D	S											
0	0	6	− 2,4 +	− [illegible],4 +	− 0,4 +	− 1,1 +	− [illegible],6 +	+ 0,4 −	− 1,1 +	− 0,1 +	− 0,4 +	+ [illegible],0 −	+ 0,4 −
	10		5,[illegible]	6,5	3,3	4,[illegible]	6,6	− [illegible] +	4,6	3,1	[illegible],9	− 1,6 +	− 2,6 +
	20		7,8	9,3	6,1	7,3	10,4	4,9	7,9	6,0	5,3	5,1	5,5
1	0	7	10,2	11,9	8,7	10,1	13,9	7,4	11,0	8,7	7,5	8,5	8,[illegible]
	10		12,3	14,1	11,0	1[illegible],6	1[illegible],0	9,5	1[illegible],8	11,[illegible]	9,5	11,7	10,7
	20		14,1	15,8	1[illegible],0	14,7	19,6	11,[illegible]	16,1	1[illegible],[illegible]	11,[illegible]	14,5	12,8
2	0	8	15,4	17,1	14,6	16,[illegible]	21,6	13,1	18,0	15,0	12,6	16,8	14,6
	10		16,2	17,9	15,8	17,[illegible]	[illegible],9	14,[illegible]	19,[illegible]	16,3	13,6	18,7	15,9
	20		16,[illegible]	18,1	16,5	[illegible]8,1	[illegible],5	15,0	[illegible]0,0	17,1	14,[illegible]	20,0	16,7
3	0	9	16,4	17,8	16,6	18,2	[illegible],4	15,4	[illegible]0,1	17,3	14,[illegible]	20,6	17,6
	10		15,7	16,9	16,3	1[illegible],8	2[illegible],6	15,[illegible]	19,6	17,0	14,0	20,7	16,8
	20		14,0	15,5	1[illegible],5	16,7	[illegible],1	14,[illegible]	18,5	[illegible]6,[illegible]	1[illegible],3	[illegible]0,1	16,1
4	0	10	1[illegible],0	13,7	14,[illegible]	15,2	10,0	1[illegible],[illegible]	16,9	14,0	[illegible],2	18,9	14,9
	10		11,0	11,4	1[illegible],5	1[illegible],2	16,3	1[illegible],0	14,7	13,2	10,7	1[illegible],1	13,3
	20		8,7	8,8	0,4	10,9	[illegible],1	10,1	1[illegible],1	11,0	8,9	14,8	11,[illegible]
5	0	11	6,1	[illegible],9	8,0	8,1	9	8,0	9,1	8,6	6,8	12,1	8,8
	10		3,4	− 2,8 +	[illegible]	[illegible]	[illegible],6	5,6	5,9	[illegible],8	[illegible],5	9,0	[illegible],2
	20		− [illegible],5 +	+ 0,[illegible] −	− [illegible],5 +	− 2,1 +	− 1,[illegible] +	[illegible],0	− [illegible] +	− [illegible],0 +	− [illegible],1 +	5,6	3,[illegible]
6	0	12	+ [illegible],4 −	[illegible]	+ 0,4 −	+ 1,1 −	+ [illegible],6 −	0,4	+ 1,1 −	+ 0,1 −	+ 0,4 −	2,0	0,4

TABLE VIII *Continued*

Argument Longitude of the ☽'s ☊			Pollux	α Hydræ	Regulus	β Leonis	β Virginis	Spica Virginis	Arcturus	α Libræ	α Coronæ Borealis	α Serpentis
S	D	S										
0	0	6	+ 20 −	− 10 +	+ 19 −	+ 26 −	+ 05 −	− 16 +	+ 30 −	− 20 +	+ 31 −	+ 07 −
	10		− 15 +	37	− 11 +	− 03 +	− 23 +	43	04	50	07	− 20 +
	20		4,9	63	4,1	32	51	72	− 23 +	79	− 17 +	4,7
1	0	7	83	87	70	60	78	98	49	105	4,0	72
	10		113	109	96	87	102	120	73	128	63	95
	20		14,0	127	120	110	122	139	95	14,7	83	115
2	0	8	163	14,1	14,0	131	14,0	15	115	162	101	132
	10		181	151	155	14,7	153	103	131	172	116	14,5
	20		193	157	166	159	161	168	143	177	128	153
3	0	9	200	158	172	166	164	168	150	17,6	135	137
	10		200	134	173	168	163	162	153	170	138	156
	20		194	143	168	165	156	152	15,1	15,9	137	130
4	0	10	183	132	159	157	14,3	137	145	142	132	139
	10		166	114	144	14,4	129	118	135	122	122	124
	20		144	94	125	127	110	96	120	98	111	106
5	0	11	11,7	70	103	10,5	87	0	101	71	93	85
	10		87	4,5	77	81	61	42	80	42	76	60
	20		54	− 18 +	49	54	33	− 14 +	56	− 11 +	54	34
6	0	2	20	+ 10 −	19	26	05	+ 6 −	30	+ 20 −	31	07

TABLE VIII *Continued*

ARGUMENT Longitude of the ☽ s ☊			Antares	α Ophiuchi	α Lyræ	α Aquilæ	α Capricorni	α Cygni	α Aquarii	Fomalhaut	α Pegasi	α Andromedæ
S	D	S	″									
0	0	6	− 0 +	+ 03 −	− 09 +	− 06 +	+ 1 −	− 58 +	+ 02 −	+ 54 −	− 23 +	− 50 +
	10		54	− +	8	32	− 19 +	76	− 6 +	3	5,0	78
	20		86	4,7	46	58	0	9,2	54	− 10 +	7	10
1	0	7	11,5	71	6,	82	79	105	8,1	4,	9,9	12,5
	10		14,1	92	76	104	106	11,	104	73	11,9	14,3
	20		16	11,1	89	1,2	129	1,1	1,	10,	13,6	157
2	0	8	179	1 6	98	1	148	124	14,2	127	14,9	166
	10		190	13,8	10,4	147	163	1,	154	14,8	1 7	1 0
	20		195	14,	108	1	173	117	162	165	60	169
3	0	9	19,5	14,8	10,8	1,4	1,8	109	16	7,8	5,9	16,3
	10		188	14,6	104	5,1	17,7	9,7	16,3	18,4	15,2	15,2
	20		1 6	14,0	98	14,	171	8	156	185	141	136
4	0	10	1 8	1 0	88	131	160	6	14,1	181	126	116
	10		136	11	6	114	14,4	4,6	12	1 1	07	3
	20		09	98	6	9	123	6	10	156	85	66
5	0	11	8,0	7	46	,2	99	− 04 +	,4	1 6	60	8
	10		4,7	4	8	4,7		+ 1, −	,6	11,1	3	− 1,0 +
	20		− 14 +	9	− 09 +	− ,1 +	4,	3	1	84	− 8 +	+ 2, −
6	0	12	+ 2,0 −	03	+ 09 −	+ 06 −	1	,8	02	54	+ 2,3 −	5,0

Day of the Month		γ Pegaſi	α Arietis	α Ceti	Aldebaran	Capella	Rigel	β Tauri	α Orionis	Sirius	Caſtor	Procyon	Pollux	α Hydræ	Regulus	β Leonis	μ Virginis
January	0	−0 19	+0 50	+0 13	+1 -1	+1 17	+1 -5	+1 44	+1 3-	+1 41	+1 59	+1 33	+1 51	+1 0/	+0 9-	+0 39	+0 39
	c	0 30	0 8	0 6	1 13	1 70	1 -+	1 43	1 36	1 4/	1 /+	1 47	1 6/	-9	1 9	0 72	0 70
	~	0 41	0 -3	c 3	1 0	1 69	1 18	1 -	1 35	1, -6	1 83	55	1 7/	1 47	1 41	01	0 96
	30	0 50	9 11	0 ‡1	0 97	1 33	1 09	1, 4	1 29	1 43	1 96	1 58	1 81	1 59	39	1 2	1
F b	9	0 37	−0 03	0 -	0 83	1 9	0 9	1 -1	1 20	1 3/	1 84	1 55	1 79	1 64	1 71	1 50	1 4
	19	0 6-	0 1/	0 1	0 69	1 8	0 81	1 03	1 07	1 24	1 //	1 30	1 74	1 71	1 /9	1 64	1 0
March	1	0 6	0 -8	−0 0	0 3-	0 93	0 63	0 89	0 93	1 09	1 63	1 40	1 63	1 70	1 83	1 81	1 /3
	11		0 38	0 13	0 33	0 70	0 48	0 69	0 /6	0, 9-	1 49	1 -7	1 49	1 65	1 8-	1 91	
	-1		0 -	0 -3	0 19	0 -6	0 31	0 31	0, 59	0 /+	1 31	1-	1 3-	1 56	1 //	1 95	1 89
	1		0 -/	0 3-	0, 06	0 -+	0 14	0 33	0 43	0 56	1 13	0 96	1 15	1 43	1 69	1 9/	1 91
April	10	0 /	0 46	0 33	−0 03	0 04	0 00	0 18	0 -7	0 38	0 93	0 80	0 96	1 3-	1 39	1 9	1 69
	-0	0 -	0 +-		0 1	−0 00	−0 11	0 03	0 15	0 -	0 /3	0 63	0 79	1 19	1 48	1 90	1 8
	30	0 04	0 3		0 17	0 13	0 19	−0 03	0 05	0 08	0 59	0 51	0 6	1 05	1 34	1 83	1 /9
May	0	−0 -	0 18		0 13	0 -	0 -4	0 0/	−0 01	−0 04	0 -6	0 39	0 30	0 91	1 -2	1 74	1 /
	20	0 49	+0 0-	0, 1-	0, 11	0, -0	0 +	0 0/	0 0-	0 13	0 30	0 0	0 40	0 79	1 10	1 64	1 6/
	0	0 /9	0 -8	+0 0/	0 00	0 13	0 19	0 0-	0 00	0 17	0 30	0 -3	0 33	0 69	0 99	1 53	1 3
June	9	1 09	0 33	0 2/	+0 15	0 00	0 09	−0 11	+0 06	0, 1/	0 -8	0 --	0 30	0 59	0 90	1, 43	1 +
	9	1 41	0 63	0 3+	0 33	+0 20	+0 0+	0 -6	0 17	0 13	0 30	0 23	0 3-	0 5	0 8-	1 33	1 34
	-9	1 /	1 17	0 81	0 33	0, 43	0 -0	0 +7	0 31	0 06	0, 3/	0 -8	0 3/	0 48	0 76	1 -3	-+
July	9	- 03	1, 49	1 09	0, 61	0 /3	0 40	0 69	0 49	+0 03	0, 48	0 33	0 46	0 45	0 /-	1 11	
	10	- 33	1 8-	1 39	1 08	1 08	0 63	0, 96	0 70	0 -1	0 6+	0 46	0 61		0 /1	1 06	
	-9	- 60	- 13	1 0	1 37	1 44	0 8/	1 -3	0 93	0, 38	0 8-	0 6-	0 /6		0 71	1 01	
Auguſt	8	2 8	- 47	- 00	1 67	1 8-	1 14	1 57	1 19	0, 58	1 04	0 80	0 9/		0 7+	2 93	
	18	3 06	- /6	- -9	1, 9/	- --	1 11	1 89	1 46	0 81	1 -9	1 00	1 19		0 81	0 90	
	-8	3 2	3 03	2 3/	- -9	- 63	1 /0	- --	1 74	1 06	1 56	1 --	1 44	0 /9	0 91	0 89	
Sept	/	3 3	3 27	- 8	- 38	3 04	1 98	- 53	2 03	1 -	1 86	1 45	1 71	0 94	1 03	0 0-	
	17	3 40	3, +9	3 0/	- 88	3, 44	2, -/	- 89	2 32	1, 60	- 1/	1 /1	- 01	1, 1-	1, 19	0, 96	
	-	3 42	3 6/	3 29	3 16	3 84	- 53	3 21	2 61	1 89	- 50	1 99	- 3-	1 -3	1 3/	1 03	
Oᶜtob r	/	33	3 8-	3 -/	3 43	4, -3	- 81	3 53	-, 90	-, 18	- 64	2 -7	- 64	1, 3/	1 59	1 1/	
	1/	3 33	3 94	3 63	6/	4 39	3 0/	3 83	3 18	- +6	3 19	- 3/	- 08	1 83	1 6	+	
	-/	3 31	4, 0-	3 /0	3 90	4, 9	3 31	4, 1	45	- 73	3 34	- 57	3 3-	- 11	2 11	1 3	1 3-
No em	6	43	-, 0/	3 86	4, 09	5 3	3 2	4, 3/	3 /0	3 0	3 89	3 10	3 66	2 41	- 10	1 /0	/-
	16	3 3/	4, 09	3 9	4, -/	5 5-	3 /1	4, 61	3 93	3 -8	4 -+	3 43	4, 00	- 7-	2 /1	- 06	- 0
	-6	3 -8	4 08	3 97	4, 41	5 73	3 6/	4, 9-	4 13	31	4, 36	3 /3	4, 31	3 03	3 04	0	+
Decem	6	3 18	4, 05	3 99	4, 51	5 94	3 99	5 00	4 31	3 /-	-, 86	3 99	4 61	3 3+	3 3/	2 03	- 6
	16	3 0/	3 97	3, 96	4, 57	6 07	4, 08	5 1	4, 43	3 88	5 13	4, --	-, 8/	3 63	3 6	3 0-	2 9
	26	- 93	3 89	3 91	4, 61	6, 14	4, 13	5, -1	4, 53	4, 01	5, 33	-, 41	5, 10	3, 91	3, 98	3 33	3 31
	31	-, 89	3 83	3, 89	4, 61	6, 16	4, 13	5 -3	4, 58	4, 06	5 43	4 +9	5 -0	4, 03	4, 1-	0	3 4

TABLE IX *Continued*

Day of the Month		Spica Virginis	Arcturus	α Libræ	α Coronæ Borealis	α Serpentis	Antares	α Ophiuchi	α Lyræ	α Aquilæ	2 α Capricorni	α Cygni	α Aquarii	Fomalhaut	α Pegasi	α Andromedæ
January	0	−0	−0 [illegible]	−0 6	−0 9	−0 [illegible]	−1 13	−1 28	−1 6	−1 [illegible]		−1 3[illegible]	−0 80	−0	−0 38	−0 [illegible]3
	10	+0 10	0 1	0	0 6	0 30	0 83	1 10	1 53	1 [illegible]0		1 6[illegible]	0 90	0 [illegible]	0 6[illegible]	
	20	0 [illegible]	+[illegible]	+[illegible]	0 30	0 3	0 33	0 83	1 38	1 09		1 0[illegible]	0 0	0 [illegible]	0 3	0 43
	30	0 [illegible]	0 4	[illegible]	0 [illegible]	0 0[illegible]	0 [illegible]0	0 38	1 [illegible]	0 93		1, 38		0 91	0 9	0 [illegible]
Feb	0	08	0	0 6[illegible]	+0 [illegible]	+0 3	+0 1	0 3	0 93	0 [illegible]7		1, 4[illegible]		0 [illegible]	0 80	0 6
		1 3	1 0	[illegible] 9[illegible]	0	0 6	0 48	0 03	0 63	0 3[illegible]		1 31		0 [illegible]9	0 [illegible]	0
March	1	53	1 [illegible]	[illegible]	0 9[illegible]	0 90	0 61	+0 [illegible]7	0 33	0 36		1 11		0 [illegible]	0 [illegible]	0 0
	11	1 [illegible]3	1 6	1 30	1 [illegible]1	[illegible]	13	0 33	0 0[illegible]	0 11		0 8[illegible]		0 1		0 9
	21	1 [illegible]	1 [illegible]3	1 [illegible]	[illegible]	1 4[illegible]	1 44	0 84	−0 [illegible]9	+0 1[illegible]	+0 13	0 38		0 38		0 0
	31	1 00	01	1 [illegible]	1 [illegible]0	1 63	1 [illegible]3	1 1[illegible]	0 6	0 44	0 41	0 [illegible]		0 40	0 [illegible]1	0 6
April	10	[illegible]0	[illegible] 03	[illegible] 06	[illegible]9	83	1 99	1 39	0 9	0 7[illegible]	0 [illegible]0	+0 0[illegible]	−0 06	0 19	0 [illegible]3	0 [illegible]8
	20	[illegible] 1	[illegible]	[illegible] [illegible]1	2 0	[illegible] 0[illegible]	[illegible]3	63	1 [illegible]	1 0[illegible]	0 09	0 4	0 33	+0 06	0 [illegible]	0 3[illegible]
	30	[illegible] 3	[illegible] 16	33	[illegible]	7	[illegible]	1 [illegible]8	1 7	1 31	1 31	0 [illegible]8	0 61	0 53	+0 [illegible]4	0 09
May	10	[illegible] 13	19	[illegible]1	[illegible] 30	2 [illegible]9	2 63	[illegible] 09	1 85	1 60	1 61	1 13	0 93	0 63	0 31	+0 [illegible]
	20	[illegible] 10	[illegible] 6	[illegible] [illegible]3	[illegible] 37	[illegible] 37	[illegible] [illegible]9	[illegible] [illegible]8	2 10	1 87	1 91	1 30	1 [illegible]1	0 96	0 63	0 [illegible]
	30	06	[illegible] 3	[illegible]	2 40	[illegible] 43	[illegible] 9	2 44	2 3[illegible]	[illegible] 1	2 20	83	1	3[illegible]	[illegible]	0
June	9	[illegible] 00	2, 10	[illegible] 4[illegible]	[illegible] 40	[illegible] 46	3 00	2, 36	2 [illegible]9	[illegible] 37	[illegible] 4[illegible]	2 14	81	1 67	1 46	1 1
	19	1 9[illegible]	[illegible] 03	[illegible] [illegible]5	[illegible] 3[illegible]	[illegible] 46	3 03	2 63	[illegible] 63	[illegible] 36	[illegible] 71	[illegible] [illegible]1	[illegible]	[illegible] 01	1 0	1 [illegible]
	29	1 8	1 9	[illegible] 39	[illegible] 30	2 43	3 07	[illegible] [illegible]0	[illegible] 7[illegible]	[illegible] 73	[illegible] 93	[illegible] 63	40	[illegible] 35	[illegible] 0[illegible]	[illegible]
July	9	1 [illegible]3	1 8[illegible]	2, 31	[illegible] [illegible]1	[illegible] 3[illegible]	3 03	[illegible] [illegible]1	[illegible], 76	2 89	3 10	[illegible] 8	[illegible] 04	[illegible] 6[illegible]	[illegible] 3	[illegible]
	19	, 63	1 [illegible]0	[illegible], [illegible]1	2 09	2 [illegible]9	[illegible], 99	[illegible] [illegible]0	[illegible] [illegible]3	[illegible] 99	3 [illegible]4	[illegible] 97	[illegible] 8[illegible]	2 95	[illegible] 01	[illegible] 4
	20		1 5[illegible]	[illegible] 09	1 01	2, 19	2 90	[illegible] 63	[illegible] 70	3 03	3 33	3 0[illegible]	3 04	3 [illegible]1	[illegible] 8[illegible]	[illegible]0
August	[illegible]	[illegible]	1 41	1 96	1 7[illegible]	[illegible] 0[illegible]	2 [illegible]	2 33	[illegible] 59	3 06	3 38	3 0[illegible]	1	3 [illegible]	3 0	[illegible] 93
	[illegible]	1 3[illegible]	1 [illegible]	1 84	1 60	1 31	[illegible] 61	[illegible] 41	[illegible] 44	3 0	3 38	3 03	3, [illegible]8	3, 59	3 18	1[illegible]
	[illegible]	[illegible]4	1 1	[illegible]	1 1	1 [illegible]	[illegible] 47	[illegible] [illegible]6	2 [illegible]	[illegible] 96	3 [illegible]	[illegible] 93	3 33	3 [illegible]1	3, [illegible]	3 [illegible]
Sept		1 10	1 03	1 59	1 [illegible]	1 6[illegible]	[illegible] 0	[illegible] 09	[illegible] 03	2 83	3 [illegible]6	[illegible] 81	3 33	3 [illegible]	3 33	4[illegible]
		1 1	0 91	[illegible]8	00	1 4[illegible]	2 1[illegible]	1 91	1 [illegible]9	[illegible]	3 13	[illegible] 64	3 33	3 80	3 3	3
	[illegible]	1 1	0 [illegible]4	1, 39	0 86	1 33	1 98	1 [illegible]4	1, 34	[illegible] 38	3 02	[illegible] [illegible]3	3 [illegible]	3 [illegible]8	3 3[illegible]	
October		1 4	0 [illegible]0		0 [illegible]	1 [illegible]3	83	1, 37	1 [illegible]9	[illegible] 4[illegible]	[illegible] 87	[illegible] 9	3 17	[illegible]3	3[illegible]	61
	[illegible]	[illegible]3	0 80		0 69	1, 16	1, [illegible]3	1, 43	1 04	2 [illegible]	[illegible] 71	1 01	3, 0[illegible]	3, 63	[illegible]3	6
	[illegible]	1	0, 83		0, 64	1 14	1, 6[illegible]	1 30	0, 8	[illegible] 09	[illegible], 53	1 68	[illegible] 93	3 3	3 16	3 3[illegible]
November	[illegible]	1 [illegible]	0 93		0 6[illegible]	1 13	1 60	1 [illegible]1	0 62	1, 94	[illegible], 4	1 [illegible]3	[illegible] 81	3 7	3 03	3 50
	16	1 0	1 09		0 69		69	1 15	0 4[illegible]	1, 8[illegible]	[illegible] 29	1, 19	[illegible] 68	3, [illegible]3	[illegible], 93	3 40
	26	1 9	1 [illegible]		0 8	1 31	1 [illegible]8	1, 10	0 35	1 7[illegible]	[illegible] 19	0 97	[illegible] 36	3 0[illegible]	[illegible] 61	3 [illegible]
December	6	[illegible] 21	1, 50	1 93	0 9[illegible]	1 [illegible]	1 93	1 [illegible]0	0 [illegible]8	66	[illegible] 1[illegible]	0 [illegible]8	[illegible] 45	[illegible] 9[illegible]	[illegible] 65	3 1[illegible]
	16	2, 51	1 7[illegible]	[illegible], 18	1, 18	1 66	[illegible], 13	1, [illegible]8	0 [illegible]7	1 63		0, 6[illegible]	[illegible] 33	[illegible] [illegible]	[illegible] 5[illegible]	3 03
	26	[illegible] 83	2 07	2 48	1 4[illegible]	9[illegible]	[illegible] 37	1 4[illegible]	0 30	1, 63		0 50	[illegible] [illegible]3	2 67	[illegible] [illegible]6	[illegible] 9
	31	3 00	[illegible]	2 6	0	[illegible] 0	[illegible] 51	0	0 33	68		0 4		[illegible] 61	11	[illegible] 8

N B An allowance is made for the proper motion of Capella Sirius Caſtor Procyon Pollux Regulus Arcturus and α Aquilæ

TABLE X

Corrections in Sidereal Time *of the Right Aſcenſions of Thirty two principal Fixed Stars to every Tenth Degree of Longitude of the Moon's Aſcending Node*

ARGUMENT Longitude of the ☽'s ☊			γ Pegaſi	α Arietis	α Ceti	Aldebaran	Capella	Rigel	β Tauri	α Orionis	Sirius	Caſtor	Procyon
S	D	S											
0	0	6	−0 06+	−0 [illegible]+	−0 0[illegible]+	−0 07+	−0 17+	+0 0[illegible]−	−0 0[illegible]+	−0 01+	−0, 0[illegible]+	+0 13−	+0 0[illegible]−
	10		0 35	0 4[illegible]	0 [illegible]	0 29	0, 44	−0 1[illegible]+	0 [illegible]1	0 [illegible]1	0 19	−0 11+	−0 17+
	20		0 [illegible]	0 6[illegible]	0 41	0 49	0 69	0 [illegible]	0 5[illegible]	0 40	0 35	0 [illegible]4	0 [illegible]
1	0	7	0 68	0 [illegible]9	0 [illegible]8	0 6[illegible]	0 93	0, 49	0 7[illegible]	0 58	0, 50	0 [illegible]7	0 55
	10		0, 8[illegible]	0 94	0 [illegible]	0, 84	1 13	0 6[illegible]	0, 9[illegible]	0 [illegible]	0 63	0, [illegible]8	0, [illegible]1
	20		0 94	1, 0[illegible]	0 87	0 98	1, 31	0 77	1 07	0 89	0 [illegible]	0 9[illegible]	0 8[illegible]
2	0	8	1 03	1 14	0 97	1 09	1 44	0 87	1 [illegible]0	1 00	0 84	1 1[illegible]	0 97
	10		1, 08	1, 19	1 05	1 17	1, 5[illegible]	0 9[illegible]	1 [illegible]9	1, 09	0 91	[illegible]	1, 06
	20		1 10	1, [illegible]1	1, 10	1 [illegible]1	1, 57	1 00	1, [illegible]3	1, 14	0, 95	1, [illegible]3	1, 11
3	0	9	[illegible], 09	1, 19	1 1	1 [illegible]1	1, 56	1 0[illegible]	1, [illegible]4	1 1[illegible]	0, 9[illegible]	1, [illegible]7	1 1[illegible]
	10		1, 0[illegible]	1, 13	1 09	1 19	1, 51	1 01	1 [illegible]1	1 [illegible]	0, 9[illegible]	1 [illegible]	1 1[illegible]
	20		0 9	1 0[illegible]	1, 0[illegible]	[illegible], 11	1 41	0 9[illegible]	1 2[illegible]	1 08	0 89	1 [illegible]4	1 0[illegible]
4	0	10	0 8[illegible]	0 91	0 95	1 01	1, 27	0 90	1 13	0 99	0, 81	1 [illegible]6	0 99
	10		0 [illegible]	0 [illegible]6	0 83	0 88	1 09	0 [illegible]	0 96	0 [illegible]8	0 71	1 14	0 [illegible]
	20		0 [illegible]	0 59	0 69	0, 75	0, 87	0 67	0 81	0 7[illegible]	0 [illegible]9	0 09	0 [illegible]
5	0	11	0 41	0 [illegible]9	0 53	0 54	0 6[illegible]	0, [illegible]	0, 61	0 5[illegible]	0 4[illegible]	0, 81	0 [illegible]
	10		0 [illegible]	−0 19+	0 [illegible]	0 [illegible]5	0, 37	0 [illegible]	0 [illegible]9	0 [illegible]9	0 [illegible]0	0 60	0 [illegible]
	20		−0, 0[illegible]+	+0 0[illegible]−	−0 17+	−0 14+	−0 10+	0 [illegible]0	−0 16+	−0 19+	−0 14+	0 [illegible]	0 [illegible]
6	0	12	+0 16−	0 [illegible]	+0, 0[illegible]−	+0 07−	+0 1[illegible]−	0 0[illegible]	+0 0[illegible]−	+0 0[illegible]−	+0 0[illegible]−	0 1[illegible]	0 03

TABLE X *Continued*

ARGUMENT Longitude of the ☽'s ☊			Pollux	α Hydræ	Regulus	β Leonis	β Virginis	Spica Virginis	Arcturus	α Libræ	α Coronæ Borealis	α Serpentis	Antares
s	D	s											
0	0	6	+0 13−	−0 07+	+0, 1[illegible]−	+0, 1[illegible]−	+0, 0[illegible]−	−0, 11+	+0 20−	−0 13+	+0, 21−	+0, 05−	−0, 13+
	10		−0 10+	0 25	−0 07+	−0 02+	−0, 15+	0, 30	0, 0[illegible]	0 [illegible]	0 05	−0 1[illegible]+	0 [illegible]6
	20		0 3	0 4[illegible]	0 [illegible]7	0 21	0 3[illegible]	0 48	−0 15+	0 53	−0 11+	0 [illegible]1	0 [illegible]
1	0	7	0 5[illegible]	0 [illegible]8	0 47	0 40	0 [illegible]2	0, 65	0 [illegible]	0 70	0 27	0 48	0 77
	10		0 75	0 73	0 64	0 58	0 68	0 80	0 [illegible]9	0 85	0 4[illegible]	0 6[illegible]	0 94
	20		0 93	0 85	0 80	0 7[illegible]	0 81	0 9[illegible]	0 63	0 98	0, 55	0 77	1 08
[illegible]	0	8	1 09	0 94	0 9[illegible]	0 87	0 93	1, 02	0 [illegible]7	1 08	0 67	0 88	1 19
	10		1 21	1 01	1 03	0 98	1 02	1 09	0 87	1 1[illegible]	0 77	0 97	1 27
	20		1 [illegible]9	1 05	1 11	1 06	1 07	1 1[illegible]	0 95	1 18	0 85	1, 02	1 30
[illegible]	0	9	1 [illegible]3	1 0[illegible]	1, 15	1 11	1 09	1 12	1 00	1 17	0 90	1 05	1 [illegible]0
	0		1 [illegible]	1 0[illegible]	1, 15	1, 1[illegible]	1, 09	1, 08	1, 0[illegible]	1, 1[illegible]	0 92	1 04	1 2[illegible]
	20		1 [illegible]9	0 97	1, 1[illegible]	1 10	1, 0[illegible]	1 01	1 01	1 06	0 91	1 00	1 17
4	0	10	1 [illegible]2	0 88	1 06	1 0[illegible]	0 97	0 91	0 97	0 95	0 88	0 93	1 0[illegible]
	10		1 11	0 [illegible]6	0 96	0 96	0 86	0 79	0 90	0 81	0 8[illegible]	0 83	0 91
	20		0 96	0 6[illegible]	0 8[illegible]	0 8[illegible]	0 [illegible]	0 6[illegible]	0 80	0 65	0, 74	0, 71	[illegible] 71
5	0	11	0 [illegible]8	0 47	0 69	0 70	0 [illegible]8	0 47	0 6[illegible]	0 [illegible]7	0 6[illegible]	0 [illegible]7	0 [illegible]3
	10		0 [illegible]8	0 30	0 51	0 [illegible]4	0 41	0 [illegible]8	0 [illegible]	0 [illegible]8	0 51	0 40	0 [illegible]1
	20		0 [illegible]6	−0 1[illegible]+	0 [illegible]	0 6	0 [illegible]	−0 09+	0 37	−0 0[illegible]+	0 36	0 2[illegible]	−0 09+
6	0	[illegible]	0 13	+0 07−	0 13	0 17	0 03	+0 11−	0, 20	+0 1[illegible]−	0 [illegible]1	[illegible] 0[illegible]	+0 1[illegible]−

TABLE X *Continued*

ARGUMENT Longitude of the Σ s ☊			α Ophiuchi	α Lyræ	α Aquilæ	2 α Capricorni	α Cygni	α Aquarii	Fomalhaut	α Pegasi	α Andromedæ	α Herculis
S	D	S										
0	0	6	+0 02−	−0 06+	−0 04+	+0 08−	−0 39−	+0 01−	+0 36−	−0 15+	−0 33+	+0 03−
	10		−0 15+	0 19	0 21	−0 13+	0 51	−0 17+	0 15	0 33	0 52	−0, 14+
	20		0 31	0 31	0 39	0 33	0 61	0 6	−0 07+	0 50	0 69	0 31
1	0	7	0 47	0 41	0 55	0 53	0 70	0 54	0 28	0 66	0 83	0 47
	10		0 61	0 51	0 69	0 71	0 77	0 69	0 49	0, 79	0 95	0 61
	20		0 74	0 59	0, 81	0 86	0, 81	0 85	0 67	0, 91	1 05	0 74
2	0	8	0 84	0 65	0 91	0 99	0 85	0 95	0 85	0 99	1 1	0 84
	10		0 92	0 69	0 98	1 09	0 81	1 03	0 99	1, 05	1 15	0 92
	20		0 97	0 72	1 02	1 15	0 78	1 08	1 10	1 07	1 13	0 97
3	0	9	0 99	0 72	1 03	1 19	0, 7[illegible]	1, 10	1 19	1 06	1 09	0 99
	10		0 98	0, 69	1, 01	1, 18	0 65	1 09	1 23	1, 01	1 01	0 98
	20		0 95	0 65	0 95	1 14	0 55	1, 04	1, 23	0 94	0 9	0 94
4	0	10	0 87	0 59	0 87	1 07	0 43	0 96	1 21	0 84	0 77	0 88
	10		0 77	0	0 76	0 96	0 31	0 85	1 14	0 71	0 62	0 78
	20		0 65	0 41	0 65	0 82	0 17	0 72	1 04	0 57	0 44	0 66
5	0	11	0 51	0 31	0 48	0, 66	−0 05−	0 56	0, 91	0, 40	0 [illegible]	0 53
	10		0 36	0 19	0 31	0 48	−0 11−	0 39	0 74	0 22	−0 07+	0 37
	20		0 19	−0 06+	−0, 14+	0 28	0 [illegible]	0 21	0 56	−0, 05+	+0 14−	0 20
6	0	12	0 02	+0 06−	+0 04−	0 08	0 39	0 01	0 36	+0 15−	0 33	−0 03+

TABLE XI

The Correction of the Obliquity of the Ecliptic for Days of the Year

Day of the Month		′	Day of the Month		′
January	10	−0,4	July	16	−0,6
	18	−0,3		26	−0,5
	25	−0,2	August	1	−0,4
	31	−0,1		8	−0,3
February	6	0,0		16	−0,2
	14	+0,1		21	−0,1
	1[illegible]	+0,2		31	0,0
March	[illegible]	+0,3	September	21	+0,1
	21	+0,4			
April	2	+0,3	October	11	0,0
	13	+0,2		19	−0,1
	20	+0,1		25	−0,[illegible]
	26	0,0		31	−0,3
May	2	−0,1	November	6	−0,4
	7	−0,2		1[illegible]	−0,5
	13	−0,3		17	−0,6
	19	−0,4		[illegible]3	−0,7
	26	−0,5		30	−0,8
June	[illegible]	−0,6	December	9	−0,9
	25	−0,7		[illegible]6	−1,0

TABLE XII

The Equation of the Obliquity of the Ecliptic

ARGUMENT Long of ☽'s ☊.

Long of ☊'s ☾	0 Sig + VI Sig −	I Sig + VII Sig −	II Sig + VIII Sig −	
0	9,5	8,3	4,8	30
1	9,5	8,2	4,6	29
2	9,5	8,1	4,5	28
3	9,5	8,0	4,3	27
4	9,5	7,9	4,2	26
5	9,5	7,8	4,0	25
6	9	7,7	3,9	24
7	9,5	7,6	3,7	23
8	9,5	7,5	3,6	22
9	9,4	7,4	3,4	21
10	9,4	7,3	3,2	20
11	9,4	7,2	3,1	19
12	9,3	7,1	3,0	18
13	9,3	7,0	2,8	17
14	9,3	6,9	2,6	16
15	9,2	6,8	2,5	15
16	9,2	6,6	2,3	14
17	9,1	6,5	2,1	13
18	9,1	6,4	2,0	12
19	9,0	6,3	1,8	11
20	9,0	6,1	1,7	10
21	8,9	6,0	1,5	9
22	8,9	5,9	1,3	8
23	8,8	5,7	1,2	7
24	8,7	5,6	1,0	6
25	8,7	5,5	0,8	5
26	8,6	5,3	0,7	4
27	8,5	5,2	0,5	3
28	8,4	5,1	0,3	2
29	8,4	4,9	0,2	1
30	8,3	4,8	0,0	0
	V Sig − XI Sig +	IV Sig − X Sig +	III Sig − IX Sig +	Long of ☾'s ☊

TABLE XIII

The mean aſtronomical Refractions to every Ten Minutes of apparent Zenith Diſtances

App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff
0 0	0 00	0 17	4 30	4 50	0 17	9 0	9 00	0 17	13 30	13 65	0 18	18 0	18 50	0 18
0 10	0 17	0 16	4 40	4 67	0 16	9 10	9 7	0 16	13 40	1 83	0 9	8 10	18 68	0 9
0 20	0 3	0 17	4 50	4 83	0 17	9 20	9 33	0 17	13 50	14 02	0 18	18 20	18 87	0 18
0 30	0 50	0 17	5 0	5 00	0 17	9 30	9 50	0 17	14 0	14 20	0 8	18 30	19 05	0 18
0 40	0 67	0 16	5 10	5 17	0 16	9 40	9 67	0 16	4 10	14 38	0 19	18 40	19 23	0 19
0 50	0 8	0 17	5 20	5 3	0 17	9 50	9 8	0 17	14 20	14 57	0 18	18 50	19 5	0 18
1 0	1 00	0 7	5 30	5 50	0 17	10 0	10 00	0 18	14 30	14 75	0 8	19 0	19 60	0 18
1 10	1 7	0 16	5 40	5 67	0 16	10 0	10 18	0 19	4 40	14 9	0 9	19 10	19 78	0 19
1 20	1 33	0 17	5 50	5 83	0 17	10 20	10 37	0 18	14 50	15 12	0 19	9 20	19 97	0 18
30	1 50	0 17	6 0	6 00	0 17	10 30	10 55	0 18	15 0	15 30	0 17	19 30	20 5	0 8
1 40	1 67	0 16	6 10	6 17	0 16	10 40	10 73	0 19	15 10	5 47	0 16	19 40	20 33	0 19
1 50	83	0 17	6 20	6 33	0 17	0 0	10 92	0 18	15 20	15 63	0 1	19 50	20 52	0 18
2 0	2 00	0 17	6 30	6 50	0 17	11 0	11 10	0 17	15 30	15 80	0 17	20 0	20 70	0 20
2 10	2 17	0 16	6 40	6 67	0 16	10	1 27	0 16	15 40	15 97	0 16	20 10	20 90	0 20
2 20	2 33	0 17	6 50	6 83	0 17	11 20	11 43	0 17	15 50	16 13	0	20 20	21 10	0 20
2 30	2 50	0 17	7 0	7 00	0 17	11 30	11 60	0 17	16 0	6 30	0 8	20 30	21 0	0 20
2 40	2 67	0 16	7 0	7 17	0 16	11 40	1 77	0 16	16 10	16 48	0 19	20 40	21 30	0 20
2 50	2 83	0 17	7 20	7 3	0 17	1 50	11 93	0 17	6 20	16 67	0 18	0 50	21 70	0 0
3 0	3 00	0 17	7 30	7 50	0 17	12 0	12 10	0 17	16 30	16 83	0 18	21 0	21 90	0 18
3 10	3 17	0 16	7 40	7 67	0 16	12 10	12 27	0 16	6 40	17 03	0 19	1 10	22 08	0 9
3 20	3 33	0 17	7 50	7 83	0 17	12 20	12 43	0 17	16 50	17 22	0 18	21 20		0 8
3 30	3 50	0 17	8 0	8 00	0 1	12 30	2 60	0 17	7 0	17 40	0 18	21 30	2 45	0 18
3 40	3 67	0 16	8 10	8 17	0 16	12 40	12 7	0 16	17 10	17 58	0 19	1 40	22 63	0 19
50	3 83	0 17	8 20	8 33	0 17	12 50	12 93	0 17	7 20	17 77	0 18	21 50	22 [illegible]	0 18
4 0	4 00	0 17	8 30	8 50	0 7	13 0	13 0	0 18	17 30	17 93	0 18	22 0	23 00	0 20
4 10	4 17	0 6	8 40	8 6	0 0	3 10	13 28	0 9	40	8 3	0 19	22 10	23 20	0 20
4 20	4 33	0	8 50	8 8	0	3 20	13 27	0 [illegible]	50	9 2	0	0	2 40	0 0

TABLE XIII *Continued*

App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff
[illegible]	[illegible] 00	0 20	27 0	29 00	0 [illegible]	31 30	34 90	0 25	36 0	41 40	0 25	40 30	48 65	0 28
[illegible]	[illegible] 80	[illegible]	27 10	[illegible]	0 21	31 40	35 15	0 24	36 10	41 65	0 25	40 40	48 93	0 29
[illegible]	[illegible]	[illegible]	27 20	29 43	0 [illegible]	31 50	35 37	0 [illegible]	36 20	41 90	0 25	40 50	49 [illegible]	0 28
[illegible]	[illegible]	0 20	27 30	29 65	0 [illegible]	32 0	35 60	0 23	36 30	42 15	0 25	41 0	49 50	0 30
[illegible]	[illegible]	0 20	27 40	29 87	0 21	32 10	35 83	0 24	36 40	42 40	0 25	41 10	49 80	0 30
[illegible]	[illegible]	[illegible]	27 50	30 08	0 [illegible]	32 20	36 07	0 23	36 50	42 65	0 25	41 20	50 10	0 30
[illegible]	[illegible]	[illegible]	28 0	30 30	0 [illegible]	32 30	36 30	0 23	37 0	42 90	0 27	41 30	50 40	0 30
[illegible]	[illegible] 00	[illegible]	28 10	30 52	0 21	32 40	36 53	0 24	37 10	43 17	0 26	41 40	50 70	0 30
[illegible]	[illegible]	[illegible]	28 20	30 73	0 [illegible]	32 50	36 77	0 23	37 20	43 43	0 27	41 50	51 00	0 30
[illegible]	[illegible]	0 20	28 30	30 95	0 [illegible]	33 0	37 00	0 23	37 30	43 70	0 27	42 0	51 30	0 30
[illegible]	[illegible]	0 20	28 40	31 17	0 21	33 10	37 23	0 [illegible]	37 40	43 97	0 26	42 10	51 60	0 30
[illegible]	[illegible]	0 20	28 50	31 38	0 [illegible]	33 20	37 47	0 23	37 50	44 23	0 27	42 20	51 90	0 30
[illegible]	[illegible]	0 20	29 0	31 60	0 [illegible]	33 30	37 70	0 23	38 0	44 50	0 27	42 30	52 20	0 30
[illegible]	[illegible]	0 20	29 10	31 82	0 21	33 40	37 93	0 23	38 10	44 77	0 26	42 40	52 50	0 30
[illegible]	[illegible] 40	0 20	29 20	32 03	0 [illegible]	33 50	38 16	0 24	38 20	45 03	0 27	42 50	52 80	0 30
[illegible]	[illegible]	0 20	29 30	32 25	0 [illegible]	34 0	38 40	0 25	38 30	45 30	0 27	43 0	53 10	0 [illegible]
[illegible]	[illegible]	0 20	29 40	32 47	0 [illegible]	34 10	38 65	0 25	38 40	45 57	0 26	43 10	53 [illegible]	0 31
[illegible]	[illegible]	0 20	29 50	32 68	0 [illegible]	34 20	38 90	0 25	38 50	45 83	0 27	43 20	53 73	0 [illegible]
[illegible]	[illegible] 20	[illegible]	30 0	32 90	0 [illegible]	34 30	39 15	0 25	39 0	46 10	0 28	43 30	54 05	0 [illegible]
[illegible]	[illegible] 40	[illegible]	30 10	33 12	0 [illegible]	34 40	39 40	0 25	39 10	46 38	0 29	43 40	54 37	0 31
[illegible]	[illegible] 60	0 20	30 20	33 33	0 22	34 50	39 65	0 25	39 20	46 67	0 28	43 50	54 60	0 32
[illegible]	[illegible]	0 20	30 30	33 55	0 [illegible]	35 0	39 90	0 25	39 30	46 95	0 28	44 0	55 00	0 32
[illegible]	[illegible]	0 20	30 40	33 77	0 21	35 10	40 15	0 25	39 40	47 23	0 29	44 10	55 [illegible]	0 31
[illegible]	[illegible]	0 20	30 50	33 98	0 [illegible]	35 20	40 40	0 25	39 50	47 52	0 28	44 20	55 63	0 32
[illegible]	[illegible]	0 20	31 0	34 20	0 23	35 30	40 65	0 25	40 0	47 80	0 28	44 30	55 95	0 [illegible]
[illegible]	[illegible]	0 20	31 10	34 43	0 24	35 40	40 90	0 25	40 10	48 08	0 29	44 40	[illegible]	0 31
[illegible]	[illegible]	0 20	31 20	34 67	0 23	35 50	41 15	0 25	40 20	48 37	0 28	44 50	56 58	0 [illegible]

TABLE XIII *Continued*

App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff	App Z D	Refraction	Diff
45 0	0 56 90	0 33	49 30	1 6 59	0 40	54 0	1 18 30	0 47	58 30	1 32 78	0 60	63 0	1 51 50	0 80
45 10	0 57 23	0 33	49 40	1 6 99	0 40	54 10	1 18 77	0 48	58 40	1 33 38	0 61	63 10	1 52 30	0 80
45 20	0 57 56	0 33	49 50	1 7 39	0 41	54 20	1 19 25	0 48	58 50	1 33 99	0 61	63 20	1 53 10	0 81
45 30	0 57 89	0 33	50 0	1 7 80	0 39	54 30	1 19 73	0 49	59 0	1 34 60	0 62	63 30	1 53 91	0 82
45 40	0 58 22	0 34	50 10	1 8 19	0 40	54 40	1 20 22	0 49	59 10	1 35 22	0 62	63 40	1 54 73	0 83
45 50	0 58 56	0 34	50 20	1 8 59	0 40	54 50	1 20 71	0 49	59 20	1 35 84	0 63	63 50	1 55 56	0 84
46 0	0 58 90	0 35	50 30	1 8 99	0 40	55 0	1 21 20	0 51	59 30	1 36 47	0 64	64 0	1 56 40	0 85
46 10	0 59 25	0 34	50 40	1 9 39	0 40	55 10	1 21 71	0 51	59 40	1 37 11	0 64	64 10	1 57 25	0 87
46 20	0 59 59	0 35	50 50	1 9 79	0 41	55 20	1 22 22	0 51	59 50	1 37 75	0 65	64 20	1 58 12	0 88
46 30	0 59 94	0 35	51 0	1 10 20	0 42	55 30	1 22 73	0 52	60 0	1 38 40	0 67	64 30	1 59 00	0 89
46 40	1 0 29	0 36	51 10	1 10 62	0 43	55 40	1 23 25	0 52	60 10	1 39 07	0 67	64 40	1 59 89	0 90
46 50	1 0 65	0 35	51 20	1 11 05	0 43	55 50	1 23 77	0 53	60 20	1 39 74	0 68	64 50	2 0 79	0 91
47 0	1 1 00	0 36	51 30	1 11 48	0 44	56 0	1 24 30	0 52	60 30	1 40 42	0 69	65 0	2 1 70	0 92
47 10	1 1 36	0 37	51 40	1 11 92	0 44	56 10	1 24 82	0 52	60 40	1 41 11	0 69	65 10	2 2 62	0 93
47 20	1 1 73	0 36	51 50	1 12 36	0 44	56 20	1 25 34	0 53	60 50	1 41 80	0 70	65 20	2 3 55	0 94
47 30	1 2 09	0 37	52 0	1 12 80	0 44	56 30	1 25 87	0 54	61 0	1 42 50	0 70	65 30	2 4 49	0 96
47 40	1 2 46	0 37	52 10	1 13 24	0 45	56 40	1 26 41	0 54	61 10	1 43 20	0 70	65 40	2 5 45	0 97
47 50	1 2 83	0 37	52 20	1 13 69	0 45	56 50	1 26 95	0 55	61 20	1 43 90	0 71	65 50	2 6 42	0 98
48 0	1 3 20	0 36	52 30	1 14 14	0 45	57 0	1 27 50	0 57	61 30	1 44 61	0 72	66 0	2 7 40	1 00
48 10	1 3 56	0 36	52 40	1 14 59	0 45	57 10	1 28 07	0 57	61 40	1 45 33	0 73	66 10	2 8 40	1 01
48 20	1 3 92	0 37	52 50	1 15 04	0 46	57 20	1 28 64	0 58	61 50	1 46 06	0 74	66 20	2 9 41	1 03
48 30	1 4 29	0 37	53 0	1 15 50	0 46	57 30	1 29 22	0 59	62 0	1 46 80	0 76	66 30	2 10 44	1 04
48 40	1 4 66	0 37	53 10	1 15 96	0 46	57 40	1 29 81	0 59	62 10	1 47 56	0 77	66 40	2 11 48	1 05
48 50	1 5 03	0 37	53 20	1 16 42	0 47	57 50	1 30 40	0 60	62 20	1 48 33	0 78	66 50	2 12 53	1 07
49 0	1 5 40	0 39	53 30	1 16 89	0 47	58 0	1 31 00	0 59	62 30	1 49 11	0 79	67 0	2 13 60	1 08
49 10	1 5 79	0 40	53 40	1 17 36	0 47	58 10	1 31 59	0 59	62 40	1 49 90	0 80	67 10	2 14 68	1 10
49 20	1 6 19	0 40	53 50	1 17 83	0 47	58 20	1 32 18	0 60	62 50	1 50 70	0 80	67 20	2 15 78	1 11

TABLE XIII *Continued*

App Z D	Refraction	Diff
[illegible] 0	[illegible] 16 [illegible]	1 [illegible]
[illegible] [illegible]0	[illegible] 18 0	[illegible]
[illegible] [illegible]0	[illegible] 19 [illegible]	
		[illegible]
58 0	[illegible]0 [illegible]	1 16
58 0	[illegible] [illegible]0	1 1[illegible]
58 [illegible]	[illegible] 6[illegible]	
		1 19
[illegible] [illegible]	[illegible] 8[illegible]	1 [illegible]1
[illegible] 10	[illegible] 0	1 [illegible]
[illegible]	[illegible]0 [illegible]	
		1, [illegible]
59 0	[illegible] [illegible]0	1 [illegible]8
10	[illegible]8 [illegible]8	[illegible]0
60 0	30 08	
		1 [illegible]
[illegible]9 [illegible]0	[illegible] 40	
[illegible]9 [illegible]0	[illegible]	1 [illegible]
[illegible]9 0	[illegible] 11	
		1 [illegible]9
0 0	[illegible] [illegible]0	1 41
0 0	[illegible]6 9	1, 4[illegible]
0 [illegible]	[illegible]8 [illegible]4	
		1 46
0 [illegible]0	[illegible]9 80	1 48
0 40	41 [illegible]8	1 [illegible]0
0 50	[illegible] 78	
		, 52
[illegible]0	[illegible]4 [illegible]0	1 5[illegible]
10	[illegible]	1, 56
[illegible]	[illegible]9	1 [illegible]8

App Z D	Refraction	Diff
[illegible] [illegible]0	[illegible] 40 9	1 6[illegible]
[illegible]1 40	[illegible] 50 [illegible]9	1 64
[illegible]1 [illegible]0	[illegible] 5[illegible] [illegible]	
		1, 67
[illegible] 0	[illegible] [illegible] 90	1 7[illegible]
[illegible] 0	[illegible] 6	1 [illegible]
[illegible] [illegible]0	[illegible] [illegible]7 [illegible]	
		1 78
[illegible] [illegible]0	[illegible] 50 1[illegible]	1 8[illegible]
[illegible] 40	[illegible] 0 9	1 8[illegible]
[illegible] [illegible]0	[illegible] [illegible] 8[illegible]	
		1 88
[illegible] 0	[illegible] 4, 70	01
73 10	[illegible] 6 6	1 94
[illegible] [illegible]0	[illegible] 9 [illegible]	
		1 98
[illegible] [illegible]0	10 [illegible]	[illegible] 0[illegible]
[illegible] 4[illegible]	[illegible] 1[illegible] [illegible]	[illegible] 06
[illegible] [illegible]0	14, 61	
		[illegible] 09
[illegible] 0	[illegible] 16 [illegible]0	[illegible] 13
[illegible] 10	18, 8[illegible]	2 16
[illegible] [illegible]0	[illegible] [illegible] 0	
		[illegible]
[illegible]4 [illegible]0	[illegible] [illegible] [illegible]	2 [illegible]7
[illegible] 40	3 2[illegible] [illegible]1	[illegible]
[illegible] [illegible]0	[illegible] 2[illegible] 8[illegible]	
		[illegible] 37
[illegible] 0	[illegible] 0 20	2 41
7[illegible] 0	[illegible] [illegible] 61	[illegible] 4
[illegible] [illegible]0	[illegible] 08	[illegible]

App Z D	Refraction	Diff
75 30	3 37,60	2 58
75 40	3 40 18	[illegible] 63
75 50	3 4[illegible] 81	
		[illegible] 69
76 0	[illegible] 4[illegible] [illegible]0	[illegible] 7[illegible]
76 10	3 48 [illegible]	[illegible] 81
76 [illegible]0	3 51 06	
		[illegible] 88
76 30	[illegible] 5 94	[illegible] 9[illegible]
76 40	[illegible] 56 89	[illegible] 02
76 50	3 59 91	
		3 09
77 0	4 3 00	3 1[illegible]
77 10	4 6 15	[illegible], 2[illegible]
77 20	4 9 40	
		[illegible] 31
77 [illegible]0	4 12, 71	3 4[illegible]
77 40	4 16 1[illegible]	3 49
77 50	4 19 62	
		[illegible] [illegible]6
78 0	4 [illegible] 18	[illegible] 69
78 10	4 [illegible]6 8[illegible]	3 79
78 [illegible]0	4 30 66	
		[illegible] 88
78 [illegible]0	4 [illegible]4 [illegible]4	3 99
78 40	4 38 [illegible]	4, 11
78 50	4 [illegible], 6[illegible]	
		4, 21
79 0	4 46 8[illegible]	4, [illegible]
79 [illegible]0	4 51 16	4 [illegible]
79 [illegible]0	4 55 63	
		[illegible] 56

App Z D	Refraction	Diff
79 [illegible]0	5 0,21	4 72
79 40	5 4 93	4 88
79 50	5 9 81	
		5 0[illegible]
80 0	5 14 83	5 16
80 10	5 19 99	5 [illegible]2
80 20	5 [illegible]5 [illegible]1	
		5 50
80 [illegible]0	5 [illegible]0 81	5 68
80 40	5 [illegible]6 49	5, 89
80 50	5 42 [illegible]8	
		6 07
81 0	5 48 4[illegible]	6 25
81 10	5 54 [illegible]0	6 47
8[illegible] [illegible]0	6 1, 1[illegible]	
		6 71
81 [illegible]0	6 7, 88	6 95
81 40	6 14, 8[illegible]	7 [illegible]2
81 50	6 2[illegible] 05	
		7 50
8[illegible] 0	6 [illegible]9 55	[illegible]
8[illegible] 10	6 [illegible]	8, 0[illegible]
8[illegible] [illegible]0	6 4[illegible] 29	
		8, 35
8[illegible] [illegible]0	6 5[illegible], 6[illegible]	8 69
8[illegible] [illegible]0	7 [illegible] [illegible]	9 06
8[illegible] [illegible]0	7 11 [illegible]9	
		9, 45
8[illegible] 0	7 20 [illegible]4	[illegible] 74
8[illegible] 10	7 30 58	10, 17
83 [illegible]0	7 40 7	10 64

App Z D	Refraction	Diff
83 30	7 51 [illegible]9	11 12
83 40	8 [illegible] [illegible]1	11 67
83 50	8 14, 18	
		[illegible] 23
84 0	8 26 41	1[illegible] 60
84 10	8 [illegible]9 01	13 [illegible]4
84 20	8 52 25	
		13 94
84 30	9 6 19	14, 66
84 40	9 20 8[illegible]	5 [illegible]6
84 50	9 [illegible]6 31	
		16 19
85 0	9 5[illegible] 50	

TABLE XIV

Decimal Numbers for computing the Corrections of the mean astronomical Refractions depending on the Barometer & Thermometer

Enter the Table with the Height of the Barometer expreſſed in Inches and Decimals at Top and with the Height of FAHRENHEIT's Thermometer expreſſed in Degrees on the Side

Bar / Ther	29,0	29,1	29,2	29,3	29,4	29,5	29,6	29,7	29,8	29,9	30,0
20	+ 059	+ 0[illegible]	+ 067	+ 070	+ 0[illegible]4	+,077	+ 081	+ 085	+ 088	+,092	+ 096
21	+ 056	+ 060	- 064	+ 067	+ 071	+ 075	+ 078	+ 082	+ 0[illegible]5	+ 089	+ 09[illegible]
22	+,054	+ 057	+,061	+ 064	+ 068	+ 072	+,075	+ 079	+ [illegible]	+ 086	+,090
23	+ 051	+ 054	+,058	+ 062	+ 065	- 069	+ 072	+,076	+ 080	+ 083	+,0[illegible]7
24	+,048	+ 051	+ 055	+ 05[illegible]	+ 0[illegible]2	+,066	+ 070	+ 073	+,077	+,0[illegible]0	+ 084
25	+,045	+,049	+,052	+,056	+,060	+,063	+,067	+,070	+,074	+ 078	+ [illegible]
26	+ 04[illegible]	+ 046	+ 049	+ 053	+ 057	+ 060	+ 064	+ 067	+,071	+,075	+ 078
27	+ 040	+ 043	+ 04[illegible]	+ 050	+ 054	+ 057	+,061	- 065	+ 068	+ 072	+ 075
28	+ 0[illegible]7	+ 040	+ 044	+ 048	+ 051	+ 055	+ 058	+,062	+ 065	+ 069	+ 073
29	+ 034	+,036	+ 04[illegible]	+ 045	+ 048	+ 052	+ 055	+ 0[illegible]0	+ 063	+ 066	+ 0[illegible]0
30	+ 031	+ 035	+,038	+ 042	+ 046	+ 049	+ 053	+ 056	+ 060	+ 063	+ 067
31	+ 029	+,032	+ 0[illegible]6	+ 039	- 043	+ 046	+,050	+ 0[illegible]3	+ 057	+ 061	+ 06[illegible]
32	+ 026	+ 029	+ 033	+ 037	+ 040	+ 042	+ 047	+ 051	+,054	+ 058	+ 061
33	+ 02[illegible]	+ 027	+,030	+ 03[illegible]	+ 037	+ 041	+ 044	+ 048	- 051	+ 055	+ 059
34	+ 021	+ 024	+ 028	+ 031	+ 0[illegible]5	+ 038	+ 042	+ 045	+ 040	+ 052	- 056
35	+ 018	+ 021	+ 025	+ 028	+ 032	+ 036	+ 039	+ 043	+ 046	+ 050	+ 053
36	+ 015	+ 0[illegible]9	+ 02[illegible]	+,026	+ 029	+ 033	+ 036	+ 0[illegible]0	+ 043	+ 047	+ 050
37	+ 013	+ 016	+ 020	+ 02[illegible]	+ 027	+ 030	+ 034	- 037	- 041	+ 04[illegible]	+,048
38	+ 010	+ 014	+ 017	+ 020	- 024	+ 027	+ 031	+,03[illegible]	+,038	+ 041	+,045
39	+,007	+ 011	+ 014	- ,018	+ 021	+ 025	+ 028	+ 03[illegible]	+ 035	+ 039	+ 0[illegible]2
40	+,005	+ 008	- 012	+ 015	+ 019	+ [illegible]	+ 026	+ 029	+ 033	+ 03[illegible]	+ 040
41	+,002	+ 006	- 009	+ 013	+ 016	+ 020	+ 023	+ 0[illegible]6	+ 030	+ 033	+ 0[illegible]7
42	,000	+,003	+,007	+ 010	+,014	+,017	+,020	+,0[illegible]4	+,027	+,031	+,03[illegible]
43	- 003	+,001	+,004	+ 008	+ 011	+ 0[illegible]4	+ 018	+ 021	- 02[illegible]	+ 028	+ 03[illegible]
44	-,005	-,002	+,002	+ 005	+,008	+ 012	+,0[illegible]5	+ 0[illegible]9	- 0[illegible]	- ,026	- 029
45	-,008	-,004	- 00	+ 002	+ 006	+ 009	+,0[illegible]3	- ,010	+ 020	+ 0[illegible]3	+ 02[illegible]
46	- 010	- 007	- 002	[illegible]	+ 00[illegible]	+ 007	+ 010	+ 014	+ 0[illegible]7	+ 020	+ 0[illegible]
47	- 013	- 009	- 006	- 003	+ 001	+ 004	+,008	+ 011	+,014	+ 018	+ 0[illegible]1
48	- 0[illegible]5	- 012	- 009	- 00[illegible]	- 00[illegible]	+ 002	+ 005	+ 008	+ 0[illegible]	+ 015	+ 010
49	- 018	- 0[illegible]5	- 011	- 006	- 004	- 001	+ 003	+ 006	+ 009	+ 0[illegible]3	+ 010
50	- 020	- 017	[illegible]	- 0[illegible]0	- 00	- 00	000	+ 00	+ 00	+ 010	+ 0[illegible]

TABLE XIV *Continued*

Bar	0	30 2	30 3	30 4	30 5	30 6	30 7	30 8	30 9	31 0
Ther										
20	+ 099	+ [illegible]	+ 107	+ 110	+ 114	+ 118	+ 121	+ 125	+ 129	+ 132
21	+ 096	+ [illegible]	+ 104	+ 107	+ [illegible]	+ 115	+ 118	+ 122	+ 126	+ 129
22	+ 094	+ 097	+ [illegible]	+ 104	+ 106	+ 112	+ 115	+ 119	+ 12	+ 126
23	+ 091	+ 094	+ 098	+ 101	+ 05	+ 09	+ 112	+ 116	+ 120	+ 123
24	+ 088	+ 091	+ 095	+ 098	+ 102	+ 106	+ 109	+ 113	+ 117	+ 120
25	+ 085	+ 088	+ 092	+ 096	+ 0 0	+ 05	+ 100	+ 110	+ 114	+ 117
26	+ 082	+ 08	+ 089	+ 093	+ 096	+ 100	+ 103	+ 107	+ 111	+ 114
27	+ 079	+ 083	+ 086	+ 090	+ 093	+ 097	+ 100	+ 104	+ 108	+ 111
28	+ 076	+ 080	+ 083	+ 087	+ 0 0	+ 094	+ 098	+ 101	+ 05	+ 108
29	+ 073	+ 07	+ 080	+ 084	+ [illegible]	+ 091	+ 095	+ 098	+ 02	+ 105
30	+ 070	+ 074	+ 078	+ 081	+ 0	+ 088	+ 092	+ 095	+ 99	+ 102
31	+ 068	+ 0 1	+ 075	+ 078	+ [illegible]	+ 085	+ 089	+ 095	+ 096	+ 100
32	+ 065	+ 00	+ 072	+ 075	+ 079	+ 083	+ 086	+ 090	+ 09	+ 097
33	+ 062	+ 066	+ 009	+ 073	+ 070	+ 080	+ 083	+ 087	+ 000	+ 094
34	+ 0 9	+ 06	+ 006	+ 070	+ 0	+ 077	+ 080	+ 084	+ 088	+ 091
35	+ 057	+ 060	+ 004	+ 007	+ 071	+ 074	+ 078	+ 081	+ 085	+ 088
36	+ 054	+ 057	+ 061	+ [illegible]	+ 068	+ 071	+ 075	+ 078	+ 08	+ 085
37	+ 05	+ 055	+ 058	+ [illegible]	+ 065	+ 009	+ 072	+ 076	+ 079	+ 085
38	+ 048	+ 052	+ 055	+ 0 0	+ 06	+ 066	+ 069	+ 073	+ 0 6	+ 080
39	+ 046	+ 049	+ 05	+ 0 0	+ 000	+ 063	+ 067	+ 070	+ 074	+ 077
40	+ 043	+ 046	+ 050	+ 055	+ 057	+ 060	+ 064	+ 067	+ 071	+ 074
41	+ 040	+ 044	+ 047	+ 051	+ 054	+ 058	+ 061	+ 065	+ 068	+ 071
42	+ 038	+ 041	+ 045	+ 048	+ 051	+ 055	+ 058	+ 06	+ 065	+ 009
43	+ 035	+ 0 8	+ 042	+ 045	+ 049	+ 05	+ 0 6	+ 0 9	+ 003	+ 060
44	+ 03	+ 036	+ 039	+ 043	+ 046	+ 050	+ 055	+ 056	+ 060	+ 05
45	+ 030	+ 033	+ 037	+ 040	+ 044	+ 04	+ 050	+ 054	+ 05	+ 001
46	+ 0	+ 031	+ 034	+ 037	+ 041	+ 044	+ 048	+ 05	+ 054	+ 058
47	+ 025	+ 0 8	+ 0 1	+ 035	+ 038	+ 0	+ 045	+ 049	+ 05	+ 0 5
48	+ 022	+ 025	+ 029	+ 0	+ 036	+ 039	+ 01	+ 046	+ 049	+ 055
49	+ 019	+ 0	+ 026	+ 030	+ 0	+ 0 6	+ 040	+ 043	+ 017	+ 0 0
50	+ 01	+ 020	+ 024	+ 027	+ 030	+ 034	+ 03	+ 041	+ 044	+ 047

TABLE XIV *Continued*

Bar	-90	-91	-92	-93	-94	-95	-96	97	-98	299	300
Ther											
50	− 020	− 017	− 014	− 010	− 007	− 003	000	+ 003	+ 00	− 010	+ 014
51	− 023	− 019	− 016	− 0	− 009	− 006	− 003	+ 001	+ 004	+ 008	+ 01
52	− 025	− 022	− 019	− 015	− 012	− 008	− 005	− 002	+ 002	+ 005	+ 008
53	− 028	− 024	− 021	− 018	− 014	− 011	− 008	− 004	− 001	+ 00	+ 006
54	− 030	− 027	− 023	− 020	− 017	− 013	− 010	− 007	− 00	000	+ 004
55	− 032	− 029	− 026	− 022	− 019	− 016	− 012	− 009	− 006	− 002	+ 001
56	− 035	− 032	− 028	− 025	− 022	− 018	− 015	− 012	− 008	− 005	− 002
57	− 037	− 034	− 031	− 027	− 024	− 021	− 017	− 014	− 011	− 007	− 004
58	− 040	− 036	− 033	− 030	− 026	− 023	− 020	− 016	− 013	− 010	− 006
59	− 042	− 039	− 035	− 032	− 029	− 025	− 022	− 019	− 015	− 012	− 009
60	− 044	− 041	− 038	− 034	− 031	− 028	− 024	− 021	− 018	− 015	− 011
61	− 047	− 043	− 040	− 037	− 033	− 030	− 027	− 024	− 020	− 017	− 014
62	− 049	− 046	− 042	− 039	− 036	− 032	− 029	− 026	− 023	− 019	− 016
63	− 051	− 048	− 045	− 041	− 038	− 035	− 032	− 028	− 025	− 022	− 018
64	− 053	− 050	− 047	− 044	− 040	− 037	− 034	− 031	− 027	− 024	− 021
65	− 056	− 052	− 049	− 046	− 043	− 039	− 036	− 033	− 030	− 026	− 023
66	− 058	− 055	− 052	− 048	− 045	− 042	− 039	− 035	− 032	− 029	− 026
67	− 060	− 057	− 054	− 051	− 047	− 044	− 041	− 038	− 034	− 031	− 028
68	− 063	− 059	− 056	− 053	− 050	− 046	− 043	− 040	− 037	− 033	− 030
69	− 065	− 062	− 058	− 055	− 052	− 049	− 045	− 042	− 039	− 036	− 032
70	− 067	− 064	− 061	− 057	− 054	− 051	− 048	− 044	− 041	− 038	− 035
71	− 069	− 066	− 063	− 060	− 056	− 053	− 050	− 04	− 044	− 040	− 037
72	− 07	− 068	− 065	− 062	− 059	− 055	− 052	− 049	− 046	− 043	− 039
73	− 074	− 070	− 067	− 064	− 061	− 058	− 054	− 05	− 048	− 045	− 041
74	− 076	− 073	− 069	− 066	− 063	− 060	− 057	− 053	− 050	− 047	− 044
75	− 078	− 075	− 072	− 068	− 065	− 062	− 059	− 056	− 053	− 049	− 046
76	− 080	− 077	− 074	− 071	− 067	− 064	− 061	− 058	− 055	− 052	− 048
77	− 082	− 079	− 076	− 073	− 070	− 066	− 06	− 060	− 057	− 054	− 051
78	− 084	− 081	− 078	− 075	− 072	− 069	− 065	− 062	− 059	− 056	− 053
79	− [illegible]	− 08	− [illegible]	− 0	− 0	− 0	− [illegible]	− 065	− 061	− 058	− 055
80	− [illegible]	− 0	− [illegible]	− [illegible]	− [illegible]	− [illegible]	− [illegible]	− [illegible]	− 064	− [illegible]	− [illegible]

TABLE XIV *Continued*

Bar	30 1	30 2	30 3	30 4	30 5	30 6	30 7	30 8	30 9	31 0
Ther										
50	+ 017	+ 020	+ ,024	+ 027	+ 030	+ 034	+ 037	+ ,041	+ 044	+ 047
51	+ 014	+ ,018	+ 021	+ 025	+ 028	+ 031	+ 035	+ 038	+ ,041	+ 045
52	+ 012	+ 015	+ ,019	+ 022	+ ,025	+ 029	+ 032	+ 035	+ 039	+ 042
53	+ 009	+ 013	+ 016	+ 019	+ 022	+ 026	+ 030	+ 033	+ 036	+ 040
54	+ 007	+ 010	+ 014	+ 017	+ 020	+ 024	+ 027	+ 030	+ 034	+ 037
55	+ 004	+ 008	+ 011	+ 014	+ 018	+ 021	+ 024	+ 028	+ 031	+ 034
56	+ 002	+ 005	+ 009	+ 012	+ 015	+ 019	+ 022	+ ,025	+ 029	+ 032
57	− 001	+ 003	+ 006	+ 009	+ 013	+ 016	+ 019	+ 023	+ 026	+ 029
58	− ,003	000	+ 004	+ 007	+ 010	+ 014	+ 017	+ 020	+ 024	+ 027
59	− 006	− 002	+ 001	+ ,004	+ 008	+ 011	+ 014	+ 018	+ 022	+ 024
60	− 008	− 005	− 001	+ ,002	+ 005	+ 009	+ 012	+ 015	+ 019	+ 022
61	− 010	− 007	− 004	− 001	+ 003	+ 006	+ 009	+ 013	+ 016	+ 019
62	− 013	− 009	− 006	− ,003	000	+ 004	+ 007	+ 010	+ 014	+ 017
63	− 015	− 012	− 009	− ,005	− 002	+ 001	+ ,005	+ 008	+ 011	+ 014
64	− 018	− 014	− 011	− 008	− ,004	− 001	+ 002	+ ,005	+ 009	+ 012
65	− 020	− 017	− 013	− 010	− 007	− 004	000	+ 003	+ 006	+ 010
66	− 022	− 019	− 016	− 013	− 009	− 006	− 003	+ 001	+ 004	+ 007
67	− 025	− 021	− 018	− 015	− 012	− 008	− 005	− 002	+ ,001	+ 005
68	− 027	− 024	− 020	− 017	− 014	− 011	− 008	− 004	001	+ 002
69	− 029	− 026	− 023	− 020	− 016	− 013	− 010	− 007	− ,004	000
70	− 032	− 028	− 025	− 022	− 019	− 015	− 012	− 009	− 006	− 003
71	− 034	− ,031	− 027	− 024	− 021	− 018	− 015	− ,011	− ,008	− 005
72	− 036	− 033	− 030	− 027	− 023	− 020	− 017	− 014	− 011	− 007
73	− ,038	− 035	− 032	− 029	− 026	− 022	− 019	− 016	− 013	− 010
74	− 041	− 038	− 034	− 031	− 028	− 025	− 022	− ,018	− 015	− 012
75	− 043	− ,040	− 037	− 033	− 030	− 027	− ,024	− 021	− 017	− 014
76	− 045	− 042	− 039	− 036	− 032	− 029	− 026	− 023	− 020	− 017
77	− 047	− 044	− 041	− 038	− 035	− 032	− 028	− 025	− 022	− 019
78	− 050	− 047	− 043	− 040	− 037	− 034	− 031	− 028	− 024	− 021
79	− 052	− 049	− 046	− ,043	− 039	− ,036	− 033	− 030	− 027	− 023
80	− 055	− 051	− 048	− 045	− 042	− 038	− 035	− 033	− 029	− 026

TABLE XV

Augmentation of the Semi Diameter of the Moon

Apparent Altitude	HORIZONTAL DIAMETER OF THE MOON													App Zen Distance
	29 30	29 50	30 10	30 30	30 50	31 10	31 30	31 50	32 10	32 30	32 50	33 10	33 30	
Deg														Deg
0	0,3	[illegible]	0,3	0,3	0,3	0,3	0,3	[illegible]	[illegible]	0,4	0,4	0,4	0,4	90
2	1,2	1,3	[illegible]	1,3	1,4	1,4	1,4	1,5	1,5	1,5	1,6	1,6	1,6	88
4	[illegible]	[illegible]	2,3	2,4	2,4	2,5	2,5	[illegible]	2,6	2,7	[illegible]	2,9	[illegible]	86
6	[illegible]	[illegible]	3,3	3,4	3,5	3,6	3,6	3,7	[illegible]	3,9	4,0	4,0	4,1	84
8	4,1	4,2	4,3	4,4	4,5	4,6	4,7	[illegible]	[illegible]	5,0	5,2	5,2	5,4	82
10	5,1	5,2	[illegible]	5,5	5,6	5,7	[illegible]	6,0	6,1	6,2	6,3	6,5	6,6	80
11	5,6	5,7	5,8	6,0	6,1	[illegible]	6,4	6,5	6,7	6,8	6,9	7,1	7,2	79
12	6,0	6,2	6,4	6,5	6,6	6,8	6,9	7,1	7,2	[illegible]	7,5	7,7	7,8	78
13	6,5	6,6	6,8	7,0	7,1	[illegible]	[illegible]	7,6	[illegible]	7,9	8,1	8,3	8,4	77
14	7,0	7,1	7,3	7,5	7,7	[illegible]	8,0	8,2	8,3	[illegible]	8,7	8,9	9,0	76
15	7,1	7,6	7,5	8,0	8,2	8,4	8,5	8,7	[illegible]	9,1	9,3	9,5	9,6	75
16	7,9	8,1	8,3	8,5	8,7	8,9	9,1	[illegible]	[illegible]	9,7	9,9	10,1	10,3	74
17	8,4	8,6	8,8	9,0	9,2	9,4	9,6	[illegible]	10,0	[illegible]	10,4	10,6	10,8	73
18	8,6	9,1	9,3	9,5	9,7	9,9	10,1	10,4	10,6	[illegible]	11,0	[illegible]	11,4	72
19	9,3	9,5	9,8	10,0	10,2	10,4	10,7	10,9	11,1	11,4	11,6	11,8	[illegible]	71
20	9,8	10,0	10,2	10,5	10,7	11,0	11,2	11,4	11,7	11,9	[illegible]	[illegible]	[illegible]	70
21	10,2	10,5	10,7	11,0	11,2	11,5	11,7	[illegible]	12,2	12,5	[illegible]	[illegible]	13,2	69
22	10,7	10,9	11,2	11,5	11,7	12,0	12,2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13,5	68
23	11,1	11,4	11,7	11,9	12,2	12,5	12,8	13,0	[illegible]	13,6	13,9	14,1	14,4	67
24	11,6	11,9	12,1	12,4	12,7	[illegible]	13,3	13,5	13,6	14,1	[illegible]	14,7	15,0	66
25	[illegible]	12,3	12,6	12,9	13,2	13,5	13,8	14,1	[illegible]	14,7	[illegible]	[illegible]	[illegible]	65
26	12,4	12,8	13,0	[illegible]	13,7	[illegible]	14,3	14,6	14,9	15,2	[illegible]	[illegible]	16,1	64
27	12,9	[illegible]	13,5	[illegible]	14,1	14,5	[illegible]	15,1	15,1	15,7	[illegible]	[illegible]	16,7	63
28	13,3	[illegible]	14,0	[illegible]	14,6	15,0	15,3	15,6	15,9	[illegible]	[illegible]	[illegible]	[illegible]	62
29	13,7	14,1	14,4	14,7	15,1	15,4	15,8	16,1	[illegible]	[illegible]	[illegible]	17,1	17,8	61
30	14,2	14,5	14,9	[illegible]	15,6	15,9	16,2	[illegible]	[illegible]	17,3	17,6	[illegible]	[illegible]	60
31	14,6	14,9	[illegible]	[illegible]	16,0	16,1	16,7	17,1	17,4	[illegible]	[illegible]	[illegible]	[illegible]	59
32	[illegible]	15,4	[illegible]	16,1	[illegible]	16,5	17,2	17,6	17,9	18,3	[illegible]	[illegible]	[illegible]	58
33	[illegible]	15,5	16,2	16,5	16,9	[illegible]	17,7	[illegible]	[illegible]	[illegible]	19,2	[illegible]	[illegible]	57
34	[illegible]	[illegible]	16,6	17,0	17,4	17,8	[illegible]	18,5	[illegible]	[illegible]	19,7	20,1	[illegible]	56
35	16,2	16,6	17,0	17,4	[illegible]	[illegible]	18,6	19,0	19,4	19,6	20,2	[illegible]	[illegible]	55
36	16,6	17,0	17,4	[illegible]	[illegible]	18,6	19,1	19,5	19,9	20,3	20,7	[illegible]	[illegible]	54
37	17,0	17,4	[illegible]	[illegible]	18,7	19,1	19,5	19,9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	53
38	17,4	17,8	[illegible]	18,7	19,1	19,5	20,0	20,4	[illegible]	[illegible]	21,6	22,1	[illegible]	52
39	[illegible]	[illegible]	18,6	19,1	19,5	20,0	20,4	20,8	[illegible]	21,7	[illegible]	[illegible]	[illegible]	51
40	[illegible]	[illegible]	19,0	19,5	19,9	20,4	[illegible]	21,2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	50

TABLE XV *Continued*

Apparent Altitude	HORIZONTAL DIAMETER OF THE MOON													App Zenith Diſtance
	29 30	29 50	30 10	30 30	30 50	31 10	31 30	31 50	32 10	32 30	3– 50	33 10	33 30	
Deg														Deg
40	18, 2	18, 6	19, 0	19, 5	19, 9	–0 4	–0, 8	21 –	21, 7	22 –	22 6	–3, 0	23 5	50
41	18, 5	19, 0	19 4	19 9	–0 3	20, 8	21 –	–1, 7	– 1	22 6	23, 0	23 5	24, 0	49
42	18 9	19 3	19, 8	20 3	20 7	–1, –	21 7	–2 1	– 6	23, 0	–3 5	24 0	–4 4	48
43	19, 2	19 7	–0, 2	20 7	21 1	–1, 6	2–, 1	22 6	23 0	23, 5	23, 9	24 4	4 9	47
44	19, 6	20 1	20, 6	21, 0	21 5	2 0	–– 5	–3 0	23, 4	23, 9	24 4	4 9	5 4	46
45	19 9	–0 4	–0 9	1 4	21, 9	–– 4	2– 9	23 4	–3 8	24 3	–4, 8	–5 3	5 6	4
46	20 3	20, 8	21 3	21, 8	22, 3	2– 8	23, 8	–3 8	–4 3	24 8	25, 3	–5 8	26 3	44
47	20, 6	–1, 1	21 6	2 1	22 6	–3, 1	23 6	–4, 1	–4, 7	25 –	25, 7	–6 –	–6 7	43
48	–0 9	21 5	–2, 0	–2 5	–3 0	–3 5	24 0	–4, 5	–5 1	25 5	26 1	–6 6	–7 1	42
49	–1 3	21, 8	22, 3	22, 8	–3, 3	23, 9	24, 4	24, 9	–5 4	26 0	26, 5	27 0	–7 5	41
50	21, 6	––, 1	22, 6	23 –	23, 7	24, –	24 8	25, 3	25, 8	6 4	26, 9	–7 4	27, 9	40
51	21, 9	2– 4	23 0	23 5	24, 0	24, 6	–5, 1	25 7	26, 2	26, 7	27, 3	–7 8	28 3	39
52	2–, 2	22, 7	–3, 3	23, 8	24, 4	24, 9	–5 5	26, 0	26, 5	–7, 1	–7, 6	28 2	–8, 7	38
53	22, 5	23, 0	–3, 6	24, 1	24 7	25, 2	–5 8	26, 3	26 9	27, 5	28, 0	28, 6	29 1	37
54	22, 8	23 4	23, 9	24 5	25, 0	25, 6	26 1	26 7	27, 3	–7 8	–8 4	–8 9	–9 5	36
55	3, 1	23, 6	24, 2	24, 8	–5, 3	25 9	26, 5	27, 0	27 6	28, –	28 7	29 3	29 8	35
56	23, 4	23 9	24, 5	25, 1	25, 6	26, 2	26, 8	7 3	27 9	–6 5	29, 1	29 6	30 2	34
57	23, 6	24 2	24 6	25 3	–5 9	6 5	7, 1	27, 7	28	–8 6	–9 4	30 0	30 5	33
58	23, 9	24, 5	25, 1	25, 6	26 –	–6 8	7, 4	28 0	26, 6	29 1	–9, 7	30 3	30 9	3–
59	24 1	24 7	–5 3	–5 9	26 5	27, 1	27 7	28, 3	–8, 9	29, 5	30, 0	30 6	31 –	31
60	24 4	–5 0	25 6	–6 2	–6 8	27 4	28, 0	28 6	29 –	–9, 8	30 4	31 0	31 6	30
62	24, 7	25 5	26, 1	26 7	–7 3	27, 9	28 5	–9, 1	29, 7	30 3	30, 9	31, 6	3–, –	–8
64	25, 3	25 9	26, 5	27, 2	27, 8	28, 4	29 0	29, 6	30, 3	30, 9	31, 5	32, 1	32, 7	26
66	25, 9	26 3	–7, 0	–7 6	28 2	28, 9	29 5	30 1	30, 7	31, 4	3– 0	3– 6	33 3	–4
68	–6 1	26, 7	–7, 4	28, 0	28, 7	29, 3	9, 9	30 6	31 2	31, 8	32, 5	33, 1	33 7	––
70	26, 5	27 1	–7 7	28, 4	29, 3	29 7	30 3	31, 0	31, 6	3 , 3	32, 9	33, 6	34,	–0
7–	26, 6	27 4	–8 1	–8, 7	29, 4	30 0	30, 7	31, 3	32 0	32	33, 3	34 0	34 6	18
74	27 0	27 7	–8 4	–9 0	29 7	30 4	31, 0	31, 7	32, 3	33 0	33 7	34 3	35, 0	16
76	–7 3	27, 9	28 6	9 3	30 0	30, 6	31, 3	32 0	32, 6	33 3	34, 0	34, 7	35 3	14
78	–, 5	28 2	–8, 9	29 5	30, –	30, 9	31 6	32 2	32, 9	33 6	34 –	34, 9	35 6	1–
80	, 7	28, 4	–9 1	9 7	30, 4	31, 1	31 8	32, 4	33, 1	33, 6	34, 5	35, –	35 6	10
82	–7, 9	28, 5	–9, 2	9 –	30, 6	31, 3	31 9	32 6	33 3	34, 0	34 7	35 3	36 0	8
84	28, 0	28, 7	29 3	30 0	30, 7	31, 4	3 1	32, 8	33 4	34, 1	34 8	35 5	36 2	6
86	28, 1	–6 7	–9, 4	30 1	30 9	31 5	3 , 2	32 9	33 6	34 2	34 9	35, 6	36 3	4
88	28 1	25 8	29, 5	30, –	30 9	31 5	32, 2	32, 9	33, 6	34, 3	35 0	35 –	36 4	
90	28 1	28 8	–9, 3	30	30, 9	31, 6	3 3	33 0	33 6	34, 3	35, 0	35, 7	36, 4	0

TABLE XVI

Mean Preceſſion of the Equinoctial Points in Longitude, for complete Years.

Years	Preceſſion	Years	Preceſſion	Years	Preceſſion	Years	Preceſſion
1	0 50 3	31	26 0 8	61	0 51 11 3	91	1 16 21 8
2	1 40 7	32	26 51 2	62	0 52 1 7	92	1 17 12 2
3	2 31, 0	33	27 41, 5	63	0 52 52 0	93	1 18 2, 5
4	3 21 4	34	28 31 9	64	0 53 42 4	94	18 52, 9
5	4 11, 7	35	29 22, 2	65	0 54 32, 7	95	1 19 43, 2
6	5 2 1	36	30 12 6	66	0 55 23 1	96	1 20 33, 6
7	5 52, 4	37	31 2, 9	67	0 56 13, 4	97	1 21 23, 9
8	6 42 8	38	31 53	68	0 57 3 8	98	1 22 14, 3
9	7 33 1	39	32 43 6	69	0 57 54 1	99	1 23 4, 0
10	8 23 5	40	33 34, 0	70	0 58 44 5	100	1 23 55, 0
11	9 13 8	41	34 24, 3	71	0 59 34 8	200	2 47 49 9
12	10 4, 2	42	35 14, 7	72	1 0 25 2	300	4 11 44 9
13	10 54, 5	43	36 5 0	73	1 1 15 5	400	5 35 39 9
14	11 44, 9	44	36 55 4	74	1 2 5 9	500	6 59 34 8
15	12 35, 2	45	37 45, 7	75	1 2 56 2	600	8 23 29, 8
16	13 25 6	46	38 36, 1	76	1 3 46 6	700	9 47 24, 8
17	14 15 9	47	39 26, 4	77	1 4 36 9	800	11 11 19, 7
18	15 6 3	48	40 16, 8	78	1 5 27 3	900	12 35 14 7
19	15 56 6	49	41 7, 1	79	1 6 17 6	1000	13 59 9 7
20	16 47, 0	50	41 57, 5	80	1 7 8 0	2000	27 58 19, 4
21	17 37, 3	51	42 47 8	81	1 7 58, 3	3000	41 57 20, 0
22	18 27, 7	52	43 38, 2	82	1 8 48, 7	4000	55 56 38, 6
23	19 18 0	53	44 28 5	83	1 9 39, 0	5000	69 55 48, 3
24	20 8 4	54	45 18 9	84	1 10 29 4		
25	20 58 7	55	46 9, 2	85	1 11 19 7		
26	21 49, 1	56	46 59, 6	86	1 12 10, 1		
27	22 39, 4	57	47 49 9	87	1 13 0 4		
28	23 29, 8	58	48 40 3	88	1 13 50, 8		
29	24 20, 1	59	49 30 6	89	1 14 41, 1		
30	25 10 5	60	50 21 0	90	1 15 31 5		

TABLE XVII

Precession of the Equinoctial Points in Longitude to every Day in the Year, including the Solar Equation of Precession

Days	January	February	March	April	May	June	July	August	September	October	November	December
	0 6	5 6	9 0	12 1	15, 5	20, 2	25 5	30 5	34, 4	37 [illegible]	40 9	45 4
[illegible]	0 8	5 7	9 1	[illegible]	15 7	20 4	25 6	30 6	34 5	37 5	41, 0	45 6
3	1, 0	5 8	9 [illegible]	12, 3	15, 8	20 6	25 8	30 8	34, 6	37 6	41 2	45 8
[illegible]	[illegible] 1	6 0	9 3	[illegible] 4	15 9	20, 7	26 0	30 9	34 7	37 7	41 3	45 9
5	1, 3	6 1	9 4	12 5	16 1	20 9	26 [illegible]	31 1	34 8	37 8	41 4	46 1
6	1 [illegible]	6 3	9 5	12, 6	16, 2	21 1	26 3	31 [illegible]	34 9	37, 9	41 6	46 3
	[illegible] 6	6 4	9, 6	12 7	16, 3	21 3	26, 5	31 3	35 0	38, 0	41, 7	46 4
8	[illegible] 8	6 5	9 7	12 8	16 5	21 4	26 7	31 5	35 [illegible]	38 2	41 9	46 6
9	[illegible] 0	6 7	9 8	12, 9	16 6	21 6	26 8	31 6	35 3	38 3	42 0	46 8
10	[illegible] 1	6 8	9 9	13 0	16 8	21 8	27 0	31 7	35, 4	38 4	42 1	47, 0
	2 3	6 9	10 0	13 1	16 9	22, 0	27 [illegible]	31 9	35 5	38, 5	42 3	47 1
12	2 5	7 0	10 1	13 3	17 1	22, 1	27, 4	32 0	35 6	38 6	42 4	47 3
13	[illegible] 6	7 [illegible]	10 [illegible]	13 4	17 2	22 3	27 5	32 2	35 7	38 7	42 6	47, 5
14	[illegible] 8	7 3	10, 3	13, 5	17, 4	22 5	27, 7	32 3	35 8	38 8	42 [illegible]	47, 7
15	3 0	7 4	10 4	13 6	17 5	22, 7	27 8	32 4	35 9	38 9	42 9	47 8
16	3 1	7, 5	10, 5	13, 7	17 7	22 8	28, 0	32 5	36, 0	39 0	43 0	48, 0
17	3 3	7 6	10 6	13 8	17 8	23 0	28 [illegible]	32 7	36 1	39 1	43 2	48 [illegible]
18	3 4	7 8	10 7	13 9	18, 0	23 2	28 3	32 8	36 2	39, 2	43 3	48 4
19	3 6	7, 9	10 8	14 0	18 1	23 4	28 5	32 9	[illegible] 6	39 3	43 5	48 6
20	3 8	8, 0	10 9	14, [illegible]	18 3	23 5	28 7	33, 0	36 4	39 4	43 6	48 7
[illegible]	3 9	8 1	11 0	14 3	18 4	23 7	28 8	33 [illegible]	36 5	39 6	43 8	48 9
22	[illegible]	8 2	11 1	14 4	18 6	23 9	29, 0	33 3	36, 6	39 7	43 9	49 1
23	4 [illegible]	8 3	11, [illegible]	14 5	18 8	24 1	29, 1	33 4	36 7	39 8	44 1	49 3
24	4, 4	8 4	11 3	14, 6	18 9	24 [illegible]	29 3	33 5	36 8	39 9	44 3	49 5
25	[illegible] 5	8 6	11 4	14, 8	19 1	24 4	29 4	33 6	36 9	40, 0	44 4	49, 6
26	4, 7	8 7	11 5	14, 9	19 2	24, 6	29 6	33 7	37 0	40 [illegible]	44, 6	49, 8
27	4, 8	8, 8	11 6	15 0	19 4	24, 8	29 7	33 9	37 1	40 3	44 8	50 0
28	5, 0	8 9	11 7	15 1	19, 6	24 9	29 9	34 0	[illegible]	40 4	44 9	50 [illegible]
29	5 1		11 8	15 3	19 7	25 1	30 0	34, 1	37 [illegible]	40, [illegible]	45 1	50 4
30	5 3		11 9	15, 4	19 9	25 3	30 [illegible]	34 [illegible]	37 3	40, 7	45 3	50 5
31	5 4		12 0		20 1		30 3	34, 3		40 8		50 7

In the months of January and February in leap years take out for the day preceding the given day

TABLE XVIII

The Equation of the Equinoxes in Longitude

ARGUMENT Long of ☽ s ☊

Degrees	O Sig − VI Sig +	I Sig − VII Sig +	II Sig − VIII Sig +	
0	0 0	8,9	15,5	30
1	0 3	9 2	15 6	29
2	0,6	9,5	15,8	28
3	0,9	9 7	15,9	27
4	1,2	10 0	16 1	26
5	1,6	10,2	16 2	25
6	1 9	10,5	16,3	24
7	2,2	10,8	16,4	23
8	2 5	11,0	16,6	22
9	2 8	11 2	16,7	21
10	3 1	11 5	16,8	20
11	3,4	11,7	16,9	19
12	3,7	12,0	17 0	18
13	4,0	12,2	17,1	17
14	4,3	12,4	17,2	16
15	4,6	12 6	17,3	15
16	4,9	12,8	17,3	14
17	5,2	13,1	17 4	13
18	5 5	13,3	17 5	12
19	5 8	13,5	17,5	11
20	6,1	13,7	17,6	10
21	6 4	13,9	17 6	9
22	6 7	14,1	17 7	8
23	7,0	14,3	17,7	7
24	7 3	14,5	17 8	6
25	7 5	14 6	17 8	5
26	7 8	14,8	17,8	4
27	8,1	15,0	17,8	3
28	8,4	15,1	17 9	2
29	8,7	15,3	17 9	1
30	8,9	15 5	17 9	0
	V Sig − XI Sig +	IV Sig − X Sig +	III Sig − IX Sig +	Degrees

TABLE XIX

The Mean Motions of the Sun in Right Aſcenſion in Time, *to every Day in the Year*

Days	January			February			March			April			May			June		
	H	M	S	H	M	S	H	M	S	H	M	S	H	M	S	H	M	S
1	0	3	56 6		6	9 8	3	56	33, 3	5	58	46, 5	7	57	3, 2	9	59	16,4
2	0	7	53, 1	2	10	6 3	4	0	29 9	6	2	43, 1	8	0	59 8	10	3	1,0
3	0	11	49 7	2	14	2, 9	4	4	26 4	6	6	39 7	8	4	56, 3	10	7	9 5
4	0	15	46 2	2	17	59 4	4	8	23, 0	6	10	36 2	8	8	52 9	10	11	6 1
5	0	19	42, 8	2	21	56, 0	4	12	19, 5	6	14	32, 8	8	12	49, 4	10	15	2,6
6	0	23	39, 3		25	5, 5	4	16	16 1	6	18	29 3	8	16	46, 0	10	18	59 2
7	0	27	35, 9	2	29	49, 1	4	20	12, 7	6	22	25 9	8	20	4, 5	10		55,7
8	0	31	32, 4	2	33	45 7	4	24	9 2	6	26	22, 4	8	24	39 1	10	26	5 3
9	0	35	29 0		37	42, 2	4	28	5 6	6	30	19, 0	8	28	35 6	10	30	48,9
10	0	39	25, 6	2	41	38, 8	4	32	2, 3	6	34	15 5	8	32	32 [illegible]	10	34	45 4
11	0	43	22, 1	2	45	35 3	4	35	58, 9	6	38	12, 1	8	36	28, 8	10	38	4 ,0
12	0	47	18 7	2	49	31, 9	4	39	55 4	6	42	8, 6	8	40	25, 3	10	42	38, 5
13	0	51	15, 2	2	53	28 4	4	43	52, 0	6	46	5, 2	8	44	21, 9	10	46	35 1
14	0	55	11, 8	2	57	25, 0	4	47	48, 5	6	50	1, 8	8	48	18, 4	10	50	31,6
15	0	59	8, 3	3	1	21, 5	4	51	45, 1	6	53	58, 3	8	52	15, 0	10	54	28,2
16	1	3	4, 9	3	5	18, 1	4	55	41, 6	6	57	54, 9	8	56	11 5	10	58	24,7
17	1	7	1, 4	3	9	14, 7	4	59	38 [illegible]	7	1	51, 4	9	0	8 1	11	2	21,3
18	1	10	58 0	3	13	11, 2	5	3	34, 8	7	5	46, 0	9	4	4, 6	11	6	17 9
19	1	14	54 5	3	17	7, 8	5	7	31 3	7	9	44, 5	9	8	1 [illegible]	11	10	14, 4
20	1	18	51 1	3	21	4 3	5	11	27 9	7	13	41, 1	9	11	57, 7	11	14	11,0
21	1	22	47, 7	3	25	0 9	5	15	24, 4	7	17	37 6	9	15	54 3	11	18	7 5
22	1	26	44, 2	3	8	57 4	5	19	21, 0	7	21	34, 2	9	19	50, 9	11		4 1
23	1	30	40, 8	3	32	54, 0	5	23	17, 5	7	25	30, 8	9	23	47 4	11	26	0 6
24	1	34	37, 3	3	36	50, 5	5	27	14 1	7	29	27 3	9	27	44, 0	11	29	57 [illegible]
25	1	38	33 9	3	40	47 1	5	31	10, 6	7	33	23 9	9	31	40, 5	11	33	53,7
26	1	42	30 4	3	44	43, 6	5	35	7, [illegible]	7	37	0, 4	9	35	37 1	11	37	50,3
27	1	46	27, 0	3	48	40 [illegible]	5	39	3, 8	7	41	17, 0	9	39	33, 6	11	41	46,9
28	1	50	23 5	3	52	36 8	5	43	0, 3	7	45	13 5	9	43	30, 2	11	45	43 4
29	1	54	20 1				5	46	56 9	7	49	10, 1	9	47	26, 7	11	49	40,0
30	1	58	16, 7				5	50	53 4	7	53	6, 6	9	51	23 3	11	53	36,5
31	2	2	13, 2				5	54	50, 0				9	55	19, 9			

In the months of January and February in leap year, take out for the day preceding the given day

TABLE XIX *Continued*

Days	July			Auguſt			September			October			November			December		
	H	M		H	M	S	H	M		H	M	S	H	M	S	H	M	S
1	11	57	33 1	13	59	46,3	16	1	59	18	0	16,2	0	2	29 4	2	0	46 0
2	1	1	9 6	14	3	4 5	16	5	56 1	18	4	1 7	0	6	26 0	2	4	42 6
3	1	5	26,	14	7	39,4	16	9	52,6	18	8	9,3	20	10	22 5	2	8	39 2
4	12	9	7	14	11	36,0	16	13	49	18	12	5 9	0	14	19 1	2	12	35 7
5	1	13	19,3	14	15	32 5	16	17	4, 7	1	16	2,4	20	18	15 6	2	16	32 3
6	12	17	15,9	14	19	9,1	16	21	4	1	19	5,0	20	22	1 2	2	20	28 8
7	1	21	1 ,4	14	3	5,6	16	25	3 8	1	3	55 5	0	26	8 7	2	24	25 4
8	12	25	9 0	14	27	22 2	16	29	35 4	1	27	52,1	0	30	5,3	2	28	21 9
9	1	9	5,5	14	31	18 7	16	33	3 0	1	31	48 6	20	34	1 5		3	18 5
10	12	33	2 1	14	35	15 3	16	37	25 5	18	35	45,2	0	37	58 4	2	36	15 0
11	1	36	58,6	14	39	11,8	16	41	25 1	18	39	41 7	20	41	54 9		40	11 6
12	12	40	55 2	14	43	8 4	16	45	21 6	1	43	38 3	20	45	51 5	2	44	8,2
13	12	44	51,7	14	47	5 0	16	49	18 2	18	47	34 5	0	49	48 1	2	48	4,
14	12	48	43,	14	51	1 5	16	53	14 7	18	51	31 4	0	53	41,6	22	52	1 3
15	12	52	44,	14	54	58 1	16	57	11,3	18	55	7 9	0	57	41 2	2	55	57
16	1	56	41 1	14	58	54 6	17	1	7 8	18	59	24,5	21	1	37 7		59	51 4
17	13	0	38,0	15	2	51,2	17	5	4,4	19	3	21,1	21	5	34,3	3	3	50,9
18	13	4	31,5	15	6	47,7	17	9	1 0	19	7	17,6	21	9	30,8	23	7	47,5
19	13	8	31 1	15	10	44 3	17	12	57,5	19	11	14 2	1	13	27,1	23	11	44 0
20	13	12	27 6	15	14	40 8	17	16	54 1	19	15	10 7	1	17	2 9	23	15	40 6
1	13	16	24,2	15	18	37,4	17	20	50 6	19	19	7,3	21	1	0,5	23	19	37,1
22	13	20	0,7	15	2	34 0	17	24	47,2	19	23	3,8	1	25	17,1	3	23	33 7
23	13	24	17 3	15	26	30 5	17	28	43 7	19	27	0,4	21	29	13,6	3	7	30 3
24	13	8	13,6	15	30	27 1	17	32	40 3	19	30	56,9	21	3	10,2	23	31	2 6
25	13	32	10,4	15	34	3,6	17	36	36 8	19	34	53,5	21	37	6 7	23	35	2 ,4
26	13	36	7 0	15	38	0 2	17	40	33,4	19	38	50,1	1	41	3,3	23	39	19 7
27	13	40	3,5	15	42	16 7	17	44	29,9	19	42	46,0	21	11	59 9	23	43	16,5
8	13	44	0 1	15	46	13 3	17	48	26 5	19	46	43 2	21	48	0 4	3	47	13 0
29	13	47	56,6	15	50	9,8	17	5	23,1	19	50	39,7	1	52	5 ,9	23	51	9 6
30	13	51	53,	15	51	6 4	17	56	19,6	19	54	36,3	21	56	49,5	23	55	6,1
31	13	55	49 7	15	55	3 0				19	58	32,9				23	59	2,7

TABLE XX

Mean Motions of the Sun's Right Aſcenſion in Time, to Hours and Minutes of Sidereal Time

Sidereal Time	Sun's mean Motion in AR in Time	Sidereal Time	Sun's mean Motion in AR in Time	Sidereal Time	Sun's mean Motion in AR in Time
H	M S	M	S	M	S
1	0 9, 83	1	0 16	31	5 08
2	0 19 66	2	0 33	32	5, 24
3	0 9 49	3	0, 49	33	5, 41
4	0 39 3	4	0 66	34	5, 57
5	0 49 15	5	0 82	35	5 73
6	0 58 98	6	0 98	36	5 90
7	1 8 81	7	1 15	37	6 06
8	1 18 61	8	1, 31	38	6, 23
9	1 28, 47	9	1, 47	39	6, 39
10	1 38 30	10	1, 64	40	6, 55
11	1 48, 13	11	1, 80	41	6, 72
12	1 57 95	12	1, 97	42	6, 88
13	2 7, 78	13	2, 13	43	7, 05
14	2 17, 61	14	2, 29	44	7, 21
15	2 27, 44	15	2, 46	45	7, 37
16	2 37, 27	16	2, 62	46	7, 54
17	2 47 10	17	2, 78	47	7, 70
18	2 56 93	18	2, 95	48	7, 86
19	3 6 76	19	3, 11	49	8, 03
20	3 16, 59	20	3, 28	50	8 19
21	3 26 4	21	3, 44	51	8, 36
22	3 36 25	22	3, 60	52	8, 52
23	3 46 08	23	3, 77	53	8, 68
24	3 55, 91	24	3, 93	54	8, 85
		25	4, 10	55	9, 01
		26	4, 26	56	9, 17
		27	4, 42	57	9, 34
		28	4, 59	58	9, 50
		29	4, 75	59	9 67
		30	4, 91	60	9 83

TABLE XXI

Equation of the Equinoxes in Right Ascension, common to all the Stars

ARGUMENT Long of ☽'s ☊

Long of ☽'s ☊	0 Sig −	I Sig −	II Sig −	
	VI Sig +	VII Sig +	VIII Sig +	
0	0,0	8,2	14,2	30
1	0,3	8,4	14,3	29
2	0 6	8,7	14,5	28
3	0,9	8,9	14,6	27
4	1,1	9,2	14 7	26
5	1,4	9,4	14,8	25
6	1,7	9,6	15 0	24
7	2,0	9,8	15,1	23
8	2,3	10,1	15,2	22
9	2,6	10,3	15,3	21
10	2,8	10,5	15,4	20
11	3,1	10,7	15,5	19
12	3 4	11 0	15,6	18
13	3,7	11,2	15,7	17
14	4,0	11,4	15,7	16
15	4,2	11,6	15,8	15
16	4,5	11,8	15,9	14
17	4 8	12,0	16,0	13
18	5,1	12,2	16,0	12
19	5,3	12,4	16,1	11
20	5,6	12,5	16,1	10
21	5,9	12,7	16 2	9
22	6,1	12 9	16,2	8
23	6 4	13 1	16,2	7
24	6,7	13,2	16 3	6
25	6,9	13,4	16,3	5
26	7 2	13,6	16,3	4
27	7,4	13 7	16 3	3
28	7,7	13,9	16,4	2
29	7,9	14,0	16,4	1
30	8,2	14,2	16,4	0
	V Sig −	IV Sig −	III Sig	Long of ☊'s
	XI Sig +	X Sig +	IX Sig +	

TABLE XXII

Equation of the Equinoxes in Right Aſcenſion in Time, common to all the Stars

ARGUMENT Long of ☽'s ☊

Degrees	0 Sig −	I Sig −	II Sig −	
	VI Sig +	VII Sig +	VIII Sig +	
0	0 00	0 ,55	0 ,95	30
1	0 02	0 56	0, 95	29
2	0, 04	0, 58	0, 96	28
3	0 06	0 59	0 97	27
4	0 08	0, 61	0 98	26
5	0 10	0 63	0 99	25
6	0, 11	0 64	1 00	24
7	0 13	0 66	1, 00	23
8	0, 15	0, 67	1, 01	22
9	0, 17	0, 69	1, 02	21
10	0, 19	0, 70	1, 03	20
11	0, 21	0 72	1, 03	19
12	0, 23	0, 73	1, 04	18
13	0, 25	0, 74	1, 04	17
14	0 26	0, 76	1, 05	16
15	0, 28	0 77	1, 05	15
16	0 30	0 79	1, 06	14
17	0, 32	0, 80	1, 06	13
18	0, 34	0 81	1 07	12
19	0, 36	0, 82	1, 07	11
20	0, 37	0, 84	1, 08	10
21	0, 39	0, 85	1, 08	9
22	0, 41	0, 86	1, 08	8
23	0, 43	0, 87	1 08	7
24	0, 44	0 88	1 09	6
25	0, 46	0 89	1, 09	5
26	0 48	0, 90	1 09	4
27	0, 50	0 92	1, 09	3
28	0, 51	0, 93	1 09	2
29	0 53	0 94	1 09	1
30	0 55	0 95	1, 09	0
	− V Sig	− IV Sig	− III Sig	Degrees
	+ XI Sig	+ X Sig	+ IX Sig	

TABLE XXIII

Deviati n of Stars ı ı North Polar Distan e c m no ı to all the Stars

Long of moon s node	RIGHT ASCENSION OF THE STAR																		
	O VI						I VII						II VIII						III IX
	0	5	10	15	20	25	0	5	10	15°	20	25	0	5°	0°	15	20	25	0
D																			
O VI 0	+0 00	−0 8ɔ	−1 66	− +7	−ɔ −7	−+ 04	−+,/8	−ς −8	−6 1	−6 7	−ʼ 32	−/ 83	−8 −7	−8 6ɔ	−8 98	−9 −3	−9 41	−9 ɔ−	−9 ςɔ
5	0 62	0 −	1 04	1 0	6	−6	+ 2−	4,9	ɔ 64	6 −ᶜ	6 9	/ 44	7 0	8 6	8 /ɔ	9 0	9 6	9 4ɔ	9 ɔ−
10	1 −	+0 41	0 +−	1 −4	06	− 2 ⁹6	−ɔ 03	4,−8	ɔ 10	5 /8	6 −	6 99	/ ɔɔ	8 00	8 +	8 /6	9 0	0 −6	9 41
15	64	1 0	+0 −1	0 61	1 4	−−	ɔ,0	ɔ /8	4 52	5 22	, 88	6 ɔ0	/ 07	7 8	9 04	9 11	9 76	0 0	9 −ɔ
20	−,4	1 6+	0,84	+0 0,	0 8	1 ɔ9	−, 9	ɔ 16	ɔ 9c	+6ɔ	ɔ ɔ1	5 96	6 ɔ6	7,11	7 60	8,04	8,+	8, ɔ	8 98
25	01	− 4	1 46	0 56	0 ɔ	0 93	,/3	− ɔ0	−6	+ 00	− /0	ɔ ɔ7	ɔ 99	6 8	/ 1−	7 ɔ6	8 00	8 36	8 6ɔ
I VII 0	ɔ 56	−,52	,07	1 20	+0 ɔ1	0,27	1,0ɔ	1,83	− ɔ9	ɔ 3ɔ	10	4,/4	ɔ ɔ9	5,ɔ9	6 ɔ6	/ 07	/,5ɔ	/,9ɔ	8 7
5	4 08	ɔ ɔ8	− 66	1 0−	1 16	+0 ɔ9	0,28	1 ɔ	1 90	−,6ɔ	ɔ ɔ	4 07	+ /+	5 ɔ/	5 96	6 ɔ0	7 ɔ0	7 44	/,8ɔ
10	+ 57	ɔ 92	ɔ,−3	− ɔ2	1 9	1 0ɔ	+0, 0	0 4	1 −0	1 94	2 67	ɔ	4 0ɔ	4 70	5 1	^8	6 41	6 89	7 −
15	5 0ɔ	4 +−	3 /8	11	2 4−	1 /0	0 98	+0 −ɔ	0 +9	1 −−	1 ɔ−	− 6ɔ	ɔ ɔɔ	00	4 0ɔ		5 ɔ9	6 ɔ	6 /ɔ
20	5 4ɔ	4 89	4 ɔ0	ɔ 67	3 02	2 ɔ4	1,6ɔ	0,0+	+0,2ɔ	0 +9	1,−0	1,90	− ɔ9	−0	90	4 −	5 10	5 64	6 −
25	5 8ɔ	5 ɔɔ	4,/9	4,−1	3 60	− 9	,31	1 6ɔ	0 0+	+0 ɔ	0 4	1 1ɔ	1 ɔ	− ɔ0	ɔ 1ɔ	ɔ 8	4 ɔ8	+9ɔ	ς 48
II VIII 0	6 16	ς,7−	5,−4	4,71	4,76	ɔ ɔ	−,98	− ɔ1	1 0ɔ	0 98	+0 ɔ0	0 8	1 0ɔ	1 /ɔ	− ɔ8	ɔ 0−	ɔ 6ɔ	4 −	4 /8
5	6 44	6 07	ɔ,6ɔ	4 18	4,68	4 14	ɔ 7	2 97	− ɔ+	1 70	1 05	+0 ɔ9	0 7	0 03	1,ɔ9	− −ɔ	− 87	+0	4 04
10	6 68	6 /	6 0−	5 61	16	4,66	4 16	ɔ 60	ɔ 0−	2 4−	1 /9	1 16	+0 51	0 1	0 6	+3	− 00	2 6/	ɔ −/
15	6 8/	6,6ɔ	6,ɔ+	6,00	5,61	5 18	4,71	−−1	ɔ 6/	11	− ɔ−	1 9−	1 −9	−0 66	+0 0ɔ	0 61	−+	1 96	− 47
20	7 01	6 8ɔ	6 61	6,ɔ4	6 0−	ɔ 6ɔ	ɔ −4	4 9	4,ɔ0	ɔ, 8	ɔ ɔ	− 66	− 0/	1,−6	0 8−	−0 −	0 +	1 04	1,06
25	7 00	6 99	6,83	6,63	6 ɔ7	6 07	ɔ 72	5 ɔ3	4,0c	++−	,9−	ɔ ɔ6	− 8−	− −+	1 6−	1 0ɔ	−0 41	0 −1	0 8ɔ

Ir the lon_itude of tne moon s node and the right aſcenſion of t e ſtar be bo mor or l ſs th r i i grs uſe ın ıg c gn or t Tab e but on b more and the ot r l ſs than ſıx ſıgns change the ſign o he T bl

TABLE XXIII *Continued*

Long of moon node S D	RIGHT ASCENSION OF THE STAR 0 VI 0	0 VI 5	0 VI 10	0 VI 15	0 VI 20	0 VI 25	I VII 0	I VII 5	I VII 10	I VII 15	I VII 20	I VII 25	II VIII 0	II VIII 5	II VIII 10	II VIII 15	II VIII 20	II VIII 25	III IX 0
III X 0	+ 11	+ 09	+ 0	+6 87	+6,68	−6 44	+6 16	+5 85	+5,45	+5,03	+4 57	+4 08	+3 56	+3 01	+2 4	+1 84	+1 23	+0 6	−0 00
5	09	1	71	06	6 94	6 78	6 55	6 28	5 96	5,60	5 19	4,75	4 26	3 75	3 21	2 64	2 05	1 45	+0 83
10	7 01	71	19	7 0	15	7 05	6 90	6 69	6 43	6 15	5,77	5 38	4 94	4 46	3 95	3 4	85	2 26	1 66
15	6 87	00	7 0	7 8	7 30	7 77	7 19	7 05	6 85	6 61	6 31	5 97	5 58	5,14	4 67	4 17	3 65	3 06	7
20	6 68	0 9	15	0	0	7 44	7 4	7 5	7,	7 04	6 80	6 51	6 1	5 79	5 36	89	4,38	3 8	5 27
25	6 4	6 8	05	7	7 44	7 55	7 60	7 60	7 5	7 4	7	7 00	6 7	6 58	6 00	5 57	5 09	4 58	4 04
IV X 0	6, 6	6 5	6 90	7,19	7,	7 60	7 7	7 79	7,79	7 73	7,6	,44	7,	6 93	6 59	6 1	5 77	5 29	4 78
5	5	6, 5	0 69	7 05	7 35	7 60	7,79	7,9	7 99	7,99	7,9	7 85	7 66	43	7,14	6,80	6 41	5,97	5,48
10	5 5	5 93	6 45	6 85	7	7 55	7 79	7 99	8,12	8 20	8 1	8,15	8 01	7 86	7 64	7 57	6 99	6 59	6 1
15	5 05	60	6 5	6 61	7 05	7 71	7 75	7 99	8,20	8 33	8	8 4	8 36	8 5	8 06	7 85	7 5	7,17	6 75
20	4,5	9	5 7	0 5	6 80	7 5	7 6	7 94	8,21	8 41	8 31	8 6	8 6	8 36	8 44	8 5	8 00	7 69	7 32
25	4 05	4	5 5	5 9	0 5	7,00	7 44	7,53	8 15	8,4	8 6	8,75	8 8	8 8	8, 5	8 61	8 41	8 15	7 83
V XI 0	5 56	4, 6	4 91	5 58	6 1	6 7	—	7 66	8 04	8,56	8 6	8 8	8 94	9,00	8 99	8 91	8 76	8 55	8 7
5	5 01	5	4,46	5 4	5 9	6 50	6 93	3	7 86	8,5	8 56	8 8	9 05	9 1	9 16	9 14	9 05	8 69	8,65
10	4	1	3 95	6 7	5 6	6,00	6,59	7 14	7 65	8,06	8 44	8 75	8 99	9 16	9 27	9 50	9 6	0 5	8 95
15	1	5	4	4,17	4 89	5,57	6	6 80	7 54	7 82	8 5	8,01	8 91	0 11	9 30	9,39	9,41	9 55	9 5
20	5	5	55	5 6	4 58	5 09	5 7	6 41	6 99	7 52	8 00	8 41	8 0	9 05	0,6	9 41	9 45	0 45	9,41
25	0,2	5,45	0	5,06	3 8	7 5	5 9	5 97	6,59	7,17	7 69	8 15	8 5	6 89	9 15	9 55	9 48	9 53	9 5
VI O 0	0 00	0 05	1 66	2 47	5 7	4 04	4,78	5 48	6 14	6 75	7 5	7 85	8 27	8 65	8 98	9 5	9 41	9 5	9 55

If the longitude of the moon's node and the right aſcenſion of the ſtar be both more or leſs than ſix ſigns, uſe the algebraic ſign of the Table; but if one be more and the other leſs than ſix ſigns, change the ſign of the Table

TABLE XXIII *Continued*

Long of moon's node		RIGHT ASCENSION OF THE STAR																		
		III IX						IV X						V XI						VI O
		0	5	10	15	20	25	0	5	10	15	20	25	0	5	10	15	20	25	0
S	D																			
0 VI	0	−9 55	−9 52	−9 41	−9 2[illegible]	−[illegible] 98	−[illegible] 05	−8 2[illegible]	−7 85	−7 52	−6 75	−6 14	−5 48	−4,78	−4 04	−5 27	−2 47	−1 66	−0 83	−0 00
	5	9 52	9 53	9 48	9 55	9 15	8 89	[illegible]	8 15	7 69	7 17	6 59	5 97	5 [illegible]9	4 56	3 84	[illegible] 06	2 26	1 25	0 62
	10	9 41	9 4[illegible]	9 48	9 41	9 20	9 05	[illegible]5	8 11	8 00	7 52	6 99	6 41	5 77	5 09	4,38	3 63	2 85	2 05	[illegible]5
	15	9 23	9 55	9 41	9 39	9 50	9 14	[illegible] 91	8 61	8 25	7 82	7 34	6 80	6 21	5 57	4,89	4 17	3 4[illegible]	2 64	1 84
	20	8 98	9 15	9 26	9 [illegible]0	9 27	9 16	8 99	8 75	8 44	8 00	7 65	7 14	6 59	6 00	5 36	4 67	3 95	3 21	2 45
	25	8 65	8 89	9 05	9 14	9 [illegible]6	9 12	9 00	8 82	8 56	8 25	7 86	7,25	6 93	6 58	5 79	5 14	4 46	3 75	3 01
I VII	0	8 27	8 [illegible]	8 76	8 91	8 95	9 00	8 94	8 62	8 62	8 36	8,04	7 60	7 22	6 72	6 17	5 58	4 94	4,26	3 56
	5	7,83	8 15	8 41	8 61	8 75	8 82	8 02	8 75	8 02	8 42	8 15	7 85	7 44	7 00	6 51	5 97	5 [illegible]8	4 75	4 08
	10	7 32	7 69	8 00	8 25	8 44	8 56	8 02	8 62	8 54	8 11	8 21	7 94	7 02	7 25	6 80	6,51	5 77	5,19	4,57
	15	6 75	7 17	7 52	7 85	8 06	8 25	8 36	8 42	8 41	8 35	8 20	7 09	7 75	7 4[illegible]	7 05	6 61	6 13	5,60	5 03
	20	6,14	6,59	6 99	7 34	7,04	7 86	8 04	8 15	8 21	[illegible] 20	8 12	7 99	7 79	7 55	7 22	6 85	6 4[illegible]	5 96	5 45
	25	5 48	5 97	6 41	6 80	7 14	7 45	7 66	7 85	7 94	7 99	7 95	[illegible] 92	7 79	7 60	7 55	7 05	6 69	6 2[illegible]	5 83
II VIII	0	4 78	5,29	5 77	6 21	6 [illegible]9	[illegible] 95	7,22	7 44	7 62	7 [illegible]3	7 [illegible]	7 79	7 72	7 60	7 42	7 19	6,90	6 55	6 16
	5	4,04	4 58	5 09	5 5[illegible]	6 00	6 58	6 72	7 00	7 23	7 42	7 55	7 00	7 00	7 55	7 44	7 27	7 05	6 78	6 44
	10	3 27	3 84	4 58	4,89	5 56	5 79	6 17	6 51	6 80	7 04	7 22	7 [illegible]5	7 4[illegible]	7,44	7 40	7,30	7 15	6 94	6 68
	15	2,47	3 06	3 63	4,17	4,67	5 14	5 58	5 97	[illegible] 3	6 61	6 [illegible]	7 [illegible]0	7 10	7 27	7 30	7 28	7 20	7 06	6 87
	20	1 66	2,26	2 85	3 42	3 95	4 46	4,94	5 58	5,77	6 15	6 43	6 65	6 90	7 05	7 15	7 [illegible]0	7 19	7 12	7 01
	25	0 85	1,45	2 05	2 64	3 21	3 75	4,20	4, [illegible]5	5 19	5 00	5 96	6 28	6 55	6 78	6 94	7 06	7 12	7 13	7,09

If the longitude of the moon's node and the right aſcenſion of the ſtar be both more or leſs than ſix ſigns uſe the algebraic ſign of the Table but if one be more and the other leſs than ſix ſigns change the ſign of the Table

TABLE XXIII *Continued*

Long of moon's node	RIGHT ASCENSION OF THE STAR																		
	III IX						IV X						V XI						VI O
	0	5	10	15	20	25	0	5	10	15	20	25	0	5	10	15	20	25	0
S D																			
III IX 0	−0 00	−0 62	−1 22	−1 84	−2 43	−3 01	−3 56	−4 08	−4 57	−5 0	− 45	− 8	−6 16	−6 44	−6 66	−6 87	−7 01	−7 09	−7 11
5	+0 85	+0 21	0,41	1 03	1 61	2 21	2 82	3 38	92	4,42	4,89	5 33	5 2	6 07	6 37	6 6	6 85	6 99	7 09
10	1 66	1 04	+0 42	0 21	0 02	1 76	2 07	2 66	3 25	3 78	4,50	4 79	5 21	5 65	6 02	6 4	6 61	6 85	7 01
15	2 47	1 66	1 27	+0 61	0 03	0 66	1 27	1 92	2 52	3 11	67	4 21	4 71	5 18	5 61	6 00	6 54	6 65	6 87
20	3 27	2 67	2 06	1 25	+0 70	+0 13	0 51	1 10	1 79	2 42	3 02	3 60	4 16	4 68	5 16	5 61	6,02	6 57	6 66
25	4,04	3,46	87	2 23	1 59	0 95	+0 7	0 0	1,05	1 0	2 57	9	2 5	4 14	4 68	5,18	5 65	6,07	6,44
IV X 0	4,78	4 2	65	5,02	2 58	1 73	1 05	+0 58	0 50	0 98	1 65	51	98	55	4 16	4 71	5 24	5,72	6 16
5	5 48	4 0	4 58	5 78	3 6	2,50	,85	1,15	+0 45	0 25	0 04	6	2 1	2 97	3 60	4 21	4 79	5,53	5,83
10	6 14	5 24	5 10	4,52	3 90	3 26	2 59	1 90	1 20	+0,19	0 25	0 74	1 65	2 34	3 02	3 67	4,30	4,89	5 45
15	0 7	6 9	5 78	5,22	4,65	4,00	3 53	2 65	1 94	1 22	+0 49	0 25	0 98	1 70	2 42	3 11	3 78	4 42	5 03
20	7 5	6 89	6,41	5 88	5,31	4,70	4,05	3,37	2 67	94	1 20	−0 45	0 50	05	1 79	2 52	2 25	3 92	4,57
25	85	7 24	7 00	6 50	5 96	5 37	4,74	4 07	3 37	2,65	1 90	1 5	+0 55	0 59	1,16	1 92	2 66	3,38	4,08
V XI 0	8 27	7 95	55	7 07	6 56	5 99	5 59	4 74	4 05	3 35	2 59	1 83	1 05	+0 27	0 51	1 29	2 07	2 82	3 56
5	8 65	8 56	8 00	7 58	7 21	6 58	5,99	5 57	4 2	4 00	3 6	2 50	1 75	0 95	+0,15	0 66	1 46	2 24	3,01
10	8 98	8 75	8,42	8 04	7 60	7 11	6 56	5 96	5 51	4 65	3 00	3 16	2 38	1,59	0 8	0 05	0 84	1 64	2 45
15	9 25	9 05	8 76	8 44	8 04	7,58	7,07	6 50	5 88	5 22	4 52	3 78	3,02	2 23	1,43	+0 61	0 21	1 05	1 84
20	9 41	9 26	9 05	8 76	8,42	8 00	7 55	6 99	6 41	5 78	5 10	4 38	3 61	2 86	2 06	1 24	+0 42	0 41	1 25
25	9 52	9 25	9 26	9 03	8 75	8 36	7 95	7 44	6 89	6,29	5 64	9	4,22	1 46	2 67	1 86	1 04	+0 21	0 62
VI O 0	9 55	9 52	9 41	9 23	8,98	8 65	8 27	7 85	7 52	6 75	6 14	5 40	4,78	4 04	3 27	2 47	1 66	0,83	0 00

If the longitude of the moon's node and the right aſcenſion of the ſtar be both more or leſs than ſix ſigns uſe the algebraic ſign of the Table but if one be more and the other leſs than ſix ſigns change the ſign of the Table

TABLE XXIV

Time taken by Light to move over Parts of the Orbis Magnus

Parts of Orbis Magnus	Time	Parts of Orbis Magnus	Time	Parts of Orbis Magnus	Time
,001	0 0,5	,31	2 31,0	,70	5 40,9
002	0 1,0	32	2 35,8	71	5 45,8
,003	0 1,5	,33	2 40,7	,72	5 50,6
,004	0 1,9	,34	2 45,6	,73	5 55,5
,005	0 2,4	35	2 50,4	,74	6 0,4
,006	0 2,9	,36	2 55,3	,75	6 5,2
,007	0 3,4	,37	3 0,2	,76	6 10,1
,008	0 3,9	,38	3 5,1	,77	6 15,0
,009	0 4,4	,39	3 9,9	,78	6 19,9
01	0 4,9	,40	3 14,8	79	6 24,7
02	0 9,7	,41	3 19,7	,80	6 29,6
,03	0 14,6	,42	3 24,5	81	6 34,5
,04	0 19,5	,43	3 29,4	82	6 39,3
,05	0 24,4	,44	3 34,3	,83	6 44,2
,06	0 29,2	45	3 39,1	,84	6 49,1
,07	0 34,1	46	3 44,0	,85	6 53,9
,08	0 39,0	47	3 48,9	86	6 58,8
,09	0 43,8	,48	3 53,8	,87	7 3,7
,1	0 48,7	,49	3 58,6	,88	7 8,6
,11	0 53,6	,50	4 3,5	,89	7 13,4
,12	0 58,4	,51	4 8,4	,90	7 18,3
,13	1 3,3	,52	4 13,2	91	7 23,2
,14	1 8,2	,53	4 18,1	,92	7 28,0
15	1 13,0	,54	4 23,0	,93	7 32,9
16	1 17,9	55	4 27,8	94	7 37,8
,17	1 22,8	,56	4 32,7	,95	7 42,6
18	1 27,7	57	4 37,6	,96	7 47,5
,19	1 32,5	,58	4 42,5	97	7 52,4
,20	1 37,4	59	4 47,3	,98	7 57,3
,21	1 42,3	60	4 52,2	,99	8 2,1
,22	1 47,1	,61	4 57,1	1,00	8 7,0
,23	1 52,0	62	5 1,9	2	16 14,0
24	1 56,9	,63	5 6,8	3	24 21,0
,25	2 1,8	,64	5 11,7	4	32 28,0
,26	6,6	65	5 16,5	5	40 35,0
,27	11,5	,66	5 21,4	6	48 42,0
,28	16,4	,67	5 26,3	7	56 49,0
29	2 21,3	,68	5 31,2	8	64 56,0
,30	2 26,1	69	5 36,0	9	73 3,0
				10	81 10,0

TABLE XXV

To find the Nonagesimal Degree of the Ecliptic and its Altitude for the Meridian of Greenwich reduced to the Earth's Center, supposed to be 51 14 7, and the Obliquity of the Ecliptic 23 29

ARGUMENT Right Ascension of the Meridian

R of Merid	Nona	Alt Nona	R of Merid	Nona	Alt Nona	R of Merid	Nona	Alt Nona
0	0 6 2 40	44 20 25	30	1 17 47 0	53 48 10	60	2 49 10	60 2 0
1	0 27 6 40	44 41 40	31	1 18 29 10	54 2 0	61	9 1 20	60 11 10
2	0 27 50 40	45 2 30	32	1 19 11 10	54 19 0	62	2 10 13 30	60 19 30
3	0 28 4 0	45 23 50	33	1 9 53 10	54 0	63	2 0 53 40	60 27 0
4	0 29 18 10	45 44 40	34	1 0 33 20	52 3 20	64	2 11 38 0	60 35 0
5	1 0 1 30	46 5 0	35	1 21 1 20	55 5 40	65	12 23 10	60 4 30
6	1 0 43 10	46 20 10	36	1 2 39 20	55 20 30	66	2 1 2 30	60 49 40
7	1 1 28 20	46 46 30	37	22 4 20	55 35 0	67	2 13 44 30	60 56 30
8	1 2 12 0	47 6 50	38	23 2 20	5 40 0	68	2 14 27 10	61 3 0
9	1 2 53 10	47 2 0	39	1 4 7 0	0 20	69	2 15 9 30	61 9 20
10	1 3 38 30	47 47 0	40	24 47 0	6 17 30	70	2 15 51 40	61 1 10
11	1 4 21 10	48 6 30	41	1 2 20 0	50 1 0	71	2 16 34 0	61 0 30
12	1 5 3 30	48 26 20	42	26 11 30	6 2 20	72	2 7 16 30	61 26 20
13	1 5 47 50	48 4 50	43	1 26 53 20	56 37 50	73	2 17 58 50	61 31 30
14	6 20 40	49 5 10	44	1 2 0	0 40	74	2 18 41 10	61 36 10
15	7 12 20	49 24 20	45	1 2 17 30	7 2 20	75	2 9 23 30	61 40 50
16	1 7 55 0	49 43 20	46	1 28 59 30	57 3 40	76	2 20 5 50	61 45 10
17	1 8 37 30	50 0	47	1 29 41 30	5 48 0	77	2 20 48 20	61 49 0
18	1 9 20 0	50 20 30	48	2 0 23 30	55 0 0	78	2 21 30 40	61 52 40
19	1 0 2 30	50 30 0	49	2 1 5 40	58 1 40	79	2 2 13 10	61 56 0
20	1 10 43 0	50 57 10	50	2 1 47 40	58 23 0	80	2 5 30	61 59 10
21	11 27 20	5 15 10	51	2 2 29 30	58 34 10	81	2 3 8 0	62 2 0
22	1 12 9 30	5 33 0	52	2 3 11 50	58 45 0	82	4 23 30	62 4 0
23	1 12 51 30	51 50 40	53	2 3 54 0	58 53 40	83	25 2 50	6 6 10
24	1 13 34 0	52 0 0	54	2 4 6 10	59 6 0	84	2 25 45 20	6 8 30
25	1 4 16 20	52 23 10	55	2 5 18 20	59 16 10	85	2 20 2 40	62 10 10
26	14 58 30	2 2 10	56	2 6 0 0	59 26 0	86	7 10 40	6 11 0
27	1 1 40 4	52 59 0	57	2 6 22 0	59 35 40	87	2 2 52 40	6 12 30
28	1 16 22 30	53 15 40	58	7 24 0	59 45 0	88	2 8 35 10	6 13 20
29	1 17 4 50	53 32 0	59	2 8 6 30	59 54 0	89	2 29 1 30	6 15 0
30	1 1 47 0	53 48 10	60	2 8 49 10	60 2 50	90	3 0 0 0	62 13 53

TABLE XXV *Continued*

ARGUMENT Right Aſcenſion of the Meridian.

AR of Mid.					Alt Nonag.			AR of Mid.	Nonag.				Alt Nonag.			AR of Mid.	Nonag.				Alt Nonag.		
90	3	0	2	0	62	1[illegible]	[illegible]	120	3	21	10	50	60	2	50	150	4	12	13	0	53	48	10
91		0	4	0	62	15	50	121	3	21	53	10	59	54	0	151	4	12	55	10	53	32	0
			1	0	62	13	20	122	3	22	35	20	59	45	0	152	4	13	37	10	53	15	40
93				20	62	12	30	123		23	[illegible]	30	59	35	40	153	4	14	19	20	52	59	0
94	3		43	50	62	11	50	124	3	23	59	40	59	26	0	154	4	15	1	30	52	42	0
95				0	62	10	10	125	3	24	41	40	59	16	10	155	4	15	43	40	52	25	0
96		4	11	0	62	8	30	126	3	25	23	50	59	6	0	156	4	16	26	0	52	8	0
97		4		0	62	6	40	127	3	26	6	0	58	55	40	157	4	17	8	10	51	50	40
98			50	0	62	4	30	128	3	26	48	10	58	45	0	158	4	17	50	0	51	33	0
99		[illegible]	22	0	62	2	0	129	3	27	30	10	58	34	10	159	4	18	32	40	51	15	10
100	3		4	0	61	59	10	130		28	12	20	58	23	0	160	4	19	15	0	50	57	10
101	3	7	46	50	61	56	0	131	3	28	54	20	58	11	40	161	4	19	57	30	50	39	0
102		8	9	20	61	52	40	132	3	29	36	30	58	0	0	162	4	20	40	0	50	20	30
103		9	11	40	61	49	0	133	4	0	18	30	57	48	0	163	4	21	22	30	50	2	0
104	3	9	5[illegible]	10	61	45	10	134	4	1	0	30	57	35	40	164	4	22	5	0	49	43	20
105		10	[illegible]	50	61	40	50	135	4	1	42	30	57	23	20	165	4	22	47	40	49	24	20
106	3	11	18	50	61	36	10	136	4	2	24	50	57	10	40	166	4	23	30	20	49	5	10
107	3	12	1	10	61	31	50	137	4	3	6	40	56	57	50	167	4	24	13	10	48	45	50
108	3	12	4[illegible]	50	61	26	20	138	4	3	48	30	56	44	40	168	4	24	56	10	48	26	30
109	3	13	26	0	61	20	50	139	4	4	30	30	56	31	10	169	4	25	38	50	48	6	50
110	3	14	8	0	61	15	10	140	4	5	12	40	56	17	30	170	4	26	21	50	47	47	0
111	3	14	50	30	61	9	20	141	4	5	54	40	56	3	40	171	4	27	4	50	47	27	0
112	3	15	32	50	61	3	0	142	4	6	36	40	55	49	50	172	4	27	48	0	47	6	50
113	3	16	15	10	60	56	50	143	4	7	18	40	55	35	0	173	4	28	31	20	46	46	30
114	3	16	57	30	60	49	40	144	4	8	0	40	55	20	30	174	4	29	14	50	46	26	10
115	3	17	39	50	60	42	50	145	4	8	42	40	55	5	40	175	4	29	58	10	46	5	30
116	3	18	22	0	60	35	0	146	4	9	24	40	54	50	40	176	5	0	41	50	45	44	40
117	3	19	4	20	60	27	30	147	4	10	6	50	54	35	20	177	5	1	25	50	45	23	50
118	3	19	46	30	60	19	50	148	4	10	48	50	54	9	50	178	5	2	9	20	45	2	50
119	3	20	28	40	60	11	0	149	4	[illegible]	[illegible]	50	54	[illegible]	10	179	5	2	53	20	44	41	40
120	3	21	10	50	60	2	50	150	4	12	13	0	53	48	10	180	5	3	37	20	44	20	2[illegible]

TABLE XXV Continued

ARGUMENT Right Afcenfion of the Meridian

AR of Mer. d	Nonag.	Alt. Nonag.	AR of Mer. d	Nonag.	Alt. Nonag.	AR of Mer. d	Nonag.	Alt. Nonag.
180	5 3 7 20	44 20 25	210	5 27 32 10	32 52 10	240	7 0° 50 20	21 23 0
181	5 4 21 40	43 5 50	211	5 28 26 10	32 28 20	241	7 2 17 20	21 2 40
182	5 5 6 10	43 37 10	212	5 29 20 10	32 4 30	242	7 3 46 50	20 42 40
183	5 5 50 30	43 15 0	213	6 0 15 10	31 40 30	243	7 5 17 50	20 22 50
184	5 6 35 20	42 53 40	214	6 1 10 30	31 16 40	244	7 6 51 0	20 3 40
185	5 7 20 10	42 31 40	215	6 2 6 50	30 52 50	245	7 8 26 30	19 44 40
186	5 8 5 20	42 9 30	216	6 3 3 40	30 29 0	246	7 10 4 0	19 26 10
187	5 8 50 40	41 47 20	217	6 4 1 10	30 5 10	247	7 11 43 50	19 8 0
188	5 9 36 10	41 24 50	218	6 4 59 40	29 41 10	248	7 13 25 40	8 50 30
189	5 10 21 50	41 2 30	219	6 5 58 40	29 17 20	249	7 15 10 0	8 33 20
190	5 11 7 50	40 39 50	220	6 6 58 20	28 53 50	250	7 16 56 50	18 16 40
191	5 11 53 50	40 7 20	221	6 7 59 10	28 30 0	251	7 18 45 30	18 0 30
192	5 12 40 20	39 54 30	222	6 9 0 50	28 6 20	252	7 20 37 20	17 45 20
193	5 13 27 0	39 31 40	223	6 10 3 20	27 42 50	253	7 22 31 20	17 30 20
194	5 14 13 50	39 8 40	224	6 11 6 40	27 19 20	254	7 24 27 30	7 10 20
195	5 15 1 10	38 45 40	225	6 12 1 20	26 56 0	255	7 26 26 0	17 2 40
196	5 15 48 30	38 22 30	226	6 13 16 40	26 32 40	256	7 28 27 40	16 49 50
197	5 16 36 50	37 59 30	227	6 14 23 20	26 9 30	257	8 0 31 10	16 3 20
198	5 17 24 30	37 36 0	228	6 15 31 10	25 46 20	258	8 2 36 50	16 20 30
199	5 18 12 50	37 12 40	229	6 16 39 50	25 23 30	259	8 4 44 50	6 5 0
200	5 9 1 30	36 49 10	230	6 17 50 0	25 0 40	260	8 6 55 0	16 6 10
201	5 19 51 0	36 25 40	231	6 19 1 20	24 38 0	261	8 9 7 20	5 57 0
202	5 20 40 20	36 2 10	232	6 20 14 0	24 15 30	262	8 11 21 20	5 49 0
203	5 21 30 20	35 38 40	233	6 21 28 10	23 53 10	263	8 13 37 0	15 4 50
204	5 22 20 40	35 15 0	234	6 22 43 30	23 31 0	264	8 15 54 20	15 15 30
205	5 23 11 20	34 51 20	235	6 24 0 30	23 9 0	265	8 18 13 10	15 0 5
206	5 24 2 30	34 27 30	236	6 25 19 10	22 47 20	266	8 20 3 10	15 25 40
207	5 24 54 10	34 3 40	237	6 26 39 20	22 25 50	267	8 22 54 0	5 22 0
208	5 25 46 20	33 40 0	238	6 28 1 20	22 4 30	268	8 25 5 30	15 19 30
209	5 26 39 0	33 16 0	239	6 29 25 0	21 43 30	269	8 27 37 40	1 19 0
210	5 27 32 10	32 5 10	240	7 0 50 20	21 23 0	270	9 0 0 0	17 53

TABLE XXV *Continued*

ARGUMENT Right Aſcenſion of the Meridian

AR of Mid	Nonag	Alt Nonag	AR of Merid	Nonag	Alt Nonag	AR of M	Nonag	Alt Nonag
270	9 0 0 0	15 17 55	300	10 29 0 40	21 23 0	330	0 2 27 50	32 52 10
271	9 2 22 20	15 18 20	301	11 0 35 0	21 43 30	331	0 3 2 0	33 10 0
272	9 4 44 30	15 19 30	302	11 1 58 40	22 4 30	332	0 4 40	33 40 0
273	9 7 6 0	15 22 30	303	11 3 20 40	22 25 50	333	0 5 5 30	34 40
274	9 9 26 30	15 25 40	304	11 4 40 50	22 47 20	334	0 5 5 30	34 27 0
275	9 11 46 30	15 30 10	305	11 5 59 30	23 0 0	335	0 6 48 42	34 51 20
276	9 14 5 40	15 35 50	306	11 7 16 30	23 31 0	336	0 7 39 20	35 1 0
277	9 16 2 0	15 41 50	307	11 8 1 50	23 53 10	337	0 8 29 40	35 32 10
278	9 18 38 40	15 49 0	308	11 9 46 0	24 15 30	338	0 9 19 40	36 2 10
279	9 20 52 40	15 57 0	309	11 10 58 40	24 38 0	339	0 10 9 0	36 25 40
280	9 23 5 0	16 6 10	310	11 12 10 0	25 0 40	340	0 10 58 10	36 49 0
281	9 25 15 10	16 15 50	311	11 13 20 10	25 23 30	341	0 11 47 10	37 12 40
282	9 27 25 10	16 26 30	312	11 14 28 50	25 46 20	342	0 12 35 30	37 36 0
283	9 29 28 50	16 37 40	313	11 15 6 40	26 9 30	343	0 13 23 30	37 59 30
284	10 1 32 20	16 49 50	314	11 16 43 20	26 32 40	344	0 14 11 30	38 22 30
285	10 33 20	17 2 40	315	11 17 48 40	26 56 0	345	0 14 58 50	38 45 40
286	10 5 32 30	17 16 20	316	11 18 53 20	27 19 20	346	0 15 46 10	39 8 40
287	10 7 28 40	17 30 20	317	11 19 56 40	27 42 50	347	0 16 33 0	39 31 40
288	10 9 22 40	17 45 20	318	11 20 59 10	28 6 20	348	0 17 19 40	39 54 30
289	10 11 14 10	18 0 50	319	11 22 0 50	28 30 0	349	0 18 6 10	40 7 20
290	10 13 3 10	18 16 40	320	11 23 1 40	28 53 30	350	0 18 52 10	40 39 30
291	10 14 50 0	18 33 20	321	11 24 1 20	29 17 20	351	0 19 38 10	41 2 30
292	10 16 34 20	18 50 30	322	11 25 0 20	29 41 10	352	0 20 23 50	41 24 30
293	10 18 16 10	19 8 0	323	11 25 58 50	30 5 10	353	0 21 9 20	41 47 20
294	0 19 56 0	19 26 10	324	11 26 56 20	30 29 0	354	0 21 54 40	42 9 40
295	10 21 33 30	19 44 40	325	11 27 53 10	30 52 50	355	0 22 39 30	42 31 40
296	10 23 9 0	20 3 40	326	11 28 49 30	31 16 40	356	0 23 24 40	42 53 40
297	10 24 42 10	20 22 50	327	11 29 44 30	31 40 30	357	0 4 9 30	43 15 30
298	10 26 13 30	20 42 40	328	0 0 39 50	32 4 30	358	0 24 53 50	43 37 10
299	10 27 42 40	21 2 40	329	0 1 33 50	32 28 20	359	0 25 38 20	43 58 50
300	10 29 9 40	21 23 0	330	0 2 27 50	32 52 10	360	0 26 22 42	44 20 23

TABLE XXVI

Showing how much the Nonageſimal Degree, and it's Altitude, are altered by removing the Place one Degree to the North of Greenwich

AR	Nonageſ	Altitude	
	+	−	
270	0, 0	60, 0	270
280	50 9	58 2	260
290	88, 4	54 3	250
300	87, 4	50, 7	240
310	82, 0	48, 2	230
320	76, 9	46, 9	220
330	68, 6	46, 6	210
340	61, 3	47, 1	200
350	54, 5	48, 8	190
360	48, 2	49, 5	180
10	4 , 3	51, 0	170
20	36, 8	52, 7	160
30	31, 4	54, 3	150
40	26, 0	55, 9	140
50	20, 8	57, 2	130
60	15, 7	58, 4	120
70	10, 5	59, 3	110
80	5, 3	59, 8	100
90	0, 0	60, 0	90
	−	−	
	Nonageſ	Altitude	AR

TABLE XXVII

The Angle between the Ecliptic and parallel to the Equator, to the Obliquity of the Ecliptic 23° 28′, with the Variation for 10″ Variation of the Obliquity

Sun's Declinat.	Angle of Ecliptic and parallel to Equator	Diff.	Var. for 10″ Var. of Obl. of Ecliptic	Sun's Declinat.	Angle of Ecliptic and parallel to Equator	Diff.	Var. for 10″ Var. of Obl. of Ecliptic
0 0	23 28 0		10,0	16 40	16 45 38		14,4
0 30	23 27 42	0 18	10,0	17 0	16 25 20	20 18	14,7
1 0	23 26 47	0 55	10,0	17 20	16 4 8	21 12	15,0
1 30	23 25 17	1 30	10,0	17 40	15 41 58	22 10	15,4
2 0	23 23 10	2 7	10,1	18 0	15 18 46	23 12	15 8
2 30	23 20 26	2 44	10,1	18 20	14 54 28	24 18	16 3
3 0	23 17 5	3 21	10,1	18 40	14 28 57	25 31	16 8
3 30	23 13 8	3 57	10,1	19 0	14 2 7	26 50	17,3
4 0	23 8 32	4 36	10,1	19 15	13 41 2	21 5	17 8
4 30	23 3 18	5 14	10 2	19 30	13 19 4	21 58	16 3
5 0	22 57 26	5 52	10 2	19 45	12 56 7	22 57	18 9
5 30	22 50 55	6 31	10 3	20 0	12 32 8	23 59	19,5
6 0	22 43 43	7 12	10 3	20 10	12 15 30	16 38	20 0
6 30	22 35 51	7 52	10,4	20 20	11 58 19	17 11	20,4
7 0	22 27 17	8 34	10,5	20 30	11 40 32	17 47	21,0
7 30	22 18 1	9 16	10,6	20 40	11 22 6	18 26	21,5
8 0	22 8 2	9 59	10 6	20 50	11 2 59	19 7	22,2
8 30	21 57 17	10 45	10,8	21 0	10 43 6	19 53	22,7
9 0	21 45 46	11 31	10,9	21 5	10 32 52	10 14	23,3
9 30	21 33 28	12 18	11,0	21 10	10 22 4	10 28	23 7
10 0	21 20 1	13 7	11,1	21 15	10 11 43	10 41	24 1
10 30	21 6 23	13 58	11,2	21 20	10 0 16	10 57	24 6
11 0	20 51 32	14 51	11 4	21 25	9 49 35	11 11	25 0
11 30	20 35 46	15 46	11,5	21 30	9 38 8	11 27	25,5
12 0	20 19 2	16 44	11,7	21 35	9 26 23	11 45	26,0
12 30	20 1 18	17 44	11 9	21 40	9 14 19	12 4	26,6
13 0	19 42 30	18 48	12,1	21 45	9 1 57	12 22	27 3
13 30	19 22 37	19 53	12,3	21 50	8 49 13	12 44	27 9
14 0	19 1 3	21 5	12,6	21 55	8 36 7	13 6	28,6
14 30	18 39 12	22 20	12,8	22 0	8 22 37	13 30	29 4
15 0	18 15 3	23 40	13,1	22 5	8 8 40	13 5	30 3
15 20	17 58 58	16 34	13,3	22 10	7 54 15	14 25	31,
15 40	17 41 44	17 14	13,5	22 15	7 39 18	14 57	32 2
16 0	17 23 48	17 56	13,8	22 20	7 23 47	15 31	33 4
16 20	17 5 7	18 41	14,1	22 25	7 7 38	16 9	34,7
16 40	16 45 38	19 29	14 4	22 30	6 50 16	16 52	36,1

TABLE XXVIII

The Angle of Position of every Point of the Ecliptic according to the Obliquity of the Ecliptic 23° 28', with the Variation for the Variation of the Obliquity by One Minute

ARG. [illegible] Longitude of the Point of the Ecliptic

	Sig 0 +			Sig I −			Sig II +			
	Sig VI −			Sig VII −			Sig VIII −			
	Ang. Posit.	D	Var.	Ang. Posit.	Diff.	Var.	Ang. Posit.	Diff.	Var.	
0	23 28 0	0 11 5	60 0	20 6 15 2	11 33 8	54 1	12 14 28 0	21 20 8	34 0	30
1	23 27 9 3	0 33 4	60 0	20 2 30 4	11 55 2	53 7	11 53 7 2	21 36 9	33	29
2	23 27 4 1	0 5 4	[illegible] 9	20 2 41 2	12 [illegible] 4	53	11 3 10 3	22 12 7	3	28
3	23 26 10	1 2	59 9	20 0 20	12 2 7	52 8	11 [illegible] 3 6	22 28 0	3 1	27
4	23 24 56 4	1 43 3	59	19 4 38 1	13 [illegible] 7	52 3	10 46 29 6	22 4 0	30 1	26
5	23 23 13	2 6 2	59 [illegible]	19 3 33 4	13 26 8	[illegible] 9	10 23 46 6	22 57 4	29 1	25
6	23 21 6 9	2 29 2	59	19 21 6 6	13 48 6		10 0 49 2	23 11 3	28 1	24
7	23 18 7 7	2 52 1	0 6	19 7 18 0	14 0 6	50 8	9 37 37 7	23 23 1	27 0	23
8	23 5 23 0	3 15 0	[illegible] 3	18 53 7 4	14 3 2	50 3	9 14 12 6	23 38 2	26 0	22
9	23 12 30 6	3 38 0	[illegible] 4	18 38 3 2	14 53 8	[illegible] 7	8 50 4 4	23 50 7	24 9	21
10	23 8 52 6	4 0 9	59 3	18 2 2 4	15 15 2	[illegible]	8 26 4 7	24 2 9	2 8	20
11	23 4 5 7	4 2 8	9 2	18 8 0 2	15 36 6	[illegible] 5	8 2 43 8	24 13 5	22 7	19
12	23 0 2 9	4 40 8	59	17 [illegible] 29 6	[illegible]	4 0	7 38 20 3	24 3 5	21 6	18
13	22 55 4 1	5 9 7	58 9	17 30 31 9	16 18 7	[illegible]	7 14 0 6	24 36 0	20 5	17
14	22 50 31 4	5 3	58 8	17 20 33 2	16 39 6	46 7	6 49 45 8	24 46 0	19 2	16
15	22 4 9 9	5 5 5	58 6	17 [illegible] 53 6	17 0 2	[illegible]0 1	6 24 8 8	24 55 5	[illegible]	15
16	22 39 3 4	6 19 3	58 4	16 40 3 4	17 20 7	4[illegible] 4	5 59 43 3	25 4 2	17 1	14
17	3 45 1	6 41 2	58 1	16 29 3 7	17 41 0	44 7	5 34 9 3	25 12 4	15 9	13
18	22 26 3 9	7 2 0	57 9	16 11 3 7	18 1 0	4 9	5 9 20 9	25 20 1	14 7	12
19	22 18 59 9	7 26 9	[illegible] 0	15 53 0	18 20 9	[illegible]	4 44 0 6	25 27 0	13 5	11
20	22 1 33 0	7 49 6	[illegible]	15 3 29 8	18 40 3	[illegible]	4 18 9 8	25 33 5	12	10
21	22 3 43 4	8 2 3		15 16 49 3	18 59 8	41 7	3 53 6 3	25 39 3	11 1	9
22	21 55 30 9	8 33 1	[illegible]	14 57 49 3	19 18 9	40 9	3 27 27 0	25 44 5	9 9	8
23	21 43 53 8	8 58 0	[illegible]	14 38 30 6	19 37 7	4 1	[illegible] 4 5	25 40 1	8 7	7
24	21 3 5 8	9 20 6		14 18 52 9	19 50 3	39 3	2 35 53 4	25 52 9	7 4	6
25	21 28 5	9 4 2	[illegible] 0	13 58 30 6	20 14 6	[illegible]	2 10 0 5	25 56 2	6 2	5
26	21 15 4 0	10 0	[illegible] 6	13 30 4 0	20 32 4	7 0	1 44 4 3	25 58 8	5 0	4
27	21 8 4 1	10 25	[illegible]	13 18 9 6	20 50 1	36 7	1 1 5 5	26 0 7	3 7	3
28	20 58 0 6	10 50 3	54 9	12 57 19 5	21 7 3	[illegible]	0 52 4 6	26 2 1	5	2
29	20 4 26	11 15 5	54 5	12 36 1 2	21 24 2	34 9	0 26 2 7	26 2 7	2	1
30	20 6 15 2		54 1	12 14 40 0		34 0	0 0 0 0		0 0	0
	Sig V −		Sig XI +	Sig IV −		Sig X −	Sig III −		Sig IX +	

TABLE XXIX

The Angle of Poſition of any Point of the Ecliptic according to the Obliquity of the Ecliptic 23 28 with the Variation for the Variation of the Obliquity by One Minute

ARGUMENT Right Aſcenſion of the Point of the Ecliptic

	Sig O +	Sig VI −		Sig I +	Sig VII −		Sg II +	Sg VIII −		
	Ang of P	Diff	V	Ang P	Diff	V	Ang of P	Diff	V	
0	23 28 0 0	0 13 7	60 0	20 10 24 7	12 54 6	50 8	11 29 5 2	21 12 4	28 0	30
1	23 27 46 3	0 40 9	60 0	19 57 30 1	13 16 3	50 2	11 7 52 8	21 22 2	27 1	29
2	23 27 5 4	1 8 1	59 9	19 44 13 8	13 37 4	49 5	10 46 30 6	21 33 6	26 2	28
3	23 25 57 3	1 35 4	59 8	19 30 36 4	13 59 2	48 9	10 24 57 0	21 44 7	25 3	27
4	23 24 21 9	2 2 6	59 8	19 16 37 2	14 19 6	48 4	10 3 12 3	21 53 4	24 4	26
5	23 22 19 3	2 29 6	59 7	19 2 17 6	14 40 0	47 6	9 41 18 9	22 2 6	23 5	25
6	23 19 49 7	2 56 7	59 6	18 47 37 6	15 0 1	47 0	9 19 16 3	22 11 6	22 6	24
7	23 16 53 0	3 23 6	59 4	18 32 37 5	15 19 8	46 3	8 57 4 7	22 20 1	21 7	23
8	23 13 29 4	3 50 6	59 2	18 17 17 7	15 39 1	45 6	8 34 44 6	22 28 2	20 8	22
9	23 9 38 8	4 17 3	59 0	18 1 38 6	15 58 2	44 9	8 12 16 4	22 36 1	19 9	21
10	23 5 21 5	4 43 9	58 8	17 45 40 4	16 16 9	44 2	7 49 40 3	22 43 4	19 0	20
11	23 0 37 6	5 10 4	58 6	17 29 23 5	16 35 2	43 6	7 26 56 9	22 50 4	18 1	19
12	22 55 27 2	5 36 8	58 4	17 12 48 3	16 53 1	42 9	7 4 6 5	22 57 2	17 1	18
13	22 49 50 4	6 2 9	58 1	16 55 55 2	17 10 8	42 1	6 41 9 3	23 3 4	16 2	17
14	22 43 47 5	6 29 0	57 9	16 38 44 4	17 27 9	41 4	6 18 5 9	23 9 3	15 2	16
15	22 37 18 5	6 54 9	57 6	16 21 16 5	17 44 9	40 5	5 54 56 6	23 14 8	14 2	15
16	22 30 23 6	7 20 5	57 3	16 3 31 6	18 1 2	39 6	5 31 41 8	23 20 0	13 3	14
17	22 23 3 1	7 45 8	57 0	15 45 30 4	18 17 4	38 8	5 8 21 8	23 24 8	12 3	13
18	22 15 17 3	8 11 5	56 7	15 27 13 0	18 33 2	38 1	4 44 57 0	23 29 3	11 4	12
19	22 7 5 8	8 36 3	56 2	15 8 39 8	18 48 4	37 3	4 21 27 7	23 33 3	10 4	11
20	21 58 29 5	9 1 0	55 8	14 49 51 4	19 3 5	36 4	3 57 54 4	23 36 9	9 5	10
21	21 49 28 5	9 25 7	55 3	14 30 47 9	19 18 0	35 6	3 34 17 5	23 40 3	8 5	9
22	21 40 2 8	9 50 0	54 8	14 11 29 9	19 32 3	34 8	3 10 37 2	23 43 2	7 6	8
23	21 30 12 8	10 14 0	54 4	13 51 57 6	19 46 2	34 1	2 46 54 0	23 45 8	6 6	7
24	21 19 58 8	10 37 8	53 9	13 32 11 4	19 59 5	33 2	2 23 8 2	23 48 0	5 7	6
25	21 9 21 0	11 1 4	53 4	13 12 11 9	20 12 7	32 3	1 59 20 2	23 49 9	4 7	5
26	20 58 19 6	11 24 6	52 9	12 51 59 2	20 25 4	31 5	1 35 30 3	23 51 2	3 8	4
27	20 46 55 0	11 47 5	52 3	12 31 33 8	20 37 7	30 6	1 11 39 1	23 52 5	2 9	3
28	20 35 7 5	12 10 2	51 7	12 10 56 1	20 49 7	29 8	0 47 46 6	23 53 1	1 9	2
29	20 22 57 3	12 32 6	51 2	11 50 6 4	21 1 2	28 9	0 23 53 5	23 53 5	1 0	1
30	20 10 24 7		50 8	11 29 5 2		28 0	0 0 0 0		0 0	0
	Sig V −	Sig XI +		Sig IV −	Sig X +		Sig III −	Sig IX +		

TABLE XXX

Augmentation of the Angle of Poſition of a Point in the Ecliptic, for the Latitude of the Zodiacal Stars

E lipt Angl of P to	LATITUDE OF THE STAR												
	0 30	1 0	1 30	2 0	2 30	3 0	3 30	4 0	4 30	5 0	30	6 0	6 30
°													
0	0	0	0	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1	0 1	0 5	1 2	2 2	0 3 4	0 4 9	0 6 6	0 8 8	0 11 1	0 13 7	0 6 6	0 19 8	0 23 3
2	0 2	1	2 4	4 4	0 6 8	0 9 9	0 13 4	0 17 6	0 22 2	0 27 5	0 33 3	0 39 7	0 46 6
3	0 4	1 6	3 7	6 6	0 10 3	0 14 8	0 20 2	0 26 4	0 3 4	0 41 3	0 50 0	0 59 6	1 9 9
4	0 5	2 2	4 9	8 8	0 13 7	0 19 8	0 26 9	0 5 2	0 44 6	0 55 1	1 6 7	1 19 5	1 3
5	0 7	2 8	6 2	11 0	0 17 2	0 24 7	0 3 8	0 44 0	0 55 8	1 9 0	1 23 4	1 39 4	1 56
6	0 8	3 3	7 4	13 2	0 20 6	0 29 7	0 40 5	0 52 9	1 7 0	1 22 8	1 40 2	1 59 4	2 20 2
7	1 0	3 8	8 6	15 4	0 24 1	0 34 7	0 47 3	1 1 8	1 18 3	1 36 7	1 57 1	2 19 5	2 43 8
8	1 1	4 4	9 9	17 6	0 27 6	0 39 7	0 54 2	1 10 7	1 29 6	1 50 7	2 14 0	2 39 7	3 7 5
9	1 2	5 0	1 2	19 8	0 31 0	0 44 8	1 1 0	1 19 8	1 41 0	2 4 8	2 31 1	3 0 0	31 3
10	4	5 5	12 4	22 1	0 34 6	0 49 9	1 7 9	1 28 8	1 52 4	2 19 0	2 48 2	3 20 3	3 55 3
11	1 5	6 1	13 7	24 4	0 38 2	0 55 0	1 14 9	1 37 9	2 3 9	2 33 1	3 5 4	3 40 9	4 19 4
12	1 7	6 7	15 0	26 7	0 41 7	1 0 1	1 21 9	47 0	2 15 5	2 47 4	3 22 8	4 1 5	4 43 6
13	1 8	7 2	16 3	29 0	0 45 3	1 5 3	1 29 0	1 56	2 27 2	3 1 9	3 40 2	4 22 3	5 8 1
14	1 9	7 8	17 6	31 4	0 48 9	1 10 5	1 36 1	2 5 3	2 39 0	3 16 4	3 57 8	4 43 5	5 32 8
15	2 1	8 4	18 9	33 7	0 52 3	1 15 8	1 43 2	2 14 9	2 50 9	31 1	4 15 6	5 4 5	5 57 6
16	2 2	9 0	20 3	36 1	0 56 3	21 1	1 50 3	2 24 4	2 9	3 45 9	4 33 6	5 25 7	6 22
17	2 4	9 6	2 6	38 4	1 0 0	26 5	1 57 8	2 34 0	3 15 0	4 0 9	4 51 7	5 47 4	6 48 1
18	2 5	10 2	22 9	40 9	1 3 8	1 32 0	2 5	2 43 6	3 27 2	4 16 0	5 10 0	6 0 2	7 13 9
19	2 7	10 8	24 3	43 3	1 7 6	37 4	2 12 7	2 53 4	3 39 6	4 31	5 28 5	6 31 3	7 39 6
20	2 9	11 4	25 7	45 8	1 11 3	1 43 0	2 20 3	3 3 3	3 5 2	4 46 8	5 47 3	6 53 6	8 5 9
21	3 0	12 0	27 1	48 2	15 4	1 48 7	2 27 9	3 3 3	4 4 9	5 2 5	6 6 3	7 16 3	8 32 3
22	3 1	12 7	28 5	50	1 19 3	1 54 4	2 3 7	3 23 3	4 7	5 18 4	6 25 5	7 39 2	8 59 4
23	3 3	13 3	30 0	53 3	1 23 4	2 0 0	2 43 6	33	4 0 8	5 34 5	6 45 1	8 2 5	9 26 9
23 28	3 4	13 7	30 7	54 5	1 25 3	2 2 9	2 47 4	7	4 30 0	4	6 54 1	8 13 4	9 30 6

TABLE XXXI

Epochs of the mean Longitude of the Moon's Ascending Node

O S	S	D	M	O	S	D	M	N S	S	D	M	N S	S	D	M	N S	S	D	M	N S	S	D	M
1690	0	0	[illegible]	1722	3	11	19	1753	7	12	17	1785	10	23	20	1817	2	4	27	1849	5	15	31
1691	11	10	52	1723	2	21	59	1754	6	22	57	1786	10	4	1	1818	1	15	7	1850	4	26	11
1692	10	21	32	1724	2	2	36	1755	6	3	37	1787	9	14	41	1819	0	25	46	1851	4	6	51
1693	10	2	13	1725	1	13	16	1756	5	14	14	1788	8	25	18	1820	0	6	25	1852	3	17	28
1694	9	12	53	1726	0	23	56	1757	4	24	55	1789	8	5	58	1821	11	17	5	1853	2	28	8
1695	8	23	33	1727	0	4	37	1758	4	5	35	1790	7	16	39	1822	10	27	45	1854	2	8	48
1696	8	4	10	1728	11	15	14	1759	3	16	15	1791	6	27	19	1823	10	8	26	1855	1	19	29
1697	7	14	51	1729	10	25	54	1760	2	26	52	1792	6	7	56	1824	9	19	[illegible]	1856	1	0	6
1698	6	25	31	1730	10	6	34	1761	2	7	32	1793	5	18	36	1825	8	29	43	1857	0	10	46
1699	6	6	11	1731	9	17	15	1762	1	18	13	1794	4	29	17	1826	8	10	23	1858	11	21	26
1700	5	16	48	1732	8	27	52	1763	0	28	53	1795	4	9	57	1827	7	21	4	1859	11	2	7
1701	4	27	28	1733	8	8	32	1764	0	9	30	1796	3	20	34	1828	7	1	41	1860	10	12	44
1702	4	8	9	1734	7	19	12	1765	11	20	11	1797	3	1	14	1829	6	12	21	1861	9	23	24
1703	3	18	49	1735	6	29	52	1766	11	0	51	1798	2	11	55	1830	6	23	1	1862	9	4	4
1704	2	29	26	1736	6	10	30	1767	10	11	31	1799	1	22	35	1831	5	3	42	1863	8	14	45
1705	2	10	6	1737	5	21	10	1768	9	22	8	1800	1	3	16	1832	4	14	19	1864	7	25	22
1706	1	20	47	1738	5	1	50	1769	9	2	49	1801	0	13	55	1833	3	24	59	1865	7	6	2
1707	1	1	27	1739	4	12	31	1770	8	13	29	1802	11	24	36	1834	3	5	39	1866	6	16	42
1708	0	12	4	1740	3	23	8	1771	7	24	9	1803	11	5	16	1835	2	16	20	1867	5	27	22
1709	11	22	44	1741	3	3	48	1772	7	4	46	1804	10	15	53	1836	1	26	57	1868	5	8	0
1710	11	3	25	1742	2	14	28	1773	6	15	27	1805	9	26	33	1837	1	7	37	1869	4	18	40
1711	10	14	5	1743	1	25	9	1774	5	26	7	1806	9	7	14	1838	0	18	17	1870	3	29	20
1712	9	24	42	1744	1	5	46	1775	5	6	47	1807	8	17	54	1839	11	28	58	1871	3	10	1
1713	9	5	22	1745	0	16	26	1776	4	17	24	1808	7	28	31	1840	11	9	35	1872	2	20	38
1714	8	16	3	1746	11	27	6	1777	3	28	5	1809	7	9	11	1841	10	20	15	1873	2	1	18
1715	7	26	43	1747	11	7	47	1778	3	8	45	1810	6	19	52	1842	10	0	55	1874	1	11	58
1716	7	7	20	1748	10	18	24	1779	2	19	25	1811	6	0	32	1843	9	11	35	1875	0	22	39
1717	6	18	0	1749	9	29	4	1780	2	0	2	1812	5	11	9	1844	8	22	12	1876	0	3	16
1718	5	28	41	1750	9	9	44	1781	1	10	42	1813	4	21	49	1845	8	2	53	1877	11	13	56
1719	5	9	21	1751	8	20	24	1782	0	21	23	1814	4	2	30	1846	7	13	33	1878	10	24	35
1720	4	19	58	1752	8	1	2	1783	0	2	3	1815	3	13	10	1847	6	24	13	1879	10	5	17
1721	4	0	38	N S 1752	8	1	37	1784	11	12	40	1816	2	23	47	1848	6	4	51	1880	9	15	54

TABLE XXXII

Mean Retrograde Motion of the Moon's Node to every Day in the Year

Subtract from the Longitude of the Epoch

Day	January	February	March	April	May	June	July	Auguſt	Septemb	October	Novemb	Decemb
1	0 3	1 42	3 1	4 49	6 24	8 3	9 38	11 17	12 55	14 31	16 9	17 44
2	0 6	1 45	3 14	4 52	6 28	8 6	9 41	11 20	12 58	14 34	16 12	17 48
3	0 10	1 48	3 17	4 55	6 31	8 9	9 45	11 23	13 2	14 37	16 15	17 5
4	0 1	1 51	3 20	4 59	6 34	8 12	9 48	11 26	13 5	14 40	16 19	17 54
5	0 16	1 54	3 23	5 2	6 37	8 16	9 51	11 29	13 8	14 43	16 22	17 7
6	0 19	1 58	3 2	5 5	6 40	8 19	9 54	11 3	13 11	14 46	16 25	18 0
7	0 22	2	3 30	5 8	6 44	8 22	9 57	11 36	13 14	14 50	16 28	18 3
8	0 25	2 4	3 33	5 11	6 47	8 25	10 1	1 39	13 17	14 53	16 31	18 7
9	0 29	2 7	3 36	5 15	6 50	8 28	10 4	11 42	13 21	14 56	16 34	18 10
10	0 32	2 10	3 39	5 18	6 53	8 32	10 7	11 45	13 24	14 59	16 38	18 13
11	0 35	2 13	3 42	5 21	6 56	8 35	10 10	11 49	13 27	15 2	16 41	18 16
12	0 38	2 17	3 46	5 24	6 59	8 38	10 3	11 52	13 30	15 6	16 44	8 19
13	0 41	2 20	3 49	5 27	7 3	8 41	10 6	11 55	13 33	15 9	16 47	18 23
14	0 44	2 23	3 52	5 30	7 6	8 44	10 20	11 58	13 37	15 12	16 50	18 26
15	0 48	2 26	3 55	5 34	7 9	8 47	10 23	12	13 40	15 15	16 54	18 29
16	0 51	2 29	3 58	5 37	7 12	8 51	10 26	12 4	13 43	15 18	16 57	18 32
17	0 54	2 33	4 1	5 40	7 15	8 54	10 29	12 8	13 46	15 21	17 0	18 35
18	0 57	2 36	4 5	5 43	7 18	8 57	10 32	12 11	13 49	15 25	17 3	8 38
19	1 0	2 39	4 8	5 46	7 22	9 0	10 36	12 14	13 52	15 28	17 6	18 42
20	1 4	2 42	4 11	5 50	7 25	9 3	10 39	12 17	1 56	15 31	17 9	18 45
21	1 7	2 45	4 14	5 53	7 28	9 7	10 42	12 20	13 59	15 34	17 13	18 48
22	1 10	2 48	4 17	5 56	7 31	9 10	10 45	12 23	14 2	15 37	17 16	18 51
23	1 13	2 52	4 21	5 59	7 34	9 13	10 48	12 27	14 5	15 40	17 9	18 54
24	1 16	2 55	4 24	6 2	7 8	9 16	10 51	12 30	14 8	15 44	17 22	8 57
25	1 19	2 58	4 27	6 5	7 41	9 19	10 55	12 33	14 11	15 47	17 25	19 1
26	1 23	3 1	4 30	6 9	7 44	9 22	10 58	12 36	14 15	15 50	17 28	19 4
27	1 26	3 4	4 33	6 12	7 47	9 26	11 1	12 39	14 18	15 53	17 32	19 7
28	1 29	3 7	4 36	6 15	7 50	9 29	11 4	12 43	14 21	15 56	17 35	19 10
29	1 32		4 40	6 18	7 53	9 32	11 7	12 46	14 24	16 0	17 38	19 3
30	1 35		4 43	6 21	7 57	9 35	11 10	12 49	14 2	6 3	17 41	19 17
31	1 38		4 46		8 0		1 14	12 52		16 6		19 20

In the months of January and February in leap year, take out for the day preceding the given day

TABLE XXXIII

Showing the Decrease of the Diameters of the Sun or Moon which are inclined to the Horizon, supposing the apparent Diameter to be 30

Inclination to Horizon	ALTITUDE OF THE SUN OR MOON							
	10	11	12	13	14	16	18	20
0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
3	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
6	0,2	0,1	0,1	0,1	0,1	0,1	0,0	0,0
9	0,4	0,3	0,3	0,2	0,2	0,2	0,1	0,1
12	0,7	0,6	0,5	0,4	0,4	0,3	0,2	0,2
15	1,0	0,9	0,7	0,6	0,6	0,4	0,3	0,3
18	1,5	1,3	1,1	0,9	0,8	0,6	0,5	0,4
21	2,0	1,7	1,4	1,2	1,1	0,6	0,7	0,5
4	2,6	2,2	1,8	1,6	1,4	1,1	0,9	0,7
27	3,2	[illegible]	2,3	2,0	1,7	1,3	1,1	0,9
30	3,9	3,3	2,8	2,4	[illegible]	1,6	1,3	1,1
33	4,6	3,9	3,3	2,8	2,5	1,9	1,5	1,3
36	5,4	4,5	3,8	3,3	[illegible]	[illegible]	1,8	1,5
39	6,2	5,2	4,3	3,7	3,3	[illegible]	2,0	1,7
4[illegible]	7,0	5,9	4,9	4,2	3,7	2,9	2,3	1,9
45	7,7	6,5	5,4	4,7	4,2	3,2	2,6	[illegible]
48	8,5	7,1	6,0	5,2	4,6	3,5	[illegible]	[illegible]
51	9,3	7,8	6,6	5,7	5,0	3,9	3,1	2,5
54	10,1	8,5	[illegible]	[illegible]	5,4	[illegible]	3,4	[illegible]
57	10,8	9,1	7,7	6,6	5,8	4,5	3,6	2,9
60	11,5	9,7	8,1	7,0	[illegible]	4,6	3,9	3,1
63	12,2	10,3	8,6	7,4	6,6	5,1	4,1	3,3
66	12,9	10,8	9,1	7,8	6,9	5,3	4,3	3,5
69	13,4	11,3	9,6	[illegible]	[illegible]	5,6	4,5	3,7
72	13,9	11,7	9,9	8,5	7,5	5,8	4,7	3,6
75	[illegible]	12,1	[illegible]	8,8	7,7	6,0	4,8	3,9
78	14,7	12,4	10,5	9,1	7,9	6,1	4,9	4,0
81	15,0	12,6	10,6	9,3	8,1	6,2	5,0	4,1
84	15,2	12,7	10,7	9,3	6,2	6,3	5,1	[illegible]
87	15,3	12,8	10,8	9,4	6,2	6,4	5,1	[illegible]
90	15,3	[illegible]	10,9	9,4	8,3	[illegible]	5,1	

TABLE XXXIII *Continued*

Inclination to Horizon	ALTITUDE OF THE SUN OR MOON							
	2	26	30	34	40	46	5	70
0 3 6 9	0,0 0,0 0,0 0,1	0,0 0,0 0,0 0,1	0,0 0,0 0,0 0,0	0,0 0,0 0,0 0,0	0,0 0,0 0,0 0,0	0,0 0,0 0,0 0,0	0,0 0,0 0,0 0,0	0,0 0,0 0,0 0,0
12 15 18	0,1 0,[illegible] 0,3	0,1 0,2 0,2	0,1 0,1 0,2	0,1 0,1 0,2	0,0 0,1 0,1	0,0 0,1 0,1	0,0 0,1 0,1	0,0 0,0 0,0
21 24 27	0,5 0,6 0,7	0,3 0,4 0,5	0,3 0,3 0,4	0,2 0,3 0,3	0,2 0,[illegible] 0,2	0,1 0,2 0,2	0,1 0,1 0,2	0,1 0,1 0,1
30 33 36	0,9 1,0 1,2	0,6 0,8 0,9	0,5 0,6 0,7	0,4 0,5 0,5	0,3 0,3 0,4	0,2 0,3 0,3	0,2 0,2 0,3	0,1 0,2 0,2
39 42 45	1,4 1,6 1,8	1,0 1,1 1,3	0,8 0,9 1,0	0,6 0,7 0,8	0,5 0,5 0,6	0,4 0,4 0,5	0,3 0,4 0,4	0,2 0,3 0,3
48 51 54	[illegible],0 2,[illegible] 2,3	1,4 1,6 1,7	1,1 1,2 1,3	0,9 1,0 1,0	0,7 0,7 0,8	0,5 0,6 0,6	0,4 0,5 0,5	0,3 0,3 0,4
57 60 63	2,5 2,6 2,8	1,8 1,9 2,0	1,4 1,5 1,6	1,1 1,2 1,3	0,8 0,9 0,9	0,7 0,7 0,8	0,6 0,6 0,6	0,4 0,4 0,4
66 69 72	[illegible],9 3,1 3,2	2,1 2,2 2,3	1,7 1,7 1,8	1,3 1,4 1,4	1,0 1,0 1,1	0,8 0,8 0,9	0,7 0,7 0,7	0,5 0,5 0,5
75 78 81	3,3 3,4 3,4	2,4 2,5 [illegible],5	1,8 1,9 1,9	1,5 1,5 1,5	1,1 1,1 1,2	0,9 0,9 0,9	0,7 0,8 0,8	0,5 0,5 0,5
84 87 90	3,5 3,5 3,5	2,6 2,6 2,6	1,9 2,0 2,0	1,6 1,6 1,6	1, 1,2 1,2	0,9 1,0 1,0	0,8 0,8 0,8	0,6 0,6 0,6

TABLE XXXIV

For reducing Sidereal to Mean Solar Time

Hours	Min Sec	Minutes	Sec	Seconds	Sec
1	0 9,83	1	0,16	1	0,00
2	0 19,66	2	0,33	2	0,01
3	0 9,49	3	0 49	3	0,01
4	0 39,32	4	0,66	4	0 01
5	0 49,15	5	0,82	5	0 01
6	0 58,96	6	0,98	6	0 02
7	1 8 81	7	1,15	7	0 02
8	1 16 64	8	1 31	8	0,02
9	1 8 47	9	1,47	9	0 02
10	1 6 30	10	1 64	10	0 03
11	1 15 1	11	1,80	11	0,03
12	1 57 96	12	1,97	12	0,03
13	2 7,[illegible]	13	13	13	0 04
14	2 17,61	14	,29	14	0,04
15	27 44	15	,46	15	0,04
16	2 37,27	16	2 62	16	0 04
17	2 47,10	17	2,78	17	0,05
18	56,93	18	2 95	18	0 05
19	3 6 76	19	3 11	19	0 05
20	3 10 59	0	3 6	0	0,05
21	3 6 42	30	[illegible] 01	30	0 08
2[illegible]	3 [illegible] 25	40	6,55	40	0 11
23	3 46 08	50	[illegible] 1[illegible]	50	0 14
24	3 55 91	60	9,83	60	0,16

TABLE XXXV

For converting Mean Solar into Sidereal Time

Hours	Min Sec	Minutes	Sec	Seconds	Sec
1	0 9,86	1	0,16	1	0,00
2	0 19 71	2	0,33	2	0,01
3	0 29 57	3	0 49	3	0,01
4	0 39 43	4	0,66	4	0,01
5	0 49 28	5	0,82	5	0 01
6	0 59,14	6	0 99	6	0,02
7	1 8,99	7	1,15	7	0,0
8	1 18 85	8	1,31	8	0,0
9	1 28,71	9	1,48	9	0,02
10	1 38,56	10	1,64	10	0,03
11	1 48,42	11	1,82	11	0,03
12	1 58,28	12	1 97	12	0,03
13	2 8,13	13	2 14	13	0,04
14	2 17,99	14	2,30	14	0,04
15	2 27, 85	15	2 46	15	0,04
16	2 37,70	16	2 63	16	0 04
17	2 47 56	17	2,79	17	0 05
18	2 57 42	18	2 96	18	0 05
19	3 7 27	19	3 12	19	0,05
20	3 17,13	20	3 28	20	0,05
21	3 26,98	30	4 93	30	0,08
22	3 36 84	40	6 57	40	0,11
23	3 46,70	50	8 21	50	0 14
24	3 56,55	60	9,86	60	0,16

TABLE XXXVI

The apparent Semidiameter of the Sun and its Horary Motion

ARGUMENT Sun's mean Anomaly

	Sig O		Sig I		Sig II		
Deg	S h m	Sun s semid	Sun hor mot	Sun md	Sun s ho mot	Sun md	D
0	2 22 99	15 45 50	2 23 60	15 47 52	2 25 32	15 53 17	30
	2 22 99	15 45 50	2 23 64	15 47 66	2 25 39	15 53 41	29
2	2 22 99	15 45 51	2 23 68	15 47 80	2 25 46	15 53 65	28
3	2 22 99	15 45 52	2 23 73	15 47 94	2 25 53	15 53 89	27
4	2 23 00	15 45 53	2 23 77	15 48 09	2 25 61	15 54 14	26
5	2 23 00	15 45 55	2 23 82	15 48 24	2 25 68	15 54 38	25
6	2 23 01	15 45 58	2 23 86	15 48 40	2 25 76	15 54 63	24
7	2 23 02	15 45 61	2 23 91	15 48 56	2 25 84	15 54 88	23
8	2 23 03	15 45 64	2 23 96	15 48 72	2 25 91	15 55 15	22
9	2 23 04	15 45 68	2 24 01	15 48 89	2 25 99	15 55 39	21
10	2 23 05	15 45 73	2 24 06	15 49 06	2 26 07	15 55 65	20
11	2 23 07	15 45 78	2 24 12	15 49 23	2 26 15	15 55 91	19
12	2 23 09	15 45 83	2 24 17	15 49 41	2 26 23	15 56 17	18
13	2 23 10	15 45 88	2 24 23	15 49 59	2 26 31	15 56 44	17
14	2 23 12	15 45 95	2 24 28	15 49 77	2 26 39	15 56 70	16
15	2 23 14	15 46 01	2 24 34	15 49 96	2 26 47	15 56 96	15
16	2 23 16	15 46 08	2 24 40	15 50 15	2 26 56	15 57 23	14
17	2 23 18	15 46 16	2 24 46	15 50 [illegible]	2 26 64	15 57 50	13
18	2 23 21	15 46 24	2 24 52	15 50 55	2 26 72	15 57 77	12
19	2 23 [illegible]	15 46 32	2 24 58	15 50 75	2 26 80	15 58 05	11
20	2 23 26	15 46 41	2 24 64	15 50 96	2 26 89	15 58 [illegible]	10
21	2 23 29	15 46 50	2 24 70	15 51 16	2 26 97	15 58 59	9
22	2 23 32	15 46 60	2 24 77	15 51 37	2 27 06	15 58 87	8
23	2 23 35	15 46 70	2 24 84	15 51 59	2 27 14	15 59 14	7
24	2 23 38	15 46 81	2 24 90	15 51 81	2 27 23	15 59 42	6
25	2 23 41	15 46 92	2 24 97	15 52 03	2 27 [illegible]	15 59 70	5
26	2 23 45	15 47 05	2 25 03	15 52 25	2 27 40	15 59 98	4
27	2 23 48	15 47 15	2 25 10	15 52 48	2 27 48	16 0 20	3
28	2 23 52	15 47 27	2 25 17	15 52 70	2 27 57	16 0 54	2
29	2 23 56	15 47 40	2 25 24	15 52 94	2 27 66	16 0 82	1
30	2 23 60	15 47 53	2 25 32	15 53 17	2 27 74	16 1 10	0
	Sig XI		Sig X		Sig IX		

TABLE XXXVI *Continued*

	Sig III		Sig IV		Sig V		
D g	Sa n	S d	S h m	S n m d	Sn m	ap d	D
0	2 27 72	16 1 10	2 30 28	16 9 30	2 32 20	16 15 49	30
1	2 27 85	16 1 38	2 30 35	16 9 55	2 32 24	16 15 63	29
2	2 2 9	6 1 67	2 30 43	16 9 80	2 32 29	16 15 6	28
3	2 28 00	16 1 95	2 30 51	16 10 05	2 32 33	16 15 92	27
4	2 28 09	16 2 2	2 30 58	16 10 29	2 32 37	16 16 05	26
5	2 28 18	16 2 51	2 30 66	16 10 54	2 32 41	16 16 18	25
6	2 28 26	16 2 79	2 30 73	16 10 77	2 32 45	16 16 30	24
7	2 28 35	6 3 07	2 30 80	16 11 01	2 32 49	16 16 42	23
8	2 28 44	16 3 36	2 30 88	16 11 24	2 32 53	16 16 54	22
9	2 28 52	16 3 64	2 30 95	16 11 45	2 32 56	16 16 65	21
10	2 28 61	16 92	2 3 02	16 11 68	2 32 59	16 16 75	20
11	2 28 70	16 4 20	2 31 09	16 1 92	2 32 62	6 16 85	19
12	2 28 78	6 4 48	2 31 16	16 12 14	2 32 65	16 16 95	18
13	2 28 87	16 4 76	2 31 22	16 12 36	2 32 68	16 17 04	17
14	2 28 96	16 5 02	2 31 29	16 12 7	2 3 71	16 17 12	16
15	2 29 04	6 5 3	2 31 6	16 1 76	2 32 73	16 1 20	15
16	2 29 13	10 5 59	2 31 42	16 12 99	2 32 76	16 17 28	14
17	2 29 21	6 5 80	2 31 48	16 13 20	2 32 78	16 17 35	13
18	2 29 0	16 6 14	2 31 55	16 3 39	2 32 80	16 17 41	12
19	2 29 38	16 6 41	2 31 61	16 13 59	2 32 82	16 17 47	11
20	2 29 46	16 6 68	2 31 67	16 1 78	2 32 84	6 17 53	10
21	2 29 54	6 6 91	31 73	16 3 97	2 32 86	16 17 58	9
22	2 29 63	16 7 22	2 31 78	16 14 16	2 32 87	16 17 62	8
23	2 29 1	16 7 49	2 31 84	16 14 34	2 32 88	16 7 66	7
24	2 29 80	16 7 75	2 31 90	16 14 51	2 32 89	16 17 70	6
25	2 29 88	16 8 02	2 31 95	16 14 69	2 32 90	16 17 73	5
26	2 29 96	16 8 28	2 32 00	16 14 85	2 32 91	16 17 75	4
27	2 30 04	16 8 54	2 32 05	6 02	2 32 91	6 17 77	3
28	2 30 2	16 8 79	2 32 10	16 15 18	2 32 92	16 17 78	2
29	2 30 20	6 9 05	2 32 1	16 15 34	2 32 92	16 17 79	1
30	2 30 28	16 9 30	2 2 3	16 1 49	2 32 92	16 17 79	0
	Sig VIII		Sig VII		Sig VI		

TABLE XXXVII

The Reduction of the Ecliptic to the Equator according to the Obliquity of the Ecliptic 23 28 15 with the Variation of Reduction for the Variation of the Obliquity by One Minute

ARGUMENT Longitude of the Point of the Ecliptic

	Sig O	Sig VI		Sig I	Sig VII		Sig II	Si VIII		
	—	Diff	Var	—	Diff	V	—	Diff	V	
0 0	0 0 0 0	2 28,9	0 0	2 5 42 4	1 19 7	10 8	2 11 14 7	1 15 3	11 7	30 0
0 30	0 2 28 9	2 28 9	0 2	2 7 2 1	1 17 5	10 9	2 9 59 4	1 17 8	11 6	29 30
1 0	0 4 57 8	2 28 8	0 4	2 8 19 6	1 15 3	11 0	2 8 41 6	1 20 4	11 5	29 0
1 30	0 7 26 6	2 28 7	0 6	2 9 34,9	1 13 0	11 1	2 7 21 2	1 22 8	11 4	28 30
2 0	0 9 55,3	2 28,5	0 8	2 10 47 9	1 10 8	11 2	2 5 58,4	1 25 2	11 3	28 0
2 30	0 12 23 8	2 28 3	1 0	2 11 58 7	1 8,6	11,3	2 4 33 2	1 27,8	11 2	27 30
3 0	0 14 52 1	2 28 1	1,3	2 13 7 3	1 6 3	11,5	2 3 5 4	1 30 1		27 0
3 30	0 17 20 2	2 27 8	1 5	2 14 13 6	1 4,0	11 6	2 1 35,3	1 32 6	10 9	26 30
4 0	0 19 48 0	2 27 5	1 7	2 15 17 6	1 1 5	11 7	2 0 2 7	1 35 0	10 8	26 0
4 30	0 22 15 5	2 27 1	1 9	2 16 19 1	0 59 1	11 8	1 58 27 7	1 37 2	10 7	25 30
5 0	0 24 42 6	2 26 7	2 1	2 17 18 2	0 56 9	11 8	1 56 50 5	1 39 6	10 5	25 0
5 30	0 27 9 3	2 26 3	2 3	2 18 15 1	0 54 4	11 9	1 55 10 9	1 41 9	10 4	24 30
6 0	0 29 35 6	2 25 9	2 5	2 19 9 5	0 51 9	12 0	1 53 29 0	1 44,2	10 2	24 0
6 30	0 32 1 5	2 25 3	2 7	2 20 1 4	0 49 6	12 1	1 51 44 8	1 46 3	10 1	23 30
7 0	0 34 26 8	2 24,7	2 9	2 20 51 0	0 47 1	12 2	1 49 58 5	1 48 5	9 9	23 0
7 30	0 36 51 5	2 24,1	3 1	2 21 38 1	0 44,5	12 2	1 48 10 0	1 50 7	9 8	22 30
8 0	0 39 15 6	2 23 5	3 3	2 22 22 6	0 42 0	12 3	1 46 19 3	1 52 8	9 6	22 0
8 30	0 41 39 1	2 22 9	3 5	2 23 4,6	0 39 5	12 4	1 44 26 5	1 55 0	9 5	21 30
9 0	0 44 2 0	2 22 2	3 7	2 23 44,1	0 36 9	12 5	1 42 31 5	1 57 0	9 3	21 0
9 30	0 46 24 2	2 21 3	3 9	2 24 21 0	0 34,5	12 5	1 40 34 5	1 59 0	9 1	20 30
10 0	0 48 45 5	2 20 6	4 1	2 24 55 5	0 31 8	12 6	1 38 35 5	2 1,0	8 9	20 0
10 30	0 51 6 1	2 19 8	4 3	2 25 27 3	0 29 3	12 6	1 36 34 5	2 3 0	8 8	19 30
11 0	0 53 25 9	2 18 9	4 5	2 25 56 6	0 26 6	12 7	1 34 31 5	2 4,8	8 6	19 0
11 30	0 55 44,8	2 18 0	4 7	2 26 23 2	0 24,0	12 7	1 32 26 7	2 6 8	8 4	18 30
12 0	0 58 2 8	2 17 1	4 9	2 26 47 2	0 21 5	12 8	1 30 19 9	2 8 7	8 2	18 0
12 30	1 0 19 9	2 16,0	5 1	2 27 8 7	0 18 6	12 8	1 28 11 2	2 10 2	8 0	17 30
13 0	1 2 35 9	2 15 0	5 3	2 27 27 3	0 16 0	12 9	1 26 1 0	2 12 1	7 8	17 0
13 30	1 4 50 9	2 14,0	5 5	2 27 43 3	0 13 5	12 9	1 23 48 9	2 13 8	7 6	16 0
14 0	1 7 4 9	2 12,9	5 7	2 27 56 8	0 10 8	12 9	1 21 35 1	2 15 4	7 4	16 0
14 30	1 9 17 8	2 11 8	5 8	2 28 7 6	0 8 0	13 0	1 19 19 7	2 17 1	7 2	15 30
15 0	1 11 29 6		6 0	2 28 15 6		13 0	1 17 2 6		7 0	15 0
	+			+			+			
	Sig V	Sig XI		Sig IV	Sig X		Sig III	Sig IX —		

TABLE XXXVII Continued

ARGUMENT Longitude of the Point of the Ecliptic

	Si O	Sig VI		Sig I	Sig VII		Sig II	Sig VIII		
	—	D ff	Va	—	D ff	V	—	D ff	V	
15 0	1 1 23 6	2 10 6	6 0	2 28 15 6	0 5 3	13 0	1 17 2 6	2 18 6	7 0	15 0
15 30	1 3 40 2	2 9 4	6 2	2 28 20 9	0 2 7	13 0	1 14 44,0	2 20 3	6 8	14 30
16 0	1 15 19 6	2 8 2	6 4	2 28 23 6	0 0 0	13 0	1 12 23 7	2 21 8	6 6	14 0
6 30	1 37 8	2 6 8	6 6	2 28 23 6	0 2 9	13 0	1 10 1 9	2 23 0	6 4	13 30
1 0	20 46	2 5 5	6 8	2 28 20 7	0 5 7	13 0	1 7 38 9	2 24,5	6 2	13 0
17 30	10	2 4,3	7 0	2 28 15 0	0 8 1	13 0	1 5 14 5	2 25 9	6 0	12 30
18 0	1 4 13 4	2 2 9	7 1	2 28 6 9	0 10 8	13 0	1 2 48 6	2 27 2	5,7	12 0
8 30	1 26 17 3	2 1 4	7 3	2 27 56 1	0 13 8	13 0	1 0 21 4	2 28 3	5 5	11 30
19 0	1 28 18 7	2 0 0	7 5	2 27 42 5	0 16,5	13 0	0 57 53 1	2 29 6	5 3	11 0
13 0	1 30 8 7	1 58 4	7 6	2 27 25 8	0 19 2	13 0	0 55 23 5	2 30 7	5 1	10 30
20 0	1 32 17 1	1 5 0	7 8	2 27 6 6	0 21,9	13 0	0 52 52 8	2 31 8	4,8	10 0
0 0	1 34 14,1	1 5 3	8 0	2 26 44,7	0 24,6	12 9	0 50 21 0	2 32,9	4,6	9 30
21 0	6 9 4	1 53 6	8 2	2 26 20 1	0 27,5	12 9	0 47 48 1	2 33 8	4,4	9 0
1 0	36 3 2	1 52 2	8 3	2 25 52 6	0 30 0	12 9	0 45 14 3	2 34 8	4 2	8 30
0	1 30 33 4	1 50 4	8 5	2 25 22 6	0 32 7	12 9	0 42 39 5	2 35 7	3 9	8 0
0	1 41 45 6	1 48 9	8 7	2 24 49 9	0 35 7	12 8	0 40 3 8	2 36 4	3 6	7 30
2 0	1 43 34 7	47 1	8 8	2 24 14,2	0 38 4	12 8	0 37 27 4	2 37 2	3 4	7 0
30	1 4 21 8	1 45 3	9 0	2 23 35 8	0 40 9	12,7	0 34 50,2	2 38,1	3,1	6 30
4 0	1 47 7 1	43 5	9 1	2 22 54,9	0 43 6	12,7	0 32 12,1	2 38,7	2 9	6 0
1 30	1 48 0 6	1 41 7	9 3	2 22 11 3	0 46 3	12 6	0 29 33,4	2 39 2	2 7	5 30
25 0	1 50 2 3	1 39 9	9 4	2 21 24 8	0 49 2	12 6	0 26 54,2	2 39,7	2,5	5 0
5 30	1 52 12 2	1 37 9	9 6	2 20 35 6	0 51,7	12 5	0 24 14,5	2 40 4	2 2	4 30
26 0	1 53 50 1	1 36 0	9 7	2 19 43 9	0 54,3	12 4	0 21 34,1	2 40,8	2 0	4 0
26 30	1 55 26 1	1 34 2	9 9	2 18 49 6	0 57 2	12 4	0 18 53 3	2 41,2	1 8	3 30
27 0	1 57 0 3	1 32 1	10,0	2 17 52 4	0 59 8	12 3	0 16 12 1	2 41,5	1 5	3 0
2 30	1 5 4	1 30 1	10 1	2 16 52 6	1 2 4	12,2	0 13 30,6	2 41,8	1 3	2 30
8 0	2 0 2 3	1 28 1	10,3	2 15 50 2	1 5 0	12,1	0 10 48 8	2 42,0	1 0	2 0
28 30	2 1 30 6	1 26 0	10 4	2 14 45 2	1 7 5	12 0	0 8 6,8	2 42 2	0 7	1 30
20 0	2 2 36 6	1 24,0	10 5	2 13 37 7	1 10 2	11 9	0 5 24,6	2 42 3	0,5	1 0
29 30	2 4 20 6	1 21,8	10 7	2 12 27,3	1 12,8	11 8	0 2 42 3	2 42 3	0 3	0 30
30 0	2 5 42 4		10,8	2 11 14,7		11 7	0 0 0 0		0 0	0 0
	+			+			+			
	Sig V	Sig XI		Sig IV	Sig X		Sig III	Sig IX		

TABLE XXXVIII

The Declination of the Points of the Ecliptic, according to the Obliquity of the Ecliptic 23 28 15, *with the Variation of Declination for the Variation of the Obliquity of One Minute*

ARGUMENT Longitude of the Point of the Ecliptic

	Sig O +	Sig VI −		Sig I +	Sig VII −		Sig II +	Sig VIII −		
	Declin	Diff	Var t	Declin	Diff en	Var t	Declinat n	Diff	V	
0 0	0 0 0	11 57 0	0 0	11 29 12 3	10 32 1	28 1	20 10 37,4	6 19,2	50 8	30 0
0 30	0 11 57 0	11 56 8	0 5	11 39 44 4	10 29 3	28,5	20 16 56 6	6 13 5	51 1	29 30
1 0	0 23 53 8	11 56 7	1,0	11 50 13 7	10 26 4	29 0	20 23 10 1	6 8 0	51,4	29 0
1 30	0 35 50,5	11 56 6	1,4	12 0 40 1	10 23 5	29 4	20 29 18 1	6 2 4	51,7	28 30
2 0	0 47 47 1	11 56 4	1 9	12 11 3 6	10 20 5	29 9	20 35 20 5	5 56 7	51 9	28 0
2 30	0 59 43 5	11 56 3	2 4	12 21 24,1	10 17 4	30 3	20 41 17 2	5 51 0	52 2	27 30
3 0	1 11 39 8	11 55 9	2 9	12 31 41 5	10 14,3	30 7	20 47 8 2	5 45,2	52 4	27 0
3 30	1 23 35,7	11 55 6	3 4	12 41 55,8	10 11 3	31,2	20 52 53,4	5 39 5	52,7	26 30
4 0	1 35 31 3	11 55 3	3 8	12 52 7 1	10 8 1	31 6	20 58 32 9	5 33 7	53 0	26 0
4 30	1 47 26 6	11 54,8	4,3	13 2 15 2	10 4,8	32 0	21 4 6 6	5 27 8	53 2	25 30
5 0	1 59 21 4	11 54,5	4 8	13 12 20 0	10 1 5	32 4	21 9 34 4	5 21 9	53 5	25 0
5 30	2 11 15 9	11 53 8	5 3	13 22 21 5	9 58 3	32 8	21 14 56 3	5 16 0	53 7	24 30
6 0	2 23 9 7	11 53 3	5 8	13 32 19,8	9 54,9	33 3	21 20 12 3	5 10 1	54 0	24 0
6 30	2 35 3 0	11 52 7	6 2	13 42 14,7	9 51 5	33 7	21 25 22 4	5 4,1	54 2	23 30
7 0	2 46 55 7	11 52 2	6 7	13 52 6 2	9 48 0	34 1	21 30 26 5	4 58 2	54,5	23 0
7 30	2 58 47 9	11 51 3	7 2	14 1 54,2	9 44,4	34,5	21 35 24,7	4 51 9	54,7	22 30
8 0	3 10 39 2	11 50 7	7 7	14 11 38 6	9 40 8	35 0	21 40 16 6	4 45 9	54 9	22 0
8 30	3 22 29 9	11 49 8	8 2	14 21 19 4	9 37 5	35 4	21 45 2 5	4 39 8	55 2	21 30
9 0	3 34 19 7	11 48 9	8 6	14 30 56 9	9 33 1	35 8	21 49 42 3	4 33 6	55 4	21 0
9 30	3 46 8 6	11 48 3	9 1	14 40 30 0	9 30 0	36 2	21 54 15 9	4 27 6	55,6	20 30
10 0	3 57 56 9	11 47 3	9 6	14 50 0 6	9 26 1	36 0	21 58 43 5	4 21 3	55 8	20 0
10 30	4 9 44 2	11 46 2	10,1	14 59 26 7	9 22 5	37 0	22 3 4 8	4 15 1	56 0	19 30
11 0	4 21 30 4	11 45 3	10,6	15 8 49 2	9 18 6	37,4	22 7 19,9	4 9 0	56 2	19 0
11 30	4 33 15 7	11 44,2	11 0	15 18 7 8	9 14 8	37 8	22 11 28 9	4 2,4	56 4	18 30
12 0	4 44 59 9	11 43 1	11 5	15 27 22 6	9 10 7	38 2	22 15 31 3	3 56 1	56 6	18 0
12 30	4 56 43 0	11 41 9	12 0	15 36 33 3	9 6 8	38 6	22 19 27 4	3 49 9	56 8	17 30
13 0	5 8 24,9	11 40 8	12 4	15 45 40 1	9 2 8	39 0	22 23 17 3	3 43 5	57 0	17 0
13 30	5 20 5 7	11 39 5	12 9	15 54 42 9	8 58 7	39 4	22 27 0,8	3 37 2	57 1	16 30
14 0	5 31 45 2	11 38 2	13 4	16 3 41 6	8 54 6	39 8	22 30 38 0	3 30 6	57 3	16 0
14 30	5 43 23,4	11 36 8	13 9	16 12 36 2	8 50 4	40 2	22 34 8 6	3 24,3	57 5	15 30
15 0	5 55 0,2		14,3	16 21 26,6		40 6	22 37 32 9		57 6	15 0
	Sig V +	Sig XI −		Sig IV −	Sig X −		Sig III +	Sig IX −		

TABLE XXXVIII *Continued*

ARGUMENT Longitude of the Point of the Ecliptic

	Sig O +	Sig VI −		Sig I +	Sig VII −		Sig II +	Sig VIII −		
	D l t	Diff n e	V t	D lin u n	Diff	V rat	D lin n	Diff n e	Va at	
15 0	5 55 0 2	11 35 6	14 3	16 21 26 6	8 46 2	40 6	22 37 32 9	3 17 7	57 6	15 0
15 30	6 6 35 8	11 34 0	14 8	16 30 12 8	8 42 0	41 0	22 40 50 6	3 11 4	57 8	14 30
16 0	6 18 9 8	11 32 6	15 3	16 38 54 8	8 37 6	41 4	22 44 2 0	3 4 8	57 9	14 0
16 30	6 29 42 4	11 31 0	15 7	16 47 32 4	8 33 3	41 7	22 47 6 8	2 58 2	58 0	13 30
17 0	6 41 13 4	11 29 5	16 2	16 56 5 7	8 28 8	42 1	22 50 5 0	2 51 8	58 2	13 0
17 30	6 52 42 9	11 27 9	16 7	17 4 34 5	8 24 5	42 5	22 52 56 8	2 45 1	58 3	12 30
18 0	7 4 10 8	11 26 1	17 1	17 12 59 0	8 20 4	42 9	22 55 41 9	2 38 5	58 4	12 0
18 30	15 36 9	11 24 6	17 6	17 21 19 4	8 15 0	43 2	22 58 20 4	2 31 9	58 6	11 30
19 0	7 27 1 5	11 22 6	18 1	17 29 34 4	8 10 8	43 6	23 0 52 3	2 25 3	58 7	11 0
19 30	7 38 24 1	11 21 0	18 5	17 37 45 2	8 6 3	43 9	23 3 17 6	2 18 7	58 8	10 30
20 0	7 49 45 1	11 19 4	19 0	17 45 51 5	8 1 5	44 3	23 5 36 3	2 12 0	58 9	10 0
20 30	8 1 4 5	11 16 9	19 4	17 53 53 0	7 56 8	44 6	23 7 48 3	2 5 3	59 0	9 30
21 0	8 12 21 4	11 15 3	19 9	18 1 49 8	7 52 1	45 0	23 9 53 6	1 58 6	59 1	9 0
21 30	8 23 36 7	11 13 2	20 4	18 9 41 9	7 47 3	45 3	23 11 52 2	1 52 0	59 2	8 30
22 0	8 34 49 9	11 11 1	20 8	18 17 29 2	7 42 4	45 7	23 13 44 2	1 45 2	59 3	8 0
22 30	8 46 1 0	11 9 2	21 3	18 25 11 6	7 37 6	46 0	23 15 29 4	1 38 5	59 4	7 30
23 0	8 57 10 2	11 7 1	21 7	18 32 49 2	7 32 6	46 4	23 17 7 9	1 31 7	59 5	7 0
23 30	9 8 17 3	11 4 7	22 2	18 40 21 8	7 27 6	46 7	23 18 39 6	1 25 0	59 5	6 30
24 0	9 19 22 0	11 2 6	22 7	18 47 49 4	7 22 7	47 1	23 20 4 6	1 18 2	59 6	6 0
24 30	9 30 24 6	11 0 2	23 2	18 55 12 1	7 17 5	47 4	23 21 22 8	1 11 5	59 6	5 30
25 0	9 41 24 8	10 58 0	23 6	19 2 29 6	7 12 4	47 7	23 22 34 3	1 4 7	59 7	5 0
25 30	9 52 22 8	10 55 6	24 1	19 9 42 0	7 7 3	48 0	23 23 39 0	0 57 9	59 7	4 30
26 0	10 3 18 4	10 53 2	24 5	19 16 49 3	7 2 1	48 4	23 24 36 9	0 51 1	59 8	4 0
26 30	10 14 11 6	10 50 7	25 0	19 23 51 4	6 56 9	48 7	23 25 28 0	0 44 3	59 8	3 30
27 0	10 25 2 3	10 48 3	25 5	19 30 48 3	6 51 6	49 0	23 26 12 3	0 37 5	59 8	3 0
27 30	10 35 50 6	10 45 6	25 9	19 37 39 9	6 46 3	49 3	23 26 49 8	0 30 6	59 9	2 30
28 0	10 46 36 2	10 42 9	26 3	19 44 26 2	6 41 1	49 6	23 27 20 4	0 23 9	59 9	2 0
28 30	10 57 19 1	10 40 5	26 8	19 51 7 3	6 35 5	49 9	23 27 44 3	0 17 1	59 9	1 30
29 0	7 59 6	10 37 8	27 2	19 57 42 8	6 30 0	50 2	23 28 1 4	0 10 2	60 0	1 0
29 30	11 18 37 4	10 34 9	27 7	20 4 12 8	6 24 6	50 5	23 28 11 6	0 3 4	60 0	0 30
30 0	11 29 12 3		28 1	20 10 37 4		50 8	23 28 15 0		60 0	0 0
	Sig V +	Sig XI −		Sig IV +	Sig X −		Sig III +	Sig IX −		

TABLE XXXIX

The Equation of Second Difference for correcting the Equations of the Planetary Motions

Minute of Argument		SECOND DIFFERENCE																
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	59	0,0	0,0	0 0	0 0	0 0	0 0	0 1	0 1	0 1	0 1	0 1	0,1	0 1	0 1	0 1	0 1	0 1
2	58	0 0	0 0	0 0	0 1	0 1	0,1	0 1	0 1	0 1	0 2	0 2	0 2	0 2	0 2	0 2	0 3	0,3
3	57	0 0	0 0	0 1	0 1	0 1	0,1	0 2	0 2	0 2	0 2	0 3	0 3	0 [illegible]	0 3	0 4	0 4	0 4
4	56	0 0	0 1	0 1	0 1	0 2	0 2	0 2	0 2	0 3	0 3	0,3	0 4	0 4	0,4	0 5	0 5	0 5
5	55	0 0	0 1	0 1	0 2	0 2	0 2	0 3	0 3	0 3	0 4	0 4	0 5	0,5	0 5	0,6	0 6	0 6
6	54	0 0	0 1	0 1	0 2	0 2	0 3	0 3	0 4	0 4	0 5	0,5	0 5	0 6	0 6	0 7	0 7	0 8
7	53	0 1	0 1	0 2	0 2	0 3	0 3	0 4	0 4	0 5	0 5	0 6	0 6	0 7	0 7	0 8	0 8	0 9
8	52	0 1	0 [illegible]	0 2	0 2	0 3	0 [illegible]	0 4	0 5	0 5	0 6	0 6	0 7	0 8	0 8	0 9	0 9	1 0
9	51	0 1	0 1	0 2	0 3	0 3	0 4	0 4	0 5	0 6	0 6	0 7	0 8	0 8	0 9	1 0	1 0	1 1
10	50	0 1	0 1	0 2	0 3	0 3	0 4	0 5	0 6	0 6	0 7	0 8	0 8	0 9	1 0	1 0	1 1	1 2
11	49	0 1	0 1	0 2	0 3	0 4	0 4	0 5	0 6	0 7	0 7	0 8	0 9	1 0	1 0	1 1	1 2	1 3
12	48	0 1	0 2	0 2	0 3	0 4	0 5	0 6	0 6	0 7	0 8	0 9	1 0	1 0	1 1	[illegible] 2	1 3	1 4
13	47	0 1	0 2	0 3	0 [illegible]	0 4	0 5	0 6	0 7	0,8	0,8	0 9	1,0	1 1	1 2	[illegible],3	1 4	1 4
14	46	0 1	0 2	0 3	0 4	0 4	0 5	0 6	0 7	0 8	0 9	1 0	1 1	1 2	1 3	1 3	1 4	1 5
15	45	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 8	0 9	1 0	1 1	1 2	1 3	1 4	1 5	1 6
16	44	0 1	0 2	0,3	0 4	0 5	0 6	0 7	0 8	0 9	1 0	1 1	1 2	1 3	1 4	1 5	[illegible] 6	1 7
17	43	0,1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0	1 1	1 2	1 [illegible]	1 4	1 5	1 6	1 7
18	42	0 1	0,2	0 3	0 4	0,5	0,6	0 7	0 8	0 9	[illegible] 1	1 2	1 3	1 4	1 5	1,6	1 7	1 8
19	41	0 1	0 2	0 3	0 4	0 5	0 6	0 8	0 9	1 0	1 1	1 2	1 3	1 4	1 5	1 6	[illegible] 7	1 8
20	40	0 1	0 2	0 3	0 4	0 6	0 7	0 8	0 9	1 0	1 1	1 2	1 3	1 4	1 6	1 7	1 8	1 9
21	39	0 1	0 2	0 3	0 5	0 6	0 7	0 8	0 9	1 0	1 1	1 3	1 4	1 5	1 6	1 7	1 8	1 9
22	38	0 1	0,2	0 3	0 5	0 6	0 7	0 8	0 9	1 0	1 2	1 3	1 4	1 5	1 6	1 7	1 9	2 0
23	37	0 1	0 2	0 4	0 5	0 6	0 7	0 8	0 9	1 1	[illegible] 2	1 3	1 4	1 5	1 7	1 8	1 9	2 0
24	36	0 1	0 2	0 4	0 5	0 6	0 7	0 8	1 0	1 1	1 2	1 3	1 4	1 6	1 7	1 8	1 9	2 0
25	35	0,1	0 2	0 4	0 5	0 6	0 7	0 9	1 0	1 1	1 2	1 3	1 5	1 6	1 7	1 8	1 9	2 1
26	34	0 1	0 2	0 4	0 5	0 6	0 7	0 9	1 0	1 1	1,2	1 4	1 5	1 6	1 7	1 8	2 0	2 1
27	33	0,1	0 2	0 4	0 5	0 6	0 7	0,9	1 0	1 1	1 2	1 4	[illegible] 5	1 6	1 7	1 9	2 0	2 1
28	32	0,1	0,2	0 4	0,5	0 6	0 7	0 9	1 0	1 1	1 2	1,4	1 5	1,6	1,7	1 9	2,0	2,1
29	31	0 1	0 2	0 4	0 5	0 6	0,7	0 9	1 0	1 1	1 2	1 4	1 5	1 6	1 7	1,9	2 0	2 1
30	30	0 1	0 3	0 4	0,5	0 6	0 8	0 9	1 0	1,1	1 3	[illegible] 4	1 5	1 6	1 8	1 9	2 0	2 1

TABLE XXXIX *Continued*

Minute of Argument		SECOND DIFFERENCE																
		18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	59	01	02	02	02	02	02	02	02	02	02	02	02	02	03	03	03	03
2	58	03	03	03	03	04	04	04	04	04	04	05	05	05	05	05	05	05
3	57	04	05	05	05	05	05	06	06	06	06	07	07	07	07	08	08	08
4	56	06	06	06	07	07	07	07	08	08	08	09	09	09	10	10	10	11
5	55	07	07	08	08	08	09	09	10	10	10	11	11	11	12	12	13	13
6	54	08	09	09	09	10	10	11	11	12	12	13	13	14	14	14	15	15
7	53	09	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	18
8	52	10	11	12	12	13	13	14	14	15	16	16	17	17	18	18	19	20
9	51	11	12	13	13	14	15	15	16	17	17	18	18	19	20	20	21	22
10	50	13	13	14	15	15	16	17	17	18	19	19	20	21	22	22	23	24
11	49	13	14	15	16	16	17	18	19	19	20	21	22	22	23	24	25	25
12	48	14	15	16	17	18	18	19	20	21	22	22	23	24	25	26	26	27
13	47	15	16	17	18	19	20	20	21	22	23	24	25	25	26	27	28	29
14	46	16	17	18	19	20	21	21	22	23	24	25	26	27	28	29	30	30
15	45	17	18	19	20	21	22	23	23	24	25	26	27	28	29	30	31	32
16	44	18	19	20	21	22	22	23	24	25	26	27	28	29	30	31	32	33
17	43	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	34	35
18	42	19	20	21	22	23	24	25	26	27	28	29	30	32	33	34	35	36
19	41	19	21	22	23	24	25	26	27	28	29	30	31	32	34	35	36	37
20	40	20	21	22	23	24	26	27	28	29	30	31	32	33	34	36	37	38
21	39	20	22	23	24	25	26	27	28	30	31	32	33	34	35	36	38	39
22	38	21	22	23	24	26	27	28	29	30	31	33	34	35	36	37	38	39
23	37	21	22	24	25	26	27	28	30	31	32	33	34	35	37	38	39	40
24	36	22	23	24	25	26	28	29	30	31	32	34	35	36	37	38	40	41
25	35	22	23	24	26	27	28	29	30	32	33	34	35	36	38	39	40	41
26	34	22	23	25	26	27	28	29	31	32	33	34	36	37	38	39	41	42
27	33	22	24	25	26	27	29	30	31	32	33	35	36	37	38	40	41	42
28	32	22	24	25	26	27	29	30	31	32	34	35	36	37	39	40	41	42
29	31	22	24	25	26	27	29	30	31	32	34	35	36	37	39	40	41	42
30	30	23	24	25	26	28	29	30	31	33	34	35	36	38	39	40	41	43

TABLE XXXIX *Continued*

Minute of Argument		SECOND DIFFERENCE															
		35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	59	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
2	58	0,6	0,6	0,6	0,6	0,6	0,6	0,7	0,7	0,7	0,7	0,7	0,7	0,8	0,8	0,8	0,8
3	57	0,8	0,9	0,9	0,9	0,9	1,0	1,0	1,0	1,0	1,0	1,1	1,1	1,1	1,1	1,2	1,2
4	56	1,1	1,1	1,2	1,2	1,2	1,2	1,3	1,3	1,3	1,4	1,4	1,4	1,5	1,5	1,5	1,6
5	55	1,3	1,4	1,4	1,5	1,5	1,5	1,6	1,6	1,6	1,7	1,7	1,8	1,8	1,8	1,9	1,9
6	54	1,6	1,6	1,7	1,7	1,8	1,8	1,8	1,9	1,9	2,0	2,0	2,1	2,1	2,2	2,2	2,3
7	53	1,8	1,9	1,9	2,0	2,0	2,1	2,1	2,2	2,2	2,3	2,3	2,4	2,4	2,5	2,5	2,6
8	52	2,0	2,1	2,1	2,2	2,3	2,3	2,4	2,4	2,5	2,5	2,6	2,7	2,7	2,8	2,8	2,9
9	51	2,2	2,3	2,4	2,4	2,5	2,6	2,6	2,7	2,7	2,8	2,9	2,9	3,0	3,1	3,1	3,2
10	50	2,4	2,5	2,6	2,6	2,7	2,8	2,8	2,9	3,0	3,1	3,1	3,2	3,3	3,3	3,4	3,5
11	49	2,6	2,7	2,8	2,8	2,9	3,0	3,1	3,1	3,2	3,3	3,4	3,4	3,5	3,6	3,7	3,7
12	48	2,8	2,9	3,0	3,0	3,1	3,2	3,3	3,4	3,4	3,5	3,6	3,7	3,8	3,8	3,9	4,0
13	47	3,0	3,1	3,1	3,2	3,3	3,4	3,5	3,6	3,6	3,7	3,8	3,9	4,0	4,1	4,2	4,2
14	46	3,1	3,2	3,3	3,4	3,5	3,6	3,7	3,8	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5
15	45	3,3	3,4	3,5	3,6	3,7	3,8	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7
16	44	3,4	3,5	3,6	3,7	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9
17	43	3,6	3,7	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9	5,0	5,1
18	42	3,7	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9	5,0	5,1	5,3
19	41	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,7	4,8	4,9	5,0	5,1	5,2	5,3	5,4
20	40	3,9	4,0	4,1	4,2	4,3	4,4	4,6	4,7	4,8	4,9	5,0	5,1	5,2	5,3	5,4	5,6
21	39	4,0	4,1	4,2	4,3	4,4	4,6	4,7	4,8	4,9	5,0	5,1	5,2	5,3	5,5	5,6	5,7
22	38	4,1	4,2	4,3	4,4	4,5	4,6	4,8	4,9	5,0	5,1	5,2	5,3	5,5	5,6	5,7	5,8
23	37	4,1	4,3	4,4	4,5	4,6	4,7	4,8	5,0	5,1	5,2	5,3	5,4	5,6	5,7	5,8	5,9
24	36	4,2	4,3	4,4	4,6	4,7	4,8	4,9	5,0	5,2	5,3	5,4	5,5	5,6	5,8	5,9	6,0
25	35	4,3	4,4	4,5	4,6	4,7	4,9	5,0	5,1	5,2	5,3	5,5	5,6	5,7	5,8	6,0	6,1
26	34	4,3	4,4	4,5	4,7	4,8	4,9	5,0	5,2	5,3	5,4	5,5	5,6	5,8	5,9	6,0	6,1
27	33	4,3	4,5	4,6	4,7	4,8	5,0	5,1	5,2	5,3	5,4	5,6	5,7	5,8	5,9	6,1	6,2
28	32	4,4	4,5	4,6	4,7	4,9	5,0	5,1	5,2	5,4	5,5	5,6	5,7	5,8	6,0	6,1	6,2
29	31	4,4	4,5	4,6	4,7	4,9	5,0	5,1	5,2	5,4	5,5	5,6	5,7	5,9	6,0	6,1	6,2
30	30	4,4	4,5	4,6	4,8	4,9	5,0	5,1	5,3	5,4	5,5	5,6	5,8	5,9	6,0	6,1	6,3

TABLE XXXIX *Continued*

Minute of Argument		SECOND DIFFERENCE																
		51	52	53	54	55	56″	57	58	59	60	61	62	63	64	65	66	67″
1	59	0 4	0 4	0 4	0 4	0 5	0 5	0 5	0 5	0 5	0 5	0 5	0 5	0 5	0 5	0 5	0 5	0 5
2	58	0 8	0 8	0 9	0 9	0 9	0 9	0 9	0 9	1 0	1 0	1 0	1 0	1 0	1 0	1 0	1 1	1 1
3	57	1 2	1 2	1 3	1 3	1 3	1 3	1 4	1 4	1 4	1 5	1 5	1 5	1 5	1 6	1 6	1 6	1 6
4	56	1 6	1 6	1 6	1 7	1 7	1 7	1 8	1 8	1 8	1 9	1 9	1 9	2 0	2 0	2 0	2 1	2 1
5	55	1 9	2 0	2 0	2 1	2 1	2 1	2 2	2 2	2 3	2 3	2 3	2 4	2 4	2 4	2 5	2 5	2 6
6	54	2 3	2 3	2 4	2 4	2 5	2 5	2 6	2 6	2 7	2 7	2 7	2 8	2 8	2 9	2 9	3 0	3 0
7	53	2 6	2 7	2 7	2 8	2 8	2 9	2 9	3 0	3 0	3 1	3 1	3 2	3 2	3 3	3 3	3 4	3 5
8	52	2 9	3 0	3 1	3 1	3 2	3 2	3 3	3 4	3 4	3 5	3 5	3 6	3 6	3 7	3 8	3 8	3 9
9	51	3 3	3 3	3 4	3 4	3 5	3 6	3 6	3 7	3 8	3 8	3 9	4 0	4 0	4 1	4 1	4 2	4 3
10	50	3 5	3 6	3 7	3 8	3 8	3 9	4 0	4 0	4 1	4 2	4 2	4 3	4 4	4 4	4 5	4 6	4 7
11	49	3 8	3 9	4 0	4 0	4 1	4 2	4 3	4 3	4 4	4 4	4 5	4 6	4 7	4 7	4 8	4 9	5 0
12	48	4 1	4 2	4 2	4 3	4 4	4 5	4 6	4 6	4 7	4 8	4 9	5 0	5 0	5 1	5 2	5 3	5 4
13	47	4 3	4 4	4 5	4 6	4 7	4 8	4 8	4 9	5 0	5 1	5 2	5 3	5 3	5 4	5 5	5 6	5 7
14	46	4 6	4 7	4 7	4 8	4 9	5 0	5 1	5 2	5 3	5 4	5 5	5 5	5 6	5 7	5 8	5 9	6 0
15	45	4 8	4 9	5 0	5 1	5 2	5 3	5 3	5 4	5 5	5 6	5 7	5 8	5 9	6 0	6 1	6 2	6 3
16	44	5 0	5 1	5 2	5 3	5 4	5 5	5 6	5 7	5 8	5 9	6 0	6 1	6 2	6 3	6 4	6 5	6 6
17	43	5 2	5 3	5 4	5 5	5 6	5 7	5 8	5 9	6 0	6 1	6 2	6 3	6 4	6 5	6 6	6 7	6 8
18	42	5 4	5 5	5 6	5 7	5 8	5 9	6 0	6 1	6 2	6 3	6 4	6 5	6 6	6 7	6 8	6 9	7 0
19	41	5 5	5 6	5 7	5 8	6 0	6 1	6 2	6 3	6 4	6 5	6 6	6 7	6 8	6 9	7 0	7 1	7 2
20	40	5 7	5 8	5 9	6 0	6 1	6 2	6 3	6 4	6 6	6 7	6 8	6 9	7 0	7 1	7 2	7 3	7 4
21	39	5 8	5 9	6 0	6 1	6 3	6 4	6 5	6 6	6 7	6 8	6 9	7 1	7 2	7 3	7 4	7 5	7 6
22	38	5 9	6 0	6 2	6 3	6 4	6 5	6 6	6 7	6 9	7 0	7 1	7 2	7 3	7 4	7 5	7 7	7 8
23	37	6 0	6 1	6 3	6 4	6 5	6 6	6 7	6 9	7 0	7 1	7 2	7 3	7 4	7 6	7 7	7 8	7 9
24	36	6 1	6 2	6 4	6 5	6 6	6 7	6 8	7 0	7 1	7 2	7 3	7 4	7 6	7 7	7 8	7 9	8 0
25	35	6 2	6 3	6 4	6 6	6 7	6 8	6 9	7 0	7 2	7 3	7 4	7 5	7 7	7 8	7 9	8 0	8 1
26	34	6 3	6 4	6 5	6 6	6 8	6 9	7 0	7 1	7 2	7 4	7 5	7 6	7 7	7 9	8 0	8 1	8 2
27	33	6 3	6 4	6 6	6 7	6 8	6 9	7 1	7 2	7 3	7 4	7 5	7 7	7 8	7 9	8 0	8 2	8 3
28	32	6 3	6 5	6 6	6 7	6 8	7 0	7 1	7 2	7 3	7 5	7 6	7 7	7 8	8 0	8 1	8 2	8 3
29	31	6 4	6 5	6 6	6 7	6 9	7 0	7 1	7 2	7 4	7 5	7 6	7 7	7 9	8 0	8 1	8 2	8 4
30	30	6 4	6 5	6 6	6 8	6 9	7 0	7 1	7 3	7 4	7 5	7 6	7 8	7 9	8 0	8 1	8 3	8 4

TABLE XXXIX *Continued*

Minute of Argument		SECOND DIFFERENCE																
		68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
1	59	0,6	0,6	0,6	0,6	0,6	0,6	0 6	0 6	0,6	0 6	0 6	0 6	0 7	0 7	0 7	0 7	0,7
2	58	1 1	1 1	1 1	1 1	1 2	1 2	1 2	1 2	1,2	1 2	1 3	1 3	1 3	1 3	1,3	1,3	1 4
3	57	1 7	1 7	1 7	1 7	1,7	1 7	1 8	1 8	1 8	1 8	1 ,9	1,9	1,9	1,9	1 9	2 0	2,0
4	56	2 1	2 1	2 2	2 2	2 2	2 3	2 3	2 3	2 4	2 4	2 4	2 5	2 5	2 5	2 6	2 6	2 6
5	55	2 6	2 6	2 7	2 7	2 7	2 8	2 8	2 9	2 9	2 9	3 0	3 0	3 1	3 1	3 1	3 2	3 2
6	54	3 1	3	3 2	3 2	2	3 3	3	3 4	3 4	3 5	3 5	3 6	3 6	3 6	3 7	3 7	3 8
7	53	3 5	3 6	3 6	3 7	3 7	3 8	3 8	3 9	3 9	4,0	4 0	4,1	4 1	4,2	4 2	4 3	4,3
8	52	3 9	4 0	4 0	4,1	4 2	4,2	4 3	4 3	4 4	4 4	4 5	4,6	4,6	4 7	4,7	4 8	4,9
9	51	4 3	4 4	4 5	4,5	4,6	4,7	4 7	4 8	4 8	4,9	5 0	5 0	5 1	5 2	5 2	5 3	5 4
10	50	4 7	4 8	4 9	4,9	5 0	5 1	5 1	5 2	5 3	5 3	5 4	5 5	5 6	5 6	5 7	5 8	5 8
11	49	5,0	5,1	5,2	5,3	5,4	5,5	5,5	5,6	5,7	5,8	5,8	5,9	6,0	6,1	6,1	6 2	6,3
12	48	5 4	5 5	6	5 7	5 8	5 8	5 9	6 0	6 1	6 2	6 2	6 3	6 4	6 5	6 6	6 6	6 7
13	47	5 8	5 9	5 9	6 0	6 1	6 2	6 3	6 4	6 4	6 5	6,6	6 7	6 8	6 9	7 0	7 0	7 1
14	46	6 1	6 2	6	6 4	6 4	6 5	6 6	6 7	6 8	6 9	7 0	7 1	7 2	7 2	7 3	7 4	7,5
15	45	6 4	6 5	6 6	6 7	6 8	6 8	6 9	7 0	7 1	7 2	7	7 4	7 5	7 6	7 7	7 8	7,9
16	44	6 6	6 7	6 8	6 9	7 0	7 1	7 2	7 3	7 4	7 5	7 6	7 7	7 8	7 9	8 0	8 1	8 2
17	43	6 9	7 0	7 1	7 2	7 3	7 4	7 5	7 6	7 7	7 8	7 9	8 0	8 1	8 2	8 3	8 4	8 5
18	42	7 1	7 2	7 4	7,5	7 6	7 7	7 8	7 9	8 0	8 1	8 2	8 3	8 4	8 5	8 6	8 7	8 8
19	41	7 4	7 5	7 6	7	7 8	7 9	8 0	8 1	8 2	8 3	8 4	8 5	8 7	8 8	8 9	9 0	9 1
20	40	7 6	7 7	7 8	7 9	8 0	8 1	8 2	8 3	8 4	8 6	8 7	8 8	8 9	9 0	9 1	9 2	9 3
21	39	7 7	7 8	8 0	8 1	8 2	8 3	8 4	8 5	8 6	8 8	8 9	9 0	9 1	9 2	9 3	9 4	9 6
22	38	7 9	8 0	8 1	8 2	8 4	8 5	8 6	8 7	8 8	8,9	9 1	9 2	9 3	9 4	9 5	9,6	9 8
23	37	8 0	8 2	8 3	8 4	8 5	8 6	8 7	8 9	9 0	9 1	9 2	9 3	9 5	9 6	9 7	9 8	9 9
24	36	8 2	8 3	8,4	8 5	8 6	8 8	8 9	9 0	9 1	9 2	9 4	9 5	9 6	9 7	9 8	10 0	10 1
25	35	8 3	8 4	8 5	8 6	8 8	8 9	9,0	9 1	9 2	9 4	9 5	9 6	9 7	9 8	10 0	10 1	10,2
26	34	8 3	8 5	6	8 7	8 8	9 0	9,1	9 2	9 3	9 5	9 6	9 7	9 8	9 9	10 1	10,2	10,3
27	33	8 4	8 5	8 7	8 8	8 9	9 0	9 2	9 3	9 4	9 5	9 7	9 8	9 9	10 0	10 1	10 3	10 4
28	32	8 5	8 6	8 7	8 8	9 0	9,1	9 2	9 3	9 5	9 6	9 7	9 8	10 0	10 1	10 2	10 3	10 5
29	31	8 5	8 6	8 7	8 9	9 0	9 1	9 2	9,4	9 5	9 6	9 7	9 9	10 0	10 1	10 2	10 4	10 5
30	30	8 5	8,6	8 8	8,9	9 0	9,1	9 3	9 4	9 5	9 6	9 8	9 9	10 0	10,1	10,3	10 4	10 5

TABLE XXXIX *Continued*

Minute of Argument		SECOND DIFFERENCE															
		85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	59	0 7	0 7	0 7	0 7	0 7	0 7	0 7	0 8	0 8	0 8	0 8	0 8	0 8	0 8	0 8	0 8
2	58	1 4	1 4	1 4	1 4	1 4	1 5	1 5	1 5	1 5	1 5	1 5	1 5	1 6	1 6	1 6	1 6
3	57	2 0	2 0	2 1	2 1	2 1	2 1	2 2	2 2	2 2	2 2	2 3	2 3	2 3	2 3	2 4	2 4
4	56	2 6	2 7	2 7	2 7	2 8	2 8	2 8	2 9	2 9	2 9	3 0	3 0	3 0	3 0	3 1	3 1
5	55	3 2	3 3	3 3	3 4	3 4	3 4	3 5	3 5	3 6	3 6	3 6	3 7	3 7	3 7	3 8	3 8
6	54	3 8	3 9	3 9	4 0	4 0	4 1	4 1	4 1	4 2	4 2	4 3	4 3	4 4	4 4	4 5	4 5
7	53	4 4	4 4	4 5	4 5	4 6	4 6	4 7	4 7	4 8	4 8	4 9	4 9	5 0	5 0	5 1	5 2
8	52	4 9	5 0	5 0	5 1	5 1	5 2	5 3	5 3	5 4	5 4	5 5	5 5	5 6	5 7	5 7	5 8
9	51	5 4	5 5	5 5	5 6	5 7	5 7	5 8	5 9	5 9	6 0	6 1	6 1	6 2	6 2	6 3	6 4
10	50	5 9	6 0	6 0	6 1	6 2	6 3	6 3	6 4	6 5	6 5	6 6	6 7	6 7	6 8	6 9	6 9
11	49	6 4	6 4	6 5	6 6	6 7	6 7	6 8	6 9	7 0	7 0	7 1	7 2	7 3	7 3	7 4	7 5
12	48	6 8	6 9	7 0	7 0	7 1	7 2	7 3	7 4	7 4	7 5	7 6	7 7	7 8	7 8	7 9	8 0
13	47	7 2	7 3	7 4	7 5	7 6	7 6	7 7	7 8	7 9	8 0	8 1	8 1	8 2	8 3	8 4	8 5
14	46	7 6	7 7	7 8	7 9	8 0	8 1	8 1	8 2	8 3	8 4	8 5	8 6	8 7	8 8	8 9	8 9
15	45	8 0	8 1	8 2	8 3	8 3	8 4	8 5	8 6	8 7	8 8	9 0	9 0	9 1	9 2	9 3	9 4
16	44	8 3	8 4	8 5	8 6	8 7	8 8	8 9	9 0	9 1	9 2	9 3	9 4	9 5	9 6	9 7	9 8
17	43	8 6	8 7	8 8	8 9	9 0	9 1	9 2	9 3	9 4	9 5	9 6	9 7	9 8	9 9	10 1	10 2
18	42	8 9	9 0	9 1	9 2	9	9 5	9 6	9 7	9 8	9 9	10 0	10 1	10 2	10	10 4	10 5
19	41	9 2	9 3	9 4	9 5	9 6	9 7	9 8	10 0	10 1	10 2	10 3	10 4	10 5	10 6	19 7	10 8
20	40	9 4	9 6	9 7	9 8	9 9	0 0	10 1	10 2	10 3	10 4	10 6	10 7	10 8	10 9	11 0	11
21	39	9 7	9 8	9 9	10 0	10 1	10 2	10 4	10 5	10 6	10 7	10 8	10 9	11 0	11 1	11 3	11 4
22	38	9 9	10 0	10 1	0 2	10 3	10 5	10 6	10 7	10 8	10 9	11 0	1 1	11 3	11 4	11 5	11 6
23	37	10 0	10 2	10 3	10 4	10 5	10 0	10 8	10 9	11 0	11 1	11 2	11 3	11 5	11 6	11 7	11 8
24	36	10 2	10 3	10 4	10 6	10 7	10 8	10 9	11 0	11 2	11 3	11 4	11 5	1 6	11 8	11 9	12 0
25	35	10 3	0 5	10 6	10 7	10 8	10 9	11 1	11 2	11 3	11 4	11 5	11 7	11 8	11 9	12 0	12 2
26	34	10 4	0 6	10 7	10 8	10 9	11 1	11 2	11 3	11 4	11 5	11 7	11 8	11 9	12 0	12 2	12 3
27	33	10 5	10 6	10 8	10 9	11 0	11 1	11 3	11 4	11 5	11 6	11 8	11 9	12 0	12 1	12 3	2 4
28	32	10 6	10 7	10 8	11 0	11 1	11 2	11 3	11 4	11 6	11 7	11 8	1 9	12 1	12 2	12 3	1 4
29	31	10 6	10 7	10 0	11 0	11 1	1 2	1 4	11 5	11 6	1 7	11 9	12 0	12 1	12 2	12 4	12 5
30	30	10 6	10 8	10 9	11 0	11 1	11 3	11 4	11 5	11 6	11 8	11 9	12 0	12 1	12 3	12 4	12 5

TABLE XL

The Equation of Second Difference, useful in computing the Moon's Place from the Nautical Ephemeris

Apparent Time after Noon or Midnight				Second Difference of the Moon's Place										
				0 Minute					1 Minute					
				10	20	30	40	50	0	10	20	30	40	50
H	M	H	M											
0	0	12	0	0, 0	0, 0	0, 0	0, 0	0, 0	0 0	0, 0	0, 0	0, 0	0, 0	0 0
0	10	11	50	0, 1	0, 1	0, 2	0 3	0, 3	0, 4	0, 5	0, 5	0, 6	0, 7	0 8
0	20	11	40	0, 1	0, 3	0, 4	0, 5	0, 7	0, 8	0, 9	1, 1	1, 2	1, 4	1, 5
0	30	11	30	0 2	0, 4	0, 6	0 8	1 0	1, 2	1, 4	1, 6	1, 8	2, 0	2, 2
0	40	11	20	0, 3	0, 5	0, 8	1 0	1, 3	1, 6	1, 8	2, 1	2, 4	2, 6	2, 9
0	50	11	10	0, 3	0, 6	1, 0	1, 3	1, 6	1, 9	2, 3	2, 6	2, 9	3, 2	3, 6
1	0	11	0	0 4	0 8	1, 1	1, 5	1, 9	2, 3	2, 7	3, 1	3, 4	3, 8	4, 2
1	10	10	50	0, 4	0, 9	1, 3	1, 8	2, 2	2 6	3, 1	3, 5	3, 9	4, 4	4, 8
1	0	10	40	0, 5	1, 0	1, 5	2 0	2, 5	3 0	3, 5	4, 0	4, 4	4, 9	5, 1
1	30	10	30	0, 5	1 1	1, 6	2 2	2, 7	3, 3	3, 8	4, 4	4, 9	5 5	6 0
1	40	10	20	0 6	1, 2	1, 8	2 4	3, 0	3 6	4, 2	4, 8	5, 4	6 0	6, 6
1	50	10	10	0, 6	1, 3	1, 9	2 6	3, 2	3, 9	4, 5	5 2	5 8	6 5	7, 1
2	0	10	0	0, 7	1 4	2, 1	2, 8	3, 5	4, 2	4, 9	5 6	6, 3	6 9	7, 6
2	10	9	50	0, 7	1 5	2, 2	3, 0	3, 7	4, 4	5, 2	5, 9	6, 7	7, 4	8 1
2	20	9	40	0, 8	1, 6	2, 3	3, 1	3, 9	4, 7	5, 5	6 3	7, 0	7 8	8, 6
2	30	9	30	0, 8	1 6	2, 5	3, 3	4, 1	4, 9	5, 8	6, 6	7 4	8 2	9, 1
2	40	9	0	0, 9	1, 7	2, 6	3, 5	4, 3	5, 2	6, 0	6, 9	7, 8	8, 6	9, 5
2	50	9	10	0, 9	1, 8	2, 7	3, 6	4, 5	5, 4	6, 3	7, 2	8 1	9 0	9, 9
3	0	9	0	0, 9	1, 9	2, 8	3, 8	4, 7	5 6	6, 6	7, 5	8 4	9 4	10 3
3	10	8	50	1, 0	1, 9	2, 9	3 9	4 9	5, 8	6, 8	7, 8	8 7	9 7	10, 7
3	20	8	40	1, 0	2, 0	3, 0	4 0	5, 0	6 0	7, 0	8 0	9, 0	10 0	11 0
3	30	8	30	1 0	2, 1	3, 1	4, 1	5 2	6, 2	7, 2	8, 3	9 3	10, 3	11, 4
3	40	8	20	1 1	2, 1	3, 2	4, 2	5, 3	6, 4	7, 4	8 5	9 5	10, 6	11 7
3	50	8	10	1 1	2, 2	3, 3	4 3	5, 4	6, 5	7, 6	8, 7	9 8	10, 9	12, 0
4	0	8	0	1, 1	2, 2	3, 3	4 4	5 6	6 7	7, 8	8 9	10, 0	11, 1	12, 3
4	10	7	50	1, 1	2, 3	3 4	4 5	5, 7	6, 8	7, 9	9 1	10, 2	11, 3	12 5
4	20	7	40	1 2	2, 3	3, 5	4, 6	5, 8	6, 9	8, 1	9, 2	10, 4	11, 5	12, 7
4	30	7	30	1 2	2, 3	3, 5	4, 7	5, 9	7 0	8, 2	9, 4	10 5	11, 7	12 9
4	40	7	20	1, 2	2, 4	3, 6	4, 8	5 9	7, 1	8, 3	9, 5	10, 7	11, 9	13, 1
4	50	7	10	1, 2	2, 4	3, 6	4 8	6, 0	7, 2	8, 4	9, 6	10 8	12 0	13 2
5	0	7	0	1, 2	2, 4	3, 6	4, 9	6, 1	7, 3	8, 5	9, 7	10, 9	12, 2	13 4
5	10	6	50	1, 2	2, 5	3, 7	4 9	6, 1	7, 4	8, 6	9 6	11, 0	12, 3	13 5
5	20	6	40	1, 2	2, 5	3, 7	4, 9	6, 2	7, 4	8 6	9 9	11 1	12, 3	13, 6
5	30	6	30	1, 2	2, 5	3, 7	5, 0	6, 2	7 4	8, 7	9 9	11 2	12, 4	13, 7
5	40	6	20	1, 2	2, 5	3, 7	5, 0	6, 2	7, 5	8, 7	10, 0	11, 2	12, 5	13, 7
5	50	6	10	1, 2	2, 5	3, 7	5, 0	6 2	7 5	8 7	10, 0	11, 2	12, 5	13, 7
6	0	6	0	1, 3	2, 5	3, 8	5 0	6, 3	7, 5	8, 8	10 0	11, 3	12, 5	13 8

TABLE XL *Continued*

Apparent Time after Noon or Midnight				Second Difference of the Moon's Place											
				2 Minutes						3 Minutes					
				0	10	20	30	40	50	0	10	20	30	40	50
H	M	H	M												
0	0	12	0	0 0	0, 0	0, 0	0 0	0 0	0, 0	0 0	0, 0	0 0	0 0	0 0	0, 0
0	10	11	50	0, 8	0, 9	1, 0	1, 0	1, 1	1, 2	1, 2	1, 3	1, 4	1, 4	1 5	1, 6
0	20	11	40	1, 6	1, 8	1 9	2, 0	2, 2	2, 3	2 4	2 6	2, 7	2, 8	3, 0	3, 1
0	30	11	30	2, 4	2 6	2, 6	3, 0	3, 2	3, 4	3 6	3 6	4 0	4 2	4 4	4, 6
0	40	11	20	1	3, 4	3, 7	3, 9	4, 2	4 5	4 7	5 0	5 2	5 5	5 8	6 0
0	50	11	10	9	4 2	4 5	4, 6	5 2	5 5	5, 8	6 1	6 5	6, 3	7 1	7, 4
1	0	11	0	4 6	5 0	5 3	5 7	6 1	6 5	6, 9	7 2	7 6	8 0	8, 4	8 8
1	10	10	50	5, 3	5 7	6 1	6 6	7 0	7 5	7 9	8, 3	6 8	9, 2	9, 7	10, 1
1	20	10	40	5, 9	6, 4	6 9	7, 4	7, 9	8 4	8 9	9, 4	9 9	10, 4	10, 9	11 4
1	30	10	30	6 6	7 1	7, 7	8, 2	8, 8	9 3	9, 8	10, 4	10, 9	11 5	12, 0	12 6
1	40	10	20	7, 2	7, 8	8, 4	9, 0	9, 6	10, 2	10, 6	11, 4	12, 0	12, 6	13, 2	13, 8
1	50	10	10	7, 8	8, 4	9, 1	9 7	10, 4	11, 0	11, 6	12, 3	12, 9	13 6	14 2	14, 9
2	0	10	0	8, 3	9, 0	9, 7	10, 4	11, 1	11, 8	12, 5	13 2	13 9	14, 6	15 3	16, 0
2	10	9	50	8, 9	9, 6	10, 4	11, 1	11, 8	12, 6	13, 3	14, 1	14, 8	15, 5	16 3	17 0
2	20	9	40	9, 4	10, 2	11, 0	11, 7	12, 5	13 3	14 1	14, 9	15, 7	16, 4	17, 2	18, 0
2	30	9	30	9, 9	10, 7	11, 5	12, 4	13 2	14 0	14, 6	15 7	16, 5	17, 3	18 1	19, 0
2	40	9	20	10, 4	11, 2	12, 1	13, 0	13, 8	14, 7	15, 6	16, 4	17, 3	16, 1	19 0	19 9
2	50	9	10	10, 8	11, 7	12, 6	13, 5	14 4	15, 3	16, 2	17, 1	18, 0	18, 9	19, 8	20, 7
3	0	9	0	11, 3	12, 2	13, 1	14, 1	15, 0	15, 9	16, 9	17 8	18, 8	19, 7	20 6	21 6
3	10	8	50	11, 7	12, 6	13 6	14, 6	15, 5	16, 5	17 5	18 5	19 4	20, 4	21, 4	22 2
3	20	8	40	12, 0	13 0	14, 0	15 0	16, 0	17, 1	18 1	19, 1	20 1	21, 1	22, 1	23 1
3	30	8	30	12 4	13 4	14, 5	15 5	16 5	17 6	18 6	19, 6	20,	21 7	22 7	23, 8
3	40	8	20	12, 7	13, 8	14, 9	15, 9	17, 0	16, 0	19 1	20 2	21, 2	2 , 3	23 3	24 4
3	50	8	10	13 0	14, 1	15 2	16 3	17, 4	18, 5	19, 6	20 7	21, 7	9	23 9	25 0
4	0	8	0	13, 3	14, 4	15, 6	16, 7	17, 8	18, 9	20 0	21, 1	22, 2	3 3	24, 4	25 6
4	10	7	50	13 6	14, 7	15, 9	17, 0	18, 1	19, 3	20, 4	1 5	22, 7	2, 6	24, 9	26, 1
4	20	7	40	13, 8	15, 0	16, 1	17, 2	18, 5	19 6	20, 6	1, 9	23 1	24 2	25, 4	26 5
4	30	7	30	14, 1	15, 2	16 4	17 6	18, 8	19, 9	21, 1	, 2	23 4	4 6	25, 8	27 0
4	40	7	20	14, 3	15, 4	16, 6	17, 8	19, 0	20, 2	21, 4	22 6	23, 8	25, 0	26 1	27, 3
4	50	7	10	14, 4	15, 6	16, 6	18, 0	19 2	20, 4	21, 6	2, 9	24, 1	25 3	26, 5	7, 7
5	0	7	0	14, 6	15, 8	17, 0	18, 2	19, 4	20, 7	21 9	23, 1	24, 3	5, 5	26, 7	28 0
5	10	6	50	14, 7	15, 9	17, 2	18, 4	19, 6	20, 8	22, 1	23, 3	24, 5	5, 7	2, 0	8 2
5	20	6	40	14, 8	16, 0	17, 2	18, 5	19, 8	21, 0	22, 2	23, 5	24, 7	5, 9	2,	25 4
5	30	6	30	14, 9	6, 1	17, 4	18, 6	19 9	21, 1	22 3	23, 6	24 8	26 1	27, 3	28 6
5	40	6	20	15, 0	16, 2	17, 4	18, 7	19, 9	21, 2	2 , 4	23, 7	24 9	6,	27 1	8 7
5	50	6	10	15, 0	16, 2	17, 5	18, 7	20, 0	21, 2	22, 5	3, 7	25 0	6 2	27 5	28 7
6	0	6	0	15, 0	16, 3	17, 5	18, 8	20, 0	21, 3	2 , 5	23, 8	25, 0	6, 3	27, 5	6, 8

TABLE XL *Continued*

Apparent Time after Noon or Midnight				Second Difference of the Moon's Place											
				4 Minutes						5 Minutes					
				0	10	20	30	40	50	0	10	20	30	40	50
H	M	H	M												
0	0	12	0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0	10	11	50	1,6	1,7	1,8	1,8	1,9	2,0	2,1	2,1	2,2	2,3	2,3	2,4
0	20	11	40	3,2	3,4	3,5	3,6	3,8	3,9	4,1	4,2	4,3	4,5	4,6	4,7
0	30	11	30	4,8	5,0	5,2	5,4	5,6	5,8	6,0	6,2	6,4	6,6	6,8	7,0
0	40	11	20	6,3	6,6	6,8	7,1	7,3	7,6	7,9	8,1	8,4	8,7	8,9	9,2
0	50	11	10	7,8	8,1	8,4	8,7	9,0	9,4	9,7	10,0	10,3	10,7	11,0	11,3
1	0	11	0	9,2	9,5	9,9	10,3	10,7	11,1	11,5	11,8	12,2	12,6	13,0	13,4
1	10	10	50	10,5	11,0	11,4	11,8	12,3	12,7	13,2	13,6	14,0	14,5	14,9	15,4
1	20	10	40	11,9	12,3	12,8	13,3	13,8	14,3	14,8	15,3	15,8	16,3	16,8	17,3
1	30	10	30	13,1	13,7	14,2	14,8	15,3	15,9	16,4	17,0	17,5	18,0	18,6	19,1
1	40	10	20	14,4	14,9	15,5	16,1	16,7	17,3	17,9	18,5	19,1	19,7	20,3	20,9
1	50	10	10	15,5	16,2	16,8	17,5	18,1	18,8	19,4	20,1	20,7	21,4	22,0	22,7
2	0	10	0	16,7	17,4	18,1	18,8	19,4	20,1	20,8	21,5	22,2	22,9	23,6	24,3
2	10	9	50	17,8	18,5	19,2	20,0	20,7	21,5	22,2	22,9	23,6	24,4	25,2	25,9
2	20	9	40	18,8	19,6	20,4	21,1	21,9	22,7	23,5	24,3	25,1	25,8	26,6	27,4
2	30	9	30	19,8	20,6	21,4	22,3	23,1	23,9	24,7	25,6	26,4	27,2	28,0	28,9
2	40	9	20	20,7	21,6	22,5	23,3	24,2	25,1	25,9	26,8	27,7	28,5	29,4	30,2
2	50	9	10	21,6	22,5	23,4	24,3	25,3	26,2	27,1	28,0	28,9	29,8	30,7	31,6
3	0	9	0	22,5	23,4	24,4	25,3	26,3	27,2	28,1	29,1	30,0	30,9	31,9	32,8
3	10	8	50	23,3	24,3	25,3	26,2	27,2	28,2	29,1	30,1	31,1	32,1	33,0	34,0
3	20	8	40	24,1	25,1	26,1	27,1	28,1	29,1	30,1	31,1	32,1	33,1	34,1	35,1
3	30	8	30	24,8	25,8	26,9	27,9	28,9	30,0	31,0	32,0	33,1	34,1	35,1	36,2
3	40	8	20	25,5	26,5	27,6	28,6	29,7	30,8	31,8	32,9	34,0	35,0	36,1	37,1
3	50	8	10	26,1	27,2	28,3	29,3	30,4	31,5	32,6	33,7	34,8	35,9	37,0	38,0
4	0	8	0	26,7	27,8	28,9	30,0	31,1	32,2	33,3	34,4	35,6	36,7	37,8	38,9
4	10	7	50	27,2	28,3	29,5	30,6	31,7	32,9	34,0	35,1	36,3	37,4	38,5	39,7
4	20	7	40	27,7	28,8	30,0	31,1	32,3	33,5	34,6	35,8	36,9	38,1	39,2	40,4
4	30	7	30	28,1	29,3	30,5	31,6	32,8	34,0	35,2	36,3	37,5	38,7	39,8	41,0
4	40	7	20	28,5	29,7	20,9	32,1	33,3	34,5	35,6	36,8	38,0	39,2	40,4	41,6
4	50	7	10	28,9	30,1	31,3	32,5	33,7	34,9	36,1	37,3	38,5	39,7	40,9	42,1
5	0	7	0	29,2	30,4	31,6	32,8	34,0	35,2	36,5	37,7	38,9	40,1	41,3	42,5
5	10	6	50	29,4	30,6	31,9	33,1	34,3	35,6	36,8	38,0	39,2	40,5	41,7	42,9
5	20	6	40	29,6	30,9	32,1	33,3	34,6	35,8	37,0	38,3	39,5	40,7	42,0	43,2
5	30	6	30	29,8	31,0	32,3	33,5	34,8	36,0	37,2	38,5	39,7	41,0	42,2	43,4
5	40	6	20	29,9	31,2	32,4	33,6	34,9	36,1	37,4	38,6	39,9	41,1	42,4	43,6
5	50	6	10	30,0	31,2	32,5	33,7	35,0	36,2	37,5	38,7	40,0	41,2	42,5	43,7
6	0	6	0	30,0	31,3	32,5	33,8	35,0	36,3	37,5	38,8	40,0	41,3	42,5	43,8

TABLE XL *Continued*

Apparent Time after Noon or Midnight				Second Difference of the Moon's Place											
				6 Minutes						7 Minutes					
				0	10	20	30	40	50	0	10	20	30	40	50
H	M	H	M												
0	0	12	0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0	10	11	50	2,5	2,5	2,6	,7	2,7	2,8	2,9	2,9	3,0	3,1	3,2	3,2
0	0	11	40	4,9	5,0	5,1	5,3	5,4	5,5	5,7	5,8	5,9	6,1	6,2	6,3
0	30	11	30	7	7,4	7,6	7,8	8,0	8,2	8,4	8,6	8,8	9,0	9,2	9,4
0	40	11	20	9,4	9,7	10,0	10,2	10,5	10,8	11,0	11,3	11,5	11,8	12,1	12,3
0	50	11	10	11,6	12,0	12,3	12,6	12,9	13,2	13,6	13,9	14,2	14,5	14,9	15,2
1	0	11	0	13,6	14,1	14,5	14,9	15,	15,7	16,0	16,4	16,8	17,2	17,6	18,0
1	10	10	50	15,6	16,2	16,7	17,1	17,6	18,0	18,4	18,9	19,3	19,7	0,2	20,6
1	0	10	40	17,6	18,3	18,6	19,3	19,5	20,2	20,7	21	21,7	22,2	22,7	3,2
1	30	10	30	19,7	20,2	20,8	21,3	21,9	22,4	3,0	23,5	24,1	24,6	25,2	25,7
1	40	10	20	21,5	22,1	22,7	3,3	3,9	24,5	25,1	25,7	26,3	26,9	27,5	28,1
1	50	10	10	23,3	23,9	24,6	25,2	25,9	26,5	27,2	27,8	28,5	29,1	29,8	30,4
2	0	0	0	25,0	25,7	26,4	27,1	27,9	28,5	29,2	29,9	30,6	31,3	31,9	32,6
2	10	9	50	26,6	27,4	28,1	28,9	29,6	30,3	31,1	31,8	32,6	33,3	34,0	34,8
2	0	9	40	28,2	,0	29,8	30,5	31,3	32,1	32,9	33,7	34,5	35,2	36,0	36,8
2	30	9	30	29,7	30,5	31,3	3 ,2	33,0	33,8	4,6	35,5	36,3	37,1	37,9	38,8
2	40	9	20	31,1	3 ,0	3 ,8	33,7	34,6	35,4	36,3	37,2	38,0	38,9	39,8	40,6
2	50	9	10	3 ,5	33,4	34,3	35	30,1	37,0	37,9	38,8	39,7	40,6	41,5	4 4
3	0	9	0	33,8	34,7	35,6	36,6	37,5	38,4	39,4	40,3	41,3	42,2	43,1	44,1
3	10	8	50	35,0	35,9	36,9	37,9	38,9	39,8	40,8	41,8	42,7	43,7	44,7	45,6
3	20	6	40	36,1	37,1	38,1	39,1	40,1	41,1	4 1	43,1	44,1	45,1	46,1	47,1
3	30	8	30	37,2	38,2	39,3	40,3	41,3	4 ,4	43,4	44,4	45,5	46,5	47,5	48,6
3	40	9	20	38,2	39,3	40,3	41,4	42,4	43,5	44,6	45,6	46,7	47,7	48,8	49,9
3	50	8	10	39,1	40,2	41,3	4 ,4	43,5	44,6	45,7	46,7	47,8	48,9	50,0	51,1
4	0	8	0	40,0	41,1	4 ,2	43,3	44,4	45,6	46,7	47,8	48,9	50,0	51,1	5 ,2
4	10	7	50	40,8	41,9	43,1	44,2	45,	46,5	47,6	48,7	49,9	51,0	52,1	53,3
4	20	7	40	41,5	4 ,7	43,9	45,0	46,1	47,3	48,4	49,6	50,8	51,9	53,1	54,2
4	30	7	30	42,2	3,4	44,5	45,7	46,9	48,0	49,2	50,4	51,6	5 ,7	53,9	55,1
4	40	7	20	4 ,8	44,0	45,2	46,3	47,5	48,7	49,9	51,1	52,3	53,5	54,7	55,8
4	50	7	10	43,3	44,5	45,7	46,9	48,1	49,3	50,5	51,7	52,9	54,1	55,3	56,5
5	0	7	0	43,6	45,0	46,2	47,4	48,6	49,8	51,0	5 ,3	53,5	54,7	55,9	57,1
5	10	6	50	44,1	45,4	46,6	47,8	49,0	50,3	51,5	52,7	53,9	55,2	56,4	57,6
5	20	6	40	44,1	45,7	46,9	48,1	49,4	50,6	51,9	53,1	54,3	55,6	56,8	58,0
5	30	6	30	44,7	,9	47,2	4 ,4	49,7	50,9	5 ,1	53,4	54,6	55,9	57,1	58,3
5	40	6	20	44,9	46,1	47,4	48,6	49,8	51,1	5 ,3	53,6	54,8	56,1	57,3	58,6
5	50	6	10	45,0	46,	47,5	4 ,7	50,0	51,2	5 ,5	53,7	55,0	56,2	57,5	58,7
6	0	6	0	45,	46,3	47,5	4 ,8	50,0	51,3	52,5	53,8	55,0	56,3	57,5	58,8

TABLE XL *Continued*

Apparent Time aft. Noon, or Midnight				Second Difference of the Moon's Place											
				8 Minutes						9 Minutes					
				0	10	20	30	40	50	0	10	0	30	40	50
H	M	H	M												
0	0	12	0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0	10	11	50	3,3	3,4	3,4	3,5	3,6	3,6	3,7	3,8	3,8	3,9	4,0	4,0
0	20	11	40	6,5	6,6	6,8	6,9	7,0	7,2	7,3	7,4	7,6	[illegible],7	7,8	8,0
0	30	11	30	9,6	9,8	10,0	10,2	10,4	10,6	10,8	11,0	11,2	11,4	11,6	11,8
0	40	11	20	12,6	1[illegible],9	13,1	13,4	13,6	13,9	14,2	14,4	14,7	15,0	15,[illegible]	15,5
0	50	11	10	15,5	15,8	16,[illegible]	16,5	16,8	17,1	17,4	17,8	18,1	18,4	18,7	19,1
1	0	11	0	18,3	18,7	19,1	19,5	19,9	20,2	20,6	21,0	21,4	21,8	2[illegible],2	2[illegible],5
1	10	10	50	21,1	21,5	21,9	22,4	22,8	23,3	23,7	24,1	24,6	25,0	25,5	25,9
1	20	10	40	23,7	24,2	24,7	25,2	25,7	26,2	26,7	27,2	27,7	28,1	28,6	29,1
1	30	10	30	26,3	26,8	27,3	27,9	28,4	29,0	29,5	30,1	30,6	31,2	31,7	3[illegible],3
1	40	10	20	28,7	29,3	29,9	30,5	31,1	31,7	3[illegible],3	32,9	33,5	34,1	34,7	35,3
1	50	10	10	31,1	31,7	32,4	33,0	33,7	34,3	34,9	35,6	36,2	36,9	37,5	38,2
2	0	10	0	33,3	34,0	34,7	35,4	36,1	36,8	37,5	38,2	38,9	39,6	40,3	41,0
2	10	9	50	35,5	36,2	37,0	37,7	38,5	39,2	39,9	40,7	41,4	42,2	42,9	43,6
2	20	9	40	37,6	38,4	39,2	39,9	40,7	41,5	42,3	43,1	43,9	44,6	45,4	46,2
2	30	9	30	39,6	40,4	41,2	42,1	[illegible],9	43,7	44,5	45,4	46,2	47,0	47,8	48,7
2	40	9	20	41,5	42,3	43,2	44,1	44,9	45,8	46,7	47,5	48,4	49,3	50,1	51,0
	50	9	10	43,3	44,2	45,1	46,0	46,9	47,8	48,7	49,6	50,5	51,4	5[illegible],3	53,2
3	0	9	0	45,0	45,9	46,9	47,8	48,8	49,7	50,6	51,6	52,5	53,4	54,4	55,3
3	10	8	50	46,6	47,6	48,6	49,5	50,5	51,5	52,4	53,4	54,4	55,4	56,3	57,3
3	20	8	40	48,1	49,2	50,2	51,2	5[illegible],2	53,2	54,2	55,2	56,2	57,2	58,2	59,[illegible]
3	30	8	30	49,6	50,6	51,6	52,7	53,7	54,7	55,9	56,8	57,8	58,9	59,9	60,9
3	40	8	20	50,9	52,0	53,0	54,1	55,2	56,2	57,3	58,4	59,4	60,5	61,5	62,6
3	50	8	10	52,2	53,3	54,3	55,4	56,5	57,6	58,7	59,9	60,9	6[illegible],0	63,0	64,1
4	0	8	0	53,3	54,4	55,6	56,7	57,8	58,9	60,0	61,1	6[illegible],2	63,3	64,4	65,6
4	10	7	50	54,4	55,5	56,7	57,8	58,9	60,1	61,2	6[illegible],3	63,5	64,6	65,7	66,9
4	20	7	40	55,4	56,5	57,7	58,9	60,0	61,1	6[illegible],3	63,4	64,6	65,8	66,9	68,1
4	30	7	30	56,3	57,4	58,6	59,8	60,9	62,1	63,3	64,5	65,6	66,8	[illegible],0	69,1
4	40	7	20	57,0	58,2	59,4	60,6	61,8	63,0	64,2	65,4	66,5	67,7	68,9	70,1
4	50	7	10	57,7	58,9	60,1	61,3	62,5	63,7	64,9	66,2	67,4	68,6	69,8	71,0
5	0	7	0	58,3	59,5	60,8	62,0	63,2	64,4	65,6	66,8	68,1	69,3	70,5	71,7
5	10	6	50	58,8	60,1	61,3	62,5	63,7	65,0	66,2	67,4	68,6	69,9	71,1	7[illegible],3
5	20	6	40	59,3	60,5	61,7	63,0	64,2	65,4	66,7	67,9	69,1	70,4	71,6	[illegible]2,8
5	30	6	30	59,6	60,8	62,1	63,3	64,5	65,8	67,0	68,3	69,5	70,8	72,0	73,2
5	40	6	20	59,8	61,1	62,3	63,6	64,8	66,0	67,3	68,5	69,8	71,0	72,3	73,5
5	50	6	10	60,0	61,2	62,5	63,7	64,9	66,2	67,4	68,7	69,9	71,2	72,4	73,7
6	0	6	0	60,0	61,3	62,5	63,8	65,0	66,3	67,5	68,8	70,0	71,3	72,5	73,8

TABLE XL. Continued.

Apparent Time after Noon or Midnight				Second Difference of the Moon's Place												
				10 Minutes						11 Minutes						12 Min
				0	10	20	30	40	50	0	10	20	30	40	50	0
H	M	H	M													
0	0	12	0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0 0	0, 0	0 0	0 0
0	10	11	50	4, 1	4, 2	4 2	4 3	4, 4	4 5	4, 5	4, 6	4, 7	4, 7	4, 8	4, 9	4, 9
0	20	11	40	8, 1	8, 2	8, 4	8, 5	8 6	8, 8	8, 9	9, 0	9, 2	9 3	9, 5	9, 6	9, 7
0	30	11	30	12 0	12, 2	12 4	12 6	12 8	13, 0	13, 2	13 4	13, 6	13, 8	14, 0	14 2	14, 4
0	40	11	20	15 7	16, 0	16 3	16, 5	16 8	17, 1	17, 3	17 6	17, 8	18 1	18, 4	18 6	18, 9
0	50	11	10	19 4	19, 7	20, 0	20, 4	20 7	21 0	21 3	21, 6	22 0	22 3	22, 6	22 9	23 3
1	0	11	0	22 9	23, 3	23, 7	24, 1	24, 4	24 8	25, 2	25, 6	26 0	26, 4	26, 7	27, 1	27, 5
1	10	10	50	26 3	26 8	27, 2	27, 6	28, 1	28 5	29 0	29 4	29, 8	30, 3	30, 7	31, 2	31, 6
1	20	10	40	29, 6	30, 1	30, 6	31, 1	31, 6	32, 1	32, 6	33, 1	33, 6	34, 1	34, 6	35, 1	35, 6
1	30	10	30	32 8	33, 4	33, 9	34, 5	35, 0	35, 5	36, 1	36, 6	37, 2	37, 7	38, 3	38, 8	39, 4
1	40	10	20	35, 9	36 5	37 1	37, 7	38, 3	38, 9	39, 5	40 1	40, 7	41, 3	41, 9	42, 5	43, 1
1	50	10	10	38 8	39 5	40, 1	40, 8	41, 4	42, 1	42, 7	43, 4	44, 0	44, 7	45, 3	46, 0	46, 6
2	0	10	0	41, 7	42, 4	43, 1	43, 8	44, 4	45, 1	45, 8	46, 5	47, 2	47 9	48, 6	49, 3	50, 0
	10	9	50	44, 4	45, 1	45, 9	46 6	47 3	48, 1	48, 8	49 6	50, 3	51 0	51, 8	52, 5	53, 3
2	20	9	40	47, 0	47, 8	48, 6	49, 3	50, 1	50 9	51, 7	52, 5	53, 3	54 0	54, 8	55, 6	56, 4
	30	9	30	49, 5	50, 3	51, 1	52, 0	52, 8	53, 6	54, 4	55, 3	56 1	56, 9	57, 7	58, 6	59, 4
	40	9	20	51, 9	52, 7	53 6	54 4	55 3	56, 2	57, 0	57, 9	58, 9	59 6	60, 5	61, 4	62, 2
2	50	9	10	54, 1	55, 0	55, 9	56 8	57, 7	58, 6	59, 5	60 4	61 3	62, 2	63 1	64, 0	64, 9
3	0	9	0	56 3	57, 2	58, 1	59, 1	60 0	60 9	61, 9	62 8	63 8	64 7	65 6	66 6	67, 5
3	10	8	50	58, 3	59,	60 2	61, 2	62 2	63, 1	64 1	65, 1	66, 0	67 0	68 0	69, 0	69 9
3	20	8	40	60, 2	61 2	62, 2	63, 2	64, 2	65, 2	66 2	67 2	68, 2	69 2	70, 2	71, 2	72, 2
3	30	8	30	62, 0	63 0	64, 0	65, 1	66, 1	67, 1	68 2	69 2	70, 2	71 3	72 3	73, 3	74, 4
3	40	8	20	63 7	64, 7	65, 8	66, 8	67, 9	69, 0	70, 0	71, 1	72, 1	73, 2	74, 3	75, 3	76 4
3	50	8	10	65, 2	66, 3	67, 4	68 5	69, 6	70, 7	71 7	72, 8	73, 9	75 0	76, 1	77, 2	78 3
4	0	8	0	66 7	67 8	68 9	70, 0	71, 1	72 2	73, 3	74 4	75 6	76, 7	77 8	78, 9	80 0
4	10	7	50	68, 0	69 1	70, 3	71 4	72 5	73 7	74, 8	75 9	77, 1	78, 2	79, 3	80, 5	81 6
4	20	7	40	69, 2	70, 4	71 5	72 7	73, 8	75, 0	76 1	77 3	78, 4	79 6	80 7	81, 9	83 1
4	30	7	30	70 3	71 5	72 7	73 8	75 0	76 2	77, 3	78, 5	79, 7	80 9	82, 0	83, 2	84 4
4	40	7	20	71, 3	72 5	73, 7	74, 9	76, 0	77 2	78, 4	79, 6	80, 8	82 0	83, 2	84 4	85 6
4	50	7	10	72 2	73 4	74 6	75 8	77, 0	78, 2	79 4	80 6	81, 8	83, 0	84 2	85 4	86 6
5	0	7	0	72 9	74, 1	75 3	76 6	77, 8	79 0	80, 2	81, 4	82, 6	83 9	85 1	86, 3	87, 5
5	10	6	50	73 6	74, 8	76 0	77 2	78, 5	79 7	80 9	82, 1	83, 4	84 6	85 8	87, 0	88, 3
5	20	6	40	74 1	75, 3	76, 5	77 8	79, 0	80 2	81, 5	82 7	84, 0	85 2	86, 4	87, 7	88 9
5	30	6	30	74 5	75 7	77 0	78, 2	79, 4	80, 7	81, 9	83, 2	84 4	85 7	86, 9	88, 1	89, 4
5	40	6	20	74, 8	76, 1	77, 3	78 5	79 8	81, 0	82, 2	83, 5	84, 7	86 0	87, 2	88, 5	89 7
5	50	6	10	74, 9	76, 2	77 4	78, 7	79, 9	81, 2	82, 4	83 7	84 9	86, 2	87 4	88, 7	89 9
6	0	6	0	75 0	76 3	77 5	78, 8	80, 0	81, 3	82, 5	83, 8	85 0	86, 3	87, 5	88 8	90 0

TABLE XLI

The Equation of Second Difference, uſeful for interpolating the Moon's Diſtances from the Sun and Stars for every third Hour between thoſe computed at Noon and Midnight, for the Uſe of the Nautical Ephemeris

Second Difference	Equation at 3ʹ and 9ʹ	Equation at 6ʹ	Second Difference	Equation at 3ʹ and 9ʹ	Equation at 6ʹ	Second Difference	Equation at 3ʹ and 9ʹ	Equation at 6ʹ
M S			M S			M S		
0 0	0 0,0	0 0,0	6 0	0 33,8	0 45,0	12 0	1 7,5	1 30,0
0 10	0 0,9	0 1,3	6 10	0 34,7	0 46,3	12 10	1 8,4	1 31,3
0 20	0 1,9	0 2,5	6 20	0 35,6	0 47,5	12 20	1 9,4	1 32,5
0 30	0 2,8	0 3,8	6 30	0 36,6	0 48,8	12 30	1 10,3	1 33,8
0 40	0 3,8	0 5,0	6 40	0 37,5	0 50,0	12 40	1 11,3	1 35,0
0 50	0 4,7	0 6,3	6 50	0 38,4	0 51,3	12 50	1 12,2	1 36,3
1 0	0 5,6	0 7,5	7 0	0 39,4	0 52,5	13 0	1 13,1	1 37,5
1 10	0 6,6	0 8,8	7 10	0 40,3	0 53,8	13 10	1 14,1	1 38,8
1 20	0 7,5	0 10,0	7 20	0 41,3	0 55,0	13 20	1 15,0	1 40,0
1 30	0 8,4	0 11,3	7 30	0 42,2	0 56,3	13 30	1 15,9	1 41,3
1 40	0 9,4	0 12,5	7 40	0 43,1	0 57,5	13 40	1 16,9	1 42,5
1 50	0 10,3	0 13,8	7 50	0 44,1	0 58,8	13 50	1 17,8	1 43,8
2 0	0 11,3	0 15,0	8 0	0 45,0	1 0,0	14 0	1 18,8	1 45,0
2 10	0 12,2	0 16,3	8 10	0 45,9	1 1,3	14 10	1 19,7	1 46,3
2 20	0 13,1	0 17,5	8 20	0 46,9	1 2,5	14 20	1 20,6	1 47,5
2 30	0 14,1	0 18,8	8 30	0 47,8	1 3,8	14 30	1 21,6	1 48,8
2 40	0 15,0	0 20,0	8 40	0 48,8	1 5,0	14 40	1 22,5	1 50,0
2 50	0 15,9	0 21,3	8 50	0 49,7	1 6,3	14 50	1 23,4	1 51,3
3 0	0 16,9	0 22,5	9 0	0 50,6	1 7,5	15 0	1 24,4	1 52,5
3 10	0 17,8	0 23,8	9 10	0 51,6	1 8,8	15 10	1 25,3	1 53,8
3 20	0 18,8	0 25,0	9 20	0 52,5	1 10,0	15 20	1 26,3	1 55,0
3 30	0 19,7	0 26,3	9 30	0 53,4	1 11,3	15 30	1 27,2	1 56,3
3 40	0 20,6	0 27,5	9 40	0 54,4	1 12,5	15 40	1 28,1	1 57,5
3 50	0 21,6	0 28,8	9 50	0 55,3	1 13,8	15 50	1 29,1	1 58,8
4 0	0 22,5	0 30,0	10 0	0 56,3	1 15,0	16 0	1 30,0	2 0,0
4 10	0 23,4	0 31,3	10 10	0 57,2	1 16,3	16 10	1 30,9	2 1,3
4 20	0 24,4	0 32,5	10 20	0 58,1	1 17,5	16 20	1 31,9	2 2,5
4 30	0 25,3	0 33,8	10 30	0 59,1	1 18,8	16 30	1 32,8	2 3,8
4 40	0 26,3	0 35,0	10 40	1 0,0	1 20,0	16 40	1 33,8	2 5,0
4 50	0 27,2	0 36,3	10 50	1 0,9	1 21,3	16 50	1 34,7	2 6,3
5 0	0 28,1	0 37,5	11 0	1 1,9	1 22,5	17 0	1 35,6	2 7,5
5 10	0 29,1	0 38,8	11 10	1 2,8	1 23,8	17 10	1 36,6	2 8,8
5 20	0 30,0	0 40,0	11 20	1 3,8	1 25,0	17 20	1 37,5	2 10,0
5 30	0 30,9	0 41,3	11 30	1 4,7	1 26,3	17 30	1 38,4	2 11,3
5 40	0 31,9	0 42,5	11 40	1 5,6	1 27,5	17 40	1 39,4	2 12,5
5 50	0 32,8	0 43,8	11 50	1 6,6	1 28,8	17 50	1 40,3	2 13,8
6 0	0 33,8	0 45,0	12 0	1 7,5	1 30,0	18 0	1 41,3	2 15,0

TABLE XLI *Continued*

Second Difference	Equation at 3′ and 9′	Equation at 6′	Second Difference	Equation at 3′ and 9′	Equation at 6′	Second Difference	Equation at 3′ and 9′	Equation at 6′
M S	′ ″		M S			M S		
18 0	1 41,3	2 15,0	24 0	2 15,0	3 0,0	30 0	2 48,8	3 45,0
18 10	1 42,2	2 16,3	24 10	2 15,9	3 1,3	30 10	2 49,7	3 46,3
18 20	1 43,1	2 17,5	24 20	2 16,9	3 2,5	30 20	2 50,6	3 47,5
18 30	1 44,1	2 18,6	24 30	2 17,8	3,8	30 30	2 51,6	3 48,8
18 40	1 45,0	2 20,0	24 40	2 18,8	3 5,0	30 40	2 52,5	3 50,0
18 50	1 45,9	2 21,3	24 50	2 19,7	3 6,3	30 50	2 53,4	3 51,3
19 0	1 46,9	2 22,5	25 0	2 20,6	3 7,5	31 0	2 54,4	3 52,5
19 10	1 47,8	2 23,8	25 10	2 21,6	3 8,8	31 10	2 55,3	3 53,8
19 20	1 48,8	2 25,0	25 20	2 22,5	3 10,0	31 20	2 56,3	3 55,0
19 30	1 49,7	2 26,3	25 30	2 23,4	3 11,3	31 30	57,2	3 56,3
19 40	1 50,6	2 27,5	25 40	2 24,4	3 12,5	31 40	58,1	3 57,5
19 50	1 51,6	2 28,8	25 50	2 25,3	3 13,8	31 50	2 59,1	3 58,8
20 0	1 52,5	2 30,0	26 0	2 26,3	3 15,0	32 0	3 0,0	4 0,0
20 10	1 53,4	2 31,3	26 10	2 27,2	3 16,3	32 10	3 0,9	4 1,3
20 20	1 54,4	2 32,5	26 20	2 28,1	3 17,5	32 20	3 1,9	4 2,5
20 30	1 55,3	2 33,8	26 30	2 29,1	3 18,8	32 30	3 2,8	4 3,8
20 40	1 56,3	2 35,0	26 40	2 30,0	3 20,0	32 40	3 3,8	4 5,0
20 50	1 57	2 36,3	26 50	2 30,9	3 21,3	32 50	3 4,7	4 6,3
21 0	1 58,1	2 37,5	27 0	2 31,9	3 22,5	33 0	3 5,6	4 7,5
21 10	1 59,1	2 38,8	27 10	2 32,8	3 23,8	33 10	3 6,6	4 8,8
21 20	2 0,0	2 40,0	27 20	2 33,8	3 25,0	33 20	3 7,5	4 10,0
21 30	0,9	41,3	27 30	34,7	3 26,3	33 30	3 8,4	4 11,3
21 40	1,9	42,5	27 40	35,6	3 27,5	33 40	3 9,4	4 12,5
21 50	2 2,8	2 43,8	27 50	2 36,6	3 28,8	33 50	3 10,3	4 13,8
0	3,6	2 45,0	28 0	2 37,5	3 30,0	34 0	3 11,3	4 15,0
10	4,7	2 46,3	28 10	38,4	3 31,3	34 10	3 12,2	4 16,3
20	2 5,6	2 47,5	28 20	2 39,4	3 32,5	34 20	3 13,1	4 17,5
30	6,6	2 48,8	28 30	40,3	3 33,8	34 30	3 14,1	4 18,8
40	7,5	50,0	28 40	41,3	3 35,0	34 40	3 15,0	4 20,0
2 50	2 8,4	51,3	28 50	2 4[illegible]	3 36,3	34 50	3 15,9	4 21,3
23 0	9,4	52,5	29 0	2 43,1	3 37,5	35 0	3 16,9	4 22,5
23 10	10,3	53,8	29 10	44,1	3 38,8	35 10	3 17,8	4 23,8
2 0	11,3	2 55,0	29 20	45,0	3 40,0	35 20	3 18,8	4 25,0
23 30	1	2 56,3	29 30	2 45,9	3 41,3	35 30	3 19,7	4 26,3
23 40	13,1	2 57,5	29 40	2 46,9	3 42,5	35 40	3 20,6	4 27,5
23 50	2 14,1	2 58,8	29 50	47,8	3 43,8	35 50	3 21,6	4 28,8
21 0	15,0	0,0	30 0	2 48,8	3 45,0	36 0	3 22,5	4 30,0

TABLE XLI *Continued*

Second Difference		Equation at 3^h and 9^h	Equation at 6^h	Second Difference		Equation at 3^h and 9^h	Equation at 6^h
M	S			M	S		
36	0	3 22,5	4 30,0	42	0	3 56,3	5 15,0
36	10	3 23,4	4 31,3	42	10	3 57,2	5 16,3
36	20	3 24,4	4 32,5	42	20	3 58,1	5 17,5
36	30	3 25,3	4 33,8	42	30	3 59,1	5 18,8
36	40	3 26,3	4 35,0	42	40	4 0,0	5 20,0
36	50	3 27,2	4 36,3	42	50	4 0,9	5 21,3
37	0	3 28,1	4 37,5	43	0	4 1,9	5 22,5
37	10	3 29,1	4 38,8	43	10	4 2,8	5 23,8
37	20	3 30,0	4 40,0	43	20	4 3,8	5 25,0
37	30	3 30,9	4 41,3	43	30	4 4,7	5 26,3
37	40	3 31,9	4 4 5	43	40	4 5,6	5 27,5
37	50	3 32,8	4 43,8	43	50	4 6,6	5 28,8
38	0	3 33,8	4 45,0	44	0	4 7,5	5 30,0
38	10	3 34,7	4 46,3	44	10	4 8,4	5 31,3
38	20	3 35,6	4 47,5	44	20	4 9,4	5 3 ,5
38	30	3 36,6	4 48,8	44	30	4 10,3	5 33,8
38	40	3 37,5	4 50,0	44	40	4 11,3	5 35,0
38	50	3 38,4	4 51,3	44	50	4 12,2	5 36,3
39	0	3 39,4	4 52,5	45	0	4 13,1	5 37,5
39	10	3 40,3	4 53,8	45	10	4 14,1	5 38,8
39	20	3 41,3	4 55,0	45	20	4 15,0	5 40,0
39	30	3 42,2	4 56,3	45	30	4 15,9	5 4 ,3
39	40	3 43,1	4 57,5	45	40	4 16,9	5 42,5
39	50	3 44,1	4 58,8	45	50	4 17,8	5 43,8
40	0	3 45,0	5 0,0	46	0	4 18,8	5 45,0
40	10	3 45,9	5 1,3	46	10	4 19,7	5 46,3
40	20	3 46,9	5 2,5	46	20	4 20,6	5 47,5
40	30	3 47,8	5 3,8	46	30	4 21,6	5 48,8
40	40	3 48,8	5 5,0	46	40	4 22,5	5 50,0
40	50	3 49,7	5 6,3	46	50	4 23,4	5 51,3
41	0	3 50,6	5 7,5	47	0	4 24,4	5 52,5
41	10	3 51,6	5 8,8	47	10	4 25,3	5 53,8
41	20	3 52,5	5 10,0	47	20	4 26,3	5 55,0
41	30	3 53,4	5 11,3	47	30	4 27,2	5 56,3
41	40	3 54,4	5 12,5	47	40	4 28,1	5 57,5
41	50	3 55,3	5 13,8	47	50	4 9,1	5 58,8
42	0	3 56,3	5 15,0	48	0	4 30,0	6 0,0

TABLE XLI *Continued*

Second Difference		Equation at 3h and 9h	Equation at 6h	Second Difference		Equation at 3h and 9h	Equation at 6h
M	S			M	S		
48	0	4 30, 0	6 0, 0	54	0	5 3, 8	6 45 0
48	10	4 30, 9	6 1, 3	54	10	5 4, 7	6 46, 3
48	20	4 31, 9	6 2, 5	54	20	5 5, 6	6 47, 5
48	30	4 32, 8	6 3, 8	54	30	5 6, 6	6 48, 8
48	40	4 33, 8	6 5, 0	54	40	5 7, 5	6 50, 0
48	50	4 34, 7	6 6 3	54	50	5 8, 4	6 51, 3
49	0	4 35, 6	6 7 5	55	0	5 9, 4	6 52, 5
49	10	4 36, 6	6 8, 8	55	10	5 10, 3	6 53, 8
49	20	4 37, 5	6 10 0	55	20	5 11, 3	6 55, 0
49	30	4 38, 4	6 11, 3	55	30	5 12, 2	6 56 3
49	40	4 39 4	6 12, 5	55	40	5 13, 1	6 57, 5
49	50	4 40, 3	6 13, 8	55	50	5 14, 1	6 58, 8
50	0	4 41, 3	6 15, 0	56	0	5 15, 0	7 0, 0
50	10	4 42, 2	6 16, 3	56	10	5 15, 9	7 1, 3
50	20	4 43, 1	6 17, 5	56	20	5 16, 9	7 2, 5
50	30	4 44, 1	6 18, 8	56	30	5 17, 8	7 3 8
50	40	4 45, 0	6 20, 0	56	40	5 18, 8	7 5, 0
50	50	4 45, 9	6 21, 3	56	50	5 19, 7	7 6, 3
51	0	4 46, 9	6 22, 5	57	0	5 20 6	7 7, 5
51	10	4 47, 8	6 23, 8	57	10	5 21, 6	7 8, 8
51	20	4 48, 8	6 25, 0	57	20	5 22, 5	7 10, 0
51	30	4 49, 7	6 26 3	57	30	5 23, 4	7 11, 3
51	40	4 50 6	6 27, 5	57	40	5 24, 4	7 12, 5
51	50	4 51, 6	6 28, 8	57	50	5 25, 3	7 13, 8
52	0	4 52, 5	6 30, 0	58	0	5 26, 3	7 15, 0
52	10	4 53, 4	6 31, 3	58	10	5 27, 2	7 16, 3
52	20	4 54, 4	6 32, 5	58	20	5 28, 1	7 17 5
52	30	4 55, 3	6 33, 8	58	30	5 29, 1	7 18, 8
52	40	4 56, 3	6 35, 0	58	40	5 30, 0	7 20, 0
52	50	4 57 2	6 36 3	58	50	5 30, 9	7 21, 3
53	0	4 58, 1	6 37, 5	59	0	5 31, 9	7 22 5
53	10	4 59, 1	6 38, 8	59	10	5 32, 8	7 23, 8
53	20	5 0 0	6 40, 0	59	20	5 33, 8	7 25, 0
53	30	5 0, 9	6 41, 3	59	30	5 34, 7	7 26 3
53	40	5 1 9	6 42 5	59	40	5 35, 6	7 27 5
53	50	5 2, 8	6 43, 8	59	50	5 36, 6	7 28, 8
54	0	5 3 8	6 45, 0	60	0	5 37, 5	7 30 0

TABLE XLII.

Decimal Parts of a Degree.

Min	Dec.	Min	Dec	Sec	Dec	Sec	Dec
1	,01667	31	,51667	1	,00028	31	,00861
2	,03333	32	,53333	2	,00056	32	,00889
3	,05000	33	,55000	3	,00083	33	,00917
4	,06667	34	,56667	4	,00111	34	,00944
5	,08333	35	,58333	5	,00138	35	,00972
6	,10000	36	,60000	6	,00167	36	,01000
7	,11667	37	,61667	7	,00194	37	,01028
8	,13333	38	,63333	8	,00222	38	,01056
9	,15000	39	,65000	9	,00250	39	,01083
10	,16667	40	,66667	10	,00278	40	,01111
11	,18333	41	,68333	11	,00306	41	,01139
12	,20000	42	,70000	12	,00333	42	,01167
13	,21667	43	,71667	13	,00361	43	,01194
14	,23333	44	,73333	14	,00389	44	,01222
15	,25000	45	,75000	15	,00417	45	,01250
16	,26667	46	,76667	16	,00444	46	,01278
17	,28333	47	,78333	17	,00472	47	,01306
18	,30000	48	,80000	18	,00500	48	,01333
19	,31667	49	,81667	19	,00528	49	,01361
20	,33333	50	,83333	20	,00556	50	,01389
21	,35000	51	,85000	21	,00583	51	,01417
22	,36667	52	,86667	22	,00611	52	,01444
23	,38333	53	,88333	23	,00639	53	,01472
24	,40000	54	,90000	24	,00667	54	,01500
25	,41667	55	,91667	25	,00694	55	,01528
26	,43333	56	,93333	26	,00722	56	,01556
27	,45000	57	,95000	27	,00750	57	,01583
28	,46667	58	,96667	28	,00778	58	,01611
29	,48333	59	,98333	29	,00806	59	,01639
30	,50000	60	1,00000	30	,00833	60	,01667

TABLE XLIII.

Equations to Equal Altitudes. TABLE I.

Sun's Longitude.		Half Interval between the Obſervations.						
		H M	H M	H M	H M	H M	H M	H M
		1 30	1. 40	1 50	2 0	2 10	2 20	2 30
S	D.	S	S	S	S	S	S	S
O.	− 0	15, 47	15, 57	15, 68	15, 79	15, 92	16, 06	16, 21
	5	15, 38	15, 48	15, 58	15, 70	15, 83	15, 97	16, 12
	10	15, 19	15, 28	15, 38	15, 50	15, 62	15, 76	15, 91
	15	14, 90	14, 99	15, 09	15, 20	15, 32	15, 46	15, 61
	20	14, 51	14, 60	14, 69	14, 80	14, 92	15, 05	15, 20
	25	14, 02	14, 10	14, 20	14, 31	14, 42	14, 55	14, 69
I	− 0	13, 44	13, 53	13, 62	13, 72	13, 83	13, 95	14, 08
	5	12, 76	12, 84	12, 93	13, 03	13, 14	13, 25	13, 37
	10	11, 99	12, 06	12, 14	12, 23	12, 33	12, 44	12, 56
	15	11, 12	11, 19	11, 27	11, 35	11, 44	11, 54	11, 65
	20	10, 16	10, 22	10, 29	10, 37	10, 46	10, 55	10, 65
	25	9, 12	9, 17	9, 23	9, 30	9, 38	9, 46	9, 55
II.	− 0	7, 99	8, 04	8, 09	8, 15	8, 22	8, 29	8, 37
	5	6, 78	6, 82	6, 87	6, 92	6, 98	7, 04	7, 11
	10	5, 51	5, 55	5, 59	5, 63	5, 67	5, 72	5, 78
	15	4, 18	4, 21	4, 24	4, 27	4, 30	4, 34	4, 38
	20	2, 81	2, 83	2, 85	2, 87	2, 90	2, 92	2, 95
	25	1, 41	1, 42	1, 43	1, 44	1, 46	1, 47	1, 48
III	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 41	1, 42	1, 43	1, 44	1, 45	1, 47	1, 48
	10	2, 81	2, 83	2, 85	2, 87	2, 89	2, 92	2, 95
	15	4, 17	4, 20	4, 23	4, 26	4, 29	4, 33	4, 37
	20	5, 49	5, 52	5, 56	5, 60	5, 65	5, 70	5, 75
	25	6, 75	6, 79	6, 84	6, 89	6, 95	7, 01	7, 07
IV	+ 0	7, 94	7, 99	8, 05	8, 11	8, 17	8, 24	8, 32
	5	9, 06	9, 11	9, 17	9, 24	9, 32	9, 40	9, 49
	10	10, 09	10, 15	10, 22	10, 30	10, 38	10, 47	10, 57
	15	11, 03	11, 10	11, 18	11, 26	11, 35	11, 45	11, 56
	20	11, 89	11, 96	12, 04	12, 13	12, 23	12, 33	12, 45
	25	12, 65	12, 72	12, 81	12, 91	13, 01	13, 12	13, 25
V	+ 0	13, 31	13, 40	13, 49	13, 59	13, 70	13, 82	13, 95
	5	13, 88	13, 97	14, 07	14, 17	14, 28	14, 41	14, 54
	10	14, 35	14, 44	14, 54	14, 65	14, 77	14, 90	15, 04
	15	14, 74	14, 83	14, 93	15, 04	15, 16	15, 30	15, 41
	20	15, 03	15, 12	15, 22	15, 33	15, 46	15, 59	15, 74
	25	15, 22	15, 31	15, 41	15, 53	15, 65	15, 79	15, 94
VI	+ 0	15, 31	15, 40	15, 51	15, 62	15, 75	15, 89	16, 04

TABLE XLIII. *Continued.*

Equations to Equal Altitudes. TABLE I.

Sun's Longitude.		Half Interval between the Obſervations.						
		H M.	H M	H M	H M	H. M	H M.	H M.
		2. 40	2 50	3 0	3 10	3. 20	3 30	3 40
S.	D	S.	S.	S	S	S.	S	S
O.	− 0	16, 38	16, 56	16, 75	16, 96	17, 18	17, 42	17, 67
	5	16, 28	16, 46	16, 65	16, 86	17, 08	17, 32	17, 57
	10	16, 08	16, 25	16, 44	16, 64	16, 86	17, 10	17, 35
	15	15, 77	15, 94	16, 12	16, 32	16, 54	16, 77	17, 01
	20	15, 35	15, 52	15, 70	15, 89	16, 19	16, 32	16, 56
	25	14, 84	15, 00	15, 17	15, 36	15, 56	15, 78	16, 01
I.	− 0	14, 22	14, 37	14, 54	14, 72	14, 92	15, 13	15, 35
	5	13, 51	13, 65	13, 81	13, 98	14, 17	14, 36	14, 57
	10	12, 69	12, 83	12, 98	13, 14	13, 31	13, 49	13, 69
	15	11, 77	11, 90	12, 04	12, 19	12, 35	12, 52	12, 70
	20	10, 75	10, 87	11, 00	11, 14	11, 28	11, 44	11, 61
	25	9, 65	9, 76	9, 87	9, 99	10, 12	10, 26	10, 41
II.	− 0	8, 46	8, 55	8, 65	8, 76	8, 87	8, 99	9, 12
	5	7, 18	7, 26	7, 35	7, 44	7, 54	7, 64	7, 75
	10	5, 84	5, 90	5, 97	6, 04	6, 12	6, 21	6, 30
	15	4, 43	4, 48	4, 53	4, 59	4, 65	4, 71	4, 78
	20	2, 98	3, 01	3, 05	3, 09	3, 13	3, 17	3, 22
	25	1, 50	1, 51	1, 53	1, 55	1, 57	1, 59	1, 62
III.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 49	1, 51	1, 53	1, 55	1, 57	1, 59	1, 61
	10	2, 98	3, 01	3, 04	3, 08	3, 12	3, 16	3, 21
	15	4, 42	4, 47	4, 52	4, 57	4, 63	4, 70	4, 77
	20	5, 81	5, 87	5, 94	6, 02	6, 10	6, 18	6, 27
	25	7, 14	7, 22	7, 31	7, 40	7, 50	7, 60	7, 71
IV.	+ 0	8, 41	8, 50	8, 60	8, 71	8, 82	8, 94	9, 07
	5	9, 59	9, 69	9, 80	9, 93	10, 06	10, 20	10, 34
	10	10, 68	10, 80	10, 93	11, 06	11, 20	11, 36	11, 53
	15	11, 68	11, 81	11, 94	12, 09	12, 25	12, 42	12, 60
	20	12, 58	12, 72	12, 86	13, 02	13, 19	13, 38	13, 57
	25	13, 38	13, 53	13, 69	13, 86	14, 04	14, 23	14, 44
V	+ 0	14, 09	14, 24	14, 41	14, 59	14, 78	14, 98	15, 20
	5	14, 69	14, 85	15, 03	15, 21	15, 41	15, 62	15, 85
	10	15, 20	15, 36	15, 54	15, 73	15, 94	16, 16	16, 39
	15	15, 60	15, 77	15, 95	16, 15	16, 36	16, 59	16, 83
	20	15, 90	16, 07	16, 26	16, 46	16, 68	16, 91	17, 16
	25	16, 10	16, 28	16, 47	16, 67	16, 89	17, 12	17, 37
VI.	+ 0	16, 20	16, 38	16, 57	16, 78	17, 00	17, 23	17, 48

TABLE XLIII. *Continued.*

Equations to Equal Altitudes. TABLE I.

Sun's Longitude		Half Interval between the Obſervations.						
		H M	H M	H M	H M	H. M.	H. M.	H M
		3 50	4 0	4 10	4 20	4 30	4. 40	4. 50
S	D	S.	S	S	S	S	S	S
O	— 0	17, 94	18, 23	18, 54	18, 87	19, 22	19, 60	20, 01
	5	17, 84	18, 13	18, 44	18, 77	19, 12	19, 49	19, 89
	10	17, 62	17, 90	18, 20	18, 53	18, 88	19, 25	19, 64
	15	17, 27	17, 55	17, 85	18, 17	18, 51	18, 87	19, 26
	20	16, 82	17, 09	17, 38	17, 69	18, 02	18, 37	18, 75
	25	16, 26	16, 52	16, 80	17, 10	17, 42	17, 76	18, 12
I.	— 0	15, 59	15, 84	16, 11	16, 40	16, 70	17, 03	17, 38
	5	14, 80	15, 04	15, 29	15, 56	15, 86	16, 17	16, 50
	10	13, 90	14, 13	14, 37	14, 63	14, 90	15, 19	15, 50
	15	12, 90	13, 11	13, 33	13, 57	13, 82	14, 09	14, 38
	20	11, 79	11, 98	12, 18	12, 40	12, 63	12, 88	13, 14
	25	10, 57	10, 74	10, 93	11, 12	11, 33	11, 55	11, 79
II.	— 0	9, 26	9, 41	9, 57	9, 74	9, 93	10, 12	10, 33
	5	7, 87	8, 00	8, 14	8, 28	8, 43	8, 60	8, 77
	10	6, 40	6, 50	6, 61	6, 72	6, 85	6, 99	7, 13
	15	4, 85	4, 93	5, 01	5, 10	5, 20	5, 30	5, 41
	20	3, 27	3, 32	3, 37	3, 43	3, 50	3, 57	3, 64
	25	1, 64	1, 67	1, 70	1, 73	1, 76	1, 79	1, 83
III	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 64	1, 66	1, 69	1, 72	1, 76	1, 79	1, 83
	10	3, 26	3, 31	3, 37	3, 43	3, 50	3, 56	3, 63
	15	4, 84	4, 92	5, 00	5, 09	5, 19	5, 29	5, 40
	20	6, 37	6, 47	6, 58	6, 70	6, 82	6, 96	7, 10
	25	7, 83	7, 96	8, 10	8, 24	8, 39	8, 56	8, 73
IV.	+ 0	9, 21	9, 36	9, 52	9, 69	9, 87	10, 06	10, 27
	5	10, 50	10, 68	10, 86	11, 05	11, 26	11, 48	11, 71
	10	11, 70	11, 89	12, 10	12, 31	12, 54	12, 79	13, 05
	15	12, 79	13, 90	13, 22	13, 46	13, 71	13, 98	14, 27
	20	13, 78	14, 01	14, 25	14, 50	14, 77	15, 06	15, 37
	25	14, 66	14, 90	15, 15	15, 42	15, 71	16, 02	16, 35
V	+ 0	15, 44	15, 69	15, 96	16, 24	16, 54	16, 86	17, 21
	5	16, 10	16, 36	16, 64	16, 94	17, 26	17, 59	17, 95
	10	16, 64	16, 91	17, 20	17, 51	17, 84	18, 19	18, 56
	15	17, 09	17, 37	17, 67	17, 98	18, 31	18, 67	19, 06
	20	17, 42	17, 70	18, 00	18, 32	18, 67	19, 04	19, 43
	25	17, 64	17, 93	18, 23	18, 56	18, 91	19, 28	19, 67
VI	+ 0	17, 75	18, 04	18, 35	18, 68	19, 03	19, 40	19, 79

TABLE XLIII. *Continued.*

Equations to Equal Altitudes. TABLE I.

Sun's Longitude.		Half Interval between the Obſervations						
		H M	H M	H M	H M	H M	H M	H M
		5 0	5 10	5 20	5 30	5 40	5 50	6 0
S.	D	S	S	S	S	S.	S.	S
O.	— 0	20, 44	20, 90	21, 38	21, 90	22, 46	23, 05	23, 69
	5	20, 32	20, 77	21, 25	21, 77	22, 32	22, 91	23, 55
	10	20, 06	20, 51	20, 99	21, 50	22, 04	22, 62	23, 25
	15	19, 67	20, 11	20, 58	21, 08	21, 62	22, 19	22, 80
	20	19, 16	19, 59	20, 05	20, 54	21, 06	21, 61	22, 21
	25	18, 51	18, 93	19, 37	19, 84	20, 34	20, 88	21, 46
I.	— 0	17, 75	18, 15	18, 57	19, 02	19, 50	20, 02	20, 57
	5	16, 85	17, 23	17, 63	18, 06	18, 52	19, 01	19, 53
	10	15, 83	16, 19	16, 57	16, 97	17, 40	17, 86	18, 35
	15	14, 69	15, 02	15, 37	15, 74	16, 14	16, 57	17, 03
	20	13, 42	13, 72	14, 04	14, 38	14, 75	15, 14	15, 56
	25	12, 04	12, 31	12, 59	12, 90	13, 23	13, 58	13, 96
II	— 0	10, 55	10, 78	11, 03	11, 30	11, 59	11, 90	12, 23
	5	8, 96	9, 16	9, 37	9, 60	9, 85	10, 11	10, 39
	10	7, 28	7, 44	7, 62	7, 80	8, 00	8, 21	8, 44
	15	5, 53	5, 65	5, 78	5, 92	6, 07	6, 23	6, 41
	20	3, 72	3, 80	3, 89	3, 98	4, 08	4, 19	4, 31
	25	1, 87	1, 91	1, 95	2, 00	2, 05	2, 10	2, 16
III	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 87	1, 91	1, 95	2, 00	2, 05	2, 10	2, 16
	10	3, 71	3, 79	3, 88	3, 97	4, 07	4, 18	4, 30
	15	5, 51	5, 63	5, 77	5, 91	6, 06	6, 22	6, 39
	20	7, 25	7, 41	7, 59	7, 77	7, 97	8, 18	8, 41
	25	8, 92	9, 12	9, 33	9, 56	9, 80	10, 06	10, 34
IV	+ 0	10, 49	10, 73	10, 98	11, 25	11, 54	11, 84	12, 16
	5	11, 96	12, 23	12, 52	12, 82	13, 14	13, 49	13, 86
	10	13, 33	13, 63	13, 94	14, 28	14, 65	15, 04	15, 45
	15	14, 57	14, 90	15, 25	15, 62	16, 01	16, 43	16, 89
	20	15, 70	16, 05	16, 42	16, 82	17, 25	17, 71	18, 19
	25	16, 70	17, 07	17, 47	17, 90	18, 35	18, 84	19, 36
V.	+ 0	17, 58	17, 98	18, 40	18, 85	19, 32	19, 83	20, 38
	5	18, 34	18, 75	19, 18	19, 64	20, 14	20, 67	21, 25
	10	18, 96	19, 39	19, 84	20, 32	20, 84	21, 39	21, 97
	15	19, 47	19, 90	20, 36	20, 86	21, 39	21, 96	22, 56
	20	19, 84	20, 28	20, 76	21, 26	21, 80	22, 38	23, 00
	25	20, 09	20, 54	21, 02	21, 53	22, 08	22, 66	23, 29
VI	+ 0	20, 22	20, 67	21, 15	21, 67	22, 22	22, 81	23, 44

TABLE XLIII. *Continued.*

Equations to Equal Altitudes. TABLE I.

Sun's Longitude		Half Interval between the Obſervations.						
		H M	H. M	H M	H M	H M	H. M	H. M
		1 30	1 40	1. 50	2 0	2 10	2 20	2 30
S	D	S	S	S	S	S.	S	S
VI.	+ 0	15, 31	15, 41	15, 51	15, 62	15, 75	15, 89	16, 04
	5	15, 30	15, 40	15, 50	15, 61	15, 74	15, 88	16, 03
	10	15, 20	15, 29	15, 39	15, 51	15, 63	15, 77	15, 92
	15	14, 99	15, 08	15, 19	15, 30	15, 42	15, 56	15, 71
	20	14, 69	14, 78	14, 88	14, 99	15, 11	15, 24	15, 39
	25	14, 28	14, 37	14, 46	14, 57	14, 69	14, 82	14, 96
VII.	+ 0	13, 76	13, 85	13, 95	14, 05	14, 16	14, 29	14, 42
	5	13, 14	13, 22	13, 31	13, 41	13, 52	13, 64	13, 77
	10	12, 41	12, 49	12, 57	12, 66	12, 76	12, 87	13, 00
	15	11, 57	11, 64	11, 71	11, 80	11, 90	12, 01	12, 12
	20	10, 62	10, 68	10, 75	10, 83	10, 92	11, 02	11, 13
	25	9, 57	9, 63	9, 69	9, 76	9, 84	9, 93	10, 03
VIII.	+ 0	8, 42	8, 47	8, 53	8, 59	8, 66	8, 74	8, 82
	5	7, 18	7, 22	7, 27	7, 32	7, 38	7, 45	7, 52
	10	5, 85	5, 88	5, 92	5, 97	6, 02	6, 07	6, 13
	15	4, 45	4, 48	4, 51	4, 54	4, 58	4, 62	4, 66
	20	3, 00	3, 02	3, 04	3, 06	3, 09	3, 11	3, 14
	25	1, 51	1, 52	1, 53	1, 54	1, 55	1, 56	1, 58
IX	− 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 51	1, 52	1, 53	1, 54	1, 55	1, 57	1, 58
	10	3, 00	3, 02	3, 04	3, 07	3, 09	3, 12	3, 15
	15	4, 46	4, 49	4, 52	4, 55	4, 59	4, 63	4, 67
	20	5, 87	5, 90	5, 94	5, 99	6, 04	6, 09	6, 1[illegible]
	25	7, 20	7, 24	7, 29	7, 35	7, 41	7, 48	7, 55
X	− 0	8, 46	8, 51	8, 57	8, 63	8, 70	8, 78	8, 86
	5	9, 63	9, 69	9, 75	9, 82	9, 90	9, 99	10, 09
	10	10, 70	10, 76	10, 83	10, 91	11, 00	11, 10	11, 21
	15	11, 66	11, 73	11, 81	11, 90	12, 00	12, 10	12, 21
	20	12, 51	12, 59	12, 68	12, 77	12, 87	12, 99	13, 11
	25	13, 26	13, 34	13, 43	13, 53	13, 64	13, 76	13, 89
XI	− 0	13, 89	13, 98	14, 08	14, 18	14, 29	14, 42	14, 55
	5	14, 42	14, 51	14, 61	14, 72	14, 84	14, 97	15, 11
	10	14, 84	14, 93	15, 03	15, 14	15, 27	15, 10	15, 55
	15	15, [illegible]5	15, 25	15, 35	15, 46	15, 59	15, 73	15, 88
	20	15, 36	15, 46	15, 57	15, 68	15, 80	15, 91	16, 10
	25	15, 47	15, 56	15, 67	15, 78	15, 91	16, 05	16, [illegible]
XII	− 0	15, 48	15, 57	15, 68	15, 79	15, 92	16, 06	16, 2[illegible]

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE I

Sun's Longitude		Half Interval between the Observations						
		H M	H M	H M	H M	H M	H M	H M
		2 40	50	3 0	3 10	3 20	3 30	3 40
S	D	S	S	S	S	S	S	S
VI	+ 0	16, 21	16, 36	16, 57	16, 78	17, 00	17, 23	17 48
	5	16, 20	16, 37	16, 56	16, 77	16, 99	17, 2[illegible]	17 47
	10	16, 08	16, [illegible]6	16, 45	16 65	16, 87	17, 10	17, 35
	15	15 67	16, 04	16, 23	16, 43	16, 64	16, 87	17 1[illegible]
	20	15, 55	15, 7[illegible]	15, 90	16 09	16, 30	16, 53	16 77
	25	15, 11	15, 28	15, 46	15, 65	15 86	16, 08	16 31
VII	+ 0	14, 57	14, 73	14, 90	15, 08	15, 28	15, 49	15 72
	5	13, 91	14, 06	14, 22	14, 40	14 59	14 79	15, 01
	10	13, 13	13, 27	13, 42	13 59	13, 77	13, 96	14 17
	15	12 24	1[illegible] 37	1[illegible] 51	12 67	1[illegible] 64	13 02	13 [illegible]1
	20	11 24	11, 36	11 49	11 63	11 79	11 96	12 13
	25	10 13	10, [illegible]4	10, [illegible]6	10, 48	10 62	10, 77	10, 93
VIII	+ 0	8, 91	9, 01	9, 11	9, 2[illegible]	9 34	9 47	9, 61
	5	7, 59	7, 67	7, 76	7, 86	7 96	8, 07	8, 19
	10	6, 19	6, 26	6, 33	6 41	6 49	6, 58	6, 68
	15	4, 71	4 76	4, 82	4 8[illegible]	4, 91	5 01	5, 08
	20	3, 18	3, 21	3, 25	3, [illegible]9	3, [illegible]3	3, 38	3, 4[illegible]
	25	1, 60	1, 6[illegible]	1, 64	1, 66	1, 68	1, 70	1, 73
IX	− 0	0, 00	0 00	0, 00	0, 00	0 00	0, 00	0, 00
	5	1 60	1, 62	1, 64	1, 66	1, 68	1, [illegible]0	1, 73
	10	3, 16	3, 21	3, 25	3 28	3, 33	3, 38	3, 43
	15	4 72	4, 77	4, 83	4, 89	4, 95	5, 0[illegible]	5 09
	0	6, 21	6, 26	6, 35	6 43	6, 51	6, 60	6, 70
	25	7, 6[illegible]	7, 70	7, 79	7, 89	8, 00	8, 11	8 23
X	− 0	8, 95	9 05	9 15	9, 27	9, 39	9, 5[illegible]	9, 66
	5	10, 19	10, 30	10, 42	10 55	10, 69	10, 83	10, 99
	10	11, 3[illegible]	11, 44	11, 58	11 72	11 87	12, 04	12, [illegible]1
	15	1[illegible], 34	12, 48	1[illegible], [illegible]6	1[illegible], [illegible]7	12 94	13, 12	13 31
	20	13, [illegible]4	13, 39	13, 54	13, 71	13 89	14, 08	14, [illegible]9
	25	14 03	14, 1[illegible]	14, 35	14, 53	14 72	14, 92	15, 14
XI	− 0	14, 70	14, 86	15 04	15 [illegible]3	15, 42	15, 63	15 86
	5	14, [illegible]6	15, 43	15 61	15 80	16, 01	16 [illegible]3	16 47
	10	15 71	15, 88	16, 06	16, 26	16 48	16, 71	16 95
	15	16, 04	16 [illegible]1	16 40	16 60	16, 6[illegible]	1[illegible], [illegible]6	17, 31
	0	16, 26	16, 44	16 63	16, 83	17, 05	1[illegible] [illegible]9	17 [illegible]4
	25	16 37	16 55	16 74	16, 95	17 17	17, 41	17, 60
XII	− 0	16, 38	16, 56	16, 75	16, 96	17, 18	17, 4[illegible]	17 67

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE I

Sun's Longitude		Half Interval between the Observations						
		H M	H M	H M	H M	H M	H M	H M
		3 50	4 0	4 10	4 20	4 30	4 40	4 50
S	D	S	S	S	S	S	S	S
VI	+ 0	17, 75	18, 04	18, 35	18, 67	19, 02	19, 40	19, 80
	5	17, 74	18, 03	18 34	18 66	19, 01	19, 39	19, 79
	10	17, 62	17 91	18, 22	18, 54	18, 88	19 25	19, 65
	15	17, 39	17, 67	17, 97	18, 29	18, 63	18, 99	19, 36
	20	17, 03	17, 31	17, 60	17, 92	18 25	18 61	18 99
	25	16, 56	16, 8	17, 11	17, 42	17, 75	18, 10	18 47
VII	+ 0	15, 96	16 22	16, 49	16, 79	17, 11	17, 44	17 80
	5	15 4	15 49	15, 75	16 03	16 33	16 65	16 99
	10	14, 39	14, 62	14, 87	15, 14	15 42	15 72	16, 04
	15	13 41	13, 63	13, 86	14, 11	14, 37	14, 65	14, 95
	20	12, 31	12 51	12, 72	12 95	13, 19	13, 45	13, 73
	25	11, 09	11, 27	11, 46	11, 67	11, 89	12, 12	12, 37
VIII	+ 0	9, 76	9 92	10, 09	10, 27	10, 46	10, 66	10, 88
	5	8, 32	8, 45	8, 59	8, 75	8, 92	9, 09	9, 27
	10	6, 78	6, 89	7, 01	7 14	7 27	7, 41	7, 56
	15	5, 16	5, 24	5, 33	5, 43	5, 53	5, 64	5, 76
	20	3 48	3, 54	3, 60	3, 66	3, 73	3, 80	3 88
	25	1, 75	1, 78	1, 81	1, 84	1, 88	1, 91	1, 95
IX	− 0	0, 00	0 00	0, 00	0, 00	0, 00	0, 00	0 00
	5	1, 75	1, 78	1, 81	1, 84	1, 88	1 92	1, 95
	10	3, 48	3 54	3, 60	3, 66	3, 73	3 81	3 89
	15	5, 17	5, 6	5, 35	5 44	5, 54	5, 65	5, 77
	20	6, 80	6, 91	7, 03	7 16	7 9	7, 43	7 59
	25	8, 36	8 49	8 63	[illegible] 79	8, 95	9 12	9 31
X	− 0	9, 81	9 97	10, 14	10, 32	10, 51	10, 72	10 94
	5	11 16	11 34	11, 54	11, 74	11 96	12, 20	12, 15
	10	12 40	12, 60	12, 92	13 05	13 29	13 55	13 83
	15	13, 52	13, 74	13, 97	14, 22	14, 49	14, 77	15 07
	20	14 51	14, 74	14, 99	15, 26	15 55	15, 86	16, 18
	25	15, 37	15 62	15 89	16, 17	16, 47	16 79	17 14
XI	− 0	16 11	16, 37	16, 65	16, 95	17 26	17 60	17 96
	5	16 7	16, 99	17, 28	17 59	17 92	18 27	18, 65
	10	17, 21	17 49	17, 79	18, 10	18, 44	18, 80	19 19
	15	17 57	17, 86	18 16	18, 48	18, 83	19 20	19, 59
	20	17, 81	18, 10	18 41	18, 71	19, 09	19 46	19 86
	25	17 94	18 3	18, 54	18, 87	19, [illegible]	19 60	20 00
XII	− 0	17, 95	18, 24	18, 55	18, 66	19, 23	19 61	20 01

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE I

Sun's Longitude		Half Interval between the Obſervations						
		H M	H M	H M	H M	H M	H M	H M
		5 0	5 10	5 20	5 30	5 40	5 50	6 0
S	D	S	S	S	S	S	S	S
VI	— 0	20, 22	20, 67	21, 15	21, 67	22, 22	22, 81	23, 43
	5	20, 21	20, 66	21, 14	21, 66	22, 21	22, 80	23, 42
	10	20, 07	20, 52	21, 00	21, 51	22, 05	22, 64	23, 26
	15	19, 80	20, 24	20, 71	21, 22	21, 76	22, 33	22, 95
	20	19, 40	19, 83	20, 29	20, 79	21, 32	21 88	22, 48
	25	18, 86	19, 8	19, 73	20, 1	20, 72	21, 27	21, 85
VII	— 0	18, 18	18, 58	19, 02	19, 48	19, 97	20, 50	21, 06
	5	17, 35	17, 74	18, 15	18, 60	19, 07	19, 57	20, 11
	10	16, 38	16, 75	17 14	17 56	16, 00	18, 48	18 99
	15	15, 27	15 61	15, 98	16, 37	16, 78	17 23	17, 70
	20	14 03	14 31	14 67	15, 03	15, 41	15, 82	16 25
	25	12, 64	12, 9	13, 22	13, 54	1, 88	14, 25	14 65
VIII	— 0	11 11	11, 36	11, 63	11, 91	12, 21	12 51	12, 89
	5	9 47	9, 63	9, 91	10 15	10, 41	10 68	10, 98
	10	7 72	7, 89	8 08	9 9	6, 49	9, 71	6, 95
	15	5 98	6 01	6 15	6, 30	6, 46	6, 63	6, 81
	20	3, 96	4, 05	4 14	1, 4	1, 35	4, 47	4, 59
	25	1, 99	, 01	09	, 11	2, 19	2 25	2 31
IX	— 0	0, 00	0 00	0 00	0, 00	0 00	0, 00	0, 00
	5	1 99	2 04	2, 09	2, 14	2, 19	2, 25	2, 31
	10	3, 97	4, 06	4 15	4 25	4, 30	4, 48	4, 60
	15	5 89	6, 02	6, 17	6 32	6, 48	6, 65	0, 83
	20	7, 75	7, 92	8, 11	8 , 1	9, 52	8 74	8, 98
	25	9, 51	9 72	9, 95	10, 19	10, 43	10, 73	11, 02
X	— 0	11, 17	11, 42	11, 69	11, 97	12, 28	1 60	12, 94
	5	12, 72	13, 00	13, 33	13, 62	1, 97	14 34	14 73
	10	1 , 12	14 41	14, 76	15 14	15 52	15, 93	16, 37
	15	15, 39	15 74	16, 11	16 50	16 92	17 37	17, 84
	20	16, 52	16, 69	17 9	17 71	6 16	16 64	19, 15
	25	17, 51	17, 90	18, 32	18 76	19 24	19, 75	0 29
XI	— 0	18, 35	18, 76	19 19	19, 66	20, 16	0, 69	21 26
	5	19, 05	19 47	19 92	0 41	0, 93	21, 48	07
	10	19, 60	20, 04	20 53	21 00	1 54	22, 11	72
	15	0 01	20 45	0, 94	21, 45	21, 99	22 57	2, 20
	20	20, 8	20 73	21, 2	21 74	[illegible]	22 80	23 52
	25	20, 43	20, 89	21, 37	21 69	22, 45	23, 04	23 68
XII	— 0	20, 44	20, 90	21, 38	21, 90	22, 40	23, 05	23, 69

TABLE XLIII *Continued.*

Equations to Equal Altitudes TABLE II

Sun's Longitude		Half Interval between the Observations						
		H M	H M	H M	H M	H M	H M	H M
		1 30	1 40	1 50	2 0	2 10	2 20	2 30
S	D	S	S	S	S	S	S	S
O	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0 00	0 00
	5	0 49	0, 49	0 48	0, 17	0, 46	0 45	0 44
	10	0, 97	0, 96	0, 95	0 93	0, 91	0 90	0 [illegible]6
	15	1 43	1, 41	1, 39	1, 36	1, 34	1 31	1 28
	20	1, 84	1 8[illegible]	1, 79	1, 76	1, 73	1 70	1, 66
	25	2, [illegible]1	2, 18	2, 15	2, 12	2, 08	2, 04	1, 99
I	0	2 52	2, 49	2 45	2 41	2 37	2, 3[illegible]	2 27
	5	2, 77	2, 73	2 69	2, 65	2, 60	2, 55	2, 49
	10	2, 93	2, 89	2, 85	2, 81	2 76	2 70	2, 64
	15	3, 01	2, 97	2, 93	2, 88	2, 83	2, 77	2, 71
	20	3, 01	2, 97	2, 93	2, 88	2, 83	2, 77	2, 71
	25	2, 91	2, 87	2, 83	2, 78	2, 73	2, 67	2, 61
II	+ 0	2, 71	2, 68	2, 64	2, 60	2, 55	2, 50	2, 44
	5	2, 42	2, 39	2, 36	2, 32	2, 28	2, 23	2, 18
	10	2, 06	2, 03	2, 00	1, 97	1, 93	1, 89	1 85
	15	1, 61	1, 59	1, 57	1, 54	1, 51	1, 48	1, 45
	20	1, 11	1 10	1, 08	1, 06	1, 04	1, 02	1, 00
	25	0, 57	0, 56	0, 55	0, 54	0, 53	0, 52	0, 51
III	− 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 56	0 56	0, 55	0, 54	0, 53	0, 5[illegible]	0, 51
	10	1, 11	1, 10	1, 08	1 06	1 04	1, 0[illegible]	1, 00
	15	1, 61	1, 59	1, 57	1, 54	1, 51	1, 48	1, 45
	20	2, 05	2 02	1, 99	1, 96	1, 9[illegible]	1 88	1, 84
	25	2, 41	2, 38	2, 35	2, 31	2, 27	2, [illegible]	17
IV	− 0	2, 70	2, 66	2 6[illegible]	2, 58	2 53	2, [illegible]8	2, 43
	5	2, 89	2, 85	81	2, 76	2 71	2 66	2, 60
	10	, 99	2 95	90	[illegible]5	2, 80	, 75	2 69
	15	2, 99	, 95	, 91	2 86	2, 81	2, 75	, 69
	20	2 91	87	, 8[illegible]	2 79	2 73	2, 65	2, 6[illegible]
	25	, 74	, 70	, 66	2 6[illegible]	2, [illegible]7	, 52	2 47
V	− 0	2, 50	47	, 43	, 39	[illegible]5	[illegible]0	2 25
	5	2, 19	2 16	2 1[illegible]	09	[illegible]5	, 01	1, 97
	10	1 82	1 80	1 77	1 7[illegible]	1 [illegible]1	1 66	1, 64
	15	1, 41	1, 39	, 37	1, 35	1 33	1, 30	1 [illegible]
	20	0, 96	0, 95	0, 94	0, 9[illegible]	0 90	0 9[illegible]	0, [illegible]0
	25	0, 49	0 48	0, 47	0, 47	0 46	0, 45	0, 4[illegible]
VI	− 0	0, 00	0, 00	0, 00	0 00	0 00	0, 00	0 00

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE II

Sun's Longitude		Half Interval between the Obſervations						
		H M	H M	H M	H M	H M	H M	H M
		2 40	2 50	3 0	3 10	3 20	3 30	3 40
S	D	S	S	S	S	S	S	S
O	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 43	0, 42	0, 41	0, 40	0, 38	0, 37	0 35
	10	0, 85	0, 83	0, 81	0, 78	0, 75	0, 72	0, 69
	15	1, 25	1, 22	1, 18	1, 14	1, 10	1, 06	1, 01
	20	1, 62	1, 58	1, 53	1, 48	1, 43	1 37	1, 31
	25	1, 94	1, 89	1, 83	1 77	1, 71	1, 64	1, 57
I	+ 0	2, 21	2 15	2, 09	2, 02	1, 95	1, 87	1, 79
	5	2, 43	2, 36	2, 29	2 22	2, 14	2, 05	1, 96
	10	2, 57	2, 50	2, 43	2, 35	2, 26	2, 17	2 08
	15	2, 64	2, 57	2, 50	2, 42	2, 33	2, 24	2, 14
	20	2, 64	2 57	2 49	2, 41	2, 32	2, 23	2, 13
	25	2, 55	2 48	2, 41	2 33	2 24	2, 15	2, 06
II	+ 0	2, 38	2, 31	2, 24	2, 17	2, 09	2, 01	1, 92
	5	2, 13	2, 07	2, 01	1, 94	1, 67	1, 80	1, 72
	10	1, 80	1, 75	1, 70	1, 65	1, 59	1, 53	1, 46
	15	1, 41	1 37	1, 33	1, 29	1, 24	1, 19	1, 14
	20	0, 98	0, 95	0 92	0, 89	0, 86	0, 83	0, 79
	25	0, 50	0, 46	0 47	0, 45	0 44	0, 42	0, 40
III	− 0	0, 00	0 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 49	0, 48	0, 47	0, 45	0, 44	0, 42	0, 40
	10	0, 97	0, 94	0, 91	0, 86	0 65	0, 8[illegible]	0, 78
	15	1, 41	1, 37	1, 33	1, 29	1, 24	1, 19	1, [illegible]4
	20	1 60	1 75	1, 70	1, 64	1 58	1, 52	1, 45
	25	2 12	06	2, 00	1 93	1, 86	1 79	1, 71
IV	− 0	, 37	2, 30	2, 23	2, 16	2, 08	2 00	1, 91
	5	2, 54	2, 47	2, 39	2, 31	2, 23	2, 14	2 05
	10	2, 62	2, 55	2, 47	2, 39	2, 31	2, 22	2, 12
	15	2, 62	2, 55	2 48	40	2, 31	2, [illegible]2	2 12
	20	2, 55	2 46	2, 41	2, 33	2 25	2, 16	2, 06
	25	2 41	2, 34	2 27	2 20	2, 12	2 03	1, 94
V	− 0	, 19	2, 13	07	2, 00	1, 93	1, 85	1, 77
	5	1, 92	1, 67	1, 81	1, 75	1, 69	1 62	1 55
	10	1, 60	1, 56	1 51	1, 46	1, 41	1 35	1, 29
	15	1, 24	1 21	1, 17	1, 13	1, 09	1 05	1, 00
	20	0, 94	0, 82	0, 80	0, 77	0, 74	0, 71	0, 66
	25	0, 43	0, 42	0, 40	0, 39	0, 38	0, 36	0, 35
VI	− 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE II

Sun's Longitude		Half Interval between the Obſervations						
		H M	H M	H M	H M	H M	H M	H M
		3 50	4 0	4 10	4 20	4 30	4 40	4 50
S	D	S	S		S	S	S	S
0	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 33	0 31	0, 30	0, 28	0, 25	0, 23	0, 21
	10	0, 66	0, 62	0 58	0 54	0 50	0 46	0 41
	15	0, 96	0, 91	0, 85	0, 79	0, 73	0 67	0, 60
	20	1, 24	1, 17	1, 10	1, 03	0, 95	0, 87	0, 78
	25	1 49	1, 41	1 32	1 23	1, 14	1, 04	0 9
I	+ 0	1 70	1, 61	1, 51	1, 41	1, 30	1, 18	1, 06
	5	1, 86	1, 76	1, 65	1 54	1 42	1, 29	1 16
	10	1, 98	1 7	1, 76	1 64	1, 51	1, 37	1 23
	15	2, 03	1 9	1, 80	1, 66	1, 55	1, 41	1, 27
	20	2, 03	1, 9	1, 80	1, 68	1, 55	1, 41	1 6
	25	1, 96	1 85	1, 71	1, 62	1, 49	1, 36	1, 22
II	+ 0	1, 83	1, 73	1, 62	1, 51	1, 39	1, 27	1 14
	5	1 64	1, 55	1, 45	1, 35	1, 25	1, 14	1, 02
	10	1, 39	1, 31	1, 23	1 15	1, 06	0, 96	0, 86
	15	1, 09	1, 03	0, 97	0, 90	0, 83	0, 76	0 68
	20	0, 75	0, 71	0, 67	0, 62	0, 57	0, 52	0, 47
	25	0, 38	0, 36	0, 34	0, 32	0 29	0, 26	0, 24
III	− 0	0, 00	0, 00	0, 00	0, 00	0 00	0 00	0, 00
	5	0 38	0, 36	0, 34	0, 31	0, 29	0, 26	0, 24
	10	0 75	0, 71	0 67	0 62	0, 57	0 52	0, 47
	15	1, 08	1, 02	0, 96	0, 90	0, 83	0 76	0, 60
	20	1 38	1, 31	1, 3	1 11	1, 05	0, 96	0, 86
	25	1 63	1, 54	1, 45	1, 5	1, 24	1 13	1 02
IV	− 0	1 82	1, 72	1, 61	1, 50	1, 38	1, 26	1, 13
	5	1, 95	1, 84	1, 73	1, 61	1, 48	1, 35	1, 21
	10	2, 01	1, 90	1 79	1, 67	1, 54	1, 40	1, 6
	15	2, 02	1 91	1, 79	1, 67	1, 54	1 40	1, 6
	0	1, 96	1, 85	1, 74	1, 62	1, 49	1 36	1 22
	25	1, 85	75	1, 64	1 5	1, 41	1, 2[illegible]	1 15
V	− 0	1, 66	1, 59	1, 50	1, 40	1, 29	1, 17	1, 05
	5	1, 48	1, 40	1, 31	1, [illegible]	1, 13	1 03	0 92
	10	1, 3	1, 16	1, 09	1, 02	0, 94	0, 86	0 77
	15	0, 95	0, 90	0, 85	0, 79	0 73	0, 66	0 59
	20	0, 65	0 62	0, 58	0, 54	0, 50	0, 45	0 40
	25	0, 33	0 31	0, 29	0, 27	0 25	0, 23	0, 21
VI	− 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE II

Sun's Longitude		Half Interval between the Observations						
		H M	H M	H M	H M	H M	H M	H M
		5 0	5 10	5 20	5 30	5 40	5 50	6 0
S	D	S	S	S	S	S	S	S
0	+ 0	0, 00	0, 00	0 00	0, 00	0 00	0, 00	0, 00
	5	0 18	0, 16	0, 13	0, 10	0 07	0, 04	0, 00
	10	0 36	0, 31	0, 25	0, 19	0 13	0, 07	0, 00
	15	0, 53	0, 15	0, 37	0, 29	0, 20	0, 10	0, 00
	20	0 68	0, 58	0, 48	0, 37	0 25	0, 13	0, 00
	25	0, 82	0, 70	0, 57	0, 44	0, 30	0, 16	0, 00
I	+ 0	0, 93	0, 80	0, 66	0, 51	0, 35	0, 18	0, 00
	5	1, 0	0, 87	0, 71	0, 55	0, 38	0, 19	0, 00
	10	1, 08	0, 92	0, 76	0, 59	0, 41	0, 21	0, 00
	15	1, 12	0, 95	0, 78	0, 60	0 41	0, 21	0, 00
	20	1, 11	0, 95	0 76	0, 60	0, 41	0, 21	0, 00
	25	1 07	0, 91	0, 75	0, 58	0, 40	0, 20	0, 00
II	+ 0	1, 00	0 65	0 70	0, 54	0, 37	0 19	0, 00
	5	0, 90	0 [illegible]7	0 63	0, 48	0, 33	0 17	0 00
	10	0 76	0 65	0, 53	0 41	0 28	0, 14	0, 00
	15	0, 60	0 51	[illegible]	0, 32	[illegible]	0, 11	0, 00
	0	0 41	0 35	0, 9	[illegible]	0, 15	0 08	0, 00
	25	0 21	0, 18	0, 15	0, 11	0, 08	0, 04	0, 00
III	− 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0 21	0, 18	0, 15	0 11	0, 06	0, 04	0 00
	10	0, 41	0, 35	0, 29	0, 22	0, 15	0, 08	0, 00
	15	0 60	0, 51	0, 42	0, 32	[illegible]	0, 11	0 00
	0	0, 76	0, 65	0, 53	0, 41	0, 28	0, 14	0 00
	25	0, 90	0, 77	0, 63	0, 48	0 33	0, 17	0 00
IV	− 0	0, 99	0, 85	0, 70	0, 54	0, 37	0, 19	0, 00
	5	1, 06	0, 91	0, [illegible]5	0, 58	0 40	0, 20	0, 00
	10	1, 11	0, 95	0, 78	0, 60	0 41	0, 21	0, 00
	1[illegible]	1 11	0, 95	0, 78	0 60	0 41	0, 21	0, 00
	20	1, 10	0, 92	0, 76	0 56	0, 39	0, 20	0, 00
	25	1, 01	0 86	0 71	0 55	0 38	0, 19	0, 00
V	0	0 92	0, 79	0, 65	0, 50	0 34	0, 17	0, 00
	5	0, 81	0 60	0, 57	0, 44	0 30	0, 15	0, 00
	0	0 68	0, 58	0 47	0, 36	0, 25	0 13	0, 00
	[illegible]	0, 52	0, 45	0 [illegible]7	0, 28	0, 19	0 10	0, 00
	[illegible]	0, 35	0, 30	0 25	0, 19	0, 13	0, 07	0, 00
	5	0, 18	0, 15	0, 13	0, 10	0, 07	0, 04	0, 00
VI	− 0	0, 00	0 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE II

Sun's Longitude		Half Interval between the Obſervations						
		H M	H M	H M	H M	H M	H M	H M
		1 30	1 40	1 50	2 0	2 10	2 20	2 30
S	D	S	S	S	S	S	S	S
VI	+ 0	0, 00	0, 00	0 00	0, 00	0 00	0, 00	0, 00
	5	0, 49	0, 48	0, 48	0, 47	0 46	0, 45	0 44
	10	0, 97	0, 96	0, 95	0 93	0, 91	0, 89	0, 87
	15	1, 43	1, 42	1, 40	1 37	1 35	1, 32	1, [illegible]9
	20	1 84	1 84	1, 81	1 78	1 75	1, 72	1, 66
	25	2 [illegible]5	[illegible], 22	2, 19	2, 15	2, 11	2, 07	2, 03
VII	+ 0	2 58	55	2, 51	2, 47	2, 43	2 38	2 33
	5	2, 85	2, 81	2 77	2, 7[illegible]	2, 67	2, 62	2, 50
	10	3, 04	3 00	2, 95	, 90	2, 85	[illegible], 79	2, 73
	15	3, 14	3, 10	3, 05	3, 00	2, 94	2, 68	2, 8[illegible]
	20	3, 14	3, 10	3 06	3, 01	2, 95	2, 89	2, 83
	25	3, 05	3, 01	2 97	2, 92	2, 87	2, 81	2, 75
VIII	+ 0	2, 86	2, 82	2, 78	2, 73	2, 68	2, 63	2, 57
	5	2 57	2, 53	[illegible], 49	2 45	2, 41	2, 36	2, 31
	10	2, 18	2, 15	2, 1[illegible]	2, 08	2, 05	2, 01	1, 96
	15	1, 71	1, 69	1, 67	1 64	1, 61	1 58	1, 54
	20	1, 18	1, 17	1, 15	1, 13	1, 11	1, 09	1, 06
	25	0, 60	0, 60	0, 59	0 58	0, 57	0, 55	0 54
IX	− 0	0, 00	0, 00	0, 00	0, 00	0, 00	0 00	0, 00
	5	0, 60	0 60	0, 59	0 58	0, 57	0 55	0, 54
	10	1, 18	1, 17	1, 15	1, 13	1 11	1 09	1, 06
	15	1, 7[illegible]	1, 70	1, 67	1, 64	1 61	1, 56	1 54
	20	2, 19	2, 16	2, 13	2, 09	2, 05	[illegible] 01	1, 97
	25	2, 58	54	2, 50	[illegible], 16	[illegible], 42	2, [illegible]7	2, 32
X	− 0	2, 87	2, 83	2, 79	2, 79	[illegible]	[illegible] 64	2, 59
	5	3, 06	3 03	2, 99	2 94	, 68	2, 6[illegible]	76
	10	3, 16	3, 1[illegible]	3, 08	3, 03	9[illegible]	[illegible]	2 65
	15	3, 16	3, 12	3 07	[illegible], 02	97	[illegible], 91	[illegible], 84
	20	3, 06	3, 02	2, 98	2, 93	2 [illegible]	2, 62	2, 76
	25	2, 37	2, 84	, 80	2, 75	70	, 64	2, 58
XI	− 0	2 60	2, 57	2, 54	, 50	2, 45	[illegible], 40	2, 35
	5	2 [illegible]7	, 4	2, 21	2, 18	2, 14	10	[illegible], 05
	10	1 63	1, 86	1, 63	1, 80	1, 71	1, 73	1 [illegible]9
	15	1 45	1, 43	1, 41	1 39	1, 36	1, [illegible]3	1, 30
	20	0, 99	0, 97	0 96	0, 9[illegible]	0 92	0, 90	0, 66
	25	0 50	0, 49	0 48	0, 47	0, 47	0, 46	0, 15
XII	− 0	0, 00	0, 00	0, 00	0, 00	0, 00	0 00	0, 00

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE II

Sun's Longitude		Half Interval between the Obſervation						
		H M	H M	H M	H M	H M	H M	H M
		2 40	2 50	3 0	3 10	3 20	3 30	3 40
S	D	s	s	s	s	s	s	s
VI	+ 0	0 00	0, 00	0, 00	0, 00	0, 00	0, 00	0 00
	5	0, 43	0 42	0, 41	0, 39	0, 38	0, 36	0 35
	10	0, 85	0, 83	0, 81	0, 78	0, 75	0, 72	0, 69
	15	1, 26	1, 22	1, 19	1, 15	1, 11	1, 06	1, 02
	20	1, 64	1, 59	1, 54	1, 49	1, 44	1, 38	1 32
	25	1, 98	1, 92	1, 86	1, 80	1, 74	1, 67	1, 60
VII	+ 0	2, 27	2, 21	2, 14	2, 07	2 00	1, 92	1, 83
	5	2, 50	2, 43	2, 36	2 28	2, [illegible]0	2 11	2 02
	10	2, 66	2 59	[illegible], 55	2, 43	2 34	[illegible] 25	2, 15
	15	2 75	2 68	2, 60	[illegible], 51	2, 42	2 32	[illegible], 22
	20	2, 76	[illegible] 68	2, 60	2 5[illegible]	2, 43	[illegible], 33	2, [illegible]3
	25	2, 68	2 60	2, 52	2, 44	2 35	[illegible], [illegible]6	2, 16
VIII	+ 0	2, 51	2 44	2 37	2, 29	2, 1	2 12	2, 03
	5	[illegible], 25	2, 19	[illegible]	[illegible], 05	1 98	1, 90	1, 82
	10	1, 91	1, 86	1, 81	1, 75	1, 69	1, 62	1, 55
	15	1, 50	1, 46	1, 42	1, 37	1 32	1, 27	1, 22
	20	1, 04	1, 01	0, 98	0, 95	0, 91	0, 88	0, 84
	25	0, 53	0, 51	0, 50	0 48	0, 47	0, 45	0, 43
IX	− 0	0 00	0 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 53	0, 51	0, 50	0, 48	0, 47	0 45	0, 43
	10	1, 04	1, 01	0, 98	0, 95	0, 91	0, 88	0, 84
	15	1, 51	1, 47	1 [illegible]	[illegible], 38	1, 33	1, 27	1, 22
	20	1, 92	1, 87	1, 81	1, 75	1, 69	1, 6[illegible]	1, 55
	25	2, 26	2, 20	2, 13	2, 06	1, 99	1, 91	1 83
X	− 0	2, 52	2, 45	2, 38	2, 30	2, 22	2, 13	2, 04
	5	2, 69	2, 62	2, 54	2, 46	[illegible], 37	[illegible] 28	2, 18
	10	2, 78	2, 70	2, 62	2, 53	[illegible], 44	2, 34	2, 24
	15	2, 77	2, 70	2, 62	2, 53	2, 41	2, 34	2, 24
	20	2 69	2, 61	2, 53	2, 45	2, 36	2, 27	2, 17
	25	2, 52	2, 45	2, 38	[illegible], 30	2, 22	2, 13	2, 04
XI	− 0	2, 29	2, 23	2, 16	2, 09	2, 01	1 93	1, 8[illegible]
	5	2, 00	1, 94	1 88	1, 82	1, 76	1 69	1, 61
	10	1, 65	1, 61	1, 56	1, 51	1, 46	1, 40	1, 34
	15	1, 27	1, 24	1, 20	1 16	1, 12	1, 08	1, 03
	20	0, 86	0, 84	0, 81	0, 79	0 76	0, 73	0, 70
	25	0, 43	0, 4	0, 41	0, 40	0 38	0 37	0, 35
XII	− 0	0, 00	0, 00	0, 00	0 00	0, 00	0, 00	0, 00

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE II

Sun's Longitude		Half Interval between the Obſervations						
		H M	H M	H M	H M	H M	H M	H M
		3 50	4 0	4 10	4 20	4 30	4 40	4 50
S	D	s	s	s	s	s	s	s
VI	+ 0	0, 00	0, 00	0 00	0, 00	0, 00	0, 00	0, 00
	5	0, 33	0, 31	0, 29	0, 27	0, 25	0, 23	0, 21
	10	0, 66	0, 62	0 58	0, 54	0 50	0, 46	0 41
	15	0, 97	0, 91	0, 86	0, 80	0, 74	0, 67	0 60
	20	1, 26	1, 19	1, 12	1, 04	0 96	0, 87	0 78
	25	1, 52	1, 44	1, 35	1, 26	1, 16	1, 06	0, 95
VII	+ 0	1, 74	1, 65	1, 55	1, 44	1, 33	1, 21	1, 09
	5	1, 92	1, 82	1 71	1, 59	1, 47	1, 34	1, 20
	10	2, 05	1, 94	1, 82	1, 69	1, 56	1, 42	1, 28
	15	2, 11	2, 00	1, 88	1, 75	1, 61	1, 47	1 32
	20	2, 12	2, 00	1, 88	1, 75	1, 62	1, 47	1, 3
	25	2, 06	1, 95	1, 83	1, 70	1, 57	1, 43	1, 28
VIII	+ 0	1, 93	1, 82	1, 71	1, 59	1, 47	1, 34	1, 20
	5	1, 73	1, 63	1, 53	1, 43	1, 32	1, 20	1, 08
	10	1, 47	1, 39	1, 31	1, 22	1, 12	1, 02	0, 92
	15	1, 16	1, 09	1, 02	0 96	0, 88	0, 80	0 72
	20	0 80	0, 75	0, 71	0, 66	0, 61	0, 55	0 50
	25	0, 41	0 38	0, 36	0, 34	0, 31	0 28	0, 25
IX	− 0	0, 00	0 00	0, 00	0, 00	0 00	0, 00	0 00
	5	0, 41	0 38	0 36	0, 34	0 31	0, 28	0, 25
	10	0, 80	0, 75	0 71	0 66	0, 61	0, 55	0, 50
	15	1, 16	1, 10	1, 03	0, 96	0, 88	0, 80	0, 72
	20	1, 47	1, 39	1 31	1, 22	1, 13	1, 03	0 92
	25	1, 74	1, 64	1, 54	1, 44	1, 33	1, 21	1, 08
X	− 0	1, 94	1, 83	1, 72	1, 60	1, 48	1, 35	1, 1
	5	2, 07	1, 96	1 84	1, 71	1, 58	1, 44	1, 29
	10	2, 13	2, 0	1, 90	1 77	1, 63	1 48	1, 33
	15	2, 13	2, 01	1, 89	1, 76	1, 62	1 48	1 33
	20	2, 06	1, 95	1, 83	1, 71	1, 58	1, 44	1, 9
	25	1, 94	1, 83	1, 72	1, 60	1, 48	1 35	1 1
XI	− 0	1 76	1, 66	1, 56	1, 45	1, 34	1, 22	1 09
	5	1 53	1, 45	1, 36	1, 27	1 17	1, 07	0 96
	10	1 27	1 0	1, 13	1, 05	0, 97	0 88	0 79
	15	0 98	0, 93	0, 87	0 81	0, 75	0 68	0, 61
	20	0 66	0 63	0, 59	0, 55	0 51	0, 46	0, 41
	25	0, 33	0 32	0, 30	0, 23	0, 6	0 23	0 21
XII	− 0	0 00	0, 00	0, 00	0, 00	0 00	0, 00	0 00

TABLE XLIII *Continued*

Equations to Equal Altitudes TABLE II

Sun's Longitude		Half Interval between the Obſervations						
		H M	H M	H M	H M	H M	H M	H M
		5 0	5 10	5 20	5 30	5 40	5 50	6 0
S	D	S	S	S	S	S	S	S
VI	+ 0	0, 00	0, 00	0, 00	0 00	0 00	0 00	0, 00
	5	0, 18	0, 15	0, 13	0, 10	0, 07	0, 04	0 00
	10	0, 36	0, 31	0, 25	0, 19	0 13	0 07	0, 00
	15	0, 53	0, 45	0, 37	0 29	0, 20	0, 10	0, 00
	20	0, 69	0, 59	0, 48	0, 37	0, 26	0 13	0, 00
	25	0, 63	0, 71	0, 58	0, 45	0, 31	0, 16	0 00
VII	+ 0	0, 96	0, 82	0, 67	0, 52	0, 35	0 18	0, 00
	5	1, 05	0, 90	0, 74	0 57	0 39	0 20	0, 00
	10	1, 12	0, 96	0, 79	0 61	0, 42	0 21	0 00
	15	1 16	0, 99	0, 81	0 63	0, 43	0 22	0, 00
	20	1, 16	0, 99	0 82	0 63	0 43	0, 22	0, 00
	25	1, 13	0, 96	0, 79	0 61	0 4[illegible]	0, [illegible]1	0, 00
VIII	+ 0	1, 05	0 90	0 74	0 57	0, 39	0, 0	0, 00
	5	0 95	0, 81	0 66	0 51	0 35	0, 18	0, 00
	10	0, 81	0, 69	0, 57	0 44	0 30	0, 15	0, 00
	15	0 63	0, 54	0, 44	0, 34	0, 23	0, 12	0, 00
	0	0 44	0, 37	0, 31	0, 24	0 16	0 08	0, 00
	25	0, 22	0, 19	0, 16	0, 1[illegible]	0 08	0, 04	0, 00
IX	− 0	0 00	0 00	0, 00	0, 00	0 00	0, 00	0, 00
	5	0, 22	0, 19	0 16	0, 12	0 08	0 04	0, 00
	10	0, 44	0, 37	0, 31	0, [illegible]4	0 16	0 08	0, 00
	15	0, 64	0, 55	0, 45	0, 35	0 24	0 12	0, 00
	20	0, 81	0, 69	0, 57	0, 41	0, 30	0, 15	0, 00
	25	0, 95	0, 81	0, 67	0, 51	0, 35	0 18	0, 00
X	− 0	1, 06	0, 91	0, 75	0, 57	0, 39	0, 20	0, 00
	5	1, 13	0, 97	0, 80	0, 61	0, 4[illegible]	0 22	0 00
	10	1, 17	1, 00	0, 82	0 63	0, 43	0, [illegible]2	0, 00
	15	1, 17	1, 00	0, 82	0, 63	0, 43	0, 22	0, 00
	20	1 13	0, 96	0, 79	0, 61	0, 42	0 21	0, 00
	25	1, 06	0, 91	0, 75	0, 57	0, 39	0, 20	0, 00
XI	− 0	0, 96	0, 82	0, 67	0, 52	0, 36	0, 18	0 00
	5	0, 84	0, 72	0, 59	0, 45	0 31	0, 16	0 00
	10	0, 70	0, 60	0, 49	0 38	0 26	0, 13	0, 00
	15	0, 54	0, 46	0, 38	0 29	0, 20	0 10	0 00
	0	0, 36	0, 31	0, 25	0 19	0, 13	0, 07	0, 00
	25	0, 18	0, 16	0, 13	0, 10	0, 07	0, 04	0, 00
XII	− 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLIV

Equations to Equal Altitudes, where extreme Accuracy is not required

N or S Declination of Sun	Latitude N or S 20						Latitude N or S 30						Latitude N or S 40						
	Hours between Observations						Hours between Observations						Hours between Observations						
	9	8	7	6	5	4	9	8	7	6	5	4	10	9	8	7	6	5	4
Deg	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec
23	1	0	0	0	0	0	2	1	1	0	0	0	3	3	2	2	2	1	0
22	1	1	0	0	0	0	3	2	2	1	1	0	5	4	4	4	3	2	2
21	2	1	1	1	0	0	4	3	3	2	2	1	7	6	5	5	4	3	3
20	2	2	1	1	1	0	4	4	3	3	2	2	8	7	6	6	5	5	4
19	3	2	2	1	1	1	5	4	4	3	3	2	9	8	7	7	6	5	5
18	3	2	2	1	1	1	5	5	4	4	3	3	10	9	8	8	6	6	5
17	4	3	2	2	2	1	6	5	5	4	4	3		10	9	8	7	7	6
16	4	3	3	2	2	1	6	5	5	5	4	4	11	10	9	9	8	7	7
15	4	3	3	3	2	2	7	6	5	5	5	4	12	11	10	9	8	8	7
14	4	4	3	3	2	2	7	6	6	5	5	4	12	11	10	10	9	8	8
13	5	4	3	3	3	2	7	7	6	6	5	5	13	12	11	10	9	9	8
12	5	4	4	3	3	2	7	7	6	6	6	5	13	12	11	11	10	9	9
11	5	4	4	4	3	3	8	7	7	6	6	5	14	13	12	11	10	10	9
10	5	5	4	4	3	3	8	8	7	7	6	5	14	13	12	11	11	10	10
9	5	5	4	4	4	3	8	8	7	7	6	6	15	14	12	12	11	10	10
8	5	5	4	4	4	3	9	8	8	7	7	6	15	14	13	12	11	11	10
7	6	5	5	5	4	4	9	8	8	7	7	6	16	14	13	12	12	11	11
6	6	5	5	5	4	4	9	9	8	8	7	6	16	15	13	13	12	11	11
5	6	6	5	5	4	4	9	9	8	8	7	7	10	15	14	13	12	12	11
4	6	6	5	5	5	4	10	9	9	8	8	7	17	15	14	13	13	12	12
3	6	6	6	5	5	5	10	9	9	8	8	7	17	15	14	14	13	12	12
2	6	6	6	5	5	5	10	9	9	9	8	8	17	16	14	14	13	13	12
1	7	6	6	5	5	5	10	10	9	9	9	8	17	16	15	14	14	13	12
0	7	6	6	6	5	5	10	10	10	9	9	8	17	16	15	14	14	13	13

TABLE XLIV *Continued*

N or S Declination of Sun	Latitude N or S 50							Latitude N or S 60						
	Hours between Observation							Hours between Observations						
	10	9	8	7	6	5	4	10	9	8	7	6	5	4
Deg	Sec	Sec	Se	Sec	Sec	Sec	Sec	Sec	Sec	S c	Sec.	S	S	S
23	4	4	3	3	3	2	2	7	6	6	5	5	5	4
22	8	7	7	6	5	5	5	12	11	10	9	8	8	8
21	10	9	8	8	7	6	6	15	14	13	13	12	11	10
20	11	11	10	9	8	8	7	18	16	15	14	14	13	12
19	13	12	11	10	9	9	8	20	18	17	16	15	14	13
18	14	13	12	11	10	10	9	21	20	18	17	16	15	15
17	15	14	13	12	11	11	10	23	21	20	19	17	17	16
16	16	15	14	13	[illegible]	12	1	24	23	21	20	18	18	17
15	17	16	15	13	13	12	[illegible]	26	24	22	21	19	19	18
14	18	7	16	14	13	13	12	—	26	23	[illegible]	20	20	19
13	19	18	17	15	14	14	13	—	27	24	23	21	21	20
12	20	19	[illegible]	16	15	14	14	—	28	25	23	22	21	21
11	20	19	18	16	15	15	14	—	30	26	24	23	22	22
10	21	20	18	17	16	15	15	—	31	27	25	24	23	22
9	21	20	19	17	6	15	15	—	—	28	26	25	23	23
8	21	21	19	18	17	16	16	—	—	28	27	26	24	23
7	[illegible]	21	20	18	17	16	16	—	—	29	27	26	25	24
6	22	22	20	19	17	17	16	—	—	29	28	27	25	24
5	23	[illegible]	20	19	18	17	17	—	—	29	28	27	26	25
4	23	[illegible]	20	19	18	17	17	—	—	30	29	27	26	25
3	23	[illegible]	21	20	18	18	17	—	—	30	29	28	27	26
2	23	[illegible]	21	20	19	18	18	—	—	30	29	28	27	26
1	23	22	21	20	19	19	8	—	—	31	29	28	27	[illegible]
0	[illegible]	[illegible]	21	21	20	19	18	—	—	31	29	28	28	[illegible]

TABLE XLIV *Continued*

S or N Declination of Sun	Latitude N or S 20						Latitude N or S 30						Latitude N or S 40						
	Hours between Observations						Hours between Observations						Hours between Observations						
	9	8	7	6	5	4	9	8	7	6	5	4	10	9	8	7	6	5	4
D°	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec
0	7	6	6	6	5	5	10	10	10	9	9	8	—	16	15	14	14	13	13
1	7	6	6	6	6	6	10	10	10	9	9	9	—	16	15	14	14	13	13
2	7	7	7	6	6	6	11	10	10	9	9	9	—	16	15	15	14	14	13
3	7	7	7	6	6	6	11	10	10	10	9	9	—	16	15	15	14	14	13
4	7	7	7	6	6	6	11	10	10	10	10	9	—	16	15	15	14	14	13
5	7	7	7	6	7	6	11	10	10	10	10	10	—	16	16	15	15	14	14
6	—	7	7	7	7	7	—	10	10	10	10	10	—	—	16	15	15	14	14
7	—	7	7	7	7	7	—	10	10	10	10	10	—	—	16	16	15	14	14
8	—	7	7	7	7	7	—	10	10	10	10	10	—	—	16	16	15	14	14
9	—	7	7	7	7	7	—	10	10	10	10	10	—	—	16	15	15	14	14
10	—	8	8	7	7	7	—	11	10	10	10	10	—	—	16	15	15	14	14
11	—	7	7	8	7	7	—	11	11	10	10	10	—	—	15	15	14	14	14
12	—	7	7	7	7	7	—	10	11	10	10	10	—	—	15	15	14	14	14
13	—	7	7	7	7	7	—	10	10	10	10	9	—	—	15	14	14	14	14
14	—	7	7	7	7	7	—	10	10	10	9	9	—	—	15	14	14	14	14
15	—	7	7	7	7	7	—	10	10	9	9	9	—	—	14	14	14	14	15
16	—	—	6	6	6	6	—	—	9	9	9	9	—	—	—	15	13	15	15
17	—	—	6	6	6	6	—	—	9	9	9	8	—	—	—	13	12	12	12
18	—	—	6	6	6	6	—	—	9	9	8	8	—	—	—	12	12	12	12
19	—	—	6	6	6	6	—	—	8	8	8	8	—	—	—	11	11	11	11
20	—	—	5	5	5	5	—	—	8	8	7	7	—	—	—	10	10	10	10
21	—	—	5	5	5	5	—	—	7	7	7	7	—	—	—	—	9	9	9
22	—	—	4	4	4	4	—	—	6	6	6	5	—	—	—	—	7	7	7
23	—	—	2	2	2	2	—	—	3	3	3	3	—	—	—	—	4	4	4

TABLE XLIV *Continued*

S or N Declination of Sun	Latitude N or S 50							Latitude N or S 60						
	Hours between Observations							Hours between Observations						
	10	9	8	7	6	5	4	10	9	8	7	6	5	4
Deg	—	Sec	Sec	Sec	Sec	Sec	Sec	—	—	—	Sec	Sec	Sec	Sec
0	—	22	21	21	20	19	18	—	—	—	29	28	28	27
1	—	22	21	21	20	19	18	—	—	—	29	29	28	27
2	—	22	22	21	20	19	18	—	—	—	30	29	28	27
3	—	—	22	21	20	19	18	—	—	—	30	29	28	27
4	—	—	22	21	20	19	19	—	—	—	30	29	29	28
5	—	—	22	21	20	19	19	—	—	—	30	29	29	28
6	—	—	—	21	20	20	19	—	—	—	—	29	29	28
7	—	—	—	21	20	20	19	—	—	—	—	29	28	27
8	—	—	—	21	20	20	19	—	—	—	—	29	28	27
9	—	—	—	21	20	20	19	—	—	—	—	28	28	27
10	—	—	—	—	20	19	19	—	—	—	—	28	28	27
11	—	—	—	—	19	19	18	—	—	—	—	—	27	27
12	—	—	—	—	19	19	18	—	—	—	—	—	27	26
13	—	—	—	—	19	19	18	—	—	—	—	—	26	26
14	—	—	—	—	18	18	17	—	—	—	—	—	25	25
15	—	—	—	—	18	18	17	—	—	—	—	—	—	24
16	—	—	—	—	17	17	17	—	—	—	—	—	—	23
17	—	—	—	—	16	16	16	—	—	—	—	—	—	22
18	—	—	—	—	15	15	15	—	—	—	—	—	—	21
19	—	—	—	—	14	14	14	—	—	—	—	—	—	19
20	—	—	—	—	13	13	13	—	—	—	—	—	—	17
21	—	—	—	—	—	11	11	—	—	—	—	—	—	14
22	—	—	—	—	—	9	9	—	—	—	—	—	—	11
23	—	—	—	—	—	5	5	—	—	—	—	—	—	7

TABLE XLV

Semi Diurnal Arcs

Latitude and Declination of the *same* kind

DECLI NATION	LATITUDE																	
	1		2		3		4		5		6		7		8		9	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	[illegible]	6	2	6	2	6	2	6	2	6	3	6	3	6	3	6	3
2	6	2	6	2	6	3	6	3	6	3	6	3	6	3	6	3	6	3
3	6	[illegible]	6	3	6	3	6	3	6	3	6	3	6	4	6	4	6	4
4	6	[illegible]	6	3	6	3	6	3	6	4	6	4	6	4	6	4	6	5
5	6	2	6	3	6	3	6	4	6	4	6	4	6	5	6	5	6	5
6	6	3	6	3	6	3	6	4	6	4	6	4	6	5	6	6	6	6
7	6	3	6	3	6	4	6	4	6	5	6	5	6	6	6	6	6	7
8	6	3	6	3	6	4	6	4	6	5	6	5	6	6	6	7	6	7
9	6	3	6	3	6	4	6	5	6	5	6	6	6	7	6	7	6	8
10	6	3	6	4	6	4	6	5	6	6	6	6	6	7	6	8	6	9
11	6	3	6	4	6	4	6	5	6	6	6	7	6	8	6	8	6	9
12	6	3	6	4	6	5	6	6	6	6	6	7	6	8	6	9	6	10
13	6	3	6	4	6	5	6	6	6	7	6	8	6	9	6	10	6	11
14	6	3	6	4	6	5	6	6	6	7	6	8	6	9	6	10	6	11
15	6	3	6	4	6	5	6	6	6	8	6	9	6	10	6	11	6	12
16	6	3	6	4	6	6	6	7	6	8	6	9	6	10	6	11	6	13
17	6	3	6	5	6	6	6	7	6	8	6	10	6	11	6	12	6	13
18	6	4	6	5	6	6	6	7	6	9	6	10	6	11	6	13	6	14
19	6	4	6	5	6	6	6	8	6	9	6	11	6	12	6	13	6	15
20	6	4	6	5	6	7	6	8	6	9	6	11	6	12	6	14	6	15
21	6	4	6	5	6	7	6	8	6	10	6	12	6	13	6	15	6	16
22	6	4	6	6	6	7	6	8	6	10	6	12	6	14	6	15	6	17
23	6	4	6	6	6	7	6	9	6	11	6	13	6	14	6	16	6	18
24	6	4	6	6	6	8	6	9	6	11	6	13	6	15	6	17	6	19
25	6	4	6	6	6	8	6	10	6	12	6	14	6	15	6	17	6	19
26	6	4	6	6	6	8	6	10	6	12	6	14	6	16	6	18	6	20
27	6	4	6	6	6	8	6	11	6	13	6	15	6	17	6	19	6	2
28	6	5	6	7	6	9	6	11	6	13	6	15	6	17	6	20	6	22
29	6	5	6	7	6	9	6	11	6	14	6	16	6	18	6	20	6	23
30	6	5	6	7	6	9	6	12	6	14	6	16	6	19	6	21	6	23
31	6	5	6	7	6	10	6	12	6	15	6	17	6	19	6	22	6	24
32	6	5	6	7	6	10	6	12	6	15	6	18	6	20	6	23	6	25

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of the *ſame* kind

DECLI NATION	LATITUDE																	
	10		11		12		13		14		15		16		17		18	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	3	6	3	6	3	6	3	6	3	6	3	6	3	6	3	6	4
2	6	4	6	4	6	4	6	4	6	4	6	4	6	4	6	5	6	5
3	6	4	6	4	6	5	6	5	6	5	6	5	6	6	6	6	6	6
4	6	5	6	5	6	6	6	6	6	6	6	6	6	7	6	7	6	7
5	6	6	6	6	6	6	6	7	6	7	6	8	6	8	6	8	6	9
6	6	6	6	7	6	7	6	8	6	8	6	9	6	9	6	10	6	10
7	6	7	6	8	6	8	6	9	6	9	6	10	6	10	6	11	6	11
8	6	8	6	8	6	9	6	10	6	10	6	11	6	11	6	12	6	13
9	6	9	6	9	6	10	6	11	6	11	6	12	6	13	6	13	6	14
10	6	9	6	10	6	11	6	12	6	12	6	13	6	14	6	15	6	15
11	6	10	6	11	6	1	6	13	6	13	6	14	6	15	6	16	6	17
12	6	11	6	12	6	13	6	14	6	14	6	15	6	16	6	17	6	18
13	6	12	6	12	6	13	6	15	6	15	6	16	6	17	6	18	6	19
14	6	12	6	13	6	14	6	15	6	16	6	18	6	19	6	20	6	21
15	6	13	6	14	6	15	6	16	6	18	6	19	6	20	6	21	6	22
16	6	14	6	15	6	16	6	17	6	19	6	0	6	21	6	22	6	24
17	6	15	6	16	6	17	6	18	6	20	6	21	6	22	6	24	6	25
18	6	15	6	17	6	18	6	19	6	21	6	22	6	24	6	25	6	27
19	6	16	6	18	6	19	6	21	6	2	6	23	6	25	6	27	6	28
20	6	17	6	19	6	20	6	22	6	23	6	25	6	26	6	28	6	30
21	6	18	6	19	6	21	6	23	6	24	6	26	6	28	6	29	6	31
22	6	19	6	20	6	2	6	24	6	25	6	27	6	29	6	31	6	33
23	6	19	6	21	6	23	6	25	6	27	6	28	6	30	6	32	6	34
24	6	20	6	22	6	24	6	26	6	28	6	30	6	32	6	34	6	36
25	6	21	6	23	6	25	6	27	6	29	6	31	6	33	6	35	6	37
26	6	22	6	24	6	26	6	28	6	30	6	32	6	35	6	37	6	39
27	6	23	6	25	6	27	6	29	6	32	6	34	6	36	6	38	6	41
28	6	24	6	26	6	8	6	31	6	33	6	35	6	38	6	40	6	42
29	6	25	6	27	6	30	6	32	6	34	6	37	6	39	6	42	6	44
30	6	26	6	28	6	31	6	33	6	36	6	38	6	41	6	45	6	46
31	6	27	6	29	6	32	6	34	6	37	6	40	6	42	6	45	6	48
32	6	28	6	30	6	33	6	36	6	38	6	41	6	44	6	47	6	50

TABLE XIV *Continued*

Semi Diurnal Arcs

Latitude and Declination of the *ſame* kind

DECLINATION	LATITUDE															
	19		20		21		22		23		24		25		26	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	4	6	4	6	4	6	4	6	4	6	4	6	4	6	4
2	6	5	6	5	6	5	6	6	6	6	6	6	6	6	6	6
3	6	6	6	7	6	7	6	7	6	7	6	8	6	8	6	8
4	6	8	6	8	6	8	6	8	6	9	6	9	6	10	6	10
5	6	9	6	9	6	10	6	10	6	11	6	11	6	12	6	12
6	6	11	6	11	6	12	6	12	6	13	6	13	6	14	6	14
7	6	12	6	12	6	13	6	14	6	14	6	15	6	15	6	16
8	6	13	6	14	6	15	6	15	6	16	6	17	6	17	6	18
9	6	15	6	15	6	16	6	17	6	18	6	19	6	19	6	20
10	6	16	6	17	6	18	6	19	6	19	6	20	6	21	6	22
11	6	18	6	19	6	19	6	20	6	21	6	22	6	23	6	24
12	6	19	6	20	6	21	6	22	6	23	6	24	6	25	6	26
13	6	21	6	22	6	23	6	24	6	25	6	26	6	27	6	28
14	6	22	6	23	6	24	6	25	6	27	6	28	6	29	6	30
15	6	23	6	25	6	26	6	27	6	28	6	30	6	31	6	32
16	6	25	6	26	6	28	6	29	6	30	6	32	6	33	6	35
17	6	27	6	28	6	29	6	31	6	32	6	34	6	35	6	37
18	6	28	6	30	6	31	6	33	6	34	6	36	6	37	6	39
19	6	30	6	31	6	33	6	34	6	36	6	38	6	39	6	41
20	6	31	6	33	6	35	6	36	6	38	6	40	6	42	6	43
21	6	33	6	35	6	36	6	38	6	40	6	42	6	44	6	46
22	6	34	6	36	6	38	6	40	6	42	6	44	6	46	6	48
23	6	36	6	38	6	40	6	42	6	44	6	46	6	48	6	50
24	6	38	6	40	6	42	6	44	6	46	6	48	6	51	6	53
25	6	39	6	42	6	44	6	46	6	48	6	51	6	53	6	55
26	6	41	6	43	6	46	6	48	6	50	6	53	6	55	6	58
27	6	43	6	45	6	48	6	50	6	53	6	55	6	58	7	0
28	6	45	6	47	6	50	6	52	6	55	6	57	7	0	7	3
29	6	47	6	49	6	52	6	54	6	57	7	0	7	3	7	6
30	6	48	6	51	6	54	6	57	6	59	7	2	7	5	7	8
31	6	50	6	53	6	56	6	59	7	2	7	5	7	8	7	1
32	6	52	6	55	6	58	7	1	7	4	7	7	7	11	7	14

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of the *ſame* kind

DECLINATION	LATITUDE															
	27°		28		29		30		31		32		33		34	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	4	6	5	6	5	6	5	6	5	6	5	6	5	6	5
2	6	6	6	7	6	7	6	7	6	7	6	7	6	8	6	8
3	6	8	6	9	6	9	6	9	6	10	6	10	6	10	6	11
4	6	11	6	11	6	11	6	11	6	12	6	12	6	13	6	13
5	6	13	6	13	6	14	6	14	6	15	6	15	6	16	6	16
6	6	15	6	15	6	16	6	16	6	17	6	18	6	18	6	19
7	6	17	6	17	6	18	6	19	6	19	6	20	6	21	6	22
8	6	19	6	20	6	20	6	21	6	22	6	23	6	23	6	24
9	6	21	6	22	6	23	6	23	6	24	6	25	6	26	6	27
10	6	23	6	24	6	25	6	26	6	27	6	28	6	29	6	30
11	6	25	6	26	6	27	6	28	6	29	6	30	6	32	6	33
12	6	27	6	28	6	30	6	31	6	32	6	33	6	34	6	36
13	6	30	6	31	6	32	6	33	6	34	6	36	6	37	6	38
14	6	32	6	33	6	34	6	36	6	37	6	38	6	40	6	41
15	6	34	6	35	6	37	6	38	6	40	6	41	6	43	6	44
16	6	36	6	38	6	39	6	41	6	42	6	44	6	46	6	47
17	6	38	6	40	6	42	6	43	6	45	6	47	6	48	6	50
18	6	41	6	42	6	44	6	46	6	48	6	50	6	51	6	53
19	6	43	6	45	6	47	6	48	6	50	6	52	6	54	6	56
20	6	45	6	47	6	49	6	51	6	53	6	55	6	57	7	0
21	6	48	6	50	6	52	6	54	6	56	6	58	7	0	7	3
22	6	50	6	52	6	54	6	57	6	59	7	1	7	4	7	6
23	6	53	6	55	6	57	6	59	7	2	7	4	7	7	7	9
24	6	55	6	57	7	0	7	2	7	5	7	7	7	10	7	13
25	6	58	7	0	7	3	7	5	7	8	7	11	7	13	7	16
26	7	0	7	3	7	6	7	8	7	11	7	14	7	17	7	20
27	7	3	7	6	7	8	7	11	7	14	7	17	7	20	7	23
28	7	6	7	8	7	11	7	14	7	17	7	21	7	24	7	27
29	7	9	7	11	7	14	7	18	7	21	7	24	7	28	7	31
30	7	11	7	14	7	18	7	21	7	24	7	28	7	31	7	35
31	7	14	7	17	7	21	7	24	7	28	7	31	7	35	7	39
32	7	17	7	21	7	24	7	28	7	31	7	35	7	39	7	43

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of the *ſame* kind

DECLINATION	LATITUDE															
	35		36		37		38°		39°		40°		41°		42	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	5	6	6	6	6	6	6	6	6	6	6	6	6	6	6
2	6	8	6	8	6	9	6	9	6	9	6	9	6	10	6	10
3	6	11	6	11	6	12	6	12	6	12	6	13	6	13	6	14
4	6	14	6	14	6	15	6	15	6	16	6	16	6	17	6	17
5	6	17	6	17	6	18	6	18	6	19	6	20	6	20	6	21
6	6	19	6	20	6	21	6	22	6	22	6	23	6	24	6	25
7	6	22	6	24	6	24	6	25	6	26	6	26	6	27	6	28
8	6	25	6	26	6	27	6	28	6	29	6	30	6	31	6	32
9	6	28	6	29	6	30	6	31	6	32	6	33	6	35	6	36
10	6	31	6	32	6	33	6	34	6	36	6	37	6	38	6	39
11	6	34	6	35	6	36	6	38	6	39	6	40	6	42	6	43
12	6	37	6	38	6	40	6	41	6	42	6	44	6	45	6	47
13	6	40	6	41	6	43	6	44	6	46	6	48	6	49	6	51
14	6	43	6	44	6	46	6	48	6	49	6	51	6	53	6	55
15	6	46	6	48	6	49	6	51	6	53	6	55	6	57	6	59
16	6	49	6	51	6	53	6	55	6	57	6	59	7	1	7	3
17	6	52	6	54	6	56	6	58	7	0	7	2	7	5	7	7
18	6	55	6	57	7	0	7	2	7	4	7	6	7	9	7	11
19	6	59	7	1	7	3	7	5	7	8	7	10	7	13	7	15
20	7	2	7	4	7	7	7	9	7	12	7	14	7	17	7	20
21	7	5	7	8	7	10	7	13	7	15	7	18	7	21	7	24
22	7	9	7	11	7	13	7	17	7	19	7	22	7	25	7	29
23	7	12	7	15	7	18	7	21	7	24	7	27	7	30	7	33
24	7	16	7	19	7	21	7	25	7	28	7	31	7	34	7	38
25	7	10	7	22	7	25	7	29	7	32	7	35	7	39	7	43
26	7	23	7	26	7	29	7	33	7	36	7	40	7	44	7	48
27	7	27	7	30	7	34	7	37	7	41	7	45	7	49	7	53
28	7	31	7	34	7	38	7	42	7	45	7	49	7	54	7	58
29	7	35	7	38	7	4	7	46	7	50	7	54	7	59	8	4
30	7	39	7	43	7	47	7	51	7	55	8	0	8	5	8	9
31	7	43	7	47	7	54	7	56	8	0	8	5	8	10	8	15
32	7	47	7	51	7	56	8	1	8	5	8	10	8	16	8	21

TABLE XLV. *Continued.*

Semi Diurnal Arcs

Latitude and Declination of the *same* kind

DECLINATION	LATITUDE															
	43		44		45°		46°		47		48		49		50°	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	7	6	7	6	7	6	7	6	7	6	8	6	8	6	8
2	6	10	6	11	6	11	6	11	6	12	6	12	6	12	6	13
3	6	14	6	15	6	15	6	15	6	16	6	17	6	17	6	18
4	6	18	6	18	6	19	6	20	6	20	6	21	6	22	6	[illegible]
5	6	22	6	22	6	23	6	24	6	25	6	25	6	26	6	27
6	6	25	6	26	6	27	6	28	6	29	6	30	6	31	6	32
7	6	29	6	30	6	31	6	32	6	33	6	34	6	36	6	37
8	6	33	6	34	6	35	6	37	6	38	6	39	6	41	6	42
9	6	37	6	38	6	40	6	41	6	[illegible]	6	44	6	45	6	47
10	6	41	6	4	6	44	6	45	6	47	6	48	6	50	6	52
11	6	45	6	46	6	46	6	50	6	51	6	53	6	55	6	57
12	6	49	6	50	6	52	6	55	6	56	6	58	7	0	7	2
13	6	53	6	55	6	57	6	59	7	1	7	3	7	5	7	7
14	6	57	6	59	7	1	7	3	7	5	7	8	7	10	7	13
15	7	1	7	3	7	5	7	8	7	10	7	13	7	15	7	18
16	7	5	7	7	7	10	7	12	7	15	7	18	7	21	7	24
17	7	9	7	12	7	14	7	17	7	20	7	23	7	26	7	29
18	7	14	7	16	7	19	7	22	7	25	7	28	7	31	7	35
19	7	18	7	21	7	24	7	27	7	30	7	34	7	37	7	41
20	7	23	7	26	7	29	7	32	7	35	7	39	7	43	7	47
21	7	27	7	30	7	34	7	37	7	41	7	45	7	49	7	53
22	7	32	7	35	7	39	7	43	7	46	7	50	7	55	7	59
23	7	37	7	40	7	44	7	48	7	52	7	56	8	1	8	6
24	7	42	7	45	7	49	7	54	7	58	8	3	8	7	8	12
5	7	47	7	51	7	55	7	59	8	4	8	9	8	14	8	19
6	7	[illegible]	7	56	8	1	8	5	8	10	8	15	8	21	8	27
7	7	57	6	2	6	6	8	12	8	17	8	22	8	28	8	34
28	8	3	8	7	8	[illegible]	8	18	8	[illegible]	8	29	8	35	8	42
29	8	9	8	13	8	19	8	24	8	30	8	37	8	43	8	50
30	8	14	8	20	8	25	8	31	8	38	8	44	8	52	8	59
31	8	20	8	26	6	32	8	38	8	45	8	52	9	0	9	9
3	8	27	8	33	8	39	8	46	8	[illegible]	9	1	9	9	9	19

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of the *ſame* kind

DECLINATION	LATITUDE															
	51		52		53		54		55		56°		57		58	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	8	6	9	6	9	6	9	6	9	6	10	6	10	6	10
2	6	13	6	14	6	14	6	15	6	15	6	16	6	16	6	17
3	6	18	6	19	6	19	6	20	6	21	6	2	6	22	6	23
4	6	22	6	24	6	25	6	26	6	27	6	28	6	29	6	30
5	6	27	6	29	6	30	6	31	6	32	6	34	6	35	6	36
6	6	33	6	34	6	36	6	37	6	38	6	40	6	41	6	43
7	6	38	6	40	6	41	6	43	6	44	6	46	6	48	6	49
8	6	43	6	45	6	47	6	48	6	50	6	52	6	54	6	56
9	6	48	6	50	6	52	6	54	6	56	6	58	7	1	7	3
10	6	54	6	56	6	58	7	0	7	2	7	5	7	7	7	10
11	6	59	7	1	7	3	7	6	7	8	7	11	7	14	7	17
12	7	4	7	7	7	9	7	12	7	15	7	18	7	21	7	24
13	7	10	7	12	7	15	7	18	7	21	7	24	7	28	7	31
14	7	15	7	18	7	21	7	24	7	26	7	31	7	35	7	39
15	7	21	7	24	7	27	7	31	7	34	7	39	7	42	7	46
16	7	27	7	30	7	33	7	37	7	41	7	45	7	49	7	54
17	7	33	7	36	7	40	7	44	7	48	7	52	7	57	8	2
18	7	38	7	42	7	46	7	51	7	55		0	8	5	8	10
19	7	45	7	49	7	53	7	58	8	2	8	7	8	13	8	19
20	7	51	7	55	8	0		5	8	10	8	15	8	21	8	28
1	7	57	8	2	8	7	8	12	8	18	8	24	8	30	8	37
2	8	4	8	9	8	14	8	20	8	26	8	32	8	39	8	47
23	8	11	8	16	8	22	8	28	8	34	8	41	8	49	8	57
24	8	19	8	24	8	30	8	36	8	43	8	51	8	59	9	6
5	8	25	8	31	8	38	8	45	8	53	9	1	9	10	9	20
6	8	33	8	39	8	47	8	54	9	[illegible]	9	11	9	21	9	33
27	8	41	8	48	8	56	9	4	9	13	9	23	9	34	9	46
28	8	49	8	57	9	5	9	14	9	24	9	35	9	48	10	[illegible]
29	8	59	9	6	9	14	9	25	9	0	9	49	10	3	10	20
30	9	8	9	17	9	26	9	38	9	50	10	4	10	21	10	43
31	9	18	9	28	9	38	9	51	10	5	10	[illegible]	10	44	11	17
3	9	26	9	39	9	5	10	6	10	29	10	44	11	17		

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of the *ſame* kind

DECLINATION	LATITUDE															
	59°		60		61		62		63		64°		65		66	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	11	6	11	6	12	6	13	6	13	6	13	6	14	6	14
2	6	17	6	18	6	19	6	20	6	20	6	21	6	22	6	23
3	6	24	6	25	6	26	6	27	6	28	6	30	6	31	6	32
4	6	31	6	32	6	33	6	35	6	36	6	38	6	40	6	41
5	6	38	6	39	6	41	6	42	6	44	6	46	6	48	6	51
6	6	44	6	46	6	48	6	50	6	52	6	55	6	57	7	0
7	6	51	6	53	6	55	6	58	7	1	7	3	7	6	7	10
8	6	58	7	1	7	3	7	6	7	9	7	12	7	15	7	19
9	7	5	7	8	7	11	7	14	7	17	7	21	7	25	7	29
10	7	13	7	16	7	19	7	22	7	26	7	30	7	34	7	39
11	7	20	7	2[illegible]	7	7	7	31	7	35	7	39	7	44	7	49
12	7	27	7	31	7	35	7	39	7	44	7	49	7	54	8	0
13	7	35	7	39	7	43	7	48	7	53	7	59	8	5	8	11
14	7	43	7	47	7	52	7	57	8	3	8	9	8	15	8	23
15	7	51	7	56	8	1	8	6	8	13	8	19	8	27	8	35
16	7	59	8	4	8	10	8	16	8	23	8	30	8	38	8	48
17	8	7	8	13	8	19	8	26	8	34	8	42	8	51	9	1
18	8	16	8	22	8	29	8	37	8	45	8	54	9	4	9	16
19	8	25	8	32	8	40	8	48	8	57	9	7	9	18	9	32
20	8	35	8	42	8	50	8	59	9	10	9	21	9	34	9	49
21	8	45	8	53	9	2	9	12	9	23	9	37	9	51	10	10
22	8	55	9	4	9	14	9	25	9	38	9	53	10	1[illegible]	10	35
23	9	6	9	16	9	27	9	40	9	55	10	13	10	36	11	12
24	9	18	9	29	9	42	9	57	10	15	10	38	11	19		
25	9	31	9	44	9	58	10	16	10	39	11	14				
26	9	45	10	0	10	17	10	40	11	14						
27	10	1	10	18	10	41	11	14								
28	10	19	10	42	11	15										
29	10	42	11	16												
30	11	16														
31																
32																

TABLE XIV *Continued*

Semi Diurnal Arcs

Latitude and Declination of *different* kinds

DECLINATION	LATITUDE																	
	1°		2		3		4		5		6		7		8		9	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	2	6	2	6	2	6	2	6	2	6	2	6	2	6	1	6	1
2	6	2	6	1	6	1	6	1	6	1	6	1	6	1	6	1	6	1
3	6	2	6	1	6	1	6	1	6	1	6	1	6	1	6	0	6	0
4	6	2	6	1	6	1	6	1	6	1	6	0	6	0	6	0	6	0
5	6	2	6	1	6	1	6	1	6	0	6	0	6	0	5	59	5	59
6	6	2	6	1	6	1	6	0	6	0	6	0	5	59	5	59	5	58
7	6	2	6	1	6	1	6	0	6	0	5	59	5	58	5	58	5	58
8	6	1	6	1	6	0	6	0	5	59	5	59	5	58	5	57	5	57
9	6	1	6	1	6	0	6	0	5	59	5	58	5	57	5	56	5	57
10	6	1	6	1	6	0	5	59	5	59	5	58	5	57	5	56	5	56
11	6	1	6	1	6	0	5	59	5	58	5	57	5	57	5	56	5	55
12	6	1	6	1	6	0	5	59	5	58	5	57	5	56	5	55	5	54
13	6	1	6	0	5	59	5	56	5	57	5	57	5	56	5	55	5	54
14	6	1	6	0	5	59	5	58	5	57	5	56	5	55	5	54	5	53
15	6	1	6	0	5	59	5	58	5	57	5	56	5	55	5	54	5	52
16	6	1	6	0	5	59	5	56	5	56	5	55	5	54	5	53	5	52
17	6	1	6	0	5	59	5	57	5	56	5	55	5	53	5	52	5	51
18	6	1	6	0	5	58	5	57	5	56	5	54	5	53	5	52	5	50
19	6	1	5	59	5	56	5	57	5	55	5	54	5	52	5	51	5	50
20	6	1	5	59	5	58	5	56	5	55	5	53	5	52	5	51	5	49
21	6	1	5	59	5	58	5	56	5	55	5	53	5	51	5	50	5	48
22	6	1	5	59	5	57	5	56	5	54	5	53	5	51	5	49	5	48
23	6	1	5	59	5	57	5	55	5	54	5	52	5	50	5	49	5	47
24	6	1	5	59	5	57	5	55	5	53	5	52	5	50	5	48	5	46
25	6	1	5	59	5	57	5	55	5	53	5	51	5	49	5	47	5	45
26	6	0	5	56	5	56	5	55	5	53	5	51	5	49	5	47	5	45
27	6	0	5	56	5	56	5	54	5	52	5	50	5	48	5	46	5	44
28	6	0	5	58	5	56	5	54	5	52	5	50	5	47	5	45	5	43
29	6	0	5	57	5	56	5	54	5	51	5	49	5	47	5	45	5	42
30	6	0	5	56	5	55	5	53	5	51	5	49	5	46	5	44	5	41
31	6	0	5	58	5	55	5	53	5	50	5	48	5	46	5	43	5	41
32	6	0	5	57	5	55	5	5	5	50	5	47	5	45	5	43	5	40

TABLE XLV *Continued*.

Semi Diurnal Arcs

Latitude and Declination of *different* kinds

DECLINATION	LATITUDE																	
	10		11		12		13		14		15		16		17		18	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	1	6	1	6	1	6	1	6	1	6	1	6	1	6	1	6	1
2	6	1	6	1	6	0	6	0	6	0	6	0	6	0	6	0	6	0
3	6	0	6	0	6	0	5	59	5	59	5	59	5	59	5	59	5	58
4	5	59	5	59	5	59	5	59	5	58	5	58	5	58	5	57	5	57
5	5	59	5	58	5	58	5	57	5	57	5	57	5	56	5	56	5	56
6	5	58	5	57	5	57	5	57	5	56	5	56	5	55	5	55	5	54
7	5	57	5	57	5	56	5	56	5	55	5	55	5	54	5	54	5	53
8	5	56	5	56	5	55	5	55	5	51	5	54	5	53	5	52	5	52
9	5	56	5	55	5	54	5	54	5	53	5	[illegible]	5	52	5	51	5	50
10	5	55	5	54	5	54	5	53	5	[illegible]	5	51	5	51	5	50	5	49
11	5	54	5	54	5	53	5	52	5	51	5	50	5	[illegible]	5	49	9	48
12	5	54	5	[illegible]	5	[illegible]	5	51	5	50	5	[illegible]	5	48	5	47	5	46
13	5	[illegible]	5	52	5	51	5	50	5	49	5	[illegible]	5	47	5	46	5	45
14	5	52	5	51	5	50	5	49	5	48	5	47	5	[illegible]	5	45	5	44
15	5	51	5	50	5	49	5	[illegible]	5	47	5	46	5	45	5	43	5	42
16	5	[illegible]	5	49	5	[illegible]	5	47	5	46	5	45	5	43	5	[illegible]	5	41
17	5	50	5	49	5	47	5	46	5	[illegible]	5	[illegible]	5	[illegible]	5	41	5	[illegible]
18	5	49	5	48	5	46	5	45	5	44	5	[illegible]	5	41	5	[illegible]	5	[illegible]
19	5	46	5	47	5	45	5	44	5	43	5	41	5	[illegible]	5	[illegible]	5	37
20	5	48	5	46	5	[illegible]	5	43	5	41	5	40	5	[illegible]	5	37	5	[illegible]
21	5	47	5	45	5	44	5	42	5	40	5	[illegible]	5	37	5	35	5	34
22	5	46	5	44	5	43	5	[illegible]	5	[illegible]	5	[illegible]	5	36	5	31	5	32
23	5	[illegible]	5	43	5	[illegible]	5	40	5	35	5	36	5	34	5	33	5	31
24	5	44	5	[illegible]	5	[illegible]	5	39	5	[illegible]	5	[illegible]	5	[illegible]	5	31	5	[illegible]
25	5	[illegible]	5	[illegible]	5	[illegible]	5	[illegible]	5	36	5	34	5	32	5	[illegible]	5	[illegible]
26	5	43	5	[illegible]	5	[illegible]	5	37	5	31	5	32	5	30	5	[illegible]	5	[illegible]
27	5	[illegible]	5	40	5	[illegible]	5	36	5	33	5	31	5	29	5	[illegible]	5	[illegible]
28	5	41	5	39	5	37	5	[illegible]	5	[illegible]	5	30	5	27	5	[illegible]	5	[illegible]
29	5	40	5	[illegible]	5	36	5	33	5	[illegible]	5	[illegible]	5	26	5	24	5	[illegible]
30	5	[illegible]	5	37	5	[illegible]	5	32	5	[illegible]	5	27	5	24	5	[illegible]	5	[illegible]
31	5	38	5	36	5	[illegible]	5	[illegible]	5	[illegible]	5	26	5	[illegible]	5	[illegible]	5	18
32	6	37	5	35	5	[illegible]	5	[illegible]	5	27	5	24	5	[illegible]	5	19	5	16

TABLE XLV Continued

Semi Diurnal Arcs

Latitude and Declination of *different* kinds

DECLINATION	LATITUDE															
	19		20		21		22		23		24		25		26	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	1	6	1	6	1	6	1	6	1	6	1	6	0	6	0
2	5	59	5	59	5	59	5	59	5	59	5	59	5	59	5	58
3	5	58	5	58	5	58	5	57	5	57	5	57	5	57	5	56
4	5	57	5	56	5	56	5	56	5	55	5	55	5	55	5	55
5	5	55	5	55	5	55	5	54	5	54	5	53	5	53	5	53
6	5	54	5	53	5	53	5	53	5	52	5	5	5	51	5	51
7	5	53	5	52	5	51	5	51	5	50	[illegible]	50	5	49	5	49
8	5	51	5	51	5	50	5	49	5	49	5	16	5	17	5	47
9	5	50	5	49	5	48	5	48	5	47	5	46	5	45	5	45
10	5	48	5	48	5	47	5	46	5	45	5	44	5	43	5	43
11	5	47	5	46	5	45	5	44	5	43	5	42	5	4	5	41
12	5	45	5	45	5	44	5	43	5	42	5	41	5	10	[illegible]	39
13	5	44	5	43	5	42	5	41	5	40	5	39	5	38	5	[illegible]
14	5	43	5	41	5	40	5	39	5	3	5	37	5	36	5	1
15	5	41	5	40	5	39	5	38	5	36	5	35	5	34	5	32
16	5	40	5	38	5	37	5	36	5	34	5	33	5	32	5	30
17	5	39	5	37	5	35	5	34	5	33	5	31	5	30	5	5
18	5	37	5	35	5	31	5	32	5	31	5	29	5	28	5	6
19	5	35	5	34	5	3	5	30	5	29	5	27	5	26	5	4
20	5	34	5	[illegible]	5	30	5	29	5	27	5	25	5	4	5	[illegible]
21	5	32	5	30	5	6	5	27	5	25	5	23	5	21	5	19
22	5	30	5	29	5	7	5	25	5	2	5	1	5	19	5	17
23	5	29	5	7	5	25	5	3	5	21	5	19	5	17	5	15
24	5	7	5	25	5	3	5	21	5	19	5	17	5	15	5	12
25	5	6	5	24	5	21	5	19	5	17	5	11	5	12	5	10
26	5	4	[illegible]	[illegible]	5	19	5	17	5	15	5	12	5	10	5	3
27	5	[illegible]	5	0	5	17	5	15	5	13	5	10	5	6	5	5
28	5	[illegible]	5	15	5	15	5	13	5	10	5	5	5	5	5	[illegible]
29	5	19	5	16	5	13	5	11	5	8	5	6	5	[illegible]	5	0
30	5	17	5	14	5	11	5	9	5	6	5	3	5	0	4	57
31	5	15	5	1	5	9	5	7	5	4	5	1	4	3	[illegible]	55
32	5	13	5	10	5	7	5	1	5	1	[illegible]	56	4	5	[illegible]	[illegible]

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of *different* kinds

DECLINATION	LATITUDE															
	27°		28		29		30		31		32		33		34	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0
2	5	58	5	58	5	58	5	58	5	58	5	57	5	57	5	57
3	5	56	5	56	5	56	5	55	5	55	5	55	5	55	5	54
4	5	54	5	54	5	54	5	53	5	53	5	52	5	52	5	52
5	5	5	5	52	5	51	5	51	5	50	5	50	5	49	5	49
6	5	50	5	50	5	49	5	49	5	48	5	47	5	47	5	47
7	5	48	5	47	5	47	5	46	5	46	5	45	5	44	5	44
8	5	46	5	45	5	45	5	44	5	43	5	43	5	42	5	41
9	5	44	5	43	5	42	5	41	5	41	5	40	5	39	5	38
10	5	42	5	41	5	40	5	39	5	38	5	37	5	36	5	35
11	5	40	5	39	5	38	5	37	5	36	5	35	5	34	5	32
12	5	38	5	36	5	36	5	35	5	34	5	32	5	31	5	30
13	5	36	5	34	5	33	5	32	5	31	5	29	5	28	5	27
14	5	33	5	32	5	31	5	29	5	28	5	27	5	25	5	24
15	5	31	5	30	5	28	5	7	5	26	5	24	5	23	5	21
16	5	9	5	27	5	6	5	24	5	23	5	21	5	20	5	18
17	5	6	5	25	5	24	5	2	5	20	5	19	5	17	5	15
18	5	24	5	3	5	21	5	19	5	18	5	16	5	14	5	12
19	5		5	21	5	19	5	17	5	15	5	13	5	11	5	9
20	5	0	5	18	5	16	5	14	5	12	5	10	5	6	5	6
21	5	17	5	15	5	13	5	11	5	9	5	7	5	5	5	3
22	5	15	5	13	5	11	5	9	5	7	5	4	5	2	5	0
23	5	13	5	10	5	8	5	6	5	4	5	1	4	59	4	56
24	5	10	5	8	5	6	5	3	5	1	4	58	4	56	4	53
25	5	8	5	5	5	3	5	0	4	58	4	55	4	52	4	50
26	5	5	5	3	5	0	4	57	4	55	4	52	4	49	4	46
27	5	3	5	0	4	57	4	54	4	52	4	49	4	46	4	43
8	5	0	4	57	4	54	4	51	4	48	4	45	4	42	4	39
29	4	57	4	54	4	51	4	48	4	45	4	4	4	39	4	35
30	4	54	4	51	4	49	4	45	4	42	4	38	4	35	4	31
31	4	52	4	48	4	45	4	42	4	38	4	35	4	31	4	28
32	4	49	4	45	4	42	4	38	4	35	4	31	4	27	4	24

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of *different* kinds

DECLINATION	LATITUDE															
	35		36		37		38		39		40		41		42	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	6	0	6	0	6	0	6	0	5	59	5	59	5	59	5	59
2	5	57	5	57	5	57	5	56	5	56	5	56	5	56	5	56
3	5	54	5	54	5	54	5	53	5	53	5	53	5	52	5	52
4	5	51	5	51	5	51	5	50	5	50	5	49	5	49	5	48
5	5	49	5	48	5	46	5	47	5	46	5	46	5	45	5	45
6	5	46	5	45	5	44	5	44	5	43	5	43	5	42	5	41
7	5	43	5	4	5	41	5	41	5	40	5	39	5	35	5	37
8	5	40	5	39	5	38	5	37	5	37	5	36	5	33	5	34
9	5	37	5	36	5	35	5	34	5	33	5	32	5	31	5	30
10	5	34	5	33	5	32	5	31	5	30	5	29	5	28	5	26
11	5	31	5	30	5	29	5	28	5	27	5	25	5	24	5	23
12	5	28	5	27	5	26	5	25	5	23	5	22	5	20	5	19
13	5	25	5	24	5	3	5	21	5	20	5	15	5	17	5	15
14	5	22	5	21	5	19	5	18	5	16	5	15	5	13	5	11
15	5	19	5	18	5	16	5	14	5	13	5	11	5	9	5	7
16	5	16	5	15	5	13	5	11	5	9	5	7	5	5	5	3
17	5	13	5	11	5	10	5	8	5	6	5	4	5	1	4	59
18	5	10	5	8	5	6	5	4	5	2	4	59	4	57	4	55
19	5	7	5	5	5	3	5	0	4	58	4	6	4	53	4	51
20	5	4	5	2	4	59	4	57	4	54	4	52	4	49	4	47
21	5	1	4	58	4	56	4	53	4	51	4	48	4	45	4	42
22	4	57	4	55	4	52	4	49	4	47	4	44	4	41	4	38
23	4	54	4	51	4	49	4	46	4	43	4	40	4	37	4	33
24	4	50	4	48	4	45	4	42	4	39	4	35	4	32	4	29
25	4	47	4	44	4	41	4	38	4	34	4	31	4	28	4	24
26	4	43	4	40	4	36	4	34	4	30	4	27	4	23	4	19
27	4	39	4	36	4	33	4	29	4	26	4	22	4	18	4	14
28	4	36	4	33	4	29	4	25	4	21	4	17	4	13	4	9
29	4	32	4	28	4	5	4	21	4	17	4	13	4	8	4	4
30	4	28	4	21	4	[illegible]	4	16	4	1	4	8	4	3	3	59
31	4	24	4	20	4	15	4	12	4	7	4	8	3	58	3	53
32	4	19	4	15	[illegible]	11	4	7	4	2	3	57	3	52	3	47

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of different kinds

DECLINATION	LATITUDE															
	43		44		45		46		47		48		49		50	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	5	59	5	59	5	59	5	59	5	59	5	59	5	59	5	59
2	5	55	5	55	5	55	5	55	5	55	5	54	5	54	5	54
3	5	52	5	51	5	51	5	51	5	50	5	50	5	49	5	49
4	5	48	5	47	5	47	5	46	5	46	5	45	5	45	5	44
5	5	44	5	44	5	43	5	42	5	42	5	41	5	40	5	39
6	5	40	5	40	5	39	5	38	5	37	5	36	5	35	5	35
7	5	37	5	36	5	35	5	34	5	3	5	32	5	31	5	30
8	5	33	5	32	5	31	5	30	5	26	5	27	5	26	5	25
9	5	9	5	28	5	7	5	5	5	24	5	23	5	21	5	20
10	5	5	5	24	5	2	5	21	5	20	5	18	5	17	5	15
11	5	21	5	0	5	18	5	17	5	15	5	13	5	12	5	10
12	5	17	5	16	5	14	5	1	5	11	5	9	5	7	5	5
13	5	13	5	1	5	10	5	[illegible]	5	6	5	4	5	2	5	0
14	5	9	5	7	5	5	5	3	5	1	4	59	4	57	4	54
15	5	5	5	3	5	1	4	59	4	57	4	54	4	52	4	49
16	5	1	4	59	4	57	4	54	4	52	4	49	4	46	4	45
17	4	57	4	55	4	52	4	50	4	47	4	44	4	41	4	38
18	4	53	4	50	4	47	4	45	4	42	4	39	4	36	4	33
19	4	48	4	46	4	43	4	40	4	37	4	34	4	30	4	27
0	4	44	4	41	4	31	4	35	4	32	4	28	4	5	4	1
21	4	9	4	36	4	33	4	30	4	26	4	2	4	19	4	15
2	4	35	4	32	4	5	4	25	4	21	4	17	4	13	4	9
3	4	0	4	7	4	23	4	19	4	15	4	11	4	7	4	
4	4	[illegible]	4	2	4	13	4	14	4	10	4	5	4	1	3	56
5	4	20	4	17	4	1	4	8	4	4	3	59	3	54	3	49
6	4	15	4	11	4	7	4	3		5	3	53	3	46	3	4
7	4	10	4	6	4	1	3	57	3	52	3	46	3	41	3	5
8	4	5	4	0	3	[illegible]	3	50		43	3	40	3	34	3	26
9	3	59		54	3	49	3	44	3	38	3	33	[illegible]	[illegible]		0
30	3	54	3	18	3	43	3	37	3	31	3	25	3	13	3	11
31	3	48	3	4	3	37	3	31	3	24	3	17	3	10		
32	3	42	3	36	3	30		23	3	17	3	9	3	2	2	53

TABLE XLV *Continued*

Semi Diurnal Arcs

Latitude and Declination of *different* Lands

DECLINATION	LATITUDE															
	51		52		53		54		55		56		57		58	
D	H	M	H	M	H	M	H	M	H	M	H	M	H	M	H	M
1	5	56	5	5	5	55	5	58	5	55	5	56	5	58	5	55
2	5	53	5	53	5	53	5	53	5	5	5	52	5	5	5	51
3	5	49	5	46	5	45	5	47	5	47	5	46	5	45	5	45
4	5	44	5	43	5	4	5	42	5	41	5	40	5	39	5	3
5	5	39	5	35	5	7	5	36	5	35	5	34	5	33	5	3
6	5	31	5	3	5	31		30	5	29	5	26	5	7	5	5
7	5	29	5	7	5	26	5	25	5	23	5		5	20	5	19
	5	3	5	2	5	21	5	19	5	17	5	16	5	14	5	12
9	5	13	5	17	5	16	5	13	5	12	5	10	5	8	5	5
10	5	13	5	11	5	10	5	8	5	5	5	3	5	1	4	59
11	5	6	5	6	5	4	5	2	4	59	4	57	4	54	4	5
12	5	3	5	0	4	56	4	56	4	53	4	51		46	4	15
13	4	57	4	5	4	5	4	52	4	47	4	41	4	41	4	39
14	4	2	4	49	4	47	4	44	4	41	4	3	4	34	4	30
15	4	46	4	44	4	41	4	37	4	34	4	31	4	7	4	
16	4	41	4	3	4	34	4	35	4	27	4	4	4	0	4	15
17	4	35	4		4	3	4	23	4	21	4	17	4	1	4	5
18	4	3	4	26	4		4	16	4	11	4	9	4	5	4	0
19	4	23	4	19	4	15	4	11	4	7	4			56	3	51
20	4	17	4	13	4	9	4	4	3	53		4	3	1	3	43
21	4	11	4	6		2	3	57	3			16	3	10	3	34
22	4	4	4	0	3	55		50		41	3	33		31	3	1
23	3	5	3	5	3	47		4		30		29	3	3	3	15
24	3	51	3	43		40		4		7		20	3	13	3	3
25	[illegible]	34	[illegible]	35	3	2		25	3	18	3	11	3	3		53
26	3	37	3	30	3	4		17	3	9	3	1	2	52		42
27	3	9	3		3	1		3	2	59	2	50		40		9
28	3	1	3	14	3	6	2	53	2	9		3	2	6		13
9	3	[illegible]	[illegible]	5	2	50		47	2	37		6		11	2	0
30	3	[illegible]		15		46		36	2	25		13	1	5	1	11
31	2	51	2	45	2	35	2	24	2	1	1	57	1	41	1	19
32	2	44	2	34	[illegible]	3		11	1	37	1	40	1	10		

TABLE XLVII

The Amplitudes of the Rising and Setting of the Heavenly Bodies

LATITUDE	DECLINATION															
	1		2		3		4		5		6		7		8	
D	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M
2	1	0	2	0	3	0	4	0	5	0	6	0	7	0	8	0
4	1	0	2	0	3	0	4	1	5	1	6	1	7	1	8	[illegible]
6	1	0	2	1	3	1	4	2	5	2	6	2	7	3	8	4
8	1	0	2	1	3	2	4	3	5	3	6	4	7	5	8	6
10	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8
12	1	1	2	3	3	4	4	5	5	7	6	8	7	10	8	11
14	1	2	2	4	3	5	4	7	5	9	6	11	7	13	8	15
16	1	2	2	5	3	7	4	10	5	13	6	15	7	17	8	19
18	1	3	2	6	3	9	4	13	5	16	6	19	7	[illegible]	8	25
20	1	4	[illegible]	8	3	11	4	16	5	19	6	23	7	27	8	31
22	1	5	2	10	3	14	4	19	5	24	6	29	7	33	8	[illegible]
24	1	6	2	12	3	17	4	23	5	29	6	34	7	40	8	46
26	1	7	2	14	3	21	4	27	5	34	6	41	7	46	8	54
28	1	8	[illegible]	16	3	24	4	32	5	40	6	46	7	56	9	4
30	1	9	2	19	3	8	4	37	5	46	6	55	8	5	9	15
32	1	11	[illegible]	[illegible]	3	33	4	43	5	54	7	5	8	16	9	27
34	1	12	2	25	3	37	4	50	6	2	7	15	8	27	9	40
36	1	14	2	28	3	43	4	57	6	11	7	25	8	40	9	54
38	1	16	2	32	3	48	5	5	6	21	7	37	8	54	10	11
40	1	18	2	37	3	55	5	14	6	32	7	51	9	9	10	28
42	1	20	2	41	4	2	5	24	6	44	8	5	9	0	10	48
43	1	22	2	44	4	6	5	29	6	51	8	13	9	35	10	56
44	1	23	2	47	4	10	5	34	6	58	8	21	9	45	11	9
45	1	25	2	50	4	15	5	40	7	5	8	30	9	55	11	21
46	1	26	2	53	4	19	5	46	7	12	8	39	10	6	11	33
47	1	28	2	56	4	24	5	52	7	21	8	49	10	18	11	46
48	1	30	2	59	4	29	5	59	7	29	8	59	10	30	12	0
49	1	32	3	3	4	35	6	0	7	36	9	10	10	42	12	15
50	1	33	3	7	4	40	6	14	7	48	9	21	10	56	12	30
51	1	35	3	11	4	46	6	22	7	58	9	34	11	10	[illegible]	47
52	1	37	3	15	4	52	6	30	8	8	9	47	11	25	13	4
53	1	40	3	20	4	59	6	40	8	20	10	[illegible]	11	41	13	22
54	1	42	3	24	5	6	6	49	8	33	10	15	11	58	13	42
55	1	45	3	29	5	14	6	59	8	44	10	30	12	16	14	3
56	1	47	3	35	5	22	7	10	8	58	10	46	12	35	14	25
57	1	50	3	41	5	31	7	2	9	13	11	4	12	56	14	48
58	1	53	3	47	5	40	7	34	9	28	11	23	13	18	15	13
59	1	57	3	53	5	50	7	47	9	45	11	53	13	41	15	41
60	2	0	4	0	6	1	8	1	10	[illegible]	12	4	14	6	16	10
61	2	4	4	8	6	12	8	16	10	[illegible]	12	27	14	34	16	41
62	2	8	4	16	6	24	8	34	10	4	12	52	15	3	17	15
63	2	12	4	25	6	37	8	50	11	4	13	19	15	37	17	51

TABLE XLVII Continued

LATITUDE	DECLINATION															
	9		10		11		12		13		14		15		16	
D	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M
2	9	0	10	0	11	0	12	0	13	0	14	0	15	0	16	1
4	9	2	10	2	11	2	12	2	13	2	14	2	15	2	16	3
6	9	4	10	4	11	4	12	4	13	4	14	5	15	5	16	6
8	9	6	10	6	11	7	12	7	13	8	14	9	15	9	16	10
10	9	8	10	10	11	10	12	11	13	12	14	13	15	14	16	15
12	9	12	10	14	11	14	12	16	13	16	14	19	15	21	16	22
14	9	17	10	18	11	20	12	22	13	24	14	26	15	29	16	30
16	9	22	10	24	11	27	12	30	13	32	14	34	15	37	16	40
18	9	26	10	31	11	35	12	38	13	41	14	44	15	47	16	51
20	9	35	10	39	11	43	12	47	13	51	14	55	15	59	17	4
22	9	43	10	48	11	53	12	58	14	2	15	6	16	13	17	18
24	9	52	10	56	12	3	13	10	14	15	15	22	16	29	17	34
26	10	1	11	9	12	15	13	22	14	30	15	37	16	44	17	52
28	10	12	11	21	12	29	13	37	14	46	15	54	17	3	18	11
30	10	24	11	34	12	44	13	53	15	3	16	13	17	23	18	34
32	10	38	11	49	13	0	14	11	15	23	16	34	17	46	18	58
34	10	52	12	5	13	18	14	31	15	45	16	56	18	12	19	25
36	11	9	12	24	13	39	14	54	16	9	17	24	18	40	19	55
38	11	27	12	44	14	1	15	18	16	35	17	53	19	19	20	26
40	11	47	13	6	14	25	15	45	17	5	18	25	19	45	21	5
42	12	9	13	31	14	53	16	15	17	37	19	0	20	23	21	46
43	12	21	13	44	15	7	16	31	17	55	19	19	20	43	22	8
44	12	34	13	58	15	23	16	48	18	13	19	39	21	3	22	32
45	12	47	14	13	15	39	17	6	18	33	20	0	21	26	22	57
46	13	1	14	29	15	57	17	25	18	54	20	43	21	53	23	22
47	13	16	14	45	16	15	17	45	19	17	20	47	[illegible]	16	23	50
48	13	28	15	2	16	34	18	6	19	39	21	12	22	45	24	19
49	13	48	15	21	16	55	18	29	20	3	21	36	23	15	24	51
50	14	5	15	40	17	16	18	52	20	29	22	7	23	45	25	24
51	14	24	16	1	17	39	19	17	20	50	22	37	24	17	25	59
52	14	43	16	23	18	3	19	44	21	16	23	8	24	52	26	36
53	15	4	16	47	18	29	20	13	21	57	23	42	25	28	27	16
54	15	26	17	11	18	57	20	43	22	30	24	18	26	7	27	58
55	15	50	17	37	19	26	21	15	23	5	24	57	26	49	28	43
56	16	15	18	6	19	57	21	50	23	43	25	38	27	34	29	32
57	16	42	18	36	20	36	22	27	24	24	26	[illegible]	28	29	30	24
58	17	10	19	8	21	6	23	6	25	7	27	10	29	14	31	20
59	17	41	19	4	21	45	23	49	25	53	28	5	30	10	32	21
60	18	14	20	19	22	5	24	34	26	46	28	56	31	10	33	27
61	18	50	20	59	23	11	25	24	27	39	29	56	32	16	34	39
62	19	28	21	13	23	59	26	17	28	38	31	1	33	27	35	57
63	20	9	22	29	24	51	27	15	29	42	32	12	34	46	37	25

TABLE XLVII *Continued*

LATITUDE	DECLINATION															
	17°		18		19		20		21		22		23		23½	
D	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M
2	17	1	18	1	19	1	20	1	21	1	22	1	22	1	23	31
4	17	3	18	3	19	3	20	3	21	3	22	5	23	4	23	34
6	17	6	18	6	19	7	0	7	21	7	22	9	23	8	23	38
8	17	10	18	11	19	12	20	12	21	13	22	13	23	14	23	46
10	17	16	18	17	19	18	20	19	21	20	22	21	23	23	23	53
12	17	24	18	25	19	27	20	28	21	30	22	31	23	33	24	4
14	17	32	18	34	19	36	20	38	21	41	22	42	23	45	24	16
16	17	43	18	46	19	48	20	51	21	53	22	57	23	59	24	31
18	17	54	18	58	20	1	21	5	22	8	23	12	24	15	24	47
20	18	8	19	12	20	16	21	21	22	25	23	30	24	34	25	7
22	18	23	19	28	20	35	21	39	22	44	23	50	24	56	25	28
24	18	40	19	48	20	53	1	59	23	6	24	13	25	21	25	53
26	18	59	20	7	21	14	2	22	23	30	24	38	25	46	26	20
28	19	20	20	9	21	38	22	47	23	57	25	6	26	16	26	51
30	19	44	20	51	2	5	23	16	24	27	25	38	26	49	27	25
32	20	10	21	22	22	35	23	47	25	0	26	13	27	26	28	3
34	20	38	21	53	23	7	24	22	25	37	26	52	28	7	28	45
36	21	11	22	27	23	44	25	1	26	18	27	35	28	53	29	32
38	21	47	23	5	24	25	25	43	27	3	28	23	29	44	30	24
40	2	26	23	47	25	9	26	31	27	54	29	17	30	40	31	22
42	23	10	24	34	25	59	27	24	28	50	30	16	31	43	32	27
43	23	34	25	0	26	6	7	53	29	20	30	49	32	18	33	2
44	23	59	25	26	26	55	28	23	29	53	31	23	32	54	33	40
45	24	25	25	55	27	25	28	56	30	27	31	59	33	33	34	20
46	24	53	26	25	27	57	29	30	31	3	32	38	34	14	35	2
47	25	23	26	57	28	31	30	6	31	42	33	19	34	57	35	47
48	25	55	27	30	29	7	30	44	32	23	34	3	35	44	36	35
49	26	28	28	6	29	45	31	25	35	7	34	49	36	33	37	26
50	27	7	28	44	30	26	32	9	33	53	35	39	37	26	38	21
51	27	41	29	24	31	9	32	55	34	43	36	32	38	23	39	19
52	28	21	30	6	31	57	33	45	35	36	37	29	39	23	40	22
53	29	4	30	53	32	45	34	39	36	33	38	30	40	49	41	30
54	29	50	31	43	33	38	35	35	37	34	39	36	41	40	42	43
55	30	39	32	36	34	35	36	37	38	40	40	47	44	56	44	3
56	31	32	33	33	35	36	37	42	39	51	42	4	44	19	45	30
57	32	28	34	34	36	43	38	54	41	10	43	27	45	50	47	4
58	33	29	35	40	37	54	40	12	42	33	44	59	47	30	48	48
59	34	35	36	52	39	12	41	37	44	6	46	40	49	21	50	44
60	35	47	38	10	40	38	43	10	45	47	48	31	51	21	5	53
61	37	5	39	56	42	11	44	52	47	40	50	36	53	42	55	20
62	38	31	41	10	43	54	46	46	49	46	52	56	56	20	58	9
63	40	5	42	54	45	49	48	53	52	8	55	30	59	24	61	26

TABLE XLVIII

To find the Enlightened Part of the Diameter of the Moon, *or* Venus, *ſuppoſing the Diameter to be divided into* 12 *equal Parts*

Deg.	*For the MOON*—ARGUMENT Diſtance of the Moon from the Sun						
	0	30	60	90°	120	150	
	Parts	Parts	Parts	Parts	Parts	Parts	
0	0,000	0,804	3 000	6,000	9,000	11,196	30
1	0 001	0,857	3 092	6,104	9,090	11,247	29
2	0,004	0,912	3,184	6,209	9,179	11,297	28
3	0,009	0 969	3,277	6,314	9,267	11,346	27
4	0 015	1,0 6	3,370	6,418	9,355	11,392	26
5	0,023	1 085	3,465	6,523	9,441	11,437	25
6	0,033	1,146	3,560	6,627	9,526	11,481	24
7	0,045	1,209	3,656	6,731	9,611	11,523	23
8	0,059	1,272	3,753	6,834	9,694	11,563	22
9	0,074	1,337	3,850	6 938	9,776	11,601	21
10	0,091	1,404	3,948	7,041	9,856	11,638	20
11	0,110	1,472	4,047	7,145	9,936	11,673	19
12	0,131	1,542	4,147	7,247	10 014	11,706	18
13	0,154	1,612	4,247	7,349	10,091	11,737	17
14	0,179	1,684	4,347	7,451	10,167	11,767	16
15	0,205	1,758	4,448	7,552	10,242	11 795	15
16	0,233	1,833	4,549	7,653	10,316	11,821	14
17	0,263	1,909	4,651	7,753	10 388	11,846	13
18	0,294	1 986	4,753	7,853	10,458	11,869	12
19	0,327	2,064	4,855	7,953	10,528	11,890	11
20	0 362	2,144	4,959	8 052	10,596	11,909	10
21	0,399	2,224	5,062	8,150	10,663	11,926	9
22	0,437	2,306	5,166	8,247	10,728	11,941	8
23	0,477	2,389	5,269	8,344	10,791	11,955	7
24	0,519	2,474	5,373	8,440	10,854	11,967	6
25	0 563	2,559	5,477	8 535	10,915	11,977	5
26	0,608	2,645	5,582	8,630	10,974	11,985	4
27	0 654	2,733	5,686	8,723	11,031	11,991	3
28	0,702	2,821	5,791	8,816	11,088	11,996	2
29	0 753	2,910	5,896	8,908	11,143	11,999	1
30	0 804	3,000	6,000	9,000	11 196	1[illegible] 000	0
	150	120	90	60	30	0	Deg.
	For VENUS—ARGUMENT Angle formed by two lines drawn from Venus to the Sun and Earth						

TABLE XLIX

Showing the Hour-Angles of Jupiter *with the Meridian when 8 high to every half Degree of his possible Declination for the Latitude of Greenwich*

Jupiter's Declin. *North*		Hour Angle from Meridian in Time			Jupit. Declin. *North*		Hour Angle from Meridian in Time			Jupit. Declin. *South*		Hour Angle from Meridian in Time			Jupit. Declin. *South*		Hour Angle from Meridian in Time		
D	M	H			D		H			D		H			D		H	M	
0	0	5	8	21	13	0	6	13	55	0	0	5	8	21	13	0	3	54	51
0	30	5	10	55	13	30	6	16	28	0	30	5	5	46	13	30	3	51	36
1	0	5	13	29	14	0	6	19	1	1	0	5	3	12	14	0	3	48	19
1	30	5	16	2	14	30	6	21	33	1	30	5	0	34	14	0	3	44	59
2	0	5	18	35	15	0	6	24	10	2	0	4	57	57	15	0	3	41	35
2	30	5	21	7	15	30	6	26	45	2	30	4	55	19	15	30	3	38	8
3	0	5	23	39	16	0	6	29	21	3	0	4	52	40	16	0	3	34	37
3	30	5	26	11	16	30	6	31	58	3	30	4	50	0	16	30	3	1	2
4	0	5	28	42	17	0	6	34	36	4	0	4	47	19	17	0	3	27	24
4	30	5	31	13	17	30	6	37	15	4	30	4	44	37	17	30	3	23	41
5	0	5	33	43	18	0	6	39	54	5	0	4	41	54	18	0	3	19	54
5	30	5	36	13	18	30	6	42	34	5	30	4	39	10	18	30	3	16	2
6	0	5	38	44	19	0	6	45	16	6	0	4	36	24	19	0	3	12	5
6	30	5	41	14	19	30	6	47	59	6	30	4	33	37	19	30	3	8	2
7	0	5	43	44	20	0	6	50	42	7	0	4	30	49	20	0			3
7	30	5	46	14	20	30	6	53	27	7	30	4	27	59	20	30	2	59	38
8	0	5	48	44	21	0	6	56	13	8	0	4	25	8	21	0	2	55	16
8	30	5	51	14	21	30	6	59	1	8	30	4	22	15	21	30	2	50	47
9	0	5	53	45	22	0	7	1	50	9	0	4	19	21	22	0	2	46	9
9	30	5	56	15	22	30	7	4	41	9	30	4	16	24	22	30	2	41	23
10	0	5	58	46	23	0	7	7	32	10	0	4	13	26	23	0	2	36	26
10	30	6	1	16	23	30	7	10	26	10	30	4	10	25	23	30	2	31	19
11	0	6	3	4	24	0	7	13	22	11	0	4	7	23	24	0	2	25	59
11	30	6	6	18	24	30	7	16	20	11	0	4	4	18	24	30	2	20	26
12	0	6	8	50	25	0	7	19	19	12	0	4	1	12	25	0	2	14	59
12	30	6	11	22	25	30	7	22	19	12	30	3	58	3	25	30	2	8	31

TABLE L

Shewing the Hour-Angles of the Sun with the Meridian or Apparent Time from Noon when his Depression is 8° below the Horizon to every Half Degree of Declination for the Latitude of Greenwich

Sun's Declination North										Sun's Declination South									
Declin.		Hour Ang.			Declin.		Hour Ang.			Declin.		Hour Ang.			Declin.		Hour Ang.		
D	M	H	M	S	D	M	H	M	S	D	M	H	M	S	D	M	H	M	S
0	0	6	51	39	12	0	7	58	48	0	0	6	51	39	12	0	5	51	10
0	30	6	54	14	12	30	8	1	57	0	30	6	49	5	12	30	5	48	38
1	0	6	56	49	13	0	8	5	9	1	0	6	46	31	13	0	5	46	5
1	30	6	59	26	13	30	8	8	24	1	30	6	43	58	13	30	5	43	32
2	0	7	2	3	14	0	8	11	41	2	0	6	41	25	14	0	5	40	59
2	30	7	4	41	14	30	8	15	1	2	30	6	38	53	14	30	5	38	25
3	0	7	7	20	15	0	8	18	25	3	0	6	36	21	15	0	5	35	50
3	30	7	10	0	15	30	8	21	52	3	30	6	33	49	15	30	5	33	15
4	0	7	12	41	16	0	8	25	23	4	0	6	31	18	16	0	5	30	39
4	30	7	15	23	16	30	8	28	58	4	30	6	28	47	16	30	5	28	2
5	0	7	18	6	17	0	8	32	36	5	0	6	26	17	17	0	5	25	24
5	30		20	50	17	30	8	36	19	5	30	6	23	47	17	30	5	22	43
6	0	7	23	36	18	0		40	6	6	0	6	21	16	18	0	5	20	6
6	30	7	26	23	18	30	8	43	58	6	30	6	18	46	18	30	5	17	26
7	0	7	29	1	19	0	8	47	55	7	0	6	16	16	19	0	5	14	44
7	30	7	32	1	19	30	8	51	58	7	30	6	13	46	19	30	5	12	1
8	0	7	34	52	20	0	8	56	7	8	0	6	11	16	20	0	5	9	18
8	30	7	37	45	20	30	9	0	22	8	30	6	8	46	20	30	5	6	33
9	0	7	40	39	21	0	9	4	44	9	0	6	6	15	21	0	5		4
9	30	7	43	6	21	0	9	9	15	9	30	6	3	45	21	30	5	0	59
10	0	7	46	34	22	0	9	13	51	10	0	6	1	14	22	0	4	58	10
10	30	7	49	35	22	30	9	18	37	10	30	5	58	44	22	30	4	55	19
11	0	7	52	37	23	0	9	23	4	11	0	5	56	13	23	0	4	52	28
11	30	7	55	40	23	30	9	28	42	11	30	5	53	42	23	30	4	49	34
12	0	7	58	48						12	0	5	51	10					

TABLE LI

Logistic Logarithms

D M	0	1	2	3	4	5	6	7	8	9	D M
M S	0	60	1 0	180	240	300	360	420	480	540	M S
0		1,7782	1,4771	1,3010	1,1761	1 0792	1 0000	9331	8751	8239	0
1	3 5563	1,7710	1,4735	1,2986	1,1743	1,0777	9988	9320	8742	8231	1
2	3,2553	1,7639	1,4699	1,2962	1,1725	1,0763	9976	9310	8733	8223	2
3	3,079	1,7570	1,4664	1,2939	1,1707	1,0749	9964	9300	87-4	8215	3
4	2,9542	1,7501	1,46-9	1,2915	1,1689	1,0734	9952	9-89	8715	8-07	4
5	2,8573	1,7434	1,4504	1,2891	1,1671	1,0720	9940	9279	8706	8199	5
6	,7782	1,7368	1,4559	1,2868	1,1664	1,0706	9928	9269	8697	8191	6
7	2,7112	1,730-	1,4525	1 -845	1,1636	1,0692	9916	9259	8688	8183	7
8	2,6532	1,7-38	1,4491	1,2821	1,1619	1 0678	9905	9249	8679	8175	8
9	2,6021	1,7175	1,4457	1,2798	1,1601	1,0663	9893	9238	8670	8167	9
10	2,5563	1,7112	1,4424	1, 775	1,1584	1,0649	9681	9228	8661	8159	10
11	2,5149	1 7050	1,4390	1 2753	1,1566	1 0635	9869	9218	8652	8152	11
12	2,4771	1 6990	1,4357	1,2730	1,1549	1,0621	9858	9-08	8643	8144	12
13	2 44-4	1 6930	1,4325	1,2707	1,1532	1 0608	9846	9198	8635	8136	13
14	2,4102	1,6871	1,4292	1, 685	1,1515	1,0594	9834	9188	8626	8128	14
15	2,3802	1 6812	1,4260	1,2663	1,1498	1,0580	9823	9178	8617	8120	15
16	2,3522	1,6755	1 4228	1 2640	1,1481	1,0566	9811	9168	8608	8112	16
17	2,3259	1,6698	1,4196	1 2618	1,1464	1,055-	9800	9158	8599	8104	17
18	2,3010	1,6642	1,4165	1,2596	1,1447	1,0539	9788	9148	8591	8097	18
19	2,2775	1,6587	1,4133	1,2574	1,1430	1,0525	9777	9138	8582	8089	19
20	2,2553	1 6532	1,4102	1,2553	1 1413	1,0512	9765	9128	8573	8081	20
21	2 2341	1,6478	1,4071	1,2531	1 1397	1 0498	9754	9119	8565	8073	21
-2	2 2139	1 6425	1,4040	1,2510	1,1380	1 0484	9742	9109	8556	8066	22
23	2 1946	1,637-	1,4010	1 488	1,1363	1,0471	9731	9099	8547	80-8	23
24	2,1761	1,6320	1,3979	1,2467	1,1347	1,0458	9720	9089	8539	80 0	24
25	2,1584	1,6269	1,3949	1, 415	1,1331	1,0444	9708	9079	6530	8043	25
26	2 1413	1,6218	1 919	1,24 4	1 1314	1,0431	9697	9070	852-	8035	26
27	2,1249	1 616	1 3690	1, 403	1,1298	1,0418	9686	9060	8513	80-7	27
28	2,1091	1,6118	1 3-00	1 2362	1,1-62	1 0404	9675	9050	8504	80 0	28
29	2 0939	1,6069	1 3931	1,2362	1,1-66	1 0391	9664	9041	8496	8012	29
30	2,0792	1,60-1	1,3602	1,2341	1,1-49	1 0378	9652	9031	8487	8004	30
	0	1	2	3	4	5	6	7	8	9	

TABLE LI *Continued*

Logistic Logarithms

D M	0	1	2	3	4	5	6	7	8	9	D M
M S	0	60	1 0	180	40	300	360	420	480	540	M S
31	2,0649	1,5973	1,3773	1,23 0	1,1233	1,0365	9641	9021	8479	7997	31
32	2 0512	1,5925	1,3745	1,2300	1,1 17	1,0 52	9630	9012	8470	7989	32
33	,0378	1,5878	1 3716	1,2279	1,1201	1,0339	9619	9002	846	7981	33
34	2,0246	1 5832	1,3688	1,2259	1,1186	1,0 26	9608	8992	8453	7974	34
35	,0122	1,5786	1 3660	1,2239	1,1170	1,0313	9597	898	8445	7966	35
36	2,0000	1,5740	1,3632	1,2218	1 1154	1,0300	9586	89	8437	7959	36
37	1,9881	1,5695	1,3604	1 2198	1 1138	1 0 87	9575	8964	8428	7951	37
38	1,9 65	1 5651	1 3576	1 2178	1,1123	1,0274	9564	8954	8420	7944	38
39	1,965	1,5607	1 3549	1 2159	1,1107	1,0 61	9553	8945	8411	7936	39
40	1,954	1,5563	1,3522	1,2139	1,1091	1,0248	9542	8935	8403	79 9	40
41	1,9435	1,5520	1,3495	1,2119	1,1076	1,0235	953	89 6	8395	7921	41
42	1,9331	1 5477	1,3468	1,2099	1,1061	1,0223	9521	8917	8386	7914	42
43	1,9228	1,5435	1,3441	1 2080	1,1045	1,0210	9510	8907	8378	7906	43
44	1,9128	1,5393	1,3415	1,2061	1,1030	1,0197	9499	8898	8370	7899	44
45	1,9031	1,5351	1,3386	1,2041	1,1015	1,0185	9488	8888	8361	7891	45
46	1,8935	1,5310	1,3362	1,2022	1,0999	1,0172	9478	8879	8353	7884	46
47	1,8642	1,5269	1,3336	1,2003	1,0964	1,0160	9467	8870	8345	7877	47
48	1,6751	1,5229	1,3310	1 1984	1,0969	1,0147	9456	8861	8337	7869	48
49	1,8661	1,5189	1,3284	1,1965	1,0954	1,0135	9446	8851	83 8	786	49
50	1,8573	1,5149	1,3259	1,1946	1,0939	1,0122	9435	8842	8320	7855	50
51	1,8487	1,5110	1,3233	1,1927	1,0924	1,0110	94 5	8833	8312	7847	51
52	1,8403	1,5071	1,3208	1,1908	1,0909	1,0098	9414	8824	8304	7840	52
53	1,8320	1,5032	1,3183	1 1889	1 0894	1,0085	9404	8814	8296	783	53
54	1,8239	1,4994	1,3158	1,1871	1 0880	1,0073	9393	8805	8288	78 5	54
55	1,8159	1,49 6	1,3133	1,185	1,0865	1,0061	9383	8796	8279	7818	55
56	1,8081	1,4918	1,3108	1,1834	1,0850	1,0049	9372	8787	8271	7811	56
57	1,8004	1,4881	1,3083	1,1816	1,0835	1,0036	9362	8778	8263	7803	57
58	1,7929	1,4844	1,3059	1,1797	1 08 1	1,0024	9351	8769	8255	7796	58
59	1,7855	1,4808	1,3034	1,1779	1 0806	1,0012	9341	8760	8247	7789	59
60	1 7782	1,4771	1,3010	1 1761	1,079	1,0000	9331	8751	8239	7782	60
	0	1	2	3	4	5	6	7	8	9	

TABLE LI *Continued*

Logiſtic Logarithms

D M	10	11	12	13	14	15	16	17	18	19	D M
M S	600	660	720	780	840	900	960	1020	1080	1140	M S
0	7782	7369	6990	6642	6320	6021	5740	5477	5229	4994	0
1	7774	7361	6984	6637	6315	6016	5736	5473	5225	4990	1
2	7767	7354	6978	6631	6310	6011	5731	5469	5221	4966	2
3	7760	7348	6972	66 5	6305	6006	5727	5464	5217	4983	3
4	7753	7341	6966	6620	6300	6001	5722	5460	5213	4979	4
5	7745	7335	6960	6614	6294	5997	5718	5456	5209	4975	5
6	7738	7328	6954	6609	6289	5992	5713	5452	5205	4971	6
7	7731	7322	6948	6603	6284	5987	5709	5447	5201	4967	7
8	7724	7315	6942	6598	6279	5982	5704	5443	5197	4964	8
9	7717	7309	6936	6592	6274	5977	5700	5439	5193	4960	9
10	7710	7302	6930	6587	6269	5973	5695	5435	5169	4956	10
11	7703	7296	6924	6581	6264	5966	5691	5430	5185	4952	11
12	7696	7289	6918	6576	6259	5963	5666	5426	5181	4949	12
13	7688	7283	6912	6570	6254	5958	5682	5422	5177	4945	13
14	7681	7276	6906	6565	6248	5954	5677	5418	5173	4941	14
15	7674	7270	6900	6559	6243	5949	5673	5414	5169	4937	15
16	7669	7264	6894	6554	6238	5944	5669	5409	5165	4933	16
17	7660	7257	6888	6548	6233	5939	5664	5405	5161	4930	17
18	7653	7251	6882	6543	6228	5935	5660	5401	5157	49 6	18
19	7646	7244	6877	6538	6223	5930	5655	5397	5153	4922	19
20	7639	7238	6871	6532	6218	5925	5651	5393	5149	4918	20
21	763	7232	6865	65 7	6213	5920	5646	5389	5145	4915	21
22	7625	7225	6859	6521	6208	5916	5642	5384	5141	4911	22
23	7618	7219	6853	6516	6203	5911	5637	5380	5137	4907	23
24	7611	7212	6847	6510	6198	5906	5633	5376	5133	4903	24
25	7604	7206	6841	6505	6193	5902	5629	5372	5129	4900	25
26	7597	7200	6836	6500	6188	5897	5624	5368	5125	4896	26
27	7590	7193	6830	6494	6183	5892	56 0	5364	5122	4892	27
28	7563	7187	6824	6469	6176	5888	5615	5359	5118	4689	28
29	7577	7181	6818	6484	6173	5883	5611	5355	5114	18 5	29
30	7570	7175	6812	6478	6168	5878	5607	5351	5110	4 61	30
	10	11	12	13	14	15	16	17	18	19	

TABLE LI *Continued*

Logistic Logarithms

D M	10	11	12	13	14	15	16	17	18	19	D M
M S	600	660	720	780	840	900	960	1020	1080	1140	M S
31	7563	7168	6807	6473	6163	5874	5602	5347	5106	4877	31
32	7556	7162	6801	6467	6158	5869	5598	5343	5102	4874	32
33	7549	7156	6795	6462	6153	5864	5594	5339	5098	4870	33
34	7542	7149	6789	6457	6148	5860	5589	5335	5094	4866	34
35	7535	7143	6784	6451	6143	5855	5585	5331	5090	4863	35
36	7528	7137	6778	6446	6138	5850	5580	53 6	5086	4859	36
37	7522	7131	6772	6441	6133	5846	5576	53 2	5082	4855	37
38	7515	7124	6766	6435	6128	5841	5572	5318	5079	4852	38
39	7508	7118	6761	6430	6123	5836	5567	5314	5075	4848	39
40	7501	7112	6755	6425	6118	5832	5563	5310	5071	4844	40
41	7494	7106	6749	6420	6113	5827	5559	5306	5067	4841	41
42	7488	7100	6743	6414	6108	5823	5554	5302	5063	4837	42
43	7481	7093	6738	6409	6103	5818	5550	5 98	5059	4833	43
44	7474	7087	6732	6404	6099	5813	5546	5294	5055	4830	44
45	7467	7081	6726	6398	6094	5809	5541	5290	5051	4826	45
46	7461	7075	6721	6393	6089	5804	5537	5285	5048	48 2	46
47	7454	7069	6715	6388	6084	5800	5533	5 81	5044	4819	47
48	7447	7063	6709	6383	6079	5795	5528	5 77	5040	4815	48
49	7440	7057	6704	6377	6074	5790	5524	5273	5036	4811	49
50	7434	7056	6698	6372	6069	5786	5520	5 69	5032	4808	50
51	7427	7044	6692	6367	6064	5781	5516	5265	5028	4804	51
52	7421	7038	6687	6362	6059	5777	5511	5261	5025	4800	52
53	7414	7032	6681	6357	6055	5772	5507	5 57	50 1	4797	53
54	7407	7026	6676	6351	6050	5768	5503	5253	5017	4793	54
55	7401	7020	6670	6346	6045	5763	5498	5 49	5013	4789	55
56	7394	7014	6664	6341	6040	5758	5494	5245	5009	4786	56
57	7387	7008	6659	6336	6035	5754	5490	5 41	5005	4782	57
58	7381	7002	6653	6331	6030	5749	5486	5237	5002	4779	58
59	7374	6996	6648	6325	6025	5745	5481	5 33	4998	4775	59
60	7368	6990	6642	63 0	6021	5740	5477	5229	4994	4771	60
	10	11	12	13	14	15	16	17	18	19	

TABLE LI *Continued*

Logiſtic Logarithms

D M	20	21	22	23	24	25	26	27	28	29	D M
M S	1200	1260	1320	1380	1440	1500	1560	1620	1680	1740	M S
0	4771	4559	4357	4164	3979	3802	3632	3468	3310	3158	0
1	4768	4556	4354	4161	3976	3799	3629	3465	3307	3155	1
2	4764	4552	4351	4158	3973	3796	3626	3463	3305	3153	2
3	4760	4549	4347	4155	3970	3793	3623	3460	3302	3150	3
4	4757	4546	4344	4152	3967	3791	3621	3457	3300	3148	4
5	4753	4542	4341	4149	3964	3768	3618	3454	3297	3145	5
6	4750	4539	4338	4145	3961	3785	3615	3452	3294	3143	6
7	4746	4535	4334	4142	3958	3782	3612	3449	3292	3140	7
8	4742	4532	4331	4139	3955	3779	3610	3446	3289	3138	8
9	4739	45 8	4328	4136	3952	3776	3607	3444	3287	3135	9
10	4735	4525	4325	4133	3949	3773	3604	3441	3284	3133	10
11	4732	4522	4321	4130	3946	3770	3601	3438	3282	3130	11
12	4728	4518	4318	4127	3943	3768	3598	3436	3279	3128	12
13	4724	4515	4315	4124	3940	3765	3596	3433	3276	3125	13
14	47 1	4511	4311	4120	3937	3762	3593	3431	3274	3123	14
15	1717	4508	4308	4117	3934	3759	3590	3428	3271	3120	15
16	4714	4505	4305	4114	3931	3756	3587	3425	3269	3118	16
17	4710	4501	4302	4111	39 8	3753	3585	3423	3266	3115	17
18	4707	4498	4296	4108	39 5	3750	3582	3420	3264	3113	18
19	4703	4494	4295	4105	3922	3747	3579	3417	3261	3110	19
20	4699	4491	4292	410	3919	3745	3576	3415	3259	3108	20
21	4696	4488	4289	4099	3917	3742	3574	3412	3256	3105	21
22	4692	4484	4285	4096	3914	3739	3571	3409	3253	3103	22
23	4689	4481	4 82	409	3911	3736	3568	3407	3251	3101	23
24	4685	4477	4 79	4089	3908	3733	3565	3404	3248	3098	24
25	4682	4474	4276	4086	3905	3730	3563	3401	3246	3096	25
26	4678	4471	4273	4083	3902	37 7	3560	3399	3243	3093	26
27	4675	4467	4269	4080	3899	3725	3557	3396	3241	3091	27
28	4671	4464	4266	4077	3896	3722	3555	3393	3238	3088	28
29	4668	4460	4 63	4074	3893	3719	3552	3391	3236	3086	29
30	4664	4457	4260	4071	3890	3716	3549	3388	3233	3083	30
	20	21	2	23	24	25	26	27	28	29	

TABLE LI *Continued*

Logiſtic Logarithms

D M	20	21	22	23	24	25	26	27	28	29	D M
M S	1200	1260	1320	1380	1440	1500	1560	1620	1680	1740	M S
31	4660	4454	4256	4068	3887	3713	3546	3386	3231	3081	31
32	4657	4450	4 53	4065	3884	3710	3544	3383	3228	3078	32
33	4653	4447	4250	4062	3881	3708	3541	3380	3225	3076	33
34	4650	4444	4247	4059	3878	3705	3538	3378	3223	3073	34
35	4646	4440	4244	4055	3875	3702	3535	3375	3 20	3071	35
36	4643	4437	4240	4052	3872	3699	3533	3372	3218	3069	36
37	4639	4434	4237	4049	3869	3696	3530	3370	3215	3066	37
38	4636	4430	4234	4046	3866	3693	3527	3367	3 13	3064	38
39	4632	4427	4231	4043	3863	3691	3525	3365	3 10	3061	39
40	4629	4424	4228	4040	3860	3688	3522	3362	3 08	3059	40
41	4625	4420	4224	4037	3857	3685	3519	3359	3205	3056	41
42	4622	4417	4221	4034	3855	3682	3516	3357	3203	3054	42
43	4618	4414	4218	4031	3852	3679	3514	3354	3200	3052	43
44	4615	4410	4215	4028	3849	3677	3511	3351	3198	3049	44
45	4611	4407	4212	4025	3846	3674	3508	3349	3195	3047	45
46	4608	4404	4209	4022	3843	3671	3506	3346	3193	3044	46
47	4604	4400	4 05	4019	3840	3668	3503	3344	3190	3042	47
48	4601	4397	4202	4016	3837	3665	3500	3341	3188	3039	48
49	4597	4394	4199	4013	3834	3663	3197	3338	3185	3037	49
50	4594	4390	4196	4010	3831	3660	3495	3336	3183	3034	50
51	4590	4387	4193	4007	3828	3657	3492	3333	3180	3032	51
52	4587	4384	4189	4004	3825	3654	3489	3331	3178	3030	52
53	4584	4380	4186	4001	3622	3651	3487	3328	3175	3027	53
54	4580	4377	4183	3998	3820	3649	3484	3325	3173	3025	54
55	4577	4374	4180	3905	3817	3646	3481	3323	3170	3022	55
56	4573	4370	4177	3991	3814	3643	3479	3320	3168	3020	56
57	4570	4367	4174	3968	3811	3640	3476	3318	3165	3018	57
58	4566	4364	4171	3985	3808	3637	3473	3315	3163	3015	58
59	4563	4361	4167	398	3805	3635	3471	3313	3160	3013	59
60	4559	4357	4164	3979	3802	3632	3468	3310	3158	3010	60
	20	21	22	23	4	25	26	27	28	29	

TABLE LI. *Continued*

Logistic Logarithms

D M	30	31	32	33	34	35	36	37	38	39	D M
M S	1800	1860	1920	1980	2040	2100	2160	2220	2280	2340	M S
0	3010	2868	2730	2596	2467	2341	2218	2099	1984	1871	0
1	3008	2866	2728	2594	2465	2339	2216	2098	1982	1869	1
2	3005	2863	2725	2592	2462	2337	2214	2096	1980	1867	2
3	3003	2861	27 3	2590	2460	2335	2212	2094	1978	1865	3
4	3001	2859	2721	2588	2458	2333	2210	2092	1976	1863	4
5	2998	2856	2719	2585	2456	2331	2208	2090	1974	1862	5
6	2996	2854	2716	2583	2454	2328	2206	2088	1972	1860	6
7	2993	2852	2714	2581	2452	2326	2204	2086	1970	1858	7
8	2991	2849	2712	2579	2450	2324	2202	2084	1968	1856	8
9	2989	2847	2710	2577	2446	2322	2200	2082	1967	1854	9
10	2986	2845	2707	2574	2445	2320	2198	2080	1965	1852	10
11	2984	2842	2705	[illegible]	2443	2318	2196	2078	1963	1850	11
12	2981	2840	2703	2570	2441	2316	2194	2076	1961	1849	12
13	2979	2838	2701	2568	2439	2314	2192	2074	1959	1847	13
14	2977	2835	2698	2566	2437	2312	2190	2072	1957	1845	14
15	2974	2833	2696	2564	2435	2310	2188	2070	1955	1843	15
16	2972	2831	2694	2561	2433	2308	2186	2068	1953	1841	16
17	2969	2828	2692	2559	2431	2306	2184	2066	1951	1839	17
18	2967	2826	2689	2557	2429	2304	2182	2064	1950	1838	18
19	2965	2824	2687	2555	2426	2302	2180	2062	1948	1836	19
20	2962	2821	2685	2553	2424	2300	2178	2061	1946	1834	20
21	2960	2819	2683	2551	2422	2298	2176	2059	1944	1832	21
22	2958	2817	2681	2548	2420	2296	2174	2057	1942	1830	22
23	2955	2815	2678	2546	2418	2294	2172	2055	1940	1828	23
24	2953	2812	2676	2544	2416	2291	2170	2053	1938	1827	24
25	2950	2810	2674	2542	2414	2289	2169	2051	1936	1825	25
26	2948	2808	2672	2540	2412	2287	2167	2049	1934	1823	26
27	2946	2805	2669	2538	2410	2285	2165	2047	1933	1821	27
28	2943	2803	2667	2535	2408	2283	2163	2045	1931	1819	28
29	2941	2801	2665	2533	2405	2281	2161	2043	1929	1817	29
30	2939	2798	2663	2531	2403	2279	2159	2041	1927	1816	30
	30	31	32	33	34	35	36	37	38	39	

TABLE LI Continued

Logistic Logarithms

D M	30	31	32	33	34	35	36	37	38	39	D M
M S	1800	1860	1920	1980	2040	2100	2160	2220	2280	2340	M S
31	2936	2796	2660	2529	2401	2277	2157	2039	1925	1814	31
32	2934	2794	2658	2527	2399	2275	2155	2037	1923	1812	32
33	2931	2792	2656	2525	2397	2273	2153	2035	1921	1810	33
34	2929	2789	2654	2522	2395	2271	2151	2033	1919	1808	34
35	2927	2787	2652	2520	2393	2269	2149	2032	1918	1806	35
36	2924	2785	2649	2518	2391	2267	2147	2030	1916	1805	36
37	2922	2782	2647	2516	2389	2265	2145	2028	1914	1803	37
38	2920	2780	2645	2514	2387	2263	2143	2026	1912	1801	38
39	2917	2778	2643	2512	2384	2261	2141	2024	1910	1799	39
40	2915	2775	2640	2510	2382	2259	2139	2022	1908	1797	40
41	2912	2773	2638	2507	2380	2257	2137	2020	1906	1795	41
42	2910	2771	2636	2505	2378	2255	2135	2018	1904	1794	42
43	2908	2769	2634	2503	2376	2253	2133	2016	1903	1792	43
44	2905	2766	2632	2501	2374	2251	2131	2014	1901	1790	44
45	2903	2764	2629	2499	2372	2249	2129	2012	1899	1788	45
46	2901	2762	2627	2497	370	2247	2127	2010	1897	1786	46
47	2898	2760	2625	2494	2368	2245	2125	2009	1895	1785	47
48	2896	2757	2623	2492	2366	2243	2123	2007	1893	1783	48
49	2894	2755	2621	2490	2364	2241	2121	2005	1891	1781	49
50	2891	2753	2618	2488	2362	2239	2119	2003	1889	1779	50
51	2889	2750	2616	2486	359	2237	2117	2001	1888	1777	51
52	2887	2748	2614	2484	2357	2235	2115	1999	1886	1775	52
53	2884	2746	2612	248	2355	2233	2113	1997	1884	1774	53
54	2882	2744	2610	2480	2353	2231	2111	1995	1882	1772	54
55	2880	2741	2607	2477	351	2229	2109	1993	1880	1770	55
56	2877	2739	2605	2475	2349	2227	2107	1991	1878	1768	56
57	2875	737	2603	2473	2347	2225	2105	1989	1876	1766	57
58	2873	2735	2601	2471	2345	2223	2103	1987	1875	1765	58
59	2870	2732	2599	2469	2343	2220	2101	1986	1873	1763	59
60	2868	2730	2596	2467	2341	2218	2099	1984	1871	1761	60
	30	31	32	33	34	35	36	37	38	39	

TABLE LI *Continued*

Logiſtic Logarithms

D M	40	41	42	43	44	45	46	47	48	49	D M
M S	2400	2460	2520	2580	2640	700	2760	2820	2880	2940	M S
0	1761	1654	1549	1447	1347	1249	1154	1061	0969	0880	0
1	1759	1652	1547	1445	1345	1248	1152	1059	0968	0878	1
2	1757	1650	1546	1443	1344	1246	1151	10-7	0966	0877	2
3	1755	1648	1544	1442	134	1245	1149	1056	0965	0875	3
4	1754	1647	1542	1440	1340	1243	1148	1054	0963	0874	4
5	1752	1645	1540	1438	1339	1241	1146	1053	0962	0872	5
6	1750	1643	1539	1437	1337	1240	1145	1051	0960	0871	6
7	1748	1641	1537	1435	1335	1238	1143	1050	0959	0869	7
8	1746	1640	1535	1433	1334	1237	1141	1048	0957	0868	8
9	1745	1638	1534	1432	1332	1235	1140	1047	0956	0866	9
10	1743	1636	1532	1430	1331	1233	1138	1045	0954	0865	10
11	1741	1634	1530	1428	1329	1232	11-7	1044	0953	0863	11
12	1739	1633	1528	1427	1327	1230	1135	1042	0951	0862	12
13	1737	1631	1527	1425	1326	1-29	1134	1041	0950	0860	13
14	1736	1629	1525	14-3	1324	1227	1132	1039	0948	0859	14
15	1734	1627	1523	1422	132-	1225	1130	1037	0947	0857	15
16	1732	1626	152-	1426	1321	1-24	1129	1036	0945	0856	16
17	1730	1624	15-0	1418	1319	1222	1127	1034	0944	08-5	17
18	1728	1622	1518	1417	1317	1221	1126	1033	094-	0853	18
19	1727	1620	1516	1415	1316	1219	1124	1031	0941	0852	19
20	1725	1619	1515	1413	1314	1-17	1123	1030	0939	0850	20
21	1723	1617	1513	1412	1313	1216	1121	1028	0938	0849	21
22	1721	1615	1511	1410	1311	1214	1119	1027	0936	0847	22
23	1719	1613	1510	1408	1309	1213	1118	10-5	0935	0846	23
24	1718	1612	1508	1407	1308	1-11	1116	10-4	0933	0844	24
25	1716	1610	1506	1405	1306	1209	1115	1022	0932	0843	25
26	1714	1608	1504	1403	1304	1208	1113	1021	0930	0841	26
27	1712	1606	1503	1402	1303	1206	1112	1019	09-9	0840	27
28	1711	1605	1501	1400	1301	1205	1110	1018	0927	0838	28
29	1709	1603	1499	1398	1300	1-03	1109	1016	0926	0837	29
30	1707	1601	1498	1397	1298	1201	1107	1015	0924	0835	30
	40	41	42	43	44	45	46	47	48	49	

TABLE LI *Continued*

Logiſtic Logarithms

D M	40	41	42	43	44	45	46	47	48	49	D M
M S	2400	2460	2520	2580	2640	2700	2760	820	2880	2940	M S
31	1705	1599	1496	1395	1296	1200	1105	1013	0923	0834	31
32	1703	1598	1494	1393	1295	1198	1104	1012	09 1	0833	32
33	1702	1596	1493	1392	1293	1197	1102	1010	0920	0831	33
34	1700	1594	1491	1390	1291	1195	1101	1008	0981	0830	34
35	1698	1592	1489	1388	1290	1193	1099	1007	0917	0828	35
36	1696	1591	1487	1387	1288	1192	1098	1005	0915	08 7	36
37	1694	1589	1486	1385	1287	1190	1096	1004	0914	0825	37
38	1693	1587	1484	1383	1 85	1189	1095	1002	0912	0824	38
39	1690	1585	1482	138	1283	1187	1093	1001	0911	0822	39
40	1689	1584	1481	1380	1282	1186	1091	0999	0909	0821	40
41	1687	1582	1479	1378	1280	1184	1090	0998	0908	0819	41
42	1686	1580	1477	1377	1278	1182	1088	0996	0906	0818	42
43	1684	1578	1476	1375	1277	1181	1087	0995	0905	0816	43
44	168	1577	1474	1373	1275	1179	1085	0993	0903	0815	44
45	1680	1575	1472	1372	1274	1178	1084	0992	0902	0814	45
46	1678	1573	1470	1370	1272	1176	108	0990	0900	0812	46
47	1677	1571	1469	1368	1270	1174	1081	0989	0899	0811	47
48	1675	1570	1467	1367	1269	1173	1079	0987	0897	0809	48
49	1673	1568	1465	1365	1267	1171	1078	0986	0896	0808	49
50	1671	1566	1464	1363	1266	1170	1076	0984	0894	0806	50
51	1670	1565	1462	1362	1264	1168	1074	0983	0893	0805	51
52	1668	1563	1460	1360	1262	1167	1073	0981	0891	0803	52
53	1666	1561	1459	1359	1261	1165	1071	0980	0890	0802	53
54	1664	1559	1457	1357	1259	1163	1070	0978	0888	0801	54
55	1663	1558	1455	1395	1257	1162	1068	0977	0887	0799	55
56	1661	1556	1454	1354	1256	1160	1067	0975	0885	0798	56
57	1659	1554	1452	1352	1254	1159	1065	0974	0884	0796	57
58	1657	155	1450	1350	1253	1157	1064	0972	0883	0795	58
59	1655	1551	1449	1349	1251	1156	106	0971	0881	0793	59
60	1654	1549	1447	1347	1249	1154	1061	0969	0880	0792	60
	40	41	42	43	44	45	46	47	48	49	

TABLE LI *Continued*

Logiſtic Logarithms

D M	50	51	52	53	54	55	56	57	58	59	D M
M S	3000	3060	3120	3180	3 40	3300	3360	3420	3480	3540	M S
0	0792	0706	0621	0539	0458	0378	0300	0 23	0147	0073	0
1	0790	0704	0620	0537	0456	0377	0298	0221	0146	0072	1
2	0789	0703	0619	0536	0455	0375	0297	0220	0145	0071	2
3	0787	0702	0617	0535	0454	0374	0296	0219	0143	0069	3
4	0786	0700	0616	0533	045	0373	0294	0218	0142	0068	4
5	0785	0699	0615	053	0451	0371	0293	0216	0141	0067	5
6	0783	0697	0613	0531	0450	0370	0292	0215	0140	0066	6
7	078	0696	0612	0529	0446	0369	0291	0214	0139	0064	7
8	0780	0694	0610	0528	0447	0367	0289	0213	0137	0063	8
9	0779	0693	0609	0526	0446	0366	0268	0211	0136	0062	9
10	0777	069	0608	0525	0444	0365	0287	0210	0135	0061	10
11	0776	0690	0606	05 4	0443	0363	0285	0209	0134	0060	11
12	0774	0689	0605	0522	0442	036	0284	0208	0132	0058	12
13	0773	0687	0603	0521	0440	0361	0 83	0206	0131	0057	13
14	0772	0686	0602	0520	0439	0359	0262	0205	0130	0056	14
15	0770	0685	0601	0518	0438	0358	0280	0204	01 9	0055	15
16	0769	0683	0599	0517	0436	0357	0279	0202	0127	0053	16
17	0767	0682	0598	0516	0435	0356	0278	0291	0126	0052	17
18	0766	0680	0596	0514	0434	0354	0276	0200	0125	0051	18
19	0764	0679	0595	0513	0432	0353	0275	0199	0124	0050	19
20	0763	0678	0594	0512	0431	0352	0274	0197	0122	0049	20
21	076	0676	0592	0510	0430	0350	0 73	0 96	0121	0047	21
22	0760	0675	0591	0509	0428	0349	0 71	0195	0120	0046	22
23	0759	0673	0590	0507	0427	0348	0 70	0194	0119	0045	23
24	0757	0672	0588	0506	04 6	0346	0269	0192	0117	0044	24
25	0756	0670	0587	0505	0424	0345	0267	0191	0116	0042	25
26	0754	0669	0585	0503	04 3	0344	0266	0190	0115	0041	26
27	0753	0668	0584	0502	0422	0342	0265	0189	0114	0040	27
28	0751	0666	0583	0501	0420	0341	0264	0187	0112	0039	28
29	0750	0665	0581	0499	0419	0340	0262	0186	0111	0038	29
30	0749	0663	0580	0498	0418	0339	0261	0185	0110	0036	30
	50	51	52	53	54	55	56	57	58	59	

TABLE LI *Continued*

Logistic Logarithms

D M	50	51	52	53	54	55	56	57	58	59	D M
M S	3000	3060	3120	3180	3240	3300	3360	3420	3480	3540	M S
31	0747	0662	0579	0497	0416	0337	0260	0184	0109	0035	31
32	0746	0661	0577	0495	0415	0336	0258	0182	0107	0034	32
33	0744	0659	0576	0494	0414	0335	0257	0181	0106	0033	33
34	0743	0658	0574	0493	0412	0333	0256	0180	0105	0031	34
35	0741	0656	0573	0491	0411	0332	0255	0179	0104	0030	35
36	0740	0655	0572	0490	0410	0331	0253	0177	0103	0029	36
37	0739	0654	0570	0489	0408	0329	0252	0176	0101	0028	37
38	0737	0652	0569	0487	0407	0328	0251	0175	0100	0027	38
39	0736	0651	0568	0486	0406	0327	0250	0174	0099	0025	39
40	0734	0649	0566	0484	0404	0326	0248	0172	0098	0024	40
41	0733	0648	0565	0483	0403	0324	0247	0171	0096	0023	41
42	0731	0647	0563	0482	0402	0323	0246	0170	0095	0022	42
43	0730	0645	0562	0480	0400	0322	0244	0169	0094	0021	43
44	0729	0644	0561	0479	0399	0320	0243	0167	0093	0019	44
45	0727	0642	0559	0478	0398	0319	0242	0166	0091	0018	45
46	0726	0641	0558	0476	0396	0318	0241	0165	0090	0017	46
47	0724	0640	0557	0475	0395	0316	0239	0163	0089	0016	47
48	0723	0638	0555	0474	0394	0315	0238	0162	0088	0015	48
49	0721	0637	0554	0472	0392	0314	0237	0161	0087	0013	49
50	0720	0635	0552	0471	0391	0313	0235	0160	0085	0012	50
51	0719	0634	0551	0470	0390	0311	0234	0158	0084	0011	51
52	0717	0633	0550	0468	0388	0310	0233	0157	0083	0010	52
53	0716	0631	0548	0467	0387	0309	0232	0156	0082	0008	53
54	0714	0630	0547	0466	0386	0307	0230	0155	0080	0007	54
55	0713	0628	0546	0464	0384	0306	0229	0153	0079	0006	55
56	0711	0627	0544	0463	0383	0305	0228	0152	0078	0005	56
57	0710	0626	0543	0462	0382	0304	0227	0151	0077	0004	57
58	0709	0624	0541	0460	0381	0302	0225	0150	0075	0002	58
59	0707	0623	0540	0459	0379	0301	0224	0148	0074	0001	59
60	0706	0621	0539	0458	0378	0300	0223	0147	0073	0000	60
	50	51	52	53	54	55	56	57	58	59	

TABLE LI *Continued*

Logiſtic Logarithms

D M	60	61	62	63	64	65	66	67	68	69	D M
M S	3600	3660	3720	3780	3840	3900	3960	4020	4080	4140	M S
0	10000	9928	9858	9788	9720	9652	9586	9521	9456	9393	0
1	9999	9927	9856	9787	9719	9651	9585	9520	9455	9392	1
2	9998	9926	9855	9786	9717	9650	9584	9519	9454	9391	2
3	9996	9925	9854	9785	9716	9649	9583	9518	9453	9390	3
4	9995	9923	9853	9784	9715	9648	9582	9516	9452	9389	4
5	9994	9922	9852	9782	9714	9647	9581	9515	9451	9388	5
6	9993	9921	9851	9781	9713	9646	9579	9514	9450	9387	6
7	9992	9920	9849	9780	9712	9645	9578	9513	9449	9386	7
8	9990	9919	9848	9779	9711	9643	9577	9512	9448	9385	8
9	9989	9918	9847	9778	9710	9642	9576	9511	9447	9384	9
10	9988	9916	9846	9777	9708	9641	9575	9510	9446	9383	10
11	9987	9915	9845	9775	9707	9640	9574	9509	9445	9381	11
12	9986	9914	9844	9774	9706	9639	9573	9508	9444	9380	12
13	9984	9913	9842	9773	9705	9638	9572	9507	9443	9379	13
14	9983	9912	9841	9772	9704	9637	9571	9506	9442	9378	14
15	9982	9910	9840	9771	9703	9636	9570	9505	9440	9377	15
16	9981	9909	9839	9770	9702	9635	9569	9504	9439	9376	16
17	9980	9908	9838	9769	9701	9633	9567	9502	9438	9375	17
18	9979	9907	9837	9767	9699	9632	9566	9501	9437	9374	18
19	9977	9906	9835	9766	9698	9631	9565	9500	9436	9373	19
20	9976	9905	9834	9765	9697	9630	9564	9499	9435	9372	20
21	9975	9903	9833	9764	9696	9629	9563	9498	9434	9371	21
22	9974	9902	9832	9763	9695	9628	9562	9497	9433	9370	22
23	9972	9901	9831	9762	9694	9627	9561	9496	9432	9369	23
24	9971	9900	9830	9761	9693	9626	9560	9495	9431	9368	24
25	9970	9899	9829	9759	9692	9625	9559	9494	9430	9367	25
26	9969	9897	9827	9758	9690	9624	9558	9493	9429	9366	26
27	9968	9896	9826	9757	9689	9622	9557	9492	9428	9365	27
28	9966	9895	9825	9756	9688	9621	9555	9491	9427	9364	28
29	9965	9894	9824	9755	9687	9620	9554	9490	9426	9363	29
30	9964	9893	9823	9754	9686	9619	9553	9488	9425	9362	30
	60	61	62	63	64	65	66	67	68	69	

TABLE LI *Continued*

Logiſtic Logarithms

D M	60	61	62	63	64	65	66	67	68	69	D M
M S	3600	3660	3720	3780	3840	3900	3960	4020	4080	4140	M S
31	9963	9892	98 2	9753	9685	9618	9552	9487	9424	9361	31
32	9962	9890	9820	9751	9684	9617	9551	9486	94 2	9360	32
33	9960	9889	9819	9750	9683	9616	9550	9485	9421	9359	33
34	9959	9888	9818	9749	9681	9615	9549	9484	9420	9358	34
35	9958	9887	9817	9748	9680	9614	9548	9483	9419	9356	35
36	9957	9886	9816	9747	9679	9612	9547	9482	9418	9355	36
37	9956	9885	9815	9746	9678	9611	9546	9481	9417	9354	37
38	9954	9883	9813	9745	9677	9610	9545	9480	9416	9353	38
39	9953	9882	981	9744	9676	9609	9544	9479	9415	9352	39
40	9952	9881	9811	9742	9675	9608	9542	9478	9414	9351	40
41	9951	9880	9810	9741	9674	9607	9541	9477	9413	9350	41
42	9950	9879	9809	9740	9672	9606	9540	9476	9412	9349	42
43	9948	9877	9808	9739	9671	9605	9539	9475	9411	9348	43
44	9947	9876	9807	9738	9670	9604	95 6	9473	9410	9347	44
45	9946	9875	9805	9737	9669	9603	9537	9472	9409	9346	45
46	9945	9874	9804	9736	9668	9601	9536	9471	9408	9345	46
47	9944	9873	9803	9734	9667	9600	9535	9470	9407	9344	47
48	9942	9872	9802	9733	9666	9599	9534	9469	9406	9343	48
49	9941	9870	9801	9732	9665	9598	9533	9468	9405	9342	49
50	9940	9869	9800	9731	9664	9597	9532	9467	9404	9341	50
51	9939	9868	9798	9730	9662	9596	9530	9466	9402	9340	51
52	9938	9867	9797	9729	9661	9595	9529	9465	9401	9339	52
53	9937	9866	9796	9728	9660	9594	9528	9464	9400	9338	53
54	9935	9865	9795	9727	9659	9593	9527	9463	9399	9337	54
55	9934	9863	9794	9725	9658	9592	9526	9462	9398	9336	55
56	9933	9862	9793	9724	9657	9590	9525	9461	9397	9335	56
57	9932	9861	9792	9723	9656	9589	9524	9460	9396	9334	57
58	9931	9860	9790	9722	9655	9588	9523	9459	9395	9333	58
59	9929	9859	9789	9721	9653	9587	9522	9457	9394	9332	59
60	9928	9858	9788	9720	9652	9586	9521	9456	9393	9331	60
	60	61	62	63	64	65	66	67	68	69	

TABLE LI *Continued*

Logiſtic Logarithms.

D M	70	71	72	73	74	75	76	77	78	79	D M
M S	4200	4260	4320	4380	4440	4500	4560	46 0	4680	4740	M S
0	9331	9269	9 08	9148	9089	9031	8973	8917	8861	8805	0
1	93 9	9268	9207	9147	9088	9030	8972	8916	8860	8804	1
2	93 8	9267	9206	9146	9087	9029	8971	8915	8859	8803	2
3	9327	9266	9205	9145	9086	9028	8971	8914	8858	8802	3
4	93 6	9265	9204	9144	9085	9027	8970	8913	8857	8802	4
5	9325	9264	9203	9143	9084	9026	8969	8912	8856	8801	5
6	93 4	9263	9202	9142	9083	9025	8968	8911	8855	8800	6
7	9323	9262	9201	9141	9082	9024	8967	8910	8854	8799	7
8	9322	9261	9200	9140	9081	9023	8966	8909	8853	8798	8
9	9321	9260	9199	9139	9080	9022	8965	8908	8852	8797	9
10	9320	9259	9198	9138	9079	9021	8964	8907	8851	8796	10
11	9319	9258	9197	9137	9078	9020	8963	8906	8850	8795	11
12	9318	9257	9196	9136	9077	9019	8962	8905	8849	8794	12
13	9317	9256	9195	9135	9076	9018	8961	8904	8849	8793	13
14	9316	9 55	9194	9134	9076	9017	8960	8903	8848	8792	14
15	9315	9 54	9193	9133	9075	9016	8959	8903	8847	8792	15
16	9314	9253	9192	9132	9074	9015	8958	8902	8846	8791	16
17	9313	9252	9191	9131	9073	9015	8957	8901	8845	8790	17
18	9312	9251	9190	9130	9072	9014	8956	8900	8844	8789	18
19	9311	9250	9189	9129	9071	9013	8955	8899	8843	8788	19
20	9310	9249	9188	9128	9070	9012	8954	8898	8842	8787	20
21	9309	9248	9187	9128	9069	9011	8953	8897	8841	8786	21
22	9308	9247	9186	9127	9068	9010	8952	8896	8840	8785	22
23	9307	9246	9185	9126	9067	9009	8952	8895	8839	8784	23
24	9306	9245	9184	9125	9066	9008	8951	8894	8838	8783	24
25	9305	9 44	9183	9124	9065	9007	8950	8893	8837	8782	25
26	9304	9243	918	9123	9064	9006	8949	8892	8837	8781	26
27	9303	9 41	9181	9122	9063	9005	8948	8891	8836	8781	27
28	9302	9240	9180	9121	9062	9004	8947	8890	8835	8780	28
29	9301	9 39	9179	91 0	9061	9003	8946	8889	8834	8779	29
30	9300	9238	9178	9119	9060	9002	8945	8868	8833	8778	30
	70	71	72	73	74	75	76	77	78	79	

TABLE LI *Continued*

Logiſtic Logarithms

D M	70	71	72	73	74	75	76	77	78	79	D M
M S	4200	4260	4320	4380	4440	4500	4560	4620	4680	4740	M S
31	9299	9237	9177	9118	9059	9001	8944	8888	8832	8777	31
32	9298	9236	9176	9117	9058	9000	8943	8887	8831	8776	32
33	9297	9235	9175	9116	9057	8999	8942	8886	8830	8775	33
34	9296	9234	9174	9115	9056	8998	8941	8885	8829	8774	34
35	9294	9233	9173	9114	9055	8997	8940	8884	8828	8773	35
36	9293	9232	9172	9113	9054	8996	8939	8883	8827	8772	36
37	9292	9231	9171	9112	9053	8995	8938	8882	8826	8771	37
38	9291	9230	9170	9111	9052	8994	8937	8881	8825	8771	38
39	9290	9229	9169	9110	9051	8993	8936	8880	8825	8770	39
40	9289	9228	9168	9109	9050	8992	8935	8879	8824	8769	40
41	9288	9227	9167	9108	9049	8992	8935	8878	8823	8768	41
42	9287	9226	9166	9107	9048	8991	8934	8877	8822	8767	42
43	9286	9225	9165	9106	9047	8990	8933	8876	8821	8766	43
44	9285	9224	9164	9105	9046	8989	8932	8875	8820	8765	44
45	9284	9223	9163	9104	9045	8988	8931	8875	8819	8764	45
46	9283	9222	9162	9103	9044	8987	8930	8874	8818	8763	46
47	9282	9221	9161	9102	9043	8986	8929	8873	8817	8762	47
48	9281	9220	9160	9101	9042	8985	8928	8872	8816	8761	48
49	9280	9219	9159	9100	9042	8984	8927	8871	8815	8761	49
50	9279	9218	9158	9099	9041	8983	8926	8870	8814	8760	50
51	9278	9217	9157	9098	9040	8982	8925	8869	8813	8759	51
52	9277	9216	9156	9097	9039	8981	8924	8868	8813	8758	52
53	9276	9215	9155	9096	9038	8980	8923	8867	8812	8757	53
54	9275	9214	9154	9095	9037	8979	8922	8866	8811	8756	54
55	9274	9213	9153	9094	9036	8978	8921	8865	8810	8755	55
56	9273	9212	9152	9093	9035	8977	8920	8864	8809	8754	56
57	9272	9211	9151	9092	9034	8976	8919	8863	8808	8753	57
58	9271	9210	9150	9091	9033	8975	8918	8862	8807	8752	58
59	9270	9209	9149	9090	9032	8974	8918	8861	8806	8752	59
60	9269	9208	9148	9089	9031	8973	8917	8861	8805	8751	60
	70	71	72	73	74	75	76	77	78	79	

TABLE LI *Continued*

Logistic Logarithms

D M	80	81	82	83	84	85	86	87	88	89	D M
M S	4800	4860	49 0	4980	5040	5100	5160	5220	5280	5340	M S
0	8751	8697	8644	8591	8539	8488	8437	8387	8337	8288	0
1	8750	8696	8643	8590	8538	8487	8436	8386	8336	8287	1
2	8749	8695	8642	8589	8537	8486	8435	8385	8335	8286	2
3	8748	8694	8641	8589	8537	8485	8434	8384	8335	8286	3
4	8747	8693	8640	8588	8536	8484	8433	8383	8334	8285	4
5	8746	8692	8639	8587	8535	8484	8433	8383	8333	8284	5
6	8745	8692	8638	8586	8534	8483	8432	8382	8332	8283	6
7	8745	8691	8638	8585	8533	8482	8431	8381	8331	8282	7
8	8744	8690	8637	8584	853	8481	8430	8380	8330	8281	8
9	8743	8689	8636	8583	8531	8480	8429	8379	8330	8281	9
10	8742	8688	8635	8582	8530	8479	8428	8379	8329	8280	10
11	8741	8687	8634	858	8530	8478	8428	8378	8328	8279	11
12	8740	8686	8633	8581	8529	8477	84 7	8377	83 7	8 78	12
13	8739	8685	863	8580	85 6	8477	8426	8376	83 6	8278	13
14	8738	8684	8631	8579	8527	8476	8425	8375	8325	8277	14
15	8737	8684	8631	8578	8526	8475	8424	8374	8325	8276	15
16	8736	8683	8630	8577	8525	8474	8423	8373	8324	8275	16
17	8736	8682	86 9	8576	8524	8473	8423	8373	8323	8274	17
18	8735	8681	86 8	8575	8524	8472	84 2	8372	8322	8273	18
19	8734	8680	8627	8575	8523	8472	8421	8371	8322	8273	19
20	8733	8679	8626	8574	8522	8471	8420	8370	8321	8272	20
21	8732	8678	8625	8573	85 1	8470	8419	8369	8320	8271	21
22	8731	8677	8624	8572	85 0	8469	8418	8368	8319	8270	22
23	8730	8677	8624	8571	8519	8468	8418	8368	8318	8 69	23
24	8729	8676	86 3	8570	8518	8467	8417	8367	8317	8 68	24
5	87 8	8675	86 2	8569	8516	8467	8416	8366	8317	8268	25
26	87 7	8674	86 1	8565	8517	8466	8415	8365	8316	8267	26
7	87 7	8673	8620	8560	8516	8465	8414	8364	8315	8266	7
8	8726	8672	8619	8567	8515	8464	8413	8363	8314	8 65	28
29	8725	8671	8618	8566	8514	8463	8413	8363	8313	8 65	29
30	8724	8670	8617	8565	8513	8462	8412	8362	831	8264	30
	80	81	8	83	84	85	86	87	88	89	

TABLE LI *Continued*

Logistic Logarithms

D M	80	81	82	83	84	85	86	87	88	89	D M
M S	4800	4860	4920	4980	5040	5100	5160	5220	5280	5340	M S
31	8723	8669	8617	8564	8513	8461	8411	8361	8312	8 63	31
32	8722	8668	8616	8563	8512	8460	8410	8360	8311	8 62	32
33	8721	8668	8615	856	8511	8460	8409	8359	8310	8261	33
34	8720	8667	8614	8561	8510	8459	8408	8358	8309	8260	34
35	8719	8666	8613	8561	8509	8458	8408	8358	8308	8 60	35
36	8716	8665	8612	8560	8508	8457	8407	8357	8307	8259	36
37	8718	8664	8611	8559	8507	8456	8406	8356	8307	8258	37
38	8717	8663	8610	8558	8506	8455	8405	8355	8306	8 57	38
39	8716	8662	8610	8557	8506	8455	8404	8354	8305	8257	39
40	8715	8661	8609	8556	8505	8454	8403	8353	8304	8256	40
41	8714	8661	8608	8556	8504	8453	8403	8353	8304	8255	41
42	8713	8660	8607	8555	8503	8452	8402	8352	8303	8254	42
43	8712	8659	8606	8554	8502	8451	8401	8351	8302	8253	43
44	8711	8658	8605	8553	8501	8450	8400	8350	8301	8252	44
45	8710	8657	8604	8552	8501	8450	8399	8350	8300	8252	45
46	8709	8656	8603	8551	8500	8449	8390	8349	8299	8251	46
47	8709	8655	8603	8550	8499	8448	8397	8348	8299	8 50	47
48	8706	8654	8602	8549	8498	8447	8397	8347	8 98	8 49	48
49	8707	8654	8601	8549	8497	8446	8396	8346	8297	8 48	49
50	8700	8653	8600	8548	8496	8445	8395	8345	8 96	8247	50
51	8705	8652	8599	8547	8495	8445	8394	8345	8 95	8247	51
52	8704	8651	8598	8546	8494	8444	8393	8344	8 94	8246	52
53	8703	8650	8597	8545	8494	8443	839	8 43	8 94	8 45	53
54	8702	8649	8596	8544	8493	8442	839	8342	8 93	8 44	54
55	8702	8648	8596	8543	8492	8441	8391	8341	8 92	8244	55
56	8701	8647	8595	854	8491	8440	8390	8340	8 91	8 41	56
57	8700	8646	8594	8542	8490	8440	8389	8340	8 91	8 42	57
58	8699	8645	8593	8541	8489	8439	8388	8339	8 90	8 41	58
59	8698	8645	859	8540	8489	8438	83	8338	8 89	8 40	59
60	8697	8644	8591	8539	8488	8437	8387	337	8 88	8 39	60
	80	81	82	83	84	85	86	87	88	89	

THE USE OF THE TABLES

TABLE I

FOR converting Degrees, Minutes and Seconds into Time

RULE. Take the degrees, minutes and ſeconds from the firſt, third, and fifth columns, and againſt them you have the correſponding times; the ſum of which is the time required.

EXAMPLE. Reduce 78° 39′ 57″ into time.

70°	4^h	40′	0″
4	0	16	0
30′	0	2	0
9′	0	0	36
50″	0	0	3,333
7″	0	0	0,466
Time required	4	58	39,799

TABLE

TABLE II

For converting Time into Degrees, Minutes and Seconds

Rule. Take the time from the firſt, third and fifth columns, and againſt them you have the degrees, minutes and ſeconds correſponding the ſum of which is the quantity required

Example. Reduce 17*h* 34′ 19″ into degrees, minutes and ſeconds.

10*h*	150	0	0″
7*h*	105	0	0
30	7	30	0
4	1	0	0
10′	0	2	30
9″	0	2	15
Degrees required	163	34	45

TABLE III

For converting Minutes and Seconds into the Decimal of an Hour

Rule. Take the time from the firſt and third columns and againſt them you have the correſponding decimals, the ſum of which is the decimal required

Example. What is the decimal of 19′ 47″?

10′	,16666
9	,15000
40′	,01111
7′	,00194
Decimal required	,32971

TABLE IV.

For finding the Length of circular Arcs to Radius Unity.

Rule. Take the degrees, minutes and ſeconds from the firſt, third and fifth columns, and againſt them you have the correſponding lengths the ſum of which is the length required.

Example. What is the length of an arc of 37° 42′ 58″?

30°	0,5235988
7	0,1221730
40	0,0116355
2′	0,0005818
50	0,0002424
8′	0,0000388
Length required	0,6582703

If the radius be not unity the length may be found by proportion, by ſaying, unity : radius :: length here found : the length required.

TABLE V.

For finding the Sun's Parallax in Altitude, the apparent Altitude being given.

Rule. Find the altitude under the column *Sun's Alt.* and againſt it you have the parallax. If the apparent altitude be not found in the Table, the parallax muſt be found by proportion.

Example. What is the parallax at the apparent altitude 47° 27′ 20″?

At 40 app. alt. the parallax is 6″,70
— 50 ——————— 5, 62

10 1, 08

Hence, 10 : 7° 27′ 20″ :: 1″ 08 : 0″,8, which ſubtracted from 6″,7 leaves 5′,9 the parallax. Hence, the altitude, corrected for parallax, is 47° 27′ 25′,9.

TABLE

TABLE VI

Contains the mean Right Aſcenſions and North polar Diſtances of 36 *principal fixed Stars for the Beginning of* 1790 *together with their annual Preceſſions, and proper Motions all as ſettled by Dr* MASKELYNE, *thence to deduce their places for any other Year*

RULE Multiply the annual preceſſion by the number of years between the given year and 1790, and you get the annual preceſſion for that interval Then if the given year be *after* 1790 *add* the annual preceſſion in right aſcenſion for that interval to the right aſcenſion for 1790, and you get the mean right aſcenſion for the beginning of the given year and apply the annual preceſſion in north polar diſtance for the interval according to the ſign, to the north polar diſtance for 1790, and you get the north polar diſtance at the beginning of the given year But if the given year be *before* 1790, *ſubtract* the annual preceſſion in right aſcenſion, and apply the annual preceſſion of north polar diſtance with a *contrary* ſign

EXAMPLE What is the mean right aſcenſion and north polar diſtance of *Sirius*, at the beginning of the year 1800?

Mean right aſcenſion for 1790	-	6^h 35′ 53″,44
Motion in preceſſion for 10 years	-	+26, 79
Mean right aſcenſion for 1800	-	6 36 20 23

Or, in *degrees* it is 99 5′ 3″,45 (TAB II)

North polar diſtance for 1790	-	106° 26′ 21″,1
Motion in preceſſion for 10 years	-	+42, 5
North polar diſtance for 1800		106 27 3, 6

TABLE

TABLE VII

Contains the Sum of the Preceſſion, Aberration, and Solar Inequality of Preceſſion from the beginning of the Year of 31 principal Stars, in Degrees *being one Part of the Correction of the mean Right Aſcenſion from the beginning of the Year*

RULE Againſt the day of the month, under the given ſtar, you have the equation required If the day of the month be not found in the Table, the equation muſt be found by proportion

EXAMPLE What is the equation for *Sirius* on April 27?

April 20	-	the equation	+3″,4
—— 30	-	——————	+1,3
10			2,1

Hence, 10 7 2″,1 1″,5, which ſubtracted from + 3″,4 gives + 1″,9 the equation, or correction required

TABLE VIII

Contains the Equation of the Equinoxes, and Deviation of the ſame Stars as in the laſt Table, in Degrees, *being the other Part of the Correction in Right Aſcenſion from the beginning of the Year*

RULE Enter the firſt column with the longitude of the moon's aſcending node (TAB XXXI XXXII), this equation depending upon the place of the node, and againſt it, under the given ſtar, you have the equation required If the longitude be not found in the Table, the equation muſt be found by proportion

EXAMPLE

EXAMPLE. What is the equation of *Sirius* on April 27, 1800?

The longitude of the moon's aſcending node is 0 27′ 4′

Long. of ☽'s ☊	0 0°	-	Equation	−0,4
	0 10	-		−2,9
	0 10			2,5

Hence, 10° 27 4″ 2′ 5 0′,1, which added to −0″,4, gives −0′,5 the equation, or correction required

From this, and the laſt example, we find the whole correction of *Sirius* on April 27, in the year 1800, to be +1 ,4 but the mean right aſcenſion of *Sirius* at the beginning of 1800 we have found to be 99 5′ 3″,45, hence, the true right aſcenſion of *Sirius* on April 27, 1800, is 99 5′ 4 ,85

TABLES IX AND X

Are TABLES VII *and* VIII *expreſſed in* Sidereal Time

The uſe of theſe being exactly the ſame as that of the laſt two Tables, it is unneceſſary to add any examples

TABLE XI.

Contains that Part of the Equation of the Obliquity of the Ecliptic, which ariſes from the unequal Force of the Sun in cauſing the Preceſſion of the Equinoxes, and therefore depending upon the Sun, it muſt be the ſame every Year

RULE Take the day of the month, and againſt it you have the equation required If the given day of the month be not found in the Table, the equation muſt be found by proportion

EXAMPLE

Example What is the equation on May 24?

May 19	-	Equation −0 ,4
—— 26	-	——— −0, 5
7		0, 1

Hence, 7 5 0,1 0',07, which added to −0 ,4 gives −0',47 the equation

TABLE XII

Contains that Part of the Equation of the Obliquity of the Ecliptic, which ariſes from the unequal Force of the Moon in cauſing the Preceſſion of the Equinoxes, and which depends upon the Longitude of the Moon's aſcending Node

Rule Enter the Table with the longitude of the moon's aſcending node (Tab XXXI XXXII), and againſt it you have the equation required It the longitude be not found in the Table, the equation muſt be found by proportion

Example What is the equation on May 24, 1798?

The longitude of the moon's node is 2 4 17

Long of ☽'s ☊ 2 4	-	Equation +4 2
——— 2 5	-	——— +4, 0
0 1		0, 2

Hence, 1 17' 0",2 0',06, which ſubtracted from 4',2 leaves +4 ,14 the equation required

Hence, from this and the laſt example, the whole equation of the obliquity of the ecliptic on May 24, 1798, is +3',67

TABLE

TABLE XIII

Contains the mean Refraction of the heavenly Bodies, correſponding to their apparent Zenith Diſtances.

RULE. Find the apparent zenith diſtance, and againſt it you have the mean refraction. If the diſtance be not found in the Table, the refraction muſt be found by proportion.

EXAMPLE. What is the mean refraction at the apparent zenith diſtance 71° 15′ 48″?

At 71° 10′ zen. diſt.	-	Refraction 2′ 45″,83
— 71 20 ———	-	——— 2 47, 39
10		1, 56

Hence, 10′ : 5′ 48″ :: 1″,56 : 0″,9, which added to 2′ 45″,83 gives 2′ 46″,73, the refraction required.

This Table (given by Dr MASKELYNE with his Obſervations, 1796) is conſtructed to give the refraction when the barometer ſtands at 29,6 inches, and the thermometer at 50 degrees; the next Table is to correct this, for any variation of the barometer and thermometer from theſe altitudes.

TABLE XIV

Contains Decimals, which multiplied into the mean Refraction, gives the Correction for the Variation of the Weight and Temperature of the Air.

RULE. Find, in the upper horizontal line, the height of the barometer, and in the firſt perpendicular line the height of the thermometer, and correſponding to them you have the decimal required, which multiplied into the mean refraction, and applied to the mean refraction according to the ſign, gives the true refraction. If the altitudes be not found in the Table, the decimal muſt be found by proportion.

EXAMPLE What is the true refraction at the apparent zenith diſtance 71 15″ 48″, the barometer ſtanding at 30,35 inches, and the thermometer at 61,8 inches ?

Againſt 61 under 30,3 inches - Dec −,004
——— 30,4 ——— - - ——— −,001
0 1 ,003

Hence, 0,1 0,05 ,003 0015, the quantity by which ,004 muſt be decreaſed, becauſe the decimals decreaſe

Againſt 61° under 30,3 inches - - Dec −,004
——— 62 ——— - - Dec −,006
1 ,002

Hence, 1 0,8 ,002 ,0016 the quantity by which ,004 muſt be increaſed, becauſe the decimals increaſe, conſequently the decimal correſponding to the given altitudes of the barometer and thermometer is −,0041 Therefore the correction is −,0041 × 2 46″ 8 (the mean refraction by the laſt example) = −,68 ; hence, the true refraction is 2′ 46″,8 − ,68 = 2′ 46″,12

TABLE XV

Contains the Augmentation of the horizontal Diameter of the Moon for any apparent Altitude

RULE Enter the horizontal line at the top with the moon's horizontal diameter, and the firſt perpendicular line with the apparent altitude, and againſt them you have the augmentation required If the diameter be not found in the Table, the augmentation muſt be found by proportion

Example If the horizontal diameter of the moon be 29′ 38′, and the apparent altitude 56° 32, what is it's diameter?

At 29 30″ diameter, and 56 app alt - augment 23″,4
29 50 ——————— - - ——— 23 9

0 20 0, 5

Hence, 20″ 8″ 0″,5 0″,2 the increaſe for 8″

At 29′ 30″ diameter, and 56° app alt augment 23″,4
——————— 57 ——— ——— 23, 6

1 0, 2

Hence, 1° 32′ 0″,2 0″,1 the increaſe for 32′ Therefore 23″,4+0″,2+0″,1=23″,7 the augmentation of the diameter Hence, 29′ 38″+23″,7=30 1″,7 the diameter required

TABLE XVI

Contains the mean *Preceſſion of the Equinoctial Points for Years, in which you have the Number of Years in one Column, and the Preceſſion in another againſt them*

Rule If the number of years be found in the Table, you have the preceſſion required But if the number of years be not in the Table, divide it into ſuch parts as can be found

Example What is the mean preceſſion in 3247 years?

Years				
3000	Preceſ. 41	57′	29″,0
200	2	47	49, 9
47	0	39	26, 4
Mean preceſſion	45	24	45, 3

TABLE XVII

Contains the mean *Preceſſion for every Day of the Year, including the ſolar Equation*

Rule Enter with the month above, and the day of the month on the ſide, and againſt them you have the preceſſion, obſerving the rule for leap year

Example The preceſſion for January 19, in the year 1796 (being leap year), is 3 ,4

TABLE XVIII

Contains the Equation of the Equinoxes in Longitude

Rule Enter with the longitude of the moon's aſcending node (Tab XXXI XXXII), the ſigns above or below and the degrees on one ſide, and correſponding thereto you have the equation, to be applied according to the ſign

Example What is the equation on July 19, 1796?

The place of the moon's aſcending node is 3^s 9 59, againſt the neareſt place to which you find – 17″,6 the equation required,

The mean longitude of a ſtar, ſettled to ſome epoch, is to be corrected by the mean preceſſion and equation of the equinoxes, and if to this we apply the correction of aberration, we get the true longitude

TABLE XIX

Contains the mean *Motion of the Sun in Right Aſcenſion to every Day in the Year*

Rule Enter with the month at the head and day of the month at the ſide, and correſponding to them you have the mean motion

Example

EXAMPLE What is the mean motion of the ſun in right aſcenſion on February 16, 1796?

This being leap year, we muſt take out for the preceding day, hence, the mean motion in right aſcenſion is 3*h* 1′ 21′,5

TABLE XX

Contains the mean *Motion of the Sun in Right Aſcenſion, correſponding to* Sidereal *Time, for Hours and Minutes*

RULE Enter the columns under ſidereal time with the hours and minutes, and againſt them you have the mean motion of the ſun in right aſcenſion

EXAMPLE What is the mean motion in right aſcenſion for 17*h* 48, ſidereal time?

17*h*	2′ 47′,10
48′	0 7, 86
Mean motion in right aſcenſion	2 54, 96

TABLE XXI

Contains the Equation of the Equinoxes, in Degrees, *in Right Aſcenſion*

RULE Enter the Table with the longitude of the moon's aſcending node (TAB XXXI XXXII) the ſigns at the top or bottom and the degrees on the ſide, and correſponding to them you have the equation required

EXAMPLE What is the equation of the equinoxes in right aſcenſion on July 19, 1796?

The place of the moon's aſcending node is 3° 9 59′, againſt the neareſt place to which you find − 16″,1 the equation required

TABLE XXII.

Is the ſame as the laſt, only the quantity is expreſſed in Time.

The uniform motion of the equinoctial points being diſturbed by the nutation of the earth's axis, the true equinox will differ from the equinox computed according to the mean motion, by which the true right aſcenſion of all the ſtars will be affected. The two laſt Tables exhibit only part of this effect, except for thoſe ſtars which are in the equator; the other part, called the *Deviation* (1043), is found from the next Table.

TABLE XXIII.

Is to find the Deviation *of a Star in North Polar Diſtance, and in Right Aſcenſion.*

For the Deviation in North Polar Diſtance.

RULE. Enter with the right aſcenſion at the top, and the longitude of the moon's node in the firſt column on the left, and againſt them you have the deviation. If the right aſcenſion be not found in the Table, the deviation muſt be found by proportion.

EXAMPLE. What is the deviation of a ſtar in north polar diſtance, it's right aſcenſion being 2^s 27° 30′ 19″, and the longitude of the moon's node 3^s 7° 40′?

For *AR* 2^s 25° and long. ☽'s ☊ 3^s 5° the deviation is +1″,45
—— 3 0 —————— +0, 83
0 5 0, 62

Hence, 5° : 2° 30′ :: 0″,62 : 0″,31 to be ſubtracted, as the deviation decreaſes.

For *AR* 2^s 25° and long. ☽'s ☊ 3^s 5° the deviation is +1″,45
—————— 3 10 —— +2, 26
0 5 0, 81

Hence,

Hence, 5° 2 40' 0',81 0 ,43 to be added as the deviation increaſes, therefore the deviation in north polar diſtance is +1',45 − 0",31 + 0 ,43 = 1 ,57

For the Deviation in Right Aſcenſion

RULE *Add* 3 ſigns to the ſtar's right aſcenſion, if its *declination* be *north*, or *ſubtract* if *ſouth*, and with this, as a new right aſcenſion, find the equation from the Table as before, and multiply it by the tangent of the declination, and if the ſtar's right aſcenſion, thus corrected by 3 ſigns, and the longitude of the moon's node, are both more, or both leſs than 6 ſigns, uſe the algebraic ſign of the Table, but if one be more, and the other leſs than 6 ſigns, change the ſign of the Table

EXAMPLE If the right aſcenſion of a ſtar be 2 27 30' 19", and declination 37 28 30' N what is its deviation in right aſcenſion, the longitude of the moon's node being 3 7 40'?

The ſtar's right aſcenſion, increaſed by 3 ſigns, becomes 5 27° 30' 19'

For 5 25 *A R* and long ☽'s ☊ 3 5 the deviation is − 6',99
6 0 ———————————————— − 7, 09
0 5 0, 10

Hence, 5° 2 30' 19" 0',1 9",05, which is to be added to 6 ,99, as the deviation is increaſing

For 5ˢ 25 *A R* and long ☽'s ☊ 3ˢ 5° the deviation is − 6",99
———————————————— 3 10 ———————— − 6, 83
0 5 0, 16

Hence, 5° 2 40' 0",16 0',09, which is to be ſubtracted from 6',99 therefore − 6',99 − 0" 05 + 0',09 = − 6",95, and this multiplied by 0,766 (tang of dec) gives − 5" 32 the deviation in right aſcenſion

This Table Dr MASKELYNE gave with his Obſervations for 1796

TABLE

TABLE XXIV

Contains the Time in which Light moves over any Part or Multiple of the mean Radius of the Earth's Orbit, ſuppoſed to be Unity

Rule Enter with the given diſtance in the column of *Parts of Orbis Magnus*, and againſt it you have the time

Example How long will light be in moving over 0,784 of the earth's radius?

,78	- - -	6′ 19″,9
,004		0 1, 9
,784	- -	6 21, 8 Time

TABLE XXV

Contains the Nonageſimal Degree of the Ecliptic, and it's Altitude, for the Latitude of Greenwich, reduced (173) to the Earth's Center, ſuppoſed to be 51 14 7′, for the Obliquity of the Ecliptic 23° 28

Rule Enter with the right aſcenſion of the meridian, and againſt it you have the nonageſimal degree, and it's altitude If the right aſcenſion be not found in the Table, the required quantities muſt be found by proportion

Example If the right aſcenſion of the meridian be 37 17′ 30′, what is the nonageſimal degree, and it's altitude?

A R Mer	37°	nonag deg	1 22 41′ 20″
	38°		1 23 23 20″
	1		42 0

Hence, 1° 17 30″ 42′ 12′ 15′, which added to 1 22 41′ 20″ gives 1 22 53 35′ the nonageſimal degree

A R Mer

A R Mer	37°	alt nonag	55 35 0′
	38		55 49 30
	1		14 30

Hence, 1° 17′ 30″ 14′ 30″ 4′ 14′, which added to 55 35′ 0″, gives 55° 39′ 14″ the altitude of the nonageſimal degree

TABLE XXVI

Contains the Correction to be applied to the laſt Table, for the Latitude of one Degree north of Greenwich

Rule Enter with the right aſcenſion of the mid-heaven, and you have the corrections required If the right aſcenſion be not in the Table, the corrections muſt be found by proportion If the latitude be not one degree from that of Greenwich, change the quantities found in proportion

Example In the laſt example, what is the nonageſimal degree, and it's altitude, for a place 20′ north of Greenwich?

A R mid-heav	30°, the cor for 1° are	31′,4	54′,3
	40,	26,0	55,9
	10	5,4	1,6

Hence, 10 7 17′, 30″ $\begin{Bmatrix}5',4\\1,6\end{Bmatrix}$ $\begin{Bmatrix}3,94\\1,17\end{Bmatrix}$ the correction for the nonageſimal degree and altitude, for 1 change of latitude; therefore

1 20′ $\begin{Bmatrix}3,94\\1,17\end{Bmatrix}$ $\begin{Bmatrix}1',31\\0,39\end{Bmatrix}$

the reſpective corrections for 20′ change of latitude, hence, 31′,4 − 1′,31 = 30′,09 = 30′ 5″ the correction for the nonageſimal degree, and 54′,3 + 0′,39 = 54′,69 = 54′ 41″ the correction for the altitude Therefore 1 22° 53′ 35″ + 30′ 5″ = 1 23 23′ 40′ the nonageſimal degree, and 55 39 14″ − 54′ 41″ = 54° 44′ 33″ it's altitude

If the place be to the ſouth of Greenwich, apply the corrections with a contrary ſign

The calculation of parallaxes by the nonageſimal degree and it's altitude, being generally uſed in computing ſolar eclipſes, and occultations of the fixed ſtars and planets by the moon, theſe two Tables will be very uſeful for that purpoſe

TABLE XXVII

Contains the Angle between the Ecliptic and a Parallel to the Equator, to the Obliquity 23 28 , *with the Variation for* 10′ *Variation of the Obliquity*

Rule Enter with the ſun's declination, and you have the required angle If the declination be not found in the Table, you muſt find the angle by proportion

Example What is the angle between the ecliptic and a parallel to the equator, when the ſun's declination is 17 30′, and the obliquity 23 27′ 55″?

17° 20′ declin	the angle is	16 4′ 8″
17 40		15 41 58
20		22 10

Hence, 20′ 10′ 22′ 10″ 11′ 5″, which ſubtracted from 16 4′ 8″ leaves 15° 53′ 3″ for the angle at an obliquity 23 28 Now the variation at this point for 10″ of variation of the obliquity is 15″,2, hence, the variation for 5′ is 7′6, which ſubtracted from 15 53′ 3″ leaves 15 52′ 55″,4 for the angle required If the obliquity had been taken greater than 23 28′, the correction muſt have been added

TABLE

TABLE XXVIII

Contains the Angle of Poſition of any Point of the Ecliptic, according to the Obliquity 23 28, with the Variation for the Variation of the Ecliptic by one Minute

RULE Enter with the longitude of the given point of the ecliptic, and you have the angle required If the longitude be not found in the Table, the angle muſt be found by proportion

EXAMPLE What is the angle of poſition of that point of the ecliptic, whoſe longitude is 2 10 30 10 for the obliquity 23 28?

Long	2 10	ang of poſit	8 26′ 43′ 7
	2 11		8 2 40, 8
	1		24 2, 9

Hence, 1° 30′ 15″ 24′ 2″,9 12′ 7″,5, which ſubtracted from 8 26 43′,7 gives 8 14′ 36″,2 the angle of poſition

The variation of the angle of poſition for one minute of variation of obliquity, to this longitude, is 23 ,2 and the variation for any quantity leſs than one minute will be proportionably leſs and this variation is to be added to, or ſubtracted from the angle of poſition found for the obliquity 23 28′, according as the obliquity is greater or leſs than that quantity

EXAMPLE Let every thing be as before, except that the obliquity is 23 27′ 30″ Here the variation being half a minute, the correſponding variation of the angle of poſition is half 23″,2, or 11″,6, which ſubtracted from 12 7″,5 gives 11′ 55′,9, and this ſubtracted from 8 26′ 43″,7 gives 8 14′ 47 ,8 for the angle of poſition

TABLE XXIX

Is to find the Angle of Poſition, having given the Right Aſcenſion

The computation from his Table is exactly the ſame as from the laſt, only you enter with the right aſcenſion inſtead of the longitude

TABLE XXX

Contains the Augmentation of the Angle of Poſition of a Point in the Ecliptic for the Latitude of the Zodiacal Stars

RULE Enter with the angle of poſition, already found, at the ſide and the ſtar's latitude at the top, and you get the correction to be added If the angle of poſition be not found in the Table, the correction muſt be found by proportion

EXAMPLE In the laſt example, ſuppoſe the ſtar's longitude to be the ſame as the longitude of the point of the ecliptic there given, and it's latitude 3 20 to find the angle of poſition

8 angle poſit and 3 0′ lat	correct is 0′ 39′,7
——— 3 30 —	——— 0 54, 2
30	14, 5

Hence, 30′ 20′ 14″,5 9″,8 the firſt part of the correction, to be added to 0′ 39″,7

8° angle poſit and 3 lat	correct is 0′ 39″,7
9 ———	——— 0 44, 8
1	5, 1

Here 1 14′ 47″,8 5″,1 1″,3 the ſecond part of the correction, to be added to 0′ 39″,7, hence, the angle of poſition is 8 14′ 47″,8 + 0′ 39″,7 + 9″,8 + 1″3 = 8 15′ 38″,6

The four laſt Tables are of uſe in calculating the parallaxes in ſolar eclipſes, and occultations of fixed ſtars and planets by the moon, by the method of the parallactic angle

TABLE

TABLE XXXI

Contains the Epochs of the mean Longitude of the Moon's aſcending Node, for the Beginning of each Year

RULE Enter with the epoch, and againſt it you have the mean longitude

The epoch of the mean longitude of the moon's aſcending node for the beginning of 1784, is 11 12 40′, being the longitude of the node at that time, if it's motion had been uniform The epoch is put down to the neareſt minute, that being ſufficiently accurate for the purpoſes here wanted

TABLE XXXII

Contains the mean retrograde Motion of the Node, for every Day of the Year

RULE Enter with the month at the top, and day of the month at the ſide, and againſt them you have the motion required, which being ſubtracted from the epoch for the year, gives the mean place of the node ſufficiently accurate for the purpoſe of taking out all the equations relating to the nutation of the earth's axis

EXAMPLE To find the mean place of the node on November 26, 1799

The epoch for 1799 is -	1	22	35′
Ret mot for Nov 26, -		17	28
Mean place of the node - -	1	5	7

TABLE XXXIII

Shows the Decreaſe, by Refraction, of the Diameters of the Sun or Moon which are inclined to the Horizon, upon ſuppoſition that the apparent Diameter is 30′ If the Diameter be not 30′, the Decreaſe will vary accordingly

RULE Enter with the ſun's or moon's altitude at the top, and with the inclination of the diameter at the ſide, making proportion if you do not find the exact quantities and you get the decreaſe for 30′ diameter, and the decreaſe for any other diameter will be in proportion to the diameter

EXAMPLE

Example If the apparent diameter of the moon be 32, what is the decreaſe of that diameter which is inclined 32° to the horizon, at an altitude of 17

At 16° alt at an inclin 30, the decreaſe is 1″ 6
———————— 33 ———— 1, 9
3 0, 3

Hence, 3 2 0″,3 0″,2, therefore 1″,6 + 0″,2 = 1″,8 is the decreaſe for the inclination 32°, and altitude 16°

At 16 alt at an inclin 30 the decreaſe is 1″,6
— 18 ————————, ———— 1, 3
2 0 3

Hence, 2 1 0′,3 0′,15, therefore the whole decreaſe is 1″,8 − 0″,15 = = 1′,65 the diminution for a diameter of 30′, hence, 30 32′ 1″,65 1′,76 the decreaſe required

TABLE XXXIV

Is for reducing ſidereal to mean ſolar Time

Rule Subtract the numbers found in the Table correſponding to the given ſidereal time, from that time, and it reduces it to mean ſolar time

Example Reduce 17*h* 19′ 23″ ſidereal time, into mean ſolar time

17*h*	2′ 47″,10
19′	0 3, 11
20″	0 0, 05
3″	0 0, 01
	2 50, 27
	17*h* 19 23
Mean ſolar time	17 16 32, 73

TABLE

TABLE XXXV

Is for reducing mean ſolar Time into ſidereal Time

RULE Add the numbers found in the Table correſponding to the given mean ſolar time, to that time, and it reduces it to ſidereal time

EXAMPLE Reduce 11*h* 37′ 49″ mean ſolar time, to ſidereal time

11*h*	-	1′	48″,42
30′	-	0	4,93
7′	-	0	1,15
40″		0	0,11
9″	-	0	0,02
		1	54,63
	11*h* 37	49	
Sidereal time	-	11 39	43,63

TABLE XXXVI

Is to find the Semidiameter of the Sun, and it's horary Motion

RULE With the mean anomaly of the ſun, enter with the ſigns at the top or bottom, and degrees on the ſide, and correſponding to them you have the required quantities If you do not find the exact mean anomaly, the required quantities muſt be found by proportion

EXAMPLE Let the mean anomaly of the ſun be 9ˢ 14° 20′, to find it's ſemidiameter, and horary motion

Mean anom	9ˢ 14	gives the ſemidiameter	15′	57″,23
———	9 15	———	15	56,96
	1		0	0,27

Hence,

Hence, 1° 20′ 0″,27 0″,09, therefore the ſemidiameter = 15 57″,23 − 0″,09 = 15 57″,14

Mean anom	9 14°	gives the horary motion	2′ 26″,56
———	9 15	———	2 26, 47
	1		0, 0, 09

Hence, 1° 20′ 0″,09 0″,03, therefore the horary motion is 2′ 26″,56 − 0″,03 = 2′ 26″,53

TABLE XXXVII

Contains the Reduction of the Ecliptic to the Equator, or the Quantity to be applied to the Longitude of any Point of the Ecliptic, to give the Right Aſcenſion This Table is for the Obliquity 23 28′ 15″, *but there is the Correction to be applied for the Variation of the Obliquity of one Minute*

Rule With the given longitude enter the Table, and take out the reduction, and if the longitude be not found, take the neareſt leſs (*L*), with the difference, and the variation Then proportion for the difference between that and the given longitude, the difference taken out anſwering to a variation of 30′ of longitude, and apply the reſult to *L* Alſo, 60″ var the difference between 23° 28′ 15″ and the given obliquity a fourth number, which *added* to the reduction above found, or *ſubtracted* from it, according as the obliquity is *greater* or *leſs* than 23 28′ 15″, you have the true reduction, to be applied according to the ſign

Example Let the ſun's longitude be 1 11° 25 10″,3, and the obliquity 23 28′ 7″, to find the right aſcenſion

For 1ˢ 11 long	-	Reduction	2 25 56″,6
		Difference	26, 6
		Variation	12,7

Hence, 30′ 25′ 10″,3 26″,6 22″,3, which *added* to 2 25′ 56″,6 (as the reduction is *increaſing*) gives 2 26′ 18″,9

Again,

Again, 60′ 12′7 8′ 1″,7, which ſubtracted from 2° 26′ 18 ,9 leaves 2° 26 17″,2 the true reduction Hence, 1 11 25 10′,3 − 2 26 17′,2 = 1^s 8 58′ 53′,1 the true right aſcenſion

By means of this Table, we may find the longitude from the right aſcenſion, by the following

Rule Increaſe the right aſcenſion by 3 ſigns, and find the reduction in the very ſame manner as above, and apply it to the right aſcenſion according to the ſign, and you get the longitude

Example If the given obliquity be 23 28′ 7″, and the right aſcenſion 1 8 58′ 53″,1, add 3 ſigns to this, and it becomes 4 8 58′ 53 ,1, and with this the reduction is 2 26′ 19 , with the variation 12 ,9 Hence, 60′ 12″,9 8″ 1″,7, which ſubtracted from 2 26′ 19″ leaves 2 26 17″,2, and this added to 1 8 58 53′,1 gives 1 11 25 10″,4

The reaſon of this operation is, that when you increaſe the right aſcenſion by 3 ſigns, the difference of right aſcenſion and longitude continues the ſame, that being now greater which before was leſs of the two If therefore we conſider the right aſcenſion increaſed by 3 ſigns as the longitude, it muſt give the true reduction

TABLE XXXVIII

Contains the Declination of the Points of the Ecliptic to the Equator, when the Longitude and Obliquity are given

Rule The operation is exactly the ſame as in the laſt Table

Example Let the longitude be 1^s 11 25 10″,3, and the obliquity 23 28′ 7″, to find the declination

For 1^s 11° long	Declination	15 8′ 49″,2
	Difference	9 18, 6
	Variation	37, 7

Hence, 30′ 25′ 10″,3 9′ 18″,6 7′ 48″,7, which added to 15° 8′ 49″,2 gives 15 16′ 37″,9

Again, 60″ 37″,7 8″ 5″ which ſubtracted from 15 16′ 37″,9 leaves 15° 16 32″,9 the declination

By means of this Table, we may find the longitude from the declination

Rule Enter the Table with the declination, and take out the variation, then ſay, 60″ var the difference between the given obliquity and 23 28 15″ a fourth number, which *ſubtract* from, or *add* to the given declination, according as the obliquity is *greater* or *leſs* than 23 28′ 15″, and the reſult is the declination ſuch as it would be if the obliquity was 23 28 15′ Then with this declination, enter the Table again, and, by making a proportion, you get the longitude

Example On May 1, 1756, the ſun's declination was 15 16′ 34″,2 N and the obliquity was 23 28 7″, to find the longitude

Correſponding to this declination, the variation is 37″,7 hence, 60″ 37″,7 8″ 5″, which added to 15 16′ 34 ,2 gives 15 16 39′,2, the declination if the obliquity had been 23 28′ 15 Now the difference of declination in the Table is 9′ 18 ,6 and the difference between 15 16 39′ 2 and the next leſs declination is 7 50 , hence 9′ 18 ,6 7′ 50′ 30′ 25′ 14″ 5, which added to 1 11 , or ſubtracted from 4 11° each of which longitudes correſponds to the declination 15 8′ 49′,2 the next leſs than 15 16′ 39′,2, gives 1 11 25′ 14′,5 and 4 18° 34′ 45″,5 for the longitudes correſponding to the given declination, but it being in May the firſt muſt be the true one

For the utmoſt accuracy, we ought to interpolate

TABLE XXXIX

Contains the Equation of Second Difference, for the Purpoſe of Interpolation in various Caſes reſpecting the planetary Motions

Rule Take the two preceding and two following values of the given quantity, and their equations, and find the ſecond differences, and if they be not equal, take their mean for the ſecond difference Find the proportional part to be added to or ſubtracted from the quantity immediately preceding the given quantity as in the preceding examples Enter the Table with the mean of the ſecond differences at the top, and with the minute of argument

on

on the ſide *, and the correſponding number is the correction required, which *ſubtracted* from the proportional part, if the firſt difference be *increaſing* or *added*, if it be *decreaſing* gives the proportional part corrected, which properly applied to the equation correſponding to the quantity, gives the correct equation

EXAMPLE To find the equation of the orbit of *Mercury*, correſponding to it's mean anomaly 3ˢ 5 26′ 37″

Mean An	Equation	1ſt Diff	2d Diff	Mean of 2d Diff
3 4°	23° 15′ 52″	4′ 11″		
3 5	23 20 3	3 48	23	22½″
3 6	23 23 51	3 25	22	
3 7	23 27 16			

Now 60′ 26′ 37″ 3′ 48″ 1′ 41″ the proportional part, or the quantity which muſt have been added to 23° 20′ 3′ if the equations had increaſed by equal differences, or if the ſecond differences had been equal to nothing

Enter the Table at the head with the mean of the ſecond differences 22½″, and with 26′ 37′ at the ſide, and you get 2″,8, which added to 1′ 41″, gives 1′ 43″,8 the proportional part corrected, and this added to 23 20 3″ (becauſe the equation is increaſing) gives 23 21′ 46″,8 the equation required

In like manner you may find the value of any intermediate quantity, when the ſecond difference, and the argument, fall within the limits of the Table,

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* The *argument* is the quantity correſponding to which you get the proportional part or it is the ſecond term in the proportion uſed for that purpoſe

TABLE XL

Contains the Equation of Second Difference, for computing the Moon's Place from the Nautical Ephemeris

RULE Take two latitudes or longitudes immediately before, and two after the given time, and find the proportional part Enter the Table with the mean of the ſecond differences of the moon s motion for 12 hours, whether in longitude or latitude, at the top, and with the apparent time after noon or midnight at the ſide, and the correſponding number is the correction which *ſubtracted* from the proportional part, if the firſt difference *increaſe*, or *added*, if it *decreaſe*, gives the proportional part corrected, which properly applied to the equation, gives the correct equation

To find the proportional part, reduce the minutes and ſeconds of time, and of degrees, into decimals, by TABLES III and XLII and add the logarithms of the ſecond and third terms to 8,9208188, the ar co of log 12, and you have the log of the proportional part

EXAMPLE To find the moon's longitude on July 16, 1767, at 16*h* 22 16″

The given time is 4*h* 22′ 16″ after midnight; hence,

	Moon's Long	1ſt Diff	2d Diff	Mean of 2d Diff
July 16, noon	11ˢ 29 29′ 34″			
midnight	0 6 40 25	7 10′ 51″	3′ 28″	3′ 36′
17, noon	0 13 47 48	7 7 23	3 44	
midnight	0 20 51 27	7 3 39		

Now 4*h* 22′ 16″=4,39111, 7 7′ 23″=7,12306, hence,

12*h*	co ar log	8,9208188
4,39111*h*	log	0,6405917
7,112306	log	0,8526666
Pro pa 2,5946=2° 35′ 41″	log	0,4140771

Now

Now with the ſecond difference 3′ 36″ enter the top of the Table, and with 4*h* 22′ 16 enter the ſide, and the correſponding number is 24′, which added to 2 35 41 gives 2 36 5′ the proportional part corrected Hence the moon's true longitude at the given time is 0 6 40 25′+2 36 5′=0 9 16 30′, and this is as correct as the longitudes from which it is deduced

In like manner, the latitude of the moon may be found at any time

Dr MASKELYNE has added the following remarks, reſpecting the uſe of this Table

1 If the moon's latitude taken out of the *Ephemeris* for noon and midnight changes it's denomination from north to ſouth or from ſouth to north, the *ſum* of the two latitudes of contrary denominations, where the change happens is to be accounted the firſt difference of that place

2 If the three firſt differences firſt increaſe and then decreaſe, or *vice verſâ*, firſt decreaſe and then increaſe, half the difference of the two ſecond differences is to be taken for the mean ſecond difference

3 If the ſeries of four latitudes taken out ſhould firſt increaſe and then decreaſe about the moon's greateſt latitudes, take the ſum of the two firſt differences ſtanding on each ſide of the greateſt latitude for the ſecond difference in that place, correct the moon's latitude at noon or midnight by the ſimple proportional part firſt found, and to the latitude ſo corrected, add always, in this caſe, the equation of ſecond difference, anſwering to the mean ſecond differences

TABLE XLI

Contains the Equation of Second Difference for interpolating the Moon's Diſtance from the Sun or Stars for every third Hour, from thoſe computed for Noon and Midnight in the Nautical Ephemeris

RULE Take four diſtances, two before and two after the given time, and take their ſecond differences Enter the Table with the mean of the ſecond differences, and *ſubtract* the correſponding equation at 3 6, or 9 hours from the quarter, half, or three quarters of the change of diſtance in 12 hours, or *add* it to the ſame, according as the firſt difference is *increaſing* or *decreaſing*, and you have the moon's change of diſtance in 3, 6, or 9 hours, which *added* to, to, or *ſubtracted* from the moon's diſtance at the preceding noon or midnight, according as the diſtance increaſes or decreaſes, gives the moon's true diſtance at 3, 6, or 9 hours

EXAMPLE What is the moon's diſtance from α *Arietis*, on Dec 2, 1799, at 3 o clock?

	Diſtance	1ſt Diff	2d Diff	Mean of 2d Diff
Dec 1, midnight	81° 27′ 3″	6° 28′ 19″		
2, noon	74 58 44	6 21 34	6′ 45″	6′ 37″
midnight	68 37 10	6 15 5	6 29	
3, noon	62 22 5			

The quarter of 6 21′ 34″ is -	1 35′ 23″
Equation	+37
	1 36 0
2d Diſtance at noon - -	74 58 44
Diſtance required	73 22 44

TABLE XLII

Contains the Minutes and Seconds of a Degree, converted into Decimals of a Degree

RULE Take out the decimals correſponding to the given minutes and ſeconds, and add them together

EXAMPLE Reduce 49′ 57″ to the decimal of a degree

	Dec
49′	,81667
57″	,01583
49 57″	,83250

TABLE

TABLE XLIII

Contains the Equation to equal Altitudes of the Sun or the Quantity by which the middle Point of Time between the Times when the Sun had the ſame Altitudes on the Morning and the Afternoon, differs from the Time when the Sun was upon the Meridian

RULE Take the half interval of time between the two correſponding obſervations With the ſun s given longitude for noon, and the half interval, find from TABLE I the equation, if the longitude and half interval be not found in the Table, the equation muſt be found by proportion Multiply this equation by the natural tangent of the latitude, radius being unity, and you get the firſt part of the equation, which is additive or ſubtractive, according as the ſign is + or −, remembering that when the latitude is ſouth, the ſign of the Table muſt be changed

With the ſun s longitude, and half interval, find, in TABLE II the equation, by making proportions if the given quantities be not found in the Table

The ſum of theſe two equations, regard being had to their ſigns, is the equation required, which applied, according to its ſign, to the middle point of time between the two correſponding obſervations, gives the time by the watch when the ſun was upon the meridian

EXAMPLE Let the middle point of time between the correſponding obſervations be 23*h* 59′ 21″,13 by the watch, the half interval 2*h* 34′ 52″, and the ſun s longitude 6 14° 36′, to find the equation, and thence the time when the ſun was on the meridian, the latitude being 33 56′ S and longitude 18 23 E

TAB I	Sun's long 6 10 and half int	2ʰ 30	equat is	+ 15″,92
		2 40		+ 16 08
		10		0 16

Hence, 10′ 4′, 52″ 0,16 0′,08, which (as the equation increaſes) added to 15″,92 gives 16″ for the equation when the longitude is 6ˢ 10 Again,

TAB,

TAB I Sun's long 6' 15 and half int	2^h 30	equat is	+15 ,71
———————	2 40	———	+15 ,87
	10		0, 16

Hence, 10 $4'\ 52,_3''$ 0'',16 0'',8, which added to 15 ,71 gives 15'',79 for the equation when the longitude is 6 15 The difference between this and 16 , the equation when the ſun s longitude was 6 10 , is 0' 21

Hence, 5 4 36' 0',21 0',09, which ſubtracted from 16' (becauſe the equation decreaſes this way) gives 15',81 for the equation from TABLE I And this multiplied by 0,6728, the nat tang of 33 56', gives 10' 64 the firſt part of the equation, which is *ſubtractive,* becauſe the latitude is ſouth

TAB II Sun s long 6 10 and half int	2^h 30'	equat is	− 0'',87
———————	2 40	———	− 0 85
	10		0, 02

Hence 10 $4'\ 52,_3''$ 0',02 0',01 which ſubtracted from − 0'',87 (as the equation decreaſes) gives − 0'',86 for the equation when the longitude is 6 10 Again,

TAB II Sun s long 6 15 and half int	2^h 30'	equat is	− 1'',29
———————	2 40		− 1, 26
	10		0, 03

Hence, 10' 4 $52,_3''$ 0'',03 0'',01, which ſubtracted from − 1'',29 gives − 1'',28 for the equation when the ſun s longitude is 6 15 The difference between this and − 0',86, the equation when the ſun's longitude was 6 10°, is 0 ,42

Hence, 5 4 36 0',42 0'',39, which added to − 0'' 86 (becauſe the equation increaſes this way) gives − 1',25 for the ſecond part of the equation, and the two parts added together (becauſe they are both ſubtractive) gives − 11 ,89 for the whole equation This therefore ſubtracted from 23*h* 59' 21'',13, gives 23*h* 59' 9'',24 for the time by the watch when the ſun was upon the meridian

Hence,

Hence, we may determine how much the watch was too faſt, or too ſlow

Equation of time at noon at Greenwich was		− 12	19′,3
And 24*h* 1*h* 14′ (long in time) 16,7 (daily dif. of equat.)			0,8
Equation of time at noon at the given place		− 12	18,5
	24h	0	0
Mean time of apparent noon	23	47	41,5
Time of apparent noon by the watch	23	59	9,2
Watch too faſt for mean time		11	27,7

TABLE XLIV

Contains at once the Equation to equal Altitudes, ſufficiently accurate for moſt Purpoſes

Rule With the latitude and interval between the obſervations at the head, and the declination at the ſide, you get the equation required Whilſt the ſun moves from the tropic of Capricorn to the tropic of Cancer, the equation is to be *ſubtracted* from the middle point of time, and whilſt it moves from the tropic of Cancer to the tropic of Capricorn, it is to be *added*, in order to get the time of apparent noon by the watch

Example In latitude 51° 32′ N when the ſun's declination was 20 S on January 29, the interval of equal altitudes was found to be 5*h* 10, and the middle time by the watch was 12*h* 3′ 32″, to find the time of apparent noon by the watch

With the latitude 50 N and 5*h* at the head (the neareſt to the given quantities), and declination 20 S at the ſide, you get the equation 13′, which is to be ſubtracted becauſe the ſun was moving from the tropic of Capricorn to the tropic of Cancer, hence, 12*h* 3′ 19″ is the time of apparent noon by the watch

TABLE XLV

Contains the Semi diurnal Arcs of the heavenly Bodies whoſe Declinations do not change in Time, and therefore for all common Purpoſes, it will ſerve for the Sun

For the *Sun* they give the time of it's ſetting, and if it be ſubtracted from 12 o clock, you get the time of it's riſing

For a *Star*, add and ſubtract the equation to and from the time at which the ſtar paſſes the meridian (105), and you get the time of it's ſetting and riſing

The Table is calculated for the arcs correſponding to the time when the center of the ſun appears in the horizon, the eye being at the ſurface of the earth, thereby taking into conſideration the effect of refraction

Example In latitude 52° 12′, and declination of the ſun 23 28′, what is the time of it's riſing and ſetting?

Lat 52°, declin 23 arc 8h 16′
53 arc 8 22
1 6

Hence, 1° 12′ 6′ 1′ to be added to 8*h* 16′

Lat 52°, declin 23 arc 8h 16′
24 arc 8 24
1 8

Hence, 1 28′ 8′ 4′ to be added alſo to 8*h* 16′

Therefore the ſemi diurnal arc = 8*h* 16′ + 1′ + 4′ = 8*h* 21′ the time of ſetting, and 12*h* − 8*h* 21′ = 3*h* 39′ the time of riſing

TABLE XLVI

Is principally intended to find how long the Body of the Sun is in aſcending above the Horizon, or the Time between the upper and lower Limbs of the Sun touching the Horizon

Rule Enter with the declination at the top, and latitude at the ſide, and you have the time which the ſun is in aſcending 1 If the declination and latitude be not found in the Table, the time muſt be found by proportion Then 1 the diameter of the ſun that time the time of it's riſing, or by logiſtic logarithms, log of ſun's diameter + log of time found from the Tables = log of time required

Example How long will the ſun be in riſing at Cambridge on June 1, 1799?

Here the latitude is 52 12′,5, and declination 22 6′, and ſun's diameter 31′ 38

Lat 52, and declin 21° -	time for 1 is	8′ 4′
—— 53 ——————	———————	8 20
1		16

Hence, 1° 12′,5 , 16′ 3″ to be added, for 12′,5 of latitude

Lat 52 and declin 21° - -	time for 1 is	8′ 4′
——————— 24 - -	———————	8 43
3		40

Hence, 3 6′ 40 1″ to be added, for 6′ of declination

Therefore the time of riſing 1° is 8′ 4′+3″+1″ = 8′ 8 Hence,

Log 31′ 38′ -	2780
Log 8 8 -	8679
Log 4 17 time of riſing - -	1,1459

TABLE

TABLE XLVII

Contains the Amplitudes of the heavenly Bodies at the Time of their Riſing

Rule Enter with the declination at the head of the Table, and the latitude at the ſide, and you have the amplitude If the declination and latitude be not found in the Table, the amplitude muſt be found by proportion

Example If the declination of a ſtar be 17 30, and the latitude 57 20′, what is the amplitude at it s riſing?

Lat 57, and declin 17°, the amplitude is 32 28′
—— 58 ———————— 33 29
1 1 1

Hence, 1° 20′ 1 1′ 20′ to be added, for 20′ of latitude

Lat 57°, and declin 17, the amplitude is 32° 28′
———————— 18 ———— 34 34
1 2 6

Hence, 1 30′ 2 6′ 1 3′ to be added, for 30′ of declination

Therefore 32° 28′+20′+1° 3′=34 51′ the amplitude required

If the Table be entered with the complement of latitude inſtead of the latitude, you will get the ſun's altitude when on the prime vertical This follows from hence, that if for the ſine of latitude in the proportion which gives the altitude, you put coſ lat it gives the proportion for finding the amplitude This will appear from Articles 89, 91

TABLE XLVIII

Is to find at any Time what Proportion the enlightened Part of the Face of the Moon, *or* Venus, *bears to the Whole*

Rule Enter with the degrees at the top and on the firſt column for the moon, and at the bottom and laſt column for Venus and correſponding to them you have the enlightened part, the whole face being repreſented by 12

Example If the diſtance of the moon from the ſun be 137 12′, what is it's enlightened part?

Diſt 137° - enlightened part 10,388
138 - ——————— 10,458

1 ,070

Hence, 1 12′ ,070 ,014, therefore the enlightened part is 10,388 + ,014 = 10,402, the whole being 12

Example If the angle formed by two lines drawn from Venus to the earth and ſun be 50 20′, what is it's enlightened part?

Angle 50° enlightened part 9,856
51 - - ——————— 9,776

1 ,080

Hence, 1° 20′ ,080 ,027, therefore 9,856 − ,027 = 9,829 the enlightened part, the whole being 12

TABLE

TABLE XLIX

Is to find the Hour Angle of Jupiter from the Meridian, when it is 8° *above the Horizon, for the Latitude of Greenwich*

Rule Enter with Jupiter's declination in the column with that title, and againſt it you have the correſponding hour angle If the declination be not found in the Table, the hour angle muſt be ſound by proportion

Example If Jupiter's declination be 22° 20′ N what is it's hour angle, when it is 8 high?

Declin	22	0′	hour angle is	7^h	1′	50′
——	22	30	——	7	4	41
		30			2	51

Hence, 30° 20′ 2′ 51″ 1′ 54″, therefore the required hour angle is 7^h 1′ 50″ + 1′ 54″ = 7^h 3′ 44

TABLE L

Is to find the Hour Angle of the Sun, when it is 8° *below the Horizon, for the Latitude of Greenwich*

Rule Enter with the ſun's declination in the column with that title, and againſt it you have the hour angle If the declination be not found in the Table the hour angle muſt be found by proportion

Example If the ſun's declination be 18 15′ S what is it's hour angle, when it is 8 below the horizon?

Declin	18	0′	hour angle is	5^h	20	6″
——	18	30	——	5	17	26
		30			2	40

Hence,

Hence 30 15′ 2′ 40 1′ 20″, therefore the required hour angle is 5h 20 6″ − 1′ 20″ = 5h 18 46″

The uſe of the two laſt Tables is to find whether an eclipſe of Jupiter ſatellites will be viſible at Greenwich or on that parallel of latitude For if at the time of an eclipſe the hour angle of Jupiter be equal to, or greater than that which the Table gives, and the hour angle of the ſun be equal to, or leſs than that given in the Table, the eclipſe will not be viſible if the former be equal to, or leſs, and the latter equal to, or greater than what the Tables give, the eclipſe will be viſible

TABLE LI

Is a Table of Logiſtic Logarithms, which were firſt employed by STREET, *and are nothing but the common Logarithms ſubtracted from* 5,5563 *which is the Logarithm of* 3600, *by which means the Logarithm of* 3600″(=60′) *becomes nothing, and the Logarithm of* 360 *becomes* 1,0000 *For Numbers greater than* 3600, *the Logarithms would be negative, but inſtead of putting them down ſo, their arithmetic complements are put down*

RULE If the firſt term be 60′, or 3600″ add the logarithms of the ſecond and third terms together, but if the ſum of the two laſt figures in the addition, the index excepted, be equal to or greater than 10, you are not to carry 1 to the index, and the ſum is the logarithm of the fourth term If the ſecond term be 60, ſubtract the logarithm of the firſt term from the logarithm of the third, adding 1 to the index of the logarithm of the third, if neceſſary, and the remainder is the logarithm of the fourth term

EXAMPLE I What is a fourth proportional to 60′, 65′ 25″, and 37′ 41″?

65′ 25″	-	log 9625
37 41		log 2020
Anſwer 41 5	-	log 1645

EXAMPLE

Example II What is a fourth proportional to 60, 1′ 36″, and 27′ 28″?

1′ 36″		log 1,5740
27 38		log 3367
Anſwer 0 44	-	log 1,9107

Example III What is a fourth proportional to 16′ 47″, 60′, and 17′ 28″?

17′ 28″	-	log 5359
16 47	- -	log 5533
Anſwer 62 27	- -	log 9826

Here it was neceſſary to add 1 to the index of the logarithm of the third term, or which is the ſame thing, 10 was added to the laſt figure 5 of the upper line

Example IV What is a fourth proportional to 27′ 19″, 60′, and 5′ 9″?

5′ 9″		log 1,0663
27 19	-	log 3417
Anſwer 11 19	-	log 7246

If the firſt term be 24*h* and the ſecond term be hours and minutes, and the third term be given in time, or be an arc, we may find a fourth proportional, by conceiving the head of the Table to repreſent hours

Example What is a fourth proportional to 24*h*, 13*h* 53′, and 76′ 34″?

24*h*		ar co 6021
13 53′		6857
76′ 34″		8941
Anſwer 44 17	-	1319

We

We reject 2 in the index becauſe the firſt and third terms are arithmetic complements, and 1 is to be rejected for each

In like manner, whatever may be the three terms, whether hours and minutes, minutes and ſeconds of time, degrees and minutes, minutes and ſeconds of an arc, or two of one and one of the other, a fourth proportional may be found, provided the quantities fall within the limit of the Table

A

CATALOGUE

OF

THE PLACES OF 389 FIXED STARS,

IN

RIGHT ASCENSION, AND DECLINATION,

ADAPTED TO THE BEGINNING OF THE YEAR 1760

WITH

Their MAGNITUDES, and ANNUAL VARIATIONS
in RIGHT ASCENSION and DECLINATION

CALCULATED FROM THE LATE

Dr BRADLEY's OBSERVATIONS

THOSE ſtars whoſe right aſcenſion is between 90 and 270° with north declination, and more than 270 and leſs than 90 with ſouth declination, have their annual variation of declination *ſubtractive*, and thoſe ſtars whoſe right aſcenſion is more than 270 and leſs than 90 with north declination, and between 90 and 270 with ſouth declination have their annual variation of declination *additive*. This is to be underſtood with reſpect to a time *after* January 1, 1760: but, if the time *precede* that period, the variation of declination is to be applied with a *contrary* ſign.

Dr BRADLEY's CATALOGUE OF FIXED STARS

Numb. of Star	Names of the Stars	Magn.	Mean Right Ascension Jan. 1 1760			Ann. Preces. Right Ascension		Mean Distance from N. Pole Jan. 1760			Ann. Preces. Declination	
			D	M	S	1760	1800				1760	1800
1	γ Pegaſi	[illegible]	0	13	35,1	45,95	4[illegible]	[illegible]	9	53	0 01	–0 01
2	ι Ceti	3	1	47	59,9	45 60	4[illegible]	[illegible]0	9	20 8	20,[illegible]0	19 99
3	* d Piſcium	5	2	4	1 0	46,01	[illegible]6 0[illegible]	83	8	39,7	20,00	19 99
4	δ Andromedæ	3	6	36	1[illegible],7	47 24	47 30	6[illegible]	7	0 0	19,88	19 85
5	α Caſſiopeæ	3	6	45	35,0	49 32	49 63	[illegible]4	16	53 5	19,87	19 65
6	β Ceti	3	7	5	59 0	4[illegible] 97	[illegible]	100	1	[illegible]7 1	10,8	19 80
7	ζ Andromedæ	4	8	39	5 0	47 [illegible]	[illegible] 30	6[illegible]	[illegible]	30 5	19 [illegible]8	19 75
8	* 20 Ceti	5	10	11	19 0	45 [illegible]	[illegible]	9[illegible]	7	[illegible]0	19 [illegible]0	19,66
9	γ Caſſiopeæ	3	10	36	1 0	5[illegible]	5[illegible] 5[illegible]	[illegible]0	35	16	19,67	19,63
10	Piſcium	4	1[illegible]	37	4[illegible] 0	[illegible]	40,47	[illegible]	[illegible]4	[illegible]6 6	19 53	19,46
11	* e Piſcium	5	14	0	[illegible]0,0	46,30	4[illegible] [illegible]	5	[illegible]	30 8	19 4[illegible]	19 37
12	β Andromedæ	[illegible]	14	6	[illegible]9,2	4[illegible] 6	[illegible]9,41	55	39	32 6	19,41	19 36
13	η Ceti	3	14	7	48,0	44 94	44 9	101	[illegible]7	39,4	19,41	19,36
14	θ Caſſiopeæ	4	14	9	48,0	5[illegible] 6	[illegible] 95	36	7	56 0	19,40	19 35
15	ζ Piſcium	4	15	16	1[illegible] 0	46,52	46 55	83	4	0 3	19 30	19 [illegible]5
16	δ Caſſiopeæ	3	17	[illegible]4	49,0	55,90	56,41	31	1	9 5	19 08	19 01
17	θ Ceti	3	18	0	33 0	44,90	[illegible]4 90	99	25	40 6	19 0[illegible]	18 97
18	* μ Piſcium	5	19	[illegible]4	35,0	46 50	46 5[illegible]	[illegible]5	6	0 5	1[illegible] [illegible]8	18 [illegible]1
19	η Piſcium	5	19	40	14,0	47 6	[illegible]7 69	[illegible]5	5[illegible]	55 0	18 [illegible]5	18 78
20	* π Piſcium	5	21	6	1[illegible],0	47 3	[illegible]7 38	79	5	[illegible]9,7	18 67	18 60
21	105 Piſcium	5	[illegible]1	41	[illegible]8,0	[illegible]7 9	[illegible]3 0	74	19	11 7	18 60	18 [illegible]5
22	Piſcium	4	[illegible]	14	26,0	[illegible]6,49	46,[illegible]	85	41	[illegible]3	18 5	18 45
23	* ο Piſcium	[illegible]	2[illegible]	11	1[illegible]0	4[illegible] 03	[illegible]	[illegible]	[illegible]	[illegible]1	18 40	18 [illegible]
24	ε Caſſiopeæ	3	24	21	[illegible],0	61,[illegible]6	[illegible]	[illegible]	[illegible]1	6 8	18 3	1[illegible] 1[illegible]
25	γ Arietis	4	25	6	[illegible]	[illegible]	[illegible]	[illegible]1	[illegible]	[illegible] 0	18 1	18 04
26	β Arietis	3	[illegible]	21	26,1	4[illegible] [illegible]8	4[illegible] [illegible]	[illegible]	22	[illegible]	1[illegible]	18 00
27	Arietis	5	26	4	13 5	[illegible]9 6	4[illegible] 0[illegible]	73	[illegible]1	5[illegible] 0	1[illegible] 9[illegible]	1[illegible] 69
28	λ Arietis	5	26	[illegible]9	[illegible]0	[illegible] 57	[illegible]	[illegible]	[illegible]	[illegible]0	1[illegible] 90	[illegible]
29	γ Andromedæ		2[illegible]	19	7 0	5[illegible] [illegible]6	5[illegible] 1[illegible]	4	[illegible]0	[illegible]0	1[illegible] [illegible]8	1[illegible] 60
30	α Piſcium	3	[illegible]	4	10 0	[illegible]0,18	46 [illegible]	[illegible]	[illegible]4	18 4	17,[illegible]7	1[illegible] 6
31	α Arietis		[illegible]8	[illegible]	26,6	4[illegible] 64	4[illegible],95	6[illegible]	41	1 4	17 60	1[illegible] 50
32	19 Arietis	5	30	[illegible]	6,0	46 46	4[illegible],5	[illegible]	[illegible]1	[illegible] 5	17 [illegible]	17 3
33	1 ξ Ceti	5	30	1	35 0	47 30	47,[illegible]	[illegible]	[illegible]	3	1[illegible] [illegible]	1 [illegible]
34	1 θ Ariet.		31	1	16 0	49,45	[illegible]	[illegible]1	13	17,9	1[illegible], 1[illegible]	1[illegible],01
3[illegible]	Ceti [illegible]	2	31	40	[illegible]0	45 16	15 0	[illegible]1	4	41 [illegible]	17 01	16 91

DR BRADLEY's CATALOGUE OF FIXED STARS

Numb. of Star	Name of the Stars	Magnitude	Mean Right Ascension Jan 1 1760			An. Preces. in Right Ascension		Mean Distance from N. Pole, Jan 1 1760			An. Preces. in Declination	
						1760	1800				1760	1800
			D	M	S			D		S	S	S
36	* 2 ξ Ceti	4	33	51	24,0	47,37	47,43	82	37	41,0	16,62	16,51
37	δ Ceti	3	36	47	59,5	45,78	45,81	90	42	9,3	16,03	15,91
38	θ Persei	4	36	58	56,0	59,39	59,68	41	48	13,4	15,99	15,64
39	Ceti	3	36	59	31,0	43,17	43,18	10[illegible]	54	9,5	15,99	15,88
40	35 Arietis	4	37	[illegible]1	26,0	52,03	5[illegible],16	63	19	43,2	15,91	15,78
41	γ Ceti	3	37	43	27,0	46,40	46,45	87	47	18,4	15,83	15,72
42	* μ Ceti	4	38	0	6,0	47,90	47,96	80	54	46,8	15,77	15,65
43	π Ceti	3	38	10	39,0	42,64	42,65	104	53	14,5	15,73	15,63
44	τ Persei	5	39	20	51,0	62,03	6[illegible],37	38	14	18,4	15,48	15,3[illegible]
45	* 3 ρ Arietis	5	40	43	55,0	49,93	50,0[illegible]	7[illegible]	56	57,0	15,17	15,03
46	η Eridani	3	41	10	[illegible],0	43,64	43,66	99	51	58,0	15,06	14,95
47	* Arietis	5	41	23	[illegible]0	50,84	50,94	69	38	5,5	15,02	14,86
48	γ Persei	3	41	53	36,0	63,36	63,71	37	28	13,7	14,90	14,73
49	α Ceti	[illegible]	42	[illegible]0	[illegible]4,1	46,67	46,7[illegible]	86	51	59,8	14,77	14,64
50	β Persei	3	43	9	5,0	57,4[illegible]	57,62	49	59	15,8	14,60	14,44
51	* δ Arietis	4	4[illegible]	9	11,0	50,70	50,60	71	11	53,5	14,28	14,14
52	* ζ Arietis	5	45	17	19,0	51,14	51,24	69	51	40,2	14,08	13,93
53	12 Eridani	3	45	0	7,0	37,73	[illegible]7,70	119	56	51,6	14,04	13,93
54	ζ Eridani	3	46	2	54,0	43,46	43,48	99	43	32,8	13,89	13,77
55	α Persei	[illegible]	46	49	50,0	6[illegible],71	6[illegible],99	41	0	50,9	13,69	13,41
56	* [illegible] τ Arietis	5	47	14	48,0	51,24	51,3[illegible]	70	8	[illegible],3	13,59	1[illegible],44
57	* f Tauri	4	49	1	46,3	49,19	49,26	77	54	10,6	13,01	1[illegible],67
58	17 Eridani	4 5	49	41	4[illegible],0	[illegible] 35	41,38	95	54	46,8	1[illegible],9[illegible]	1[illegible],6[illegible]
59	δ Persei	3	51	9	11,0	6[illegible],78	62,97	43	0	9,6	12,46	1[illegible],[illegible]6
60	* b Pleiadum	5	5[illegible]	40	7,0	5[illegible],79	52,69	66	39	38,0	12,14	11,97
61	* e Pleiadum	5	5[illegible]	4	[illegible],0	5[illegible],92	53,0[illegible]	66	10	16,[illegible]	12,1[illegible]	11,95
6[illegible]	δ Eridani	[illegible]	5[illegible]	56	[illegible],0	42,94	4[illegible],96	100	35	34,4	12,06	11,9[illegible]
63	* d Pleiadum	5	5[illegible]	[illegible]	1,0	52,77	5[illegible],67	66	49	6,5	12,03	11,67
64	* η Tauri	3	5[illegible]	16	55,2	5[illegible],05	5[illegible],95	66	39	[illegible],1	11,96	11,79
65	γ Eridani	[illegible]	[illegible]6	1	3[illegible],0	41,63	41,71	104	1[illegible]	[illegible]6,5	10,98	10,35
66	1 λ Persei	4	57	1	6,0	65,75	6[illegible],99	40	19	28,9	10,84	10,6[illegible]
67	* A Tauri	4	57	3[illegible]	1,0	[illegible]2,5[illegible]	52,64	68	35	37,4	10,71	10,54
68	* φ Tauri	5	61	[illegible]4	[illegible],6	54,79	54,9[illegible]	63	4	45,0	9,58	9,[illegible]9
69	[illegible] Tauri	3	61	[illegible]2	6,[illegible]	50,65	[illegible]2,71	74	56	0,8	9,54	9,34
70	* χ Tauri	5	6[illegible]	0	16,0	54,10	54,[illegible]7	64	5[illegible]	[illegible]0,8	9,39	9,[illegible]1

DR BRADLEY's CATALOGUE OF FIXED STARS

Numb. of Stars	Names of the Stars	Magnitude	Mean Right Ascension Jan. 1, 1760			Annual Preces. in Right Ascens.		Mean Distance from N. Pole Jan. 1, 1760			Annual Preces. in Declination	
						1760	1800				1760	1800
			D	M	S	S		D	M	S	S	
71	* 1 δ Tauri	4	62	16	57 0	51 32	51 39	73	2	2 6	9 31	9,13
72	* 2 δ Tauri	4	62	34	25 0	51 31	51 38	73	7	56 9	9 2	9,04
73	* 1 κ Tauri	5	62	46	34,0	53,02	53 09	66	16	36,0	9 16	8,97
74	* 2 κ Tauri	4	62	47	18,0	52,9	53,06	66	22	13 5	9,15	8 97
75	* 3 δ Tauri	5	62	54	33 0	51 50	51,56	72	36	31,5	9,11	8 93
76	* 1 υ Tauri	5	62	59	43 0	53 22	5 30	67	[illegible]	[illegible]	9 09	8 90
77	* [illegible] Tauri	2	63	39	20 5	51 97	52 04	71		25,3	[illegible]	[illegible],70
78	* 1 θ Tauri	5	63	43	0	50 7	50 92	74	35	29 5	86	8,66
79	* 2 θ Tauri	5	63	44	47 0	50 64	50 90	74	10	59 3	8 85	[illegible] 67
80	* Aldebaran	1	65	32	38,7	51 15	51,21	73	59	37 7	8,49	8,10
81	* τ Tauri	5	66	58	5 0	53 55	53,61	67	31	33 3	7,63	7 64
82	1 π Orionis	4	69	23	16 0	48,72	48 75	81	31	55 1	7 05	6, 7
83	7 Cam. lopard.	5	69	31	48 0	71 12	71,31	36	39	16,3	7 00	6,74
84	* ι Tauri	4	72	11	37 7	53,33	53,3	66	46	31 5	6 11	5 91
85	* m Tauri	5	73	19	12 0	52 27	52,31	71	42	1,	5,74	5 55
86	* 103 Tauri	5	73	23	58,0	53 43	53,65	68	38	12 2	5 72	5,52
87	h Eridani	3	74	1	5 0	44 10	44,12	95	4	56 9	5 51	5 34
98	Capella	1	74	44	59 5	65 [illegible]	65 [illegible]	44	10	27 [illegible]	5, 6	5 02
89	Rigel	1	75	45	10 9	43,03	13 05	98	29	50 7	4 93	4,76
90	* β Tauri	2	77	47	7 0	56,49	56 54	61	37	12 0	4, 3	4 02
91	γ Orionis	2	78	4	8 0	46,02	46,05	5	53	20,7	4 14	3,95
92	* ο Tauri	5	78	18	35 0	53 73	53 6	6	17	30 7	4,06	3 75
93	2 ψ Orionis	5	78	34	9 0	46 91	46 93	[illegible]7	7	59 5	3 97	3,79
94	β Leporis	3	79	29	40,0	3[illegible] 39	36,40	110	58	4[illegible]	3 [illegible]	3 5[illegible]
95	δ Orionis	2	79	56	22 0	45 76	45 77	90	9	50 1	3,50	3 32
96	α Leporis	3	80	32	21,0	39 51	39 52	108	0	13,0	3 9	3,14
97	* ζ Tauri	3	80	49	41 8	53 50	53 53	69	1	37,4	3 19	2,96
98	Orionis	2	81	0	41 0	45 45	45 17	91	22	33,3	3,1	2,95
99	* 115 Tauri	5	81	13	[illegible],0	55,46	55 [illegible]1	64	15	44 3	3,06	2,84
100	* 132 Tauri	4	83	34	0,0	54,97	55 00	65	3	17,[illegible]	2,24	2 03
101	* γ Leporis	3	83	37	3 0	37 67	37,68	112	32	47,0	2	06
102	* 130 Tauri	5	84	33	46,0	56 31	56 33	6	26	9 0	1 90	1,68
103	δ Aurigæ	4	84	[illegible]6	44 3	73 60	73,65	35	45	53 2	1 62	1,51
104	* 1 χ Orionis	5	8[illegible]	2	14,0	53 [illegible]	53 26	69	47	29 5	1 73	1,52
105	* 2 χ Orionis	5	85	11	7 0	53,06	53,07	70	13	42 7	1 60	1 47

Dr BRADLEY's CATALOGUE OF FIXED STARS

Number of Stars	Names of the Stars	Mag.	Mean Right Ascension Jan. 1, 1760			Annual Precession in Right Ascension 1760	1800	Mean Distance from N. Pole Jan. 1, 1760			Annual Precession in Declination 1760	1800
			D					D	M	S	S	
106	α Orionis	1	85	32	47,	46,59	48,51	82	39	34,5	1,55	,37
107	θ Aurigæ	4	85	50	1,0	61,06	61,11	52	49	51,6	1,45	1,21
108	H Gemino	5	87	3	1,0	54,5	54,3	66	41	51,9	0,91	0,70
109	* κ Aurigæ	5	90	1	16,0	57,8	57,26	60	6	18,9	0,01	0,23
110	η Gemino	4	90	5	52,6	54,4	54,4	67	26	51,7	0,03	0,25
111	* μ Gemino	3	92	6	34,1	54,26	54,25	67	2	1,3	0,74	0,95
112	* Geminorum	4	93	40	39,	53,34	53,32	69	39	34,6	1,28	1,19
113	23 Gemino	5	95	32	4,0	52,08	5,06	—	—	—	1,93	2,13
114	γ Gemino		95	57	37,3	51,85	51,84	73	25	8,4	,08	,9
115	* 6 Gemino	5	97	6	20,0	5 33	5 31	7	6	33,2	2,48	,66
116	* Gemino	3	97	17	0,7	55,33	55,0	64	39	19,7	54	,5
117	* 28 Gemino	5	97	3	5,0	57,01	50,0	60	46	41,5	,57	2,79
118	Sirius	1	98	3	36,0	40,10	40,10	106	4	6,3	3,01	3,16
119	ζ Gemino	4	10	7	57,0	53,39	53,5	69	6	2,1	4,3	4,5
120	* 51 Gemino	5	104	53	40,0	51,67	51,70	73	7	1,8	5,1	5,4
121	19 Lyncis	5	105	18	8,0	74,16	73,97	34	17	49,0	5,45	5,73
12	* λ Gemino	5	106	4	22,0	51,78	51,75	73	2	54,1	5,54	5,73
123	* δ Gemino	3	106	6	36,0	53,64	53,79	67	35	53,7	5,67	5,66
124	* q Gemino	5	106	56	31,0	53,23	53,0	69	7	33,6	5,83	6,03
1 5	* ι Gemino	5	107	4	,0	56,17	56,11	61	44	51,5	6,06	6,9
1 6	p Gemino	5	100		8,0	53,57	53,51	68	5	1,0	6,31	6,50
127	η Canis min.	2	106	39	6,0	35,48	35,46	118	50	57,0	6,40	6,5
1 8	Castor	1	109	1	45,7	57,67	57,9	57	36	36,0	6,78	6,99
129	Gemino	4	110	10	31,1	55,66	55,59	62	5	36,0	6,91	7,11
130	* f Gemino	5	111	4	0	5 05	52,00	71	48	1,0	,30	7,17
131	Procyon	1	111	0	56,8	47,5	47,9	84	10	0,0	7,9	7,56
13	κ Gemino	5	11	6	54,0	54,5	54,17	65		54,0	7,5	,8
133	Pollux	1	11	0	7	55,99	55,91	01	4	56,	7,71	7,90
134	* ι Gemino	5	113	3	1,0	5 0	52,4	70	55	32,2	7,64	5,0
135	6 Lyncis	5	114	1	4,0	66,30	66,11	41	50	16,1	8,3	6,47
1 6	φ Gemino	5	114	11	34,0	55,33	55,5	62	38	6,1	8,6	6,5
137	Cancri	5	116	57	4,0	5 03	51,90	71	7	8	9,07	5
138	Cancri	5	11	0	54,0	53,54	53,46	66	41	57,	9,0	9,50
139	Cancri	4	116	59	6,0	54,55	54,46	63	46	59,2	9,70	9,86
140	β Cancri	3	1 0	52	13,0	48,93	48,88	80		35,0	10,7	10,43

Dr BRADLEY's CATALOGUE OF FIXED STARS

Numb of Stars	Names of the Stars	Magnitud	Mean Right Ascension Jan 1 1760			An Preces in Right Ascension		Mean Distance from N Pole Jan 5 1760			An Preces in Declination	
						1760	1800				1760	1800
			D	M	S	S	S		M	S		S
141	* θ Cancri	5	124	28	16,0	51,57	51 49	71	6	44 5	11,33	11 49
142	* η Cancri	5	124	41	53,0	52,32	52,24	68	45	40 5	11,39	11,56
143	* γ Cancri	4	127	20	30,0	52,46	52,36	67	41	9 1	12,14	12,30
144	* δ Cancri	4	127	45	17,0	51,38	48,30	70	58	47,9	12,25	12,41
145	ι Urſæ maj	4	130	40	2 0	63,37	63 09	41	2	3 1	13 04	13,23
146	* 1 α Cancri	4	130	41	58,3	49,30	49,23	77	28	[illegible]2,1	13,05	13,19
147	* 2 α Cancri	4	131	20	4,8	49 33	49 37	77	13	43,7	13,22	13,36
148	* κ Cancri	5	133	40	53 0	48,90	48,84	78	2[illegible]	52 5	13,82	14,02
149	* ξ Cancri	6	133	52	48,5	52,06	51,95	66	59	57,6	13,87	14,01
150	* ω Leonis	5	138	53	50,0	48,27	48,21	79	54	42,5	15,08	15,20
151	α Hydræ	2	138	56	57,0	44,17	44 15	97	37	48,0	15,09	15,20
152	θ Urſæ maj	3,4	139	10	9,0	63,14	62,79	37	14	35,0	15,14	15,30
153	* ξ Leonis	4	139	44	49,0	48,81	48,69	77	39	4,3	15,27	15,39
154	10 Leon min	4,5	139	51	36,6	55,82	55,63	52	33	6,3	15,30	15,44
155	10 Leonis	5	141	7	58,9	47,67	47 61	8[illegible]	5	58,5	15,58	15,69
156	* Leonis	4	14[illegible]	4	50,0	48,31	48,25	79	1	40,8	15,79	15,90
157	ι Leonis	3	143	2	44,0	51,50	51 39	65	7	59,5	15,99	16 11
158	* υ Leonis	5	146	19	[illegible]0 0	48,61	46 53	76	[illegible]5	16,5	16,65	16,76
159	* π Leonis	4	146	5[illegible]	41,0	47 70	47 64	60	48	51,7	16,76	16 96
160	* η Leonis	4	148	33	16,0	49,30	49 22	72	4	37 5	17,07	17,17
161	* A Leonis	5	148	47	16,0	47,97	47 91	78	50	10 9	17,12	17 [illegible]1
162	Regulus	1	148	53	32,5	46,34	18 [illegible]7	76	52	9 1	17,14	17,23
163	ζ Leonis	3	150	49	26,0	50,39	50 [illegible]8	65	[illegible]	17 8	17 47	17 56
164	γ Leonis	2	151	40	35,0	49,58	49,48	68	57	13,1	17,6[illegible]	17,70
165	μ Urſæ maj	3	151	59	9 0	54,60	54 37	47	16	10 5	17,67	17,76
166	* ρ Leonis	4	155	2	25,0	47,50	47 44	79	[illegible]7	56,6	18,14	18 19
167	* 48 Leonis	5	155	34	0 0	47 12	47,06	61	49	5,0	18 [illegible]	18, 9
168	37 Sextantis	6	155	23	42 0	46,91	46,67	82	2[illegible]	6 [illegible]	18 61	16,67
169	38 Sextantis	6	156	4	21,0	46,90	46 65	8[illegible]	[illegible]3	40,1	18 65	18,71
170	* 55 Leonis	5	160	50	19 0	46,16	46,13	87	59	[illegible]0 5	16,90	18,96
171	* 56 Leonis	6	160	53	17 0	46,79	46,74	8[illegible]	3[illegible]	24,0	18 91	18,96
172	β Urſæ maj	2	161	47	56 0	55 60	55 40	32	[illegible]0	1,9	19 01	19 07
173	d Leonis	5	16[illegible]		29 0	46 46	46 4	65	5	55 0	19 04	19 09
174	c Leonis	5	162	4	30 0	46,73	46 66	8[illegible]	36	53 1	19 04	19 09
175	* α Urſæ maj	1 2	16[illegible]	10[illegible]	35 0	57 97	57,45	26	57	35,2	19 05	19 1

Dr BRADLEY's CATALOGUE OF FIXED STARS

Number of Stars	Names of the Stars	Magn.	Mean Right Ascension Jan 1 1760			Ann Precess in Right Ascension		Mean Distance from N Pole Jan 1 1760			Ann Precess in Declination	
						1760	1800				1760	1800
			D	M		S		D	M	S	S	S
176	* χ Leonis	5	163	9	26,0	46,81	46,76	81	22	16,7	19,15	19,20
177	δ Leonis	3	165	19	38,0	47,96	47,87	68	9	53,3	19,36	19,40
178	θ Leonis	3	165	24	[illegible]3,0	47,44	47,37	73	15	46,5	19,37	19,41
179	* 75 Leonis	5	166	14	1,0	46,20	46,18	86	40	[illegible]1,8	19,44	19,48
180	* 76 Leonis	5	166	36	57,0	46,17	46,14	87	[illegible]	15,5	19,47	19,51
181	* σ Leonis	5	167	11	17,0	46,50	46,46	82	39	30,3	19,52	19,55
182	* 79 Leonis	5,6	167	55	51,0	46,13	46,10	87	16	40,1	19,57	19,60
183	τ Leonis	4	168	53	51,0	46,21	46,18	85	49	[illegible]7,5	19,64	19,67
184	* e Leonis	5	169	30	53,0	45,82	45,80	91	40	54,8	19,66	19,71
185	* υ Leonis	4	171	9	57,0	45,95	45,94	89	30	1,6	19,78	19,80
186	* 1 ξ Virginis	5	173	13	35,0	46,33	46,29	80	24	3[illegible],6	19,87	19,69
187	* ν Virginis	5	173	22	41,0	46,[illegible]5	46,21	8[illegible]	7	35,0	19,88	19,89
188	β Leonis	1,2	174	11	59,0	46,50	46,45	74	5	13,0	19,91	19,92
189	* β Virginis	3	174	32	51,3	46,03	46,02	86	5[illegible]	56,1	19,9[illegible]	19,93
190	γ Ursæ maj	2	175	16	25,0	46,29	48,00	34	56	16,1	19,95	19,96
191	* π Virginis	5	177	8	28,0	46,07	46,04	82	2	50,3	19,99	19,99
192	δ Ursæ maj	3	180	51	27,0	45,44	45,16	31	37	54,1	20,01	20,00
193	γ Corvi	3	180	5[illegible]	30,0	46,01	46,07	106	12	27,1	[illegible]0,01	[illegible]0,00
194	* n Virginis	5	181	35	43,0	45,93	45,93	89	27	1,3	[illegible]0,01	[illegible]0,00
195	* η Virginis	3	181	54	30,7	45,94	45,95	89	19	5[illegible],9	20,00	19,99
196	* c Virginis	3	18[illegible]	2	32,0	45,87	45,86	85	20	55,7	[illegible]0,00	19,99
197	κ Draconis	3	185	46	32,0	40,04	39,67	18	53	2,6	19,91	19,89
198	* χ Virginis	5	186	43	19,0	46,21	46,23	96	40	13,5	19,88	19,85
199	* γ Virginis	3	187	2	45,9	45,93	45,95	90	7	43,5	19,85	19,[illegible]
200	ψ Virginis	5	190	28	3[illegible],0	46,45	46,50	98	1[illegible]	47,8	19,68	19,64
[illegible]01	δ Virginis	3	190	52	54,0	45,62	45,62	85	17	3[illegible],5	19,65	19,61
[illegible]0[illegible]	[illegible] Virginis	3	19[illegible]	33	31,0	44,98	44,97	77	44	43,9	19,53	19,49
203	* g Virginis	5	193	50	21,0	46,72	46,77	99	27	0,8	19,43	19,36
204	* θ Virginis	4	194	23	16,0	46,30	46,33	94	15	3,2	19,39	19,34
205	* Spica Virg	1	198	8	44,1	47,01	47,07	99	54	3,0	19,02	18,96
206	* ι Virginis	4	198	31	4,0	47,22	47,27	101	27	1,4	18,99	18,91
207	ζ Ursæ maj	3	198	33	12,0	36,42	36,30	33	46	53,7	18,97	16,92
208	* 2 l Virginis	5	199	5[illegible]	44,0	46,52	46,57	95	0	6,8	15,82	18,75
209	* m Virginis	5	20[illegible]	15	44,0	46,92	46,98	97	[illegible]8	57,5	18,5[illegible]	16,45
[illegible]10	η Ursæ maj	2	204	31	1,0	35,85	35,77	39	28	51,7	18,21	18,15

DR BRADLEY's CATALOGUE OF FIXED STARS

Numb of Stars	Names of the Stars	Magn	Mean Right Ascension Jan 1 1760			An Prec in Right Ascension		Mean Distance from N Pole Jan 1 1760			An Preces in Declination	
						1760	1800				1760	1800
			D	M		s	s	D	M		s	s
211	α Draconis	2	209	28	28, 0	24,29	24,31	24	28	18, 2	17,42	17,33
212	* κ Virginis	4	210	1	50, 0	47,54	47,60	99	8	41, 8	17,33	17,23
213	Arcturus	1	211	10	53, 0	4[illegible],07	42,06	69	33	27, 1	17,12	17,04
214	* λ Virginis	4	211	32	26, 0	48,20	48,27	102	15	12, 3	17,06	16,95
215	θ Bootis	4	14	15	32, 0	30,99	30,97	37	1	48, 3	16,54	16,51
216	* μ Libræ	5	219	3	4, 0	48,87	48,95	103	8	2, 5	15,54	15,42
217	* α Libræ	2	219	24	40, 6	49,35	49,42	105	1	44, 7	15,46	15,35
218	* 2 ξ Libræ	5	220	56	46, 0	48,34	48,41	100	25	30, 5	15,12	14,99
219	* 18 Libræ	5	221	29	16, 0	48,30	48,37	100	9	47, 5	14,99	14,86
220	β Ursæ min	3	222	55	17, 0	5,43	4,78	14	51	44, 5	14,66	14,67
221	* 1 ι Libræ	5	223	19	19, 0	49,69	49,76	105	18	33, 5	14,56	14,46
222	* 1 ι Libræ	3	24	38	51, 0	50,73	50,83	108	51	58, 7	14,24	14,10
223	β Libræ	2	26	1	57, 5	48,08	48,14	98	28	51, 5	13,89	13,76
224	* 4 ζ Libræ	4	29	50	59, 8	50,32	50,40	106	1	11, 9	12,90	1[illegible],75
225	* γ Libræ	[illegible]	230	32	1, 7	49,77	49,84	103	58	14, 2	12,7[illegible]	12,57
226	α Coronæ Bor	[illegible]	231	8	6, 0	37,80	37,81	6[illegible]	[illegible]7	48, 0	1[illegible],56	12,44
227	* 42 Libræ	5	31	3[illegible]	12, 0	5[illegible],58	52,68	113	1	3, 0	1[illegible],45	12, 9
228	* κ Libræ	4	32	2	34, 0	51,32	51,41	108	52	50, 6	12,31	12,15
229	α Serpentis	2	33	7	0, 0	43,91	43,93	8[illegible]	48	11,	12,01	11,87
230	* 1 A Scorpii	5	34	48	51, 0	53,41	53,54	114	35	22, 5	11,53	11,36
231	* λ Libræ	4	234	51	40, 0	51,70	51,78	109	25	48, 6	11,52	11,35
232	* ϑ Libræ	4	235	2	58, 0	50,63	50,71	106	0	18, 1	11,47	11,30
233	ρ Serpentis	3	235	11	3, 0	39,38	39,40	66	17	9, 4	11,43	11,30
234	* π Scorpii	3	236	5	47, 0	53,81	53,91	115	[illegible]3	41	11,16	10,99
235	* ψ Libræ	4	36	11	53, 0	49,94	50,01	103	[illegible]4	0, 4	11,13	10,97
236	* δ Scorpii	3	236	32	44, 0	52,65	52,73	111	[illegible]	1, [illegible]	11,03	10,86
237	* β Scorpii	2	237	52	47, 0	51,81	51,86	109	7	39, 1	10,64	10,47
238	* 1 Scorpii	5	238	1[illegible]	9, 0	[illegible],13	[illegible]	109	59	5, 2	10,[illegible]	10,3[illegible]
239	* 2 ω Scorpii	5	38	0	37, 0	52,20	5[illegible],27	110	11	5[illegible]	10,50	10,3[illegible]
240	[illegible] Herculis	5	38	50	11, 0	27,75	[illegible],77	43	1[illegible]	9, 0	10,36	10,26
241	* Scorpii	4	239	31	18, 0	51,60	51,88	108	48	58,	10,15	9,98
242	* ο Ophiuchi	[illegible]	40	26	[illegible]4, 0	46,66	46,90	93	[illegible]	26, 9	9,67	9,71
243	* 19 Scorpii	[illegible]	41	33	35, 0	53,60	5[illegible],69	113	34	[illegible]	9,53	9,34
244	* σ Scorpii	4	41	39	40, 0	54,14	54,21	114	59	41, 1	9,50	9,31
245	* ψ Ophiuchi	5	4[illegible]	[illegible]1	[illegible], 0	[illegible]2, 0	5[illegible],27	109	[illegible]7	16, [illegible]	9,[illegible]3	9,05

Dr BRADLEY's CATALOGUE OF FIXED STARS

Numb. of Stars	Names of the Stars	Magn.	Mean Right Ascension Jan. 1 1760			An. Preces. in Right Ascension 1760	1800	Mean Distance from N. Pole, Jan. 1 1760			An. Preces. in Declination 1760	1800
				′		″		°	′		″	″
246	* g Ophiuchi	5	-42	48	34,0	53,44	53,51	112	52	18 5	9,15	8,96
247	* Antares	1	243	41	0,9	54,63	54,71	115	52	32,5	8,87	8,68
-48	* φ Ophiuchi	4	244	21	24,0	51,12	51,18	106	4	2,5	8 66	8,48
249	* ω Ophiuchi	5	244	29	14,0	52,84	52,90	110	55	51 0	8,62	8,43
250	* τ Scorpii	4	245	14	48,0	55,47	55,55	117	41	41,9	8 38	8,18
251	* 24 Scorpii	5	246	55	47,0	51 65	51,71	107	15	14,4	7,84	7,65
252	μ Draconis	4	255	5	46,3	18,51	18,56	35	12	23,0	5,15	5,08
253	* A Ophiu. dup	5	55	9	24,0	55,46	55,51	116	13	22 5	5,12	4,92
254	α Herculis	3	255	55	45,0	40,85	40 85	75	19	5,5	4 87	4,71
255	* ρ Ophiuchi	4	56	39	35,0	53,40	53,37	110	49	49,6	4,6-	4 41
256	* θ Ophiuchi	3	256	49	28 0	54,90	54 95	114	44	1 9	4 56	4,34
257	* 43 Ophiuchi	5	257	4	19 0	56 5	56 30	117	53	1,5	4,48	4,26
-58	* B Ophiuchi	4	257	56	4,0	54,61	54,65	113	55	44 3	4,18	3,97
259	* e Ophiuchi	5	259	11	56,0	54 58	54 58	113	45	7 5	3,75	3,54
260	α Ophiuchi	2	260	57	4 0	41,45	41,46	77	14	48,9	3 14	2,99
261	μ Ophiuchi	4	61	12	13,0	48,69	48,71	97	57	5 9	3,06	- 87
262	β Draconis	3	261	15	28,0	20,16	20 18	37	30	41 2	3,04	2 96
263	* D Ophiuchi	5	62	15	59,0	53,76	53 78	111	32	29,8	2,69	2 46
264	* p Sagittarii	3	263	7	5,0	56,36	56 38	117	42	45 1	2,40	2 18
265	* b Sagittarii	5	66	17	18,0	54,7-	54 79	113	46	5 5	1,30	1,08
266	γ Sagittarii	3,4	267	36	0,0	57,66	57,67	120	23	56 4	0,84	0,61
267	γ Draconis	2	267	45	50,0	20,76	20 76	38	28	29,6	0,78	0,10
268	* 1 μ Sagittarii	4	269	51	11,9	53,65	53,63	111	5	47,7	0,05	0,16
269	* 2 μ Sagittarii	4	70	13	34 0	53,52	53,52	110	46	28,4	0 08	0 -9
270	* δ Sagittarii	3	271	24	26,8	57 43	57 43	119	54	11,5	0,49	0,72
271	ε Sagittarii	2	272	3	44,0	59,66	59,65	1-4	28	12,1	0,72	0 95
272	* λ Sagittarii	4	73	17	32,0	55 47	55,47	115	31	40,8	1,15	1 36
273	α Lyræ	1	277	12	11,0	30,09	30,06	51	-5	33,7	,51	-,63
274	* φ Sagittarii	3	277	39	49 0	56,13	56 10	117	12	39,3	2,67	2,88
275	* 28 Sagittarii	5	77	57	59,0	54 19	54,16	112	37	4,5	2,77	-,98
276	c Draconis	5	279	9	44,0	17 4-	17,40	34	41	45,7	3 30	3 37
277	* 1 Sagittarii	4	79	55	9,0	54 30	54 08	113	0	56,2	3,33	3,66
278	* σ Sagittarii	3	80	5	40 0	55,78	55,74	116	31	7,3	3 51	3 7-
279	* 2 ν Sagittarii	4	280	8	59,0	54 7	54 24	112	56	47,1	3,53	3 73
280	β Lyræ	3	80	18	26,0	33 09	33,09	56	53	47,8	3,58	3 71

Dr BRADLEY's CATALOGUE OF FIXED STARS

Numb of Stars	Names of the Stars	Magnitud	Mean Right Ascension Jan 1 1760			An Precess in Right Ascension, 1760	An Precess in Right Ascension, 1800	Mean Distance from N Pole Jan 1 1760			An Precess Declination 1760	An Precess Declination 1800
			D	M	S	s		D	M		s	s
281	* 1 ξ Sagittarii	6	280	46	8 0	53 45	53 4[illegible]	110	56	44,3	3,74	3,94
282	* 2 ξ Sagittarii	5	280	51	5,0	53,63	53,60	111	23	51,8	3,77	3 97
283	1 ϑ Serpentis }	3	281	4	20,9	44,59	44,58	86	5	29,3	3 84	4,01
284	[illegible] ϑ Serpentis }		281	4	39,6	44,59	44,58	86	5	23,3	3 85	4,02
285	ζ Sagittarii	4	281	49	59,0	57,33	57,28	120	11	49,6	4,10	4,32
[illegible]86	Draconis	4	281	54	46 0	13 21	13 [illegible]5	30	53	55,4	4,13	4,18
[illegible]87	* Sagittarii	3	282	34	26 0	53,85	53 61	11[illegible]	4	8,9	4,36	4,56
288	* τ Sagittarii	3	282	59	9 0	56 29	56 25	117	59	42 4	4,50	4,72
289	ζ Aquilæ	3	283	35	50 0	41,25	41 24	76	25	32,0	4,71	4,86
290	* π Sagittarii	4	[illegible]83	5	14,0	53,53	53 49	111	22	52,7	4,80	5,00
291	* ψ Sagittarii	4	285	12	11,0	55,20	55,15	115	38	4,5	5,25	5,45
292	* d Sagittarii	4	285	53	43,0	52,69	5[illegible],65	109	21	26,0	5,48	5,73
293	* 1 χ Sagittarii	5	287	39	48,0	54,80	54,74	114	57	6,5	6,07	6,27
294	κ Cygni	4	287	53	20 0	20,71	20,69	37	3	57,0	6,15	6,22
295	δ Draconis	3	288	6	35,0	0,59	0,45	22	45	34,5	6 22	6,22
296	δ Aquilæ	3	288	20	56,0	45 05	45,03	87	20	44,3	6,30	6,46
297	* 2 h Sagittarii	5	290	31	10,0	54,82	54,76	115	23	[illegible]9,8	[illegible],02	7,21
298	ι Cygni	4	290	54	49,0	[illegible]2,65	22,63	38	46	21,0	7,14	[illegible]
299	θ Cygni	4	29[illegible]	30	0,0	24 15	[illegible]4,13	40	19	30,3	7,66	7 74
300	* f Sagittarii	5	293	5	6,0	52,74	52,68	110	19	[illegible],0	7 85	8,03
301	γ Aquilæ	3	293	42	45,3	4[illegible],68	42 67	79	57	16,6	8 05	8,[illegible]0
302	δ Cygni	3	94	22	9,0	27,98	27 97	45	[illegible]6	39,1	8 26	8,[illegible]5
303	α Aquilæ	1	294	46	0,7	43,[illegible]9	43,28	81	44	51 1	8,38	8,54
304	* ω Sagittarii	5	295	16	31,0	55 11	55 03	116	54	49 0	8 55	8,74
305	* b Sagittarii	4	295	32	52,0	55,44	55 35	117	46	58 8	8,63	8,62
306	β Aquilæ	3	295	52	51,5	44,09	44 07	81	10	28,3	8,74	8,89
307	* a Sagittarii	5	96	4	34,0	55 02	54,93	116	49	19 9	8 80	8,99
308	ι Draconis	5	297	13	20 0	[illegible],06	2,37	[illegible]0	20	6,6	9,15	9,15
309	θ Aquilæ	3	[illegible]99	43	46,0	46,39	46 35	91	30	56,9	9,9	10,08
310	[illegible] Draconis	5	300	[illegible]4	52,0	4,69	4,69	[illegible]2	48	[illegible]9,2	10,13	10,15
311	1 α Capricorni	4	301	4	55,1	49,96	49,90	103	14	44,4	10,33	10,49
312	2 α Capricorni	3	301	10	49,9	49,97	49 83	103	16	2[illegible],0	10,[illegible]8	10,52
313	* 1 σ Capricorni	5	301	22	43,0	52,10	52 [illegible]	109	50	51,5	10,42	10,59
314	* 2 σ Capricorni	5	301	23	1 0							
315	* β Capricorni	3	301	52	36,1	50 65	50 58	105	31	11,5	10,57	10,73

Dr BRADLEY's CATALOGUE OF FIXED-STARS

Numb of Stars	Names of the Stars	Magn	Mean Right Ascension Jan 1 1760			Ann Preces in Right Ascension		Mean Distance from N Pole Jan 1 1760			Ann Prece in Declinatio	
						176	800				1760	800
			D		S	S	S	D		S	S	S
316	* ρ Capricorni	5	303	47	15,0	51,52	51,44	108	35	19,0	11,13	11,29
317	* υ Capricorni	5	306	35	19,0	51,45	51,37	108	57	56 8	11,93	12 09
318	α Delphini	3	307	7	27,0	41,63	41,62	74	55	13,4	12,08	12,[illegible]0
319	α Cygni	1	308	18	51,3	30,[illegible]3	30,54	45	33	59,9	12,41	12,50
320	ι Aquarii	4	308	40	1,0	48 78	48 72	100	21	28 1	12,50	12,6[illegible]
321	ι Cygni	3	309	7	39,0	35,82	35,82	56	55	6,6	12,63	1 73
322	μ Aquarii	4	309	55	21,8	48,60	48,54	99	5[illegible]	3,3	12,84	12,98
323	* 19 Capricorni	4	310	18	10,0	51,13	51,04	108	48	58 6	12,95	13,09
324	* ψ Capricorni	5	312	40	43 0	51,51	51,42	110	47	13,2	13,66	13 71
325	* θ Capricorni	4	313	6	2 4	50 [illegible]	50 64	108	10	12,1	13 74	13,81
326	* 1 χ Capricorni	5	313	41	40,0	51,8	51 [illegible]2	11[illegible]	8	35,8	13,83	13,96
327	* ι Aquarii	5	314	7	30 0	49,07	49,00	102	19	4 8	13,93	14 07
328	* φ Capricorni	5	315	29	5,0	51,49	51,39	111	37	59 2	14,27	14,41
329	* 2[illegible]Capricorni	5	315	36	34,0	49,98	49 90	106	9	13,	14,30	14,43
330	α [illegible]lei	4	315	57	2[illegible] 0	44 89	44,66	85	43	52 8	14,39	14,50
331	* ι Capricorni	5	317	12	50,0	50,30	50,[illegible]2	107	50	30,7	14 69	14,82
332	α Cephei	3	318	12	28,0	21,29	[illegible]1,[illegible]4	[illegible]6	25	26,7	14,92	14,07
333	ζ Capricorni	4	318	13	48,4	51,71	51,58	11[illegible]	26	8,4	14,93	15,09
334	* b Capricorni	5	318	45	3,0	51 46	51,38	112	50	16,9	15,05	15,18
335	β Aquarii	3	319	43	3[illegible],8	47 43	47,[illegible]7	96	36	50 8	15,[illegible]7	15,38
336	* ε Capricorni	4	320	54	,7	50,65	50,55	110	31	39,5	15,53	15,65
337	* ξ Aquarii	5,6	321	14	24 0	47,69	47,83	98	55	2,7	15,61	15,72
338	ε Cygni	4	321	14	34,0	33,60	33 68	45	[illegible]7	38,1	15 61	15,7[illegible]
339	β Cephei	3	321	2	13,0	12,49	1[illegible]9	[illegible]0	29	22,2	15 63	15,66
340	* γ Capricorni	4	321	41	26 0	49,89	49,81	107	4[illegible]	56,5	15,70	15 8[illegible]
341	* κ Capricorni	5	[illegible]22	18	2[illegible],0	50 37	50,27	109	56	48,1	15 64	15,9[illegible]
342	* λ Capricorni	5	323	[illegible]3	57,0	48,56	48,49	10[illegible]	27	37 7	16 07	16 16
343	* δ Capricorni	3	3[illegible]3	[illegible]6	[illegible]8 3	49 62	49,53	107	12	12 9	16,08	16,18
344	2 π Cygni	5	324	[illegible]9	11,0	32 92	33,1[illegible]	41	47	33 7	16, 9	16,36
345	* μ Capricorni	5	3[illegible]5	2	44,4	48,93	48,05	104	40	7,9	16,49	16 [illegible]1
346	ο Aquarii	5	3[illegible]7	43	23 5	46 54	46,50	93	18	12 3	16 92	17,01
347	α Aquarii	3	328	[illegible]1	46,1	46,20	46 16	91	28	32,8	17,04	17,[illegible]
348	Aquarii	5	328	21	47,5	48,74	48,67	105	1	2[illegible],8	17,04	17,1[illegible]
349	* 35 Aquarii	5	328	56	50,0	49,62	49,52	109	41	0,4	17,15	1[illegible]4
350	* θ Aquarii	4	331	2	16 2	47,46	47,38	98	58	6,0	17 51	17,60

DR BRADLEY's CATALOGUE OF FIXED STARS

Numb of Stars	Names of the Stars	Magn	Mean Right Ascension Jan 1 1760			An Preces in Right Ascension		Mean Distance from N Pole Jan 1 1760			An Preces in Declination,	
						1760	1800				1760	1800
			D	M	S	S		D	M			
351	* ρ Aquarii	5	331	53	18 2	47,42	47 37	99	1	0,3	17,65	17 73
352	γ Aquarii	3	332	18	47, 2	46,35	46 31	92	35	17 2	17 72	17,80
353	π Aquarii	4	333	15	13, 1	45,90	45 87	89	49	54,6	17 67	17,95
354	ζ Aquarii	4	334	6	57, 2	46,12	46,08	91	14	23,9	18,01	18,08
355	* σ Aquarii	5	334	28	51, 9	47 74	47 53	101	53	52,2	18,06	18,14
356	7 Lacertæ	4	335	21	45 6	36,31	36 40	40	56	41,6	18,19	18,25
357	υ Aquarii	5	335	22	53 7	49,17	49,07	111	15	41,8	18 19	18,27
358	η Aquarii	4	335	45	17 1	46,12	46 09	91	20	48,2	18 25	18,31
359	* κ Aquarii	5	336	19	46, 5	46,70	46,66	95	27	-9,5	18,33	18,37
360	* 1 τ Aquarii	5	338	44	16, 2	47 91	47,84	105	18	51,7	18,65	18,71
361	* 2 τ Aquarii	4	339	12	56, 9	47 81	47,74	104	51	8,5	18,71	18,77
362	* λ Aquarii	4	340	1	15, 9	46,99	46,94	98	50	59,8	18,81	18,87
363	ι Cephei	4	340	17	54, 0	31,50	31,61	25	3	-8 9	18,84	18,88
364	δ Aquarii	3	340	28	21, 6	47,98	47,91	107	5	26,6	18,86	18,92
365	Fomalhaut	1	341	5	5, 0	49,81	49,67	120	53	14,0	18 93	18,99
366	β Piſcium	4	342	55	2, 0	45,67	45,66	87	28	1 7	19 13	19,18
367	β Pegaſi	2	343	2	39, 4	42,98	43,04	63	12	5-,0	19,14	19,19
368	* 1 h Aquarii	6	343	9	29, 0	46,84	46,80	98	59	1,8	19 15	19,20
369	* 2 h Aquarii	7	343	12	5, 5	46,85	46,80	99	2	50 1	19,16	19,-1
370	α Pegaſi	2	343	12	22, 5	44,49	44 52	76	4	54,0	19,16	19 -1
371	* 3 h Aquarii	7	343	20	30, 6	46,86	46,81	99	13	25,7	19 17	19, -
372	* φ Aquarii	4	345	28	15, 9	46,56	46,54	97	-0	1, 5	19 37	19 41
373	* 1 ψ Aquarii	5	345	49	32, 6	46,83	46,78	100	23	26 6	19,40	19,44
374	* χ Aquarii	6	346	5	59, 4	46,69	46,65	99	1	50,3	19 43	19,47
375	* 2 ψ Aquarii	5	346	21	15 7	46 80	46,75	100	29	18,5	19 45	19,49
376	* 3 ψ Aquarii	5	346	36	55 3	46,82	46 ,7	100	55	6,3	19,47	19 51
377	* 96 Aquarii	5	346	41	13, 0	46 45	46,41	96	25	55 7	19,48	19,5-
378	d Caſſiopeiæ	5	348	34	10 0	38,77	38,99	29	1	52,	19,62	19 64
379	* 1 κ Piſcium	5	348	39	34, 0	45,93	45 9-	90	3	17 9	19 6-	19,65
380	1 λ Andromedæ	4	351	28	16, 0	42,94	43,06	44	50	-3 3	19,74	19 ,6
381	* λ Piſcium	5	352	27	10, 0	45,91	45,90	89	32	1 0	19,84	19 86
382	* 19 Piſcium	5	353	32	9, 0	45,84	45,84	87	50	36 6	19,89	19,90
383	27 Piſcium	5	356	35	49, 0	46,03	46 01	—	—	—	19,98	19,96
384	* ω Piſcium	4	356	45	4, 0	45,82	45 83	84	27	53 0	19,98	19 99
385	* 29 Piſcium	5	357	2	36, 0	46,00	45,98	94	21	48,5	19,99	20 00

Dr BRADLEY's CATALOGUE OF FIXED STARS

Numb of Stars	Names of the Stars	Magnitude	Mean Right Ascension, Jan 1 1760			An Preces in Right Ascension		Mean Distance from N Pole, Jan 1 1760			An Preces in Declination	
						1760	1800				1760	1800
			D	M			s	D	M	S	s	s
386	* 30 Piſcium	5	357	24	47,0	46 04	46,02	97	20	50,7	19,99	20,00
387	* 33 Piſcium	5	358	15	47,0	46,00	45,98	97	3	0,5	20,00	20 00
388	α Andromedæ	2	359	0	25,0	45 74	45,84	62	14	5,9	20,01	20,01
389	β Caſſiopeiæ	3	359	7	40,0	45,45	45,72	32	10	25,0	20,01	20,01

The ſtars marked with Aſteriſcs, are thoſe which may be eclipſed by the moon to any part of the globe

This catalogue of the right aſcenſions and north polar diſtances of 389 fixed ſtars, deduced from the obſervations of Dr BRADLEY, is taken from the firſt volume of his *Obſervations*, publiſhed by Dr HORNSBY It was firſt printed in the *Nautical Almanac* for 1773, but Dr HORNSBY re publiſhed it with corrections, for the moſt important of which, he has aſſigned the following reaſons

2 ι *Ceti* The right aſcenſion and polar diſtance of this ſtar were both faulty The former was deduced from the apparent right aſcenſions found in 1751, and carried on by preceſſion to 1755 only the latter was the mean polar diſtance for January 1, 1758, and was deduced from ſix obſervations, the difference of the extremes amounting only to 4″,3

4 δ *Andromedæ* The polar diſtance of this ſtar was, through miſtake, given for 1758

10 ε *Piſcium* The polar diſtance ſhould have been one minute greater the miſtake was committed in reducing the obſervations of December 31, 1753, and October 28, and November 18, 1754

β *Andromedæ* Having reaſon to believe the right aſcenſion of this ſtar to be faulty, I reduced the obſervations in 1756 and 1757, and was enabled to apply the correction

33 1 ξ *Ceti* The catalogue here, at firſt ſight, was erroneous The polar diſtances were computed twice from the ſame obſervations, viz December 8, and 26, of 1753, and January 3, and 25, of the following year, but more truly in the ſecond inſtance if we except a miſtake of 3 degrees committed by the computer, in writing down the reduced zenith diſtances In the former, it may be obſerved the preceſſion was applied for ſeven years inſtead of ſix

36 2 ξ *Ceti* A fault was committed in applying the preceſſion for ſix years to this ſtar, by writing 9″,9 inſtead of 99″,8 = 1′ 39″,8

37 δ *Ceti* The obſervations of January 17, 1753, and of January 13, 1755 aſſuming the right aſcenſion of Aldebaran, and of α Orionis, as ſettled by Dr BRADLEY, give the ſame right aſcenſion exactly, the extreme obſervations differing only 4″,6

47 ε *Arietis* The right aſcenſion of this ſtar was corrected by three obſervations in 1752

48 γ *Perſei* The preceſſion applied was one minute too little

57 f *Tauri* The right aſcenſion of this ſtar was corrected by the obſervations of February 18, 1753, of January 11, and December 11, 1754, and of January 22, of the following year

58 17 *Eridani* A miſtake was committed in applying the aberration to the obſerved zenith diſtances, and it appears, from a looſe paper, that the right aſcenſion had been determined to be 49 41′ 48

79 2 θ *Tauri* The aſcenſional difference of this and the preceding ſtar, as given by Mr Zach, p cxliii muſt be faulty By the obſervations of January 7, and December 26, 1754, of January 22, 1755, of February 8, 1756, and of January 29, 1757, the mean of the aſcenſional differences is 5′,37 of time = 1′ 20″,55, if the obſervation of January 22, 1755, be corrected by reading 1^h 15′ 4′,0 inſtead of 1^h 15′ 5′,0, and the obſervation of January 29, 1757, be diminiſhed by 1 By my own obſervations in 1779 the aſcenſional difference was 5′,43 of time = 1′ 21″ 45 By comparing theſe two ſtars with Aldebaran on October 15, 1753, and on January 1, and 3, 1754, and aſſuming the place of Aldebaran as uſed by Dr Bradley himſelf (ſee the aſcenſional differences), I find the right aſcenſion of the former = 63 43 18″,7, and of the latter 63 44′ 47′,3 on January 1, 1760 and by my own obſervations 63 43 26″,6 and 63 44 49″,5 It muſt be obſerved that the obſervation of October 15, 1753, appears to be faulty, an error of 1′ having been committed in the obſervation of 2 θ

84 ι *Tauri* I found, by ſeveral compariſons, that the right aſcenſion of this ſtar was to be diminiſhed by 1 5,3

104 1 χ *Orionis* An examination of the ſeveral obſervations of this and the following ſtar ſhows, that the right aſcenſions were very nearly determined, if that of February 1, 1752 be excepted, where the ſtar marked 2 χ preceding α Orionis, with a difference of 2 2″,66, is Flamsteed's 1 χ, and was obſerved as ſuch December 29, 1752, and the preceding ſtar is perhaps, the 223d of Mayer's catalogue The error is therefore probably in the polar diſtance of 2 χ, and upon reducing the ſeveral obſervations, I find the polar diſtance of 2 χ, as given in the catalogue of 1773 to belong to Flamsteed's

4 χ,

4 χ, the right aſcenſion of which has not been calculated The order in which the zenith diſtances were obſerved (ſee the obſervations of January 18, 1754, where the book of obſervations is exactly copied, notwithſtanding the obvious error committed in the caſe of this very ſtar, and alſo of February 15 1754) prove that the right aſcenſion was greater than that of α Orionis, and very nearly the ſame with that of μ Geminorum on theſe two days therefore for 2 χ we ſhould read 3 χ Orionis

I ſubjoin the right aſcenſion of 3 χ, and alſo the polar diſtances of 3 and 4 χ, reduced to January 1, 1760

3 χ Orionis	6	87 25 3″,5	53″,25	53″,26	69 52′ 46′ 5	0″,90	0″,69
4 χ Orionis	6	— — —	53,08	53,09	70 19 46,1	0,68	0,47

129 *υ Geminorum* Having compared this ſtar with ſeveral principal ſtars obſerved on the ſame day, aſſuming their places as determined and uſed by Dr **Bradley** himſelf, I find its right aſcenſion to be

From the obſervations of	1751	110° 16 33″,9
	1752	110 16 38, 9
	1754	110 16 30, 2
	1755	110 16 32, 4
	1756	110 16 35, 1
By a mean		116 16 34, 1

An error therefore of 1′ was committed

145 10 *Leonis* The right aſcenſion and polar diſtance of this ſtar, as given in the catalogue publiſhed in 1773, were deduced each from one obſervation only and from the ſituation of the ſtar, I conjectured that the right aſcenſion belonged to 10 Leonis minoris, and would, perhaps, have been exact, if the proper corrections peculiar to that ſtar, for preceſſion, aberration, and nutation had been applied The obſervations of the 19th and 28th of March and of the 12th of April 1754, prove 10 Leonis to be the ſame with the firſt of the Sextant of **Hevelius**, and the ſame concluſion may be drawn from the Britiſh catalogue The zenith diſtances both of 10 Leonis minoris, and 10 Leonis were obſerved on the 12th of April, we are enabled therefore to determine the place of each ſtar

168 37 *Sextantis* In computing the polar diſtance, an error of 1′ was committed

194 n *Virginis* The polar diſtance of this ſtar is evidently too great, and the obſervations of March 9, and May 4, 1754, enable us to make the correction, for on thoſe days, the ſtars n and η Virginis were both obſerved, and their ſituation is ſo nearly the ſame that the differences of the zenith diſtances need only be applied to the calculated polar diſtance of η

208 2 l *Virginis* The error of 1 in the right aſcenſion was probably committed at the preſs, and alſo of the letter of reference

223 4 ζ *Libræ* By computing the obſervations of June 7, and 19, 1755, and of May 16, 1756, I applied the correction

253 μ *Draconis* The right aſcenſion of this ſtar was unqueſtionably faulty The obſervations of July 30, 1755, and of March 1 1758, enable us to make the correction I ſuſpect that the right aſcenſion firſt found was 255 5 49

333 ζ *Capricorni* In reducing the right aſcenſion of this ſtar from 1755 to 1760, the preceſſion uſed was 10″ too ſmall it ſhould have been printed therefore 311 13′ 52″,0, or more exactly 51 ,8

360 1 τ *Aquarii* The obſervations of September 28, and October 3, 1753 do not juſtify the application of any correction to the polar diſtance of this or the following ſtar

388 α *Andromedæ* In reducing ſeven obſervations of 1753 the refraction for 33 44′ was applied inſtead of 23 44 a correction therefore for the polar diſtance was neceſſary

A

CATALOGUE

OF

THE PLACES OF 515 ZODIACAL STARS,

IN

RIGHT ASCENSION, AND DECLINATION,

ADAPTED TO THE BEGINNING OF THE YEAR 1765

WITH

Their MAGNITUDES, and ANNUAL VARIATIONS
in RIGHT ASCENSION and DECLINATION

BY

M DE LA CAILLE

M. DE LA CAILLE's CATALOGUE OF ZODIACAL STARS

Number of Stars	Names of the Constellations	Bayer	Magnitude	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Declination			Annual Variation
					M		s	H	M	S	D	M		
1	Ceti		3	1	51	53,3	46,14	0	7	28	10	7	41,9S	−20,06
2	Piſcium	d	6	2	7	43,5	46,29	0	8	31	6	53	6,4N	+20,06
3	Ceti		6	3	38	39,4	46,17	0	14	35	1	21	9,2S	−20,04
4	Ceti		6	5	47	23,2	46,03	0	23	10	4	53	4,3S	−20,00
5	Piſcium		6	8	49	0,0	46,49	0	35	16	5	27	19,9N	+19,69
6	Piſcium	δ	4	9	7	36,8	46,55	0	36	30	6	18	11,0N	+19,88
7	Ceti		6	10	15	15,8	46,06	0	41	1	2	25	24,6S	−19,8
8	Piſcium		7	11	55	2,6	46,56	0	47	40	5	12	47,3N	+19,72
9	Piſcium	ε	4	12	41	25,4	46,70	0	50	46	6	37	14,5N	+19,68
10			7	12	56	11,7	46,20	0	51	45	0	6	15,6N	+19,66
11			6	13	10	52,2	46,52	0	52	43	4	23	39,1N	+19,65
12			7	13	58	29,0	46,26	0	55	54	0	45	39,5N	+19,58
13		e	5	14	4	12,6	46,54	0	56	17	4	24	13,0N	+19,57
14			7	14	37	18,4	46,33	0	58	29	1	11	30,0N	+19,53
15			7	15	7	13,6	46,29	1	0	29	1	13	27,5N	+19,49
16		ζ	4	15	2	1,0	46,79	1	1	28	6	19	40,7N	+19,47
17			6,7	15	37	41,3	46,73	1	2	31	5	45	52,8N	+19,44
18	Ceti			15	42	32,6	45,99	1	2	50	2	14	0,0S	−19,44
19	Piſcium	f	6	16	25	19,3	46,42	1	5	41	2	22	20,0N	+19,37
20	1	ρ	5	18	4	22,1	48,25	1	13	37	17	56	35,1N	+19,18
21	2	ρ	5	18	30	37,8	48,26	1	14	3	18	1	0,9N	+19,17
22				19	1	19,5	46,97	1	16	5	6	44	10,6N	+19,12
23			6,7	19	12	12,0	46,89	1	16	49	6	4	23,5N	+19,10
24		μ	5	19	8	20,0	46,78	1	17	53	4	55	39,2N	+19,07
25		η	4	19	44	4,3	47,89	1	18	57	14	7	37,4N	+19,04
26			6	20	36	8,4	47,61	1	22	25	11	20	55,9N	+18,94
27	1	π	5	21	10	1,2	47,61	1	24	40	10	55	58,3N	+18,87
28		π		21	17	52,2	47,62	1	25	12	10	52	26,5N	+18,85
29				21	39	16,6	46,94	1	26	37	5	44	50,0N	+18,81
30			6,7	21	45	[illegible],0	46,[illegible]2	1	27	[illegible]	15	12	2,4N	+18,80
31			5	22	18	19,3	46,78	1	29	13	4	17	31,7N	+18,73
32			5	23	15	7,7	47,01	1	33	1	7	58	5,1N	+18,62
33				24	4	16,9	46,56	1	36	17	2	30	23,4N	+18,51
34				24	36	28,1	48,92	1	38	26	17	53	4,0N	+18,42
35	Arietis	γ	4	25	9	57,5	49,05	1	40	40	18	8	10,1N	+18,3

M. DE LA CAILLE's CATALOGUE OF ZODIACAL STARS.

Numb. of Stars	Names of the Constellation		Bayer's Lett.	Magn.	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Declination			Annual Variation
					D	M		S	H	M	S	D	M		S
36	Piscium		ξ		25	21	13,5	46,50	1	41	25	2	1	15,8N	+18,34
37	Arietis		β	3	25	25	17,8	49,20	1	41	41	19	39	6,3N	+18,32
38	Piscium		ι	6	26	8	10,5	48,85	1	44	33	16	39	41,3N	+18,[illegible]3
39	Piscium		α	3	27	28	40,6	46,35	1	49	55	1	37	16,4N	+18,01
40	Arietis		κ	5,6	28	21	53,4	49,94	1	53	[illegible]7	21	31	24,4N	+17,88
41			α	2	28	29	34,3	50,01	1	53	58	22	20	32,3N	+17,86
42	Ceti			6	29	44	33,4	47,51	1	58	58	7	27	36,9N	+17,64
43	Arietis		η	6	29	55	17,1	49,85	1	59	41	20	5	47,5N	+17,61
44				6,7	30	4	2,9	48,73	2	0	16	14	10	7,6N	+17,59
45	Ceti	1	ξ	6	30	8	31,7	47,57	2	0	34	7	44	8,2N	+17,58
46	Arietis		θ	5,6	31	16	23,8	49,74	2	5	6	18	48	10,9N	+17,38
47	Ceti	2	ξ	4,5	33	3	40,3	48,03	2	12	15	9	32	13,2N	+17,07
48				7	33	43	50,2	48,00	2	14	55	9	9	47,1N	+16,91
49	Ceti	3	ξ	4,5	33	55	19,6	47,65	2	15	41	7	24	49,0N	+16,90
50	Arietis			6,7	34	22	30,8	50,04	2	17	30	18	48	1,0N	+16,82
51				6,7	35	1	1,2	49,07	2	20	4	13	59	6,1N	+16,69
52	Ceti		ν	4,5	35	53	5,7	4[illegible],12	2	23	34	4	3	29,6N	+16,53
53	Arietis			5,6	35	57	44,2	48,60	2	23	51	11	26	43,8N	+16,51
54			ν	4,5	36	22	41,[illegible]	50,75	2	25	31	20	55	51,5N	+16,43
55			μ	6	37	17	19,0	5[illegible],81	2	29	9	8	29	5,3N	+16,[illegible]
56	Ceti				37	27	57,1	48,18	2	29	5	9	13	43,2N	+16,21
57	Ceti		γ	3	37	47	14,6	46,80	2	31	9	2	14	6,5N	+16,14
58	Arietis			6	37	54	28,2	49,36	2	31	36	14	18	15,4N	+16,1[illegible]
59				7	38	2	4[illegible],8	48,71	2	32	11	11	26	40,8N	+16,09
60	Ceti		μ	4	38	3	58,5	46,19	2	32	16	9	6	37,7N	+16,09
61	Arietis			6	38	50	57,7	50,12	2	35	[illegible]4	17	17	31,0N	+15,92
62			π	6	39	3	12,0	49,94	2	36	13	16	28	24,5N	+15,88
63			σ	6	39	38	12,0	49,41	2	38	33	14	6	0,2N	+15,75
64				6	40	6	51,5	50,13	2	41	17	16	16	1,0N	+15,57
65		1	ρ	6,7	40	39	37,7	50,27	2	4	39	17	21	56,2N	+15,[illegible]3
66		2	ρ	6,7	40	48	4,0	50,23	2	43	12	17	4	22,[illegible]N	+15,49
67				6,7	41	10	7,1	50,93	2	44	40	19	43	0,0N	+15,41
68				5	41	[illegible]7	13,5	51,14	2	45	49	20	23	[illegible]3,3N	+15,35
69	Ceti		λ	4	41	47	10,[illegible]	48,06	2	47	9	7	57	36,0N	+15,27
70	Arietis				41	50	20,[illegible]	50,3[illegible]	2	47	2[illegible]	17	3	3,9N	+15,26

M. De la CAILLE's CATALOGUE OF ZODIACAL STARS

Numbers of Stars	Names of the Constellations	Bayer's Letters	Magnitude	Right Ascension in Degrees			Annual Variation +	Right Ascension in Time			Declination			Annual Variation
				D	M	″	S	H	M	S	°	′	″	
71			7	43	33	31 0	50,43	2	54	14	16	58	33 4N	+14 67
72			6,7	43	46	3, 8	50 67	2	55	4	17	52	45 6N	+14,81
73		δ	4	44	33	25 1	50 73	2	58	14	18	49	23,0N	+14,63
74		ζ	5	45	1	28 0	51,44	3	1	26	20	10	27 8N	+14,44
75				46	50	48 0	51 36	3	7	23	19	38	46 6N	+14,07
76	1	τ	7	46	55	6 8	51,6	3	7	42	20	16	56 9N	+14 05
77	Arietis 2	τ	6	47	19	0 5	51 54	3	9	16	19	53	9,8N	+13 95
78				47	37	3, 1	52 75	3	10	26	23	52	27 6N	+13 86
79	3	τ	7	47	43	50 2	51 59	3	10	55	19	57	11 5N	+13,83
80	Tauri	o	4	48	2	5, 6	48,30	3	12	11	8	11	26 2N	+13 76
81	Tauri	ξ	4	48	36	54 2	48 85	3	14	8	9	57	4,9N	+13 61
82	Arietis		7	48	41	7, 9	52,27	3	14	45	21	58	38 6N	+13,59
83	Tauri		6	49	23	54, 2	48,99	3	17	36	10	30	59,6N	+13,42
84		f	5	49	28	55, 6	49 56	3	17	56	12	7	2,2N	+13 40
85		t	6	49	58	36, 1	48 51	3	19	54	8	33	55 8N	+13 26
86			6	50	8	27, 4	5[illegible],83	3	20	34	23	39	30 8N	+13 21
87				51	32	36, 0	50,45	3	26	8	15	45	28 9N	+12 85
88			6	51	41	36, 1	53,39	3	26	46	24	33	0,3N	+12 81
89				52	11	56 1	51 95	3	28	47	19	55	53,9N	+12,68
90				52	33	32, 3	51 64	3	30	14	18	54	20,4N	+12,59
91	Celeno	g	7	52	43	4, 0	53,14	3	30	5[illegible]	23	31	52,0N	+12 55
92	Electra	b	5	52	44	22, 2	53 08	3	30	57	23	21	20,2N	+12 55
93	Aſterope	m	7	52	47	41, 9	53,38	3	31	11	24	5	0,2N	+12 54
94	Taygeta	e	5	5	48	50 9	53,20	3	31	15	23	4	41,0N	+12,53
95	Maia	c	6	52	58	14, 1	53,19	3	31	53	23	36	50,9N	+1 47
96	Merope	d	5	53	6	19, 0	53,07	3	32	25	23	12	56,5N	+12,43
97		η	3	53	23	6, 9	53,15	3	33	32	23	21	42,6N	+1[illegible],37
98	Atlas	f	6	5[illegible]	48	23, 9	53,19	3	35	14	23	18	54,2N	+12,25
99	Pleyone	h	7 8	53	48	41, 5	53 20	3	35	15	23	24	53,1N	+12 25
100				54	2	4, 0	52 59	3	36	8	21	30	49 5N	+12,17
101				54	3	22, 7	53,72	3	36	13	24	51	5,4N	+1 ,17
102			6	55	45	9, 1	52,81	3	43	1	21	47	50,3N	+11,68
103		λ	4	56	55	18, 0	49,71	3	47	41	11	48	39,8N	+11,34
104			7	57	35	8, 1	53,52	3	50	21	23	24	16,4N	+11,15
105	1	A	5	57	42	29, 3	52,65	3	50	50	1	25	9 7N	+11,12

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Numb of Stars	Names of the Constellations	Bayer's Letters	Magn	Right Ascension in Degrees			Annual Variation +	Right Ascension in Time			Declination			Annual Variation
				D	M		s	H	M	S	D	M	S	
106	2	A		57	51	42 4	52,82	3	51	27	21	21	18,9N	+11 07
107		ψ	5	58	7	47,3	54 93	3	52	31	28	10	44,7N	+11 00
108	1	ω	6	58	52	32 4	52 11	3	55	30	18	58	10, N	+10,77
109		p	6	59	8	26,8	54,52	3	56	34	–5	50	49,4N	+10,69
110				59	44	14,	53,11	3	58	57	21	47	20,5N	+10,51
111				60	36	42,3	50,85	4	2	27	14	47	43,8N	+10, 6
112		μ	4	60	41	57 4	48,74	4	2	48	8	17	20,3N	+10, 4
113	2	ω	6	60	52	49,2	52,58	4	3	31	19	58	50,6N	+10,19
114			7	61	7	34,1	52 87	4	4	30	20	59	6 5N	+10 11
115		φ	5	61	–8	55 3	54 88	4	5	56	26	46	3 1N	+ 9,99
116		γ	3	61	36	37 0	50,92	4	6	–6	15	2	36,2N	+ 9,95
117	Tauri		5	6	4	37,0	54,42	4	8	18	25	3	9 0N	+ 9 81
118	1	δ	4	62	21	7 –	51,63	4	9	–4	16	58	25 1N	+ 9 73
119			7	62	27	44 7	54,02	4	9	51	23	43	59,4N	+ 9,70
120	–	δ	4	62	38	36 9	51,57	4	10	34	16	5–	53 4N	+ 9 64
121	1	κ	5	62	50	55,2	53,33	4	11	24	21	44	5,5N	+ 9 56
122	2	κ	5	6	51	42 7	53,33	4	11	27	–1	37	26,7N	+ 9 58
123	3	δ	6	62	58	46,+	51 79	4	11	55	17	22	20,–N	+ 9 54
124	1	υ	5	63	4	7 1	53,50	4	12	16	––	15	31 8N	+ 9 51
1–5			7	63	14	48,5	51,01	4	12	59	15	4	10,7N	+ 9 46
126	2		6	63	18	57 7	53,53	4	13	16	22	16	41 3N	+ 9,44
127			3 4	63	43	⊢ 8	52,28	4	14	55	18	38	–7,5N	+ 9 30
128	1	θ	5	6,	47	32 4	51,17	4	15	10	15	25	21 3N	+ 9,–8
129	2	θ₁	5	63	50	2 0	51,14	4	15	–0	15	19	54 –N	+ 9,27
130			7	64	11	30,2	51,07	4	16	46	15	7	17 2N	+ 9 16
131			7	64	18	57 6	51,07	4	17	16	15	9	40,4N	+ 9,1–
13–			7	64	36	56,9	51,16	4	18	28	15	19	45 2N	+ 9,03
133		ε	5	65	7	59,2	50,83	4	20	3	14	19	55,7N	+ 6,89
134		α	1	65	36	53,7	51,45	4	22	–8	16	1	9,4N	+ 8,71
135		d	5	65	41	28 6	49,32	4	2–	46	9	39	43,8N	+ 8,69
136	Eridani	υ	4	66	8	56 1	44,97	4	24	36	–1	50	41 0S	– 8,55
137	Tauri 1	σ	6	66	26	19,–	51 14	4	25	45	15	20	7 0N	+ 8 19
138	2	σ₁	6	66	27	46,6	51,17	4	25	51	15	26	3 6N	+ 8,49
139	/	τ	5	67	2	28 –	53,85	4	28	10	–	–5	3,1N	+ 8 26
140				67	15	22,0	53,90	4	29	1	23	38	19,7N	+ 8 19

M. DE LA CAILLE's CATALOGUE OF ZODIACAL STARS

Number of Stars	Names of the Constellations	Bayer	Magnitude	Right Ascension in Degrees			Annual Variation +	Right Ascension in Time			Declination			Annual Variation
				D	M	S		H	M	S	D	M	S	S
141				67	51	21,5	49,60	4	31	25	10	41	23,1N	+7,90
142	Eridani	μ	4	68	26	22,1	45,01	4	33	45	3	41	6,98	−7,81
143	Tauri		6	69	4	39,3	51,58	4	36	19	15	28	24,4N	+7,61
144		l	6	69	24	39,3	51,38	4	37	39	15	[illegible]	16,0N	+7,50
145	Orionis 1		4 5	69	48	47,3	51,52	4	39	15	15	46	[illegible]2,0N	+7,37
146	2		4,5	70	47	35,6	51,33	4	43	10	15	7	3,3N	+7,04
147	Tauri			70	53	32,9	54,90	4	43	34	3	33	30,3N	+7,01
148		k	6	70	56	52,0	54,45	4	43	47	24	39	55,3N	+6,99
149		ι	4	72	15	53,9	53,63	4	49	4	21	14	0,6N	+6,56
150	Orionis 1	y	5	7[illegible]	47	29,7	51,33	4	51	10	15	3	23,5N	+6,38
151	Tauri	m	6	73	23	26,9	52,54	4	53	34	18	18	33,7N	+6,18
152				73	27	5,4	54,74	4	53	48	23	55	56,4N	+6,16
153		l	6	73	48	16,0	53,72	4	53	53	1	22	20,0N	+6,15
154				73	28	43,3	53,23	4	53	55	20	6	15,5N	+6,15
155	Orionis 2	y	5	74	3	57,4	51,46	4	56	16	15	16	38,7N	+5,96
156	3	y		74	33	7,8	51,65	4	58	13	15	44	9,6N	+5,79
157	Tauri 1	n	7	75	20	16,8	54,03	5	1	21	1	59	37,2N	+5,54
158	2	n	6	76	17	31,1	53,99	5	5	10	1	49	49,6N	+5,21
159		β	2	77	51	37,8	56,76	5	10	26	8	23	13,2N	+4,69
160		σ	5	78	22	54,5	54,03	5	13	32	21	43	50,5N	+4,53
161		ι	6	78	42	17,8	55,33	5	14	49	24	56	11,8N	+4,42
162	Aurigæ	χ	5,6	79	21	33,4	56,52	5	17	26	31	59	36,8N	+4,17
163	Tauri		6	80	16	33,6	54,94	5	21	6	3	51	40,9N	+3,85
164		ζ	3	80	54	7,1	53,80	5	23	36	0	58	42,7N	+3,63
165			6,7	81	17	41,4	55,74	5	[illegible]	11	25	44	29,6N	+3,50
166			5	81	55	47,5	52,04	5	[illegible]7	43	16	23	35,9N	+3,29
167			6	8	16	8,3	52,96	5	[illegible]9	5	18	50	48,3N	+3,17
168			4	83	38	58,0	55,28	5	34	36	24	27	54,0N	+[illegible],68
169				84	2	23,5	56,76	5	36	10	27	5	23,1N	+2,56
170			5	84	38	18,4	56,61	5	38	33	7	31	56,8N	+2,36
171	Orionis 1	χ	5	85	6	58,7	53,55	5	40	28	0	1[illegible]	41,1N	+2,19
172			6	85	51	7,0	55,91	5	43	24	25	54	4,4N	+1,92
173				86	52	29,0	54,43	5	47	30	22	2[illegible]	23,8N	+1,57
174	Orionis 2	χ	5	87	23	10,3	53,36	5	49	33	19	40	25,9N	+1,39
175	Geminorum	H	5	87	27	27,7	54,78	5	49	50	23	15	14,5N	+1,37

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Numb of Stars	Name of the Constellations	Bayer	Magnitude	Right Ascension in Degrees			Annual Variat +	Right Ascension Time			Declination			Annual Variation
				D	M	S		H	M	S	D	M	S	
176	Orionis 3	χ	5	87	29	–4, 4	53,54	5	49	58	20	7	23,2 N	+1,35
177	Orionis	ν	4,5	88	32	18 3	51,48	5	54	9	14	46	37,3 N	+0,99
178			6	89	31	16, 0	53,44	5	58	5	19	49	25,1 N	+0 64
179	1	f	6	89	37	32 4	52,02	5	58	30	16	9	55,8 N	+0,61
180	Geminorum	η	4,5	90	10	15 3	54,52	6	0	41	22	33	14,2 N	+0 41
181	Orionis 2	f	6	90	28	0, 3	52,03	6	1	52	16	11	45,8 N	+0 30
182	Geminorum	μ	3	92	10	57, 4	54,51	6	8	44	22	36	47,0 N	−0 29
183				92	48	41, 2	54,87	6	11	15	23	26	4,9 N	−0 51
184				92	48	51, 9	54,91	6	11	16	23	32	51,1 N	−0,51
185		ν	4	93	44	57, 7	53,62	6	15	0	20	20	27,7 N	−0,85
186			7,8	94	38	52, 0	5–,68	6	18	35	17	55	53,1 N	−1 14
187		γ	2,3	96	1	52, 1	52 02	6	24	7	16	34	49,1 N	−1,63
188			5	97	10	42 3	52,59	6	28	43	17	51	16 9 N	−2,04
189		ι	3	97	21	59 5	55,60	6	29	–8	25	20	24,7 N	−–,10
190		d	6	99	21	41, 9	54,–0	6	37	27	–2	1	2,0 N	−– 80
191			6	101	13	58, 4	55,83	6	44	56	6	12	39,9 N	−, 45
192	1	ω	6	10–	1	2, 9	55,14	6	48	4	4	31	39,2 N	−3,71
193		ζ	3,4	102	32	12 0	53,67	6	50	9	0	5[illegible]	41,8 N	−3 [illegible]
194	2		6,7	102	47	2 0	53,47	6	51	8	2	58	1–,0 N	−3,97
195		τ	5	104	–	9 5	57,74	6	56	9	30	36	23 5 N	−4,40
196		m	6	104	32	0, 9	55,05	6	58	8	21	29	57,4 N	−4,5[illegible]
197	Geminorum 1	λ	5,6	104	57	53, 1	51,97	6	59	52	16	32	26,1 N	−4,66
198		n	6,7	105	4	28, 7	55,55	7	0	18	25	27	12,9 N	−4,76
199	2	λ	5	106	8	31, 3	51,09	7	4	34	16	56	50,9 N	−5,12
200		δ	3	106	30	54, 7	54,12	7	6	4	–	23	42 5 N	−5,25
201		q	6 7	107	0	52, 3	53,52	7	8	3	0	52	0,9 N	−5,4
202		A	5 6	107	16	47, 1	55 33	7	9	4	25	28	48,6 N	−5 51
203	1	ι	6	107	28	32, 7	56,38	7	9	54	8	4	8 2 N	−5,57
204	2		4 5	107	46	24 7	56,45	7	11	6	8	15	31,6 N	−5,68
205		r	6	108	16	7, 3	53,41	7	13	4	0	42	34, N	−5,84
06		p	6	108	26	17, 4	53,86	7	13	45	1	54	15,9 N	−5,90
207	1	b	6	108	39	56, 3	56 56	7	14	10	28	34	52,3 N	−5,98
208	2	b	6	108	47	26, 2	56,45	7	15	10	28	22	40,9 N	−6 0–
209		α	1	109	53	33, 5	58,05	7	19	34	32	22	54 4 N	−6 39
210		k	6	110	2	35, 1	51,73	7	20	10	16	18	50,6 N	−6,13

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Number of Stars	Names of the Constellations	Letters	Magnitude	Right Ascension in Degrees			Annual Variation +	Right Ascension in Time			Declination			Annual Variation
				D	M		S	H	M		D	M	S	
211		υ	5	110	21	7,0	55,94	7	21	24	7	[illegible]3	48 8N	— 6,54
212		f	6	111	28	24,5	52 33	7	25	54	18	11	29 8N	— 6,91
213		σ	5	112	8	42,8	52,39	7	28	35	18	[illegible]5	49 0N	— [illegible],12
214		c	6	112	26	13,1	55,37	7	29	45	26	19	[illegible]4,4N	— 7 22
215		κ	4,5	112	34	31,6	54,80	7	30	16	24	56	28,2N	— 7 6
216		β	2	112	43	41,9	56,6	7	30	55	28	34	27 6N	— 7,31
217		g	6	113	7	21,1	52 57	7	32	[illegible]9	19	3	51 8N	— 7 43
218			6	113	37	13,1	54 8	7	34	29	23	42	15 0N	— 7,59
219		φ	5	114	46	5,3	55 61	7	39	4	27	21	4 2N	— 7 98
220		1	6	115	[illegible]8	51,6	51,90	7	41	55	20	29	14 0N	— 8 21
221				115	54	[illegible]0,4	51,51	7	43	38	16	24	9,5N	— 8 35
222	Cancri 1	ω	6	116	40	9,0	54,93	7	46	41	26	0	57,5N	— 8,59
223	2	ω	6	116	52	44,5	54,80	7	47	31	25	42	55,7N	— 8 65
224	Geminorum	χ	5	117	15	34,6	55,86	7	49	2	28	26	2,8N	— 8,78
225	Cancri	μ	5	118	28	34,6	5[illegible],41	7	53	54	2[illegible]	14	47 6N	— 9 15
226	2	ψ	4	119	3	51,2	54,84	7	56	15	26	13	6 5N	— 9,32
227	3	b	5	119	37	58,0	56 36	7	58	3[illegible]	30	20	27 2N	— 9 ,0
228		ζ	5,6	119	40	36 7	51,96	7	58	42	18	20	27,0N	— 9,51
229		β	3,4	120	56	24,4	49 21	8	3	46	9	53	35,2N	— 9 91
230		χ	6	1[illegible]1	26	5,8	55,30	8	5	44	27	57	32,8N	— 10 06
231		λ	6	1 1	37	50,3	54,04	8	6	31	24	44	48 5N	— 10 11
232		φ	6	123	7	55 0	55,03	8	1[illegible]	32	27	41	6,7N	— 10 56
233	2	υ	6	123	39	26 9	53,96	8	14	38	24	54	3 0N	— 10 68
234	3	υ	6 7	124	23	37,2	53,86	6	17	34	24	51	23 5N	— 10 94
235		θ	5,6	1[illegible]4	32	28,1	[illegible]1,85	8	18	10	18	5	[illegible]3 2N	— 10 87
236	4	′υ	6 8	124	46	6 9	53,86	8	19	4	24	52	7,2N	— 11 06
237	Cancri		6	126	28	16,2	52,34	8	25	53	20	50	17,9N	— 11,55
238		γ	4	127	25	4,8	52,72	8	29	40	22	17	56,1N	— 11 64
239		δ	4	127	49	34 3	51,75	8	31	18	19	0	19 2N	— 11,94
240			7	129	28	37,1	50 70	8	37	54	16	12	34,8N	— 12,40
241	Hydræ	ζ	4	130	43	29,0	47,79	8	4	54	6	49	40,7N	— 12,74
242	Cancri		6	131	1	46,0	50,61	8	44	7	16	12	37 5N	— 12 81
243		α	4	131	24	11,0	49,61	8	45	37	12	45	18 6N	— 12,91
244		ι	6	132	14	19,3	53,2[illegible]	8	48	57	25	[illegible]1	44 9N	— 13,13
245		τ	6,7	133	27	29,2	54,81	8	5[illegible]	50	30	34	56,9N	— 13,45

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Numb. of Stars	Names of the Constellations		Bayer's Letters	Magnitude	Right Ascension in Degrees			Annual Variation +	Right Ascension in Time			Declination			Annual Variation
					D	M	S	S	H	M	S	D	M	S	
246	Lyncis				133	35	7,6	56,24	8	54	[illegible]	34	49	[illegible]0,6N	−13,46
247	Cancri		ξ	5,6	133	56	6,0	52,[illegible]2	8	55	44	2	59	2,4N	−13,5[illegible]
248					135	7	58,1	56,33	9	0	32	35	35	15,6N	−13,68
249			π	7	135	33	22,8	50,0	9	[illegible]	13	15	54	10,8N	−13,98
250					136	42	5,4	50,86	9	6	48	18	41	22,1N	−14 7
251	Leonis		κ	4	137	43	43,5	53,13	9	10	55	27	10	51,3N	−14,[illegible]7
252			ω	5	138	58	14,0	48,54	9	15	53	10	4	6,9N	−14,84
253			ξ	4	139	48	50,2	49,03	9	19	15	12	20	52,0N	−15,03
254			h	6	139	50	7,3	48,66	9	19	20	10	44	[illegible],7N	−15,04
255				5	141	11	48,1	47,93	9	[illegible]4	4[illegible]	7	5[illegible]	[illegible]7,[illegible]N	−15,33
256				4	142	8	52,0	48,60	9	28	36	10	57	6,3N	−15,55
257			ψ	6	142	4[illegible]	35,0	49,48	9	30	54	15	5	4,2N	−15,67
258				3	143	7	0,2	51,59	9	32	28	24	50	39,2N	−15,76
259			[illegible]	4,5	146	23	22,0	48,88	9	45	33	1[illegible]	33	22,4N	−16,44
260			π	4	146	56	41,2	47,97	9	47	47	9	9	48,3N	−16,49
261	Hydræ	2	[illegible]	5	148	25	25,8	43,98	9	53	42	11	55	59,9S	+16,84
262	Leonis		η	3	148	37	15,1	49,57	9	54	29	17	54	5,4N	−16,88
263			A	5	148	51	13,8	48,23	9	55	25	11	8	28,9N	−16,93
264	Regulus		α	1	148	57	31,6	48,61	9	55	50	13	6	34,9N	−16,95
265	Sextantis				149	37	12,0	44,91	9	58	29	7	16	4,6S	+17,06
266	Sextantis			6	149	51	34,0	44,91	9	59	[illegible]6	7	16	9,1S	+17,11
267	Hydræ		λ	4	149	55	51,6	44,21	9	59	43	11	1[illegible]	7,0S	+17,12
268	Sextantis			6	151	29	31,4	45,05	10	5	56	6	54	18,3S	+17,40
269	Leonis	1	γ	6	151	43	38,3	49,78	10	6	55	20	39	19,2N	−17,49
270		2	γ	3	151	44	30,9	49,84	10	6	58	21	1	26,2N	−17,49
271	Sextantis				153	29	58,5	45,28	10	4	0	5	5[illegible]	56,2S	+17,74
272	Leonis		1	6	154	54	25,0	48,53	10	19	38	15	[illegible]0	13,4N	−17,95
273			ρ	4	155	6	13,9	47,77	10	20	[illegible]5	10	30	41,0N	−18,00
274				6	155	40	13,8	47,84	10	22	41	9	51	28,9N	−18,09
275	Hydræ		φ	5	156	47	14,3	43,97	10	27	9	15	39	53,6S	+18,26
276	Leonis		k	6	158	29	19,5	48,26	10	33	57	15	25	49,7N	−18,49
277			l	6	159	13	18,3	47,61	10	36	53	1[illegible]	47	7,8N	−18,59
278	Hydræ		[illegible]	4	159	30	43,8	44,31	10	38	3	14	58	8,5S	+18,62
279					159	37	51,5	45,26	10	38	31	7	29	35,0S	+18,64
280	Leonis			5,6	160	54	15,1	46,42	10	43	37	1	59	7,7N	−18,80

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Numb of Stars	Names of the Constellations	Bayer Lett	Magnitud	Right Ascension in Degrees			Annual Variat +	Right Ascension in Time			Declination			Annual Variation
				D	M	S	S	H	M	S	D	M	S	S
281	1	c	6,7	160	57	13,0	47 06	10	43	49	7	26	5,5N	−18,81
282		d	5,6	162	6	16,4	46,72	10	48	25	4	52	50,4N	−18,95
283	–	c	5	162	8	19,4	47,00	10	48	33	7	21	36,8N	−18 95
284				162	48	36 1	46,08	10	51	14	1	14	34 2S	+19,02
285		g	6	162	53	38 1	46 32	10	51	35	1	15	39 0N	−19 03
286		χ	4 5	163	13	16 5	47 08	10	52	5	6	36	12,6N	−19,07
287			6	16–	43	46,6	46,51	10	54	55	3	1–	30,2N	−19,14
288			5 6	165	26	–,0	46,31	11	1	44	1	12	16,9N	−19,30
289		θ	3	165	–8	15,1	47,71	11	1	53	16	42	39,9N	−19,30
290		n	6	165	53	17,0	47 46	11	3	33	14	35	7,0N	−19,34
291			6	166	17	54,9	46,47	11	5	1–	3	18	2,2N	−19,38
292	Crateris	δ	4	166	54	7,6	45 12	11	7	37	13	30	9,5S	+19,43
293	Leonis	σ	4 5	167	15	13,1	46,77	11	9	1	7	19	57,6N	−19,46
294			5,6	167	59	37,8	46,38	11	11	56	2	41	46,9N	−19,52
295	Crateris	ι	4	168	11	23,6	45,50	11	1–	4–	9	34	26,4S	+19 54
296	Leonis	τ	4	168	56	3 5	46,47	11	15	52	4	8	57 6N	−19,60
297				170	35	1,7	46,45	11	22	20	4	21	53,0N	−19,–1
298	Crateris	θ	4	171	11	42,1	45 73	11	24	4–	8	30	14,0S	+19,75
299	Virginis	ω	6	1–1	35	4,9	46 71	11	26	–0	9	25	56 7N	19,77
300	1	ε	5	173	17	30,0	46,60	11	33	10	9	34	48 3N	−19,86
301			5	173	26	51,0	46,5–	11	3–	47	7	49	46,0N	−19,87
302	2	ξ	6	173	57	33 3	46,56	11	35	50	9	3–	58,9N	−19,90
303		β	3	174	36	37,8	46,30	11	38	27	3	5	–6,1N	−19,93
304		A	6	175	44	50,0	46,48	11	4–	59	9	45	3,8N	−19,97
305		b	5,6	176	58	47,0	46,–9	11	47	55	4	57	58,6N	−20,01
306		π	5	177	12	30,8	46,34	11	49	14	7	54	35,6N	−20,01
307		o	5	178	18	35,4	46 –1	11	53	11	10	2	25,7N	−20,04
308		r	6	179	–4	50,6	46,2–	11	57	39	3	13	1–,6N	−20,05
309	Virginis	[illegible]	6	179	31	12,–	46,–4	11	58	5	7	6	56,7N	−20,05
310		n	6	181	39	33,9	46,18	12	6	38	0	31	19,8N	−20 06
311		η	3,4	181	58	– 1	46 18	12	7	53	0	38	35,3N	−20,06
312		c	3,4	169	6	20,6	46,15	12	8	25	1	37	35 0N	−20,06
313	Corvi	δ	3	184	59	5,6	46,68	12	20	0	15	12	11,4S	+20,02
314	Virginis	l	6	185	25	–,8	46,47	12	21	40	6	9	7,8S	+20 01
315		f	6	186	10	39 6	46 7	1–	24	43	4	3	1,6S	+20,0

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Numb. of Stars	Names of the Constellations	Bayer's Letters	Magnitud.	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Declination			Annual Variation
				D	M	S	S	H	M	S	D	M		S
316		χ	5	186	47	10,5	46,47	12	27	9	6	42	9,7 S	+19,98
317	Virginis	γ	3	187	26	39,6	46,[illegible]4	12	29	47	0	9	16,9 S	+19,95
318			6	190	17	39,1	46,33	12	41	11	2	16	15,6 S	+19,61
319		ψ	3	190	32	26,3	46,[illegible]2	12	42	10	8	15	33,4 S	+19,80
320		δ	3	190	56	47,7	45,89	12	43	47	4	40	5[illegible],0 N	−19,75
321	1	K	6	191	53	34,0	46,37	12	47	34	2	3[illegible]	28,2 S	+19,72
322	2	K	6	192	7	30,0	46,35	12	48	30	2	5	58,1 S	+19,71
323		g	5	193	54	20,4	46,98	12	55	37	9	29	38,3 S	+19,58
324		θ	3,4	194	27	7,3	46,58	12	57	40	4	16	37,1 S	+19,54
325				197	0	18,1	47,14	13	8	1	9	4	4,0 S	+19,3[illegible]
326		α	1,2	198	12	43,4	47,29	13	12	51	9	55	36,5 S	+19,20
327		1	4	198	35	7,6	47,49	13	14	21	11	[illegible]8	33,3 S	+19,16
328		l	6	199	56	38,6	40,80	13	19	47	5	0	39,8 S	+19,02
329			6	200	5	10,4	47,92	13	20	21	14	8	43,2 S	+19,01
330		ζ	3	200	41	5,2	46,08	13	22	44	0	36	46,5 N	−18,93
331		m	6	202	19	40,5	47,19	13	29	19	7	30	39,7 S	+18,73
332				202	57	47,5	48,31	13	31	51	14	59	8,2 S	+18,66
333			5,6	204	17	10,1	48,72	13	37	9	16	58	13,7 S	+18,48
334		p	6	205	39	51,2	46,35	13	42	39	0	20	7,6 S	+18,30
335			6	208	28	16,3	47,50	13	53	53	7	45	33,1 S	+17,85
336				209	0	47,1	47,58	13	58	3	8	10	52,8 S	+17,77
337		κ	4	210	5	56,9	47,83	14	0	24	9	10	2,1 S	+17,59
338	1	ι		210	32	12,6	47,05	14	2	9	4	50	43,2 S	+17,51
339	2	ι	4	210	55	51,3	47,06	14	3	43	4	52	9,4 S	+17,44
340		λ	4	211	36	28,0	48,48	14	6	26	12	16	40,3 S	+17,33
341		φ	4	214	1	48,3	46,43	14	16	7	1	9	38,9 S	+16,89
342				216	7	56,6	48,35	14	24	32	11	17	31,9 S	+16,48
343		μ	4	217	40	35,9	47,2[illegible]	14	30	42	4	37	2[illegible],9 S	+16,16
344				218	31	5,5	49,55	14	34	4	15	0	8,2 S	+16,00
345	Libræ	μ	5	219	7	9,0	49,15	14	36	9	13	9	29,5 S	+15,87
346	1	α	6	219	25	59,2	49,6[illegible]	14	37	44	14	59	17,7 S	+15,79
347	2	α	2,3	219	28	47,1	49,63	14	37	55	15	2	1,2 S	+15,78
348	1	ξ	6	220	24	56,7	48,73	14	41	40	10	55	37,0 S	+15,58
349	2	ξ	6	221	0	52,4	48,54	14	44	3	10	28	41,7 S	+15,45
350	3	ξ	7	221	22	53,7	48,58	14	45	32	10	11	42,0 S	+15,37

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Numb of Stars	Names of the Constellations	Bayer Letter	Magnitude	Right Ascension in Degrees D	M	S	Annual Var at + S	Right Ascension in Time H	M	S	Declination D	M	S	Annual Variation
351	4	ξ	6	221	33	15 5	48,58	14	46	13	10	11	8 S	+15 33
352		δ	4 5	222	6	46 8	47 92	14	49	27	7	34	24,4S	+15,20
353	Scorpionis	γ	3,4	222	35	35,4	52,31	14	50	22	24	20	32 6S	+15 09
354	Libræ	ι	5	223	23	27,9	49,97	14	53	34	15	19	56 4S	+14 90
355	1	ι	3,4	224	43	5,0	51,21	14	58	52	18	53	9,0S	+14 58
356	2	ι	6	224	59	41,9	51,17	14	59	59	18	44	55 4S	+14,52
357	Libræ	β	3	6	5	55,3	48,36	15	4	24	8	30	0,1S	+14,26
358			7	226	58	53 1	50 04	15	7	56	14	41	5,0S	+14 03
359			4	227	5	26 8	48 70	15	11	30	9	27	44 7S	+13 81
360	1	ζ	6	28	45	39,2	50,49	15	15	3	15	52	40 6S	+13 58
361	3	ζ	6	29	21	53 8	50,50	15	17	28	15	45	52 1S	+13,42
362	4	ζ	4	229	55	18,9	50,61	15	19	41	16	2	25,4S	+13,27
363		γ	4	230	55	29,9	49,39	15	23	42	11	26	42,8S	+13 01
364		κ	4	232	6	47,0	51,60	15	28	27	18	54	0,0S	+12,70
365		η	4	232	43	20,	50,44	15	30	53	14	54	4 5S	+12 52
366		λ	4	234	55	58,4	52,01	15	39	44	19	26	40,0S	+11 92
367		θ	4	235	7	12,1	50 90	15	40	29	16	1	7,0S	+11 87
368	Scorpionis	ρ	4	235	36	28,3	55,19	15	4	26	28	30	24,1S	+11 7,
369		π	3,4	236	10	16,6	53,74	15	44	41	25	25	2,3S	+11,56
370	Libræ	ψ	4	236	15	58,5	50,22	15	45	4	13	34	53 -S	+11 54
371	Scorpionis	δ	3	236	37	12,7	5 ,94	15	46	29	21	56	0,4S	+11 43
372		ζ	4 5	37	5	8,-	49,41	15	51	-9	10	42	2 -S	+11,10
373		β	-	237	57	11,7	52,09	15	51	49	19	8	33 4S	+11,04
374	1	ω	5	238	16	30 ,	5 ,40	15	53	6	20	0	45 5S	+10 95
375	2	ω	5	238	24	57,3	52,49	15	53	40	0	12	5 1S	+10 91
376		ν	4	239	28	4,7	5 ,59	15	57	5-	16	49	50,8S	+10,89
377		σ	3,4	241	41	15,1	54 42	16	6	57	25	0	2 7S	+ ,90
378	Serpentis	[illegible]	5	242	35	48,1	5 ,47	16	10	-4	19	27	57 -S	+ 9 66
379		ξ	5	242	52	58,5	53,74	16	11	3	22	55	10 ,S	+ 9 5,
380		χ	6	243	21	33 1	51,96	16	13	6	17	54	19 3S	+ 9 43
381	Antares	α	1	243	45	38,1	54,92	16	15	3	25	53	20,8S	+ 9,31
82	Serpentis	φ	4	244	25	45,2	51,11	16	17	43	16	4	44,0S	+ 9 10
383		ω	5	244	33	35,8	53,13	16	18	14	0	56	8 3S	+ 9 06
384	Scorpionis	τ	3,4	245	19	27,3	55,77	16	21	18	27	12	8,7S	+ 8 80
385			6	247	0	18,5	51,94	16	28	1	17	15	49,7S	+ 8 7

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Numb of Stars	Names of the Constellations		Bayer Lett	Magnitude	Right Ascension in Degrees			Annual Variat +	Right Ascension in Time			Declination			Annual Variation
									h			M			
386	Serpentis			6	52	1	59,7	52,38	16	48	6	18	31	0,3 S	+6,64
387	Scorpionis			6	25[illegible]		46,7	53,65	16	5[illegible]	11	21	12	49,3 S	+6,30
388	Serpentis		η	3	254	13	49,0	51,5[illegible]	16	56	55	15	24	49,2 S	+5,90
389			ρ	4	256	44	3,9	53,63	17	6	56	0	50	9,9 S	+5,06
390			θ	3	256	54	1,4	55,20	17	7	36	[illegible]4	44	[illegible]7,0 S	+5,06
391	Sagittarii				57	39	57,2	53,82	17	10	40	21	11	47,4 S	+4,74
39	Serpentis			6	262	9	4,5	54,1[illegible]	1[illegible]	28	39	21	44	53,0	+3,[illegible]1
393			d	6	6	0	33,5	54,04	17	29	[illegible]	[illegible]1	32	3,0 S	+3,05
394	Sagittarii			7	266	56	12,6	5[illegible],36	17	47	45	17	[illegible]	[illegible]5,3 S	+1,89
395		1	γ	4	267	30	18,6	57,57	17	50	1	9	33	56,6 S	+1,70
396		[illegible]	γ	3,4	267	10	51,4	5[illegible],97	17	[illegible]0	43	[illegible]0	[illegible]4	3,9 S	+1,64
397	Sagittarii	1	μ	4	269	55	46,4	54,0[illegible]	17	59	4[illegible]	1	5	55,4 S	+0,51
398		2	μ	6	2[illegible]0	16	5,1	53,80	16	1	12	0	46	24,5 S	+0,35
399			δ	3	271	[illegible]9	1,4	57,7[illegible]	1[illegible]	5	56	9	54	9,0 S	+0,[illegible]0
400				6	27[illegible]	50	[illegible]6,4	53,76	16	11	[illegible]2	0	36	53,5 S	−0,49
401			λ	3	273	2[illegible]	7,[illegible]	55,75	18	1[illegible]	9	5	[illegible]1	38,4 S	−0,70
402	Aquilæ		m	4	7[illegible]	36	25,[illegible]	49,15	18	[illegible]	[illegible]6	6	[illegible]3	15,6 S	−1,42
403	Sagittarii				75	57	33,[illegible]	54,08	16	[illegible]	50	21	34	8,8 S	−1,6[illegible]
404					76	13	13,6	53,95	18	4	53	[illegible]1	13	18,[illegible] S	−1,71
405				7	77	35	45,5	54,47	18	10	[illegible]3	[illegible]	[illegible]6	39,7 S	−2,19
406			φ	3,4	[illegible]77	44	39,0	56,41	18	30	50	7	1[illegible]	30,4 S	−[illegible],[illegible]4
407				6	278	55	51,6	53,63	18	35	4[illegible]	0	34	4,0 S	−[illegible],71
408		1		5	279	59	38,5	51,56	18	39	59	3	0	23,5 S	−3,0[illegible]
409			σ	[illegible]3	280	10	19,8	56,06	18	40	41	26	3[illegible]	56,5 S	−3,08
410		[illegible]		5	280	13	34,1	54,55	18	40	54	[illegible]2	56	19,6 S	−3,10
411		1	ξ	5	280	50	36,2	53,74	18	43	2[illegible]	20	56	31,5 S	−3,[illegible]1
412		2	ξ	6	280	55	37,3	53,9[illegible]	16	43	42	21	23	28,8 S	−3,34
413			ζ	3	281	54	39,5	57,63	18	47	39	30	11	31,9 S	−3,65
414				4	282	39	53,[illegible]	54,12	18	50	40	2[illegible]	[illegible]3	5,3 S	−3,82
415			τ	4	283	4	47,7	56,60	18	52	19	47	50	[illegible]4,3 S	−4,07
416	Aquilæ		λ	3	283	[illegible]6	43,7	47,97	18	53	47	5	12	59,8 S	−4,19
417	Sagittarii		π	3	283	56	43,4	53,61	18	55	47	21	2[illegible]	33,1 S	−4,38
418			d	6	285	58	14,1	52,97	19	3	53	19	20	53,6 S	−4,73
419		1	ρ	5	[illegible]87	0	3,5	5[illegible],52	19	8	2	18	16	3,0 S	−5,4[illegible]
4[illegible]0		[illegible]	ρ	6	387	1	59,3	5[illegible],79	19	8	8	9	43	19,6 S	−5,43

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					D	M	S	S	H	M	S	D	M	S	
421				6	288	4	20,5	54,00	19	12	17	2	13	1[illegible],5 S	— 5,77
422	Aquilæ		κ	3,4	291	3	44,5	48,64	19	[illegible]4	15	7	3[illegible]	1 0 S	— 6 76
423	Sagittarii	1	e	6	291	48	46 6	51 8[illegible]	19	27	15	16	48	45,4 S	— 7,01
424		2	e	6	[illegible]92	16	1,7	51,76	19	29	4	16	39	15,7 S	— 7,16
425			f	6	293	9	34,6	53,03	19	32	38	20	18	18,8 S	— 7,15
426				6	294	37	55 5	52,70	19	36	32	19	37	23 6 S	— 7,94
427			g	6	296	9	7 0	51 40	19	41	36	16	5	50,6 S	— 6,43
428	Capricorni			6	99	43	56,7	50 24	19	58	56	13	4	20 1	— 9,[illegible]4
429				6	299	49	49,7	50,32	19	59	19	13	17	26 3 S	— 9 57
430				6	300	50	[illegible]3,6	50,18	20	3	22	13	2	18,6 S	— 9,66
431			α	3	301	9	5,5	50,25	20	4	36	13	13	9,0 S	— 9,98
432			σ	6	301	27	13,6	52 37	20	5	49	19	50	[illegible],1 S	— 10 09
433		1	β	3	301	53	25,0	50 92	20	7	34	15	30	[illegible]6,6	— 10 15
434			ν	6	301	54	12,8	50 [illegible]6	20	7	37	13	[illegible]8	53 5 S	— 10,16
435		2	β		301	56	46,6	50,9[illegible]	20	7	47	15	[illegible]0	[illegible]1 7 S	— 10 19
436			π	6	303	27	42,4	51,97	20	13	51	18	57	51,6 S	— 10 67
437	Capricorni		ρ	6	303	51	31 5	51,78	20	15	26	18	34	[illegible]8,0 S	— 10 [illegible]9
438				7	304	5	58,9	5[illegible] 03	20	16	[illegible]4	19	0	31 9 S	— 10 84
439					305	33	18,[illegible]	51,[illegible]9	20	22	13	17	19	5 0	— 11 [illegible]7
440			τ	6	306	31	43,1	50,75	20	26	7	15	45	[illegible]	— 11 56
441			υ	6	306	39	44,0	51,74	20	[illegible]6	39	18	56	[illegible]6,6 S	— 11,60
442				6	308	7	40,3	52,69	20	32	31	2	0	[illegible]4 7 S	— 12 01
443	Aquarii		ι	4,5	308	44	8,6	49 05	20	34	5[illegible]	10	0	9,0 S	— 1[illegible] 19
444			μ	4 5	309	59	29,4	48 89	[illegible]0	39	[illegible]6	9	51	1 7 S	— 1[illegible] 51
445	Capricorni			6,7	310	22	26 [illegible]	51,40	[illegible]0	41	30	18	47	5[illegible] 6 S	1[illegible] 63
446			η	5	312	45	2,6	51,79	20	51	0	20	46	[illegible],7 S	— 13 6
447			θ	5	313	10	40,6	51,00	20	52	43	19	9	5 7 S	— 13 37
448			χ	6	31[illegible]	46	1,6	52 09	20	55	4	[illegible]	7	21,5 S	— 1[illegible]
449	Aquarii		ψ	5	314	11	40,4	49,34	[illegible]0	56	4[illegible]	12	16	36,4 S	— 1[illegible] 6
450	Capricorni		φ	6	31[illegible]	[illegible]3	[illegible],2	51,78	[illegible]1	2	13	1	[illegible]6	44 4 S	— 1[illegible] [illegible]6
451			ι	5	317	17	0 [illegible]	50,59	21	9	8	1[illegible]	49	1,6	— 14 [illegible]
452			b	6	318	49	20,	51 77	21	15	17		4[illegible]	16,1 S	— 1[illegible] [illegible]
453	Aquarii		β	3	319	47	45,4	47 74	21	19	11	6	[illegible]5	33 4	[illegible]
454	Capricorni			4	320	58	2,[illegible]	50 9[illegible]	[illegible]1	[illegible]3	53	0	30	[illegible]	[illegible]
455	Aquarii		ζ	6	[illegible]1	8	29,	48,17	[illegible]1	5	14	[illegible]	[illegible]4	5[illegible]	[illegible]

M DE LA CAILLE's CATALOGUE OF ZODIACAL STARS

Numb of Stars	Names of the Constellations	Bayer's Letters	Magnitud	Right Ascension in Degrees			Annual Variat +	Right Ascension in Time			Declination			Annual Variation
				D	M	S	S	H	M	S	D	M	S	S
456	Capricorni	γ	3	321	45	30 9	50,17	21	27	2	17	42	44 9S	− 15,47
457	1	d	6	322	11	15 5	49 52	21	28	45	15	5	2,5S	− 15,56
458		κ	5	322	2	36, 3	50,63	21	29	30	19	55	3[illegible],6S	− 15,60
459	2	d	6	322	33	20, 0	49,58	21	30	13	15	[illegible]7	51,7S	− 15,64
460			6	322	47	22, 8	49 64	21	31	9	15	49	1,4S	− 15,68
461		c	6	323	6	56, 0	48,38	21	32	28	10	9	5,1S	− 15,76
462		τ	5	323	27	58, 5	48,85	21	33	52	12	26	19,8S	− 15,83
463		δ	3	323	30	39, 7	49,91	21	34	3	17	10	52,6S	− 15,84
464		μ	5	325	6	57, 1	49 1	21	40	28	14	38	47,0S	− 16,18
465	Aquarii		5	327	47	23 0	46 81	21	51	10	3	16	42 S	− 16,71
466		α	3	328	25	38 6	46,47	21	53	43	1	27	6,2S	− 16,85
467		ι	4	328	25	50 1	49,00	21	53	43	15	0	0 0S	− 16,85
468		e	6	329	30	39, 4	46 19	21	58	3	12	42	4 0S	− 17,04
469		θ	4	331	6	20, 9	47,74	22	4	25	8	56	34,3S	− 17,34
470		ρ	5,6	331	57	19, 7	47,70	22	[illegible]	49	8	59	33,8S	− 17,48
471		γ	3	332	22	44, 6	46,62	22	9	31	2	33	47,5S	− 17,55
472		ζ	4	334	10	59 1	46,39	22	16	44	1	12	46,9S	− 17,84
473		σ	5	334	32	58, 6	47 01	22	18	12	11	52	24,5S	− 17,89
474		υ	5	335	27	6, 2	49,54	22	21	48	21	54	26,[illegible]S	− 18,06
475		η	4	335	49	14, 7	46,39	22	23	17	1	19	13,0S	− 18,11
476		κ	5	336	23	42, 2	46,97	22	25	35	5	[illegible]6	4,8S	− 18 20
477	Aquarii 2	τ	5	338	48	15, 5	48 20	22	35	13	15	18	9,1S	− 18 48
478		τ	5,6	339	16	58, 4	48,08	22	37	8	14	49	39,6S	− 18,59
479		λ	4	340	5	8, 2	47,27	22	40	20	8	49	23 7S	− 18,70
480		δ	3	340	32	19, 3	48,26	22	42	9	17	3	51,7S	− 18,75
481	Piſcium	β	5	342	58	53, 0	45,94	22	51	56	2	33	37 3N	+ 19,05
482	Aquarii	h	6	343	13	33, 0	47,13	22	52	54	8	57	36,0S	− 19 07
483			7	343	38	37 7	47,11	22	54	35	8	57	26,0S	− 19,12
484			6	343	44	39, 5	47,11	22	54	59	8	57	25,4S	− 19,13
485	Piſcium	A	6	344	9	45, 2	46,10	22	56	39	0	51	12,7N	+ 19 17
486	Aquarii	φ	4,5	345	32	12, 9	46,84	23	2	9	7	18	38,5S	− 19,30
487	1	ψ	5	345	53	31, 4	47,09	23	3	34	10	21	52,9S	− 19 34
488		χ	5	346	9	57, 0	46,98	23	4	40	9	0	13,3S	− 19,36
489	Piſcium	γ	4	346	14	49, 3	46,03	23	4	59	2	0	6,9N	+ 19,37
490	Aquarii 2	ψ	5	346	25	12, 1	47,05	23	5	41	10	27	41,9S	− 19,39

M DE LA CAILLE's CATALOGUE OF ZODIACAL STARS

Number of Stars	Names of the Constellations	Bayer's Letters	Magnitude	Right Ascension in Degrees			Annual Variation +	Right Ascension in Time			Declination			Annual Variation
				D	M	S	S		M		D	M	S	
491	3	ψ	5	346	40	56,0	47 02	23	6	44	10	53	29,6S	−19,41
492	Piſcium	b	5,6	347	5	45,1	45 88	23	8	23	4	6	9 2N	+19 45
493		κ	5	348	43	19,7	46,20	23	14	53	0	1	37 1S	−19 58
494		θ	5	349	0	58,0	45,87	23	16	4	5	5	31 5N	+19,60
495			6	349	21	54,7	46,34	23	17	28	2	19	34,0S	−19,6
496			6	349	58	35,8	46 33	-3	19	54	2	22	51 9S	−19,67
497			6	350	31	4,8	46 34	23	22	4	2	32	34 6S	−19 70
498			6	351	6	6,1	46 16	23	24	24	0	48	2,5N	+19 75
499		ι	6	351	58	5,9	46,00	23	-7	52	4	21	22,9N	+19,80
500		λ	5	352	30	56,1	46 18	23	30	4	0	29	16,1N	+19 82
501			5	353	35	56,5	46 11	23	34	24	2	11	6,8N	+19,83
502			5,6	353	57	58,2	46,35	23	35	52	4	3	52,5S	−19 90
503				354	12	8,7	46,20	23	36	49	0	13	21 1S	−19,91
504			6	354	-1	30,2	46 20	23	37	26	0	13	23 0S	−19 91
505			6	355	12	49,6	46,33	23	40	51	4	-7	37 4S	−19,95
506			6	355	15	48,4	46,18	23	41	3	0	47	6 9N	+19,95
507			5	356	39	40,1	46,32	23	46	39	4	51	37 6S	−20,00
508		ω	5	356	48	53,6	46,10	23	47	16	5	33	55,5N	+20 00
509			5	357	-6	46,0	46,28	23	49	47	4	20	1,4S	−20 02
510			5	357	28	36,7	46,32	-3	49	56	7	19	14,3S	−20 02
511	2	c	5,6	357	37	34,4	46,09	23	50	30	7	10	55,6N	+-0,02
512				357	40	,9	46,12	23	50	40	6	31	3,3N	+ 0 02
513				358	15	25,7	46,23	23	53	2	1	48	27,8S	−20 03
514			4	358	19	40,2	46,27	23	53	19	7	1	22,1S	−-0 04
515				359	2	47,9	46,22	23	56	11	3	45	27,0S	−20,05

A
CATALOGUE
OF THE
PRINCIPAL STARS IN THE HEAVENS,
IN
RIGHT ASCENSION, AND DECLINATION,
ADAPTED FOR THE YEAR 1750

WITH

Their MAGNITUDES, and ANNUAL VARIATIONS
in RIGHT ASCENSION and DECLINATION

BY

M. DE LA CAILLE

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STARS

Number of the Stars	Names and Places of the Stars	Magn.	Right Ascension in Degrees	Right Ascension in Time	Annual Variation +	Declination	Annual Variation
			M	H M	S	D	″
1	γ Pegasi		0 5 53,5	0 0 24	3,08	13 47 36,3 N	+ 0,04
2	β Hydri	3	3 2 56,3	0 12 1	2,7	78 39 50,5 S	− 0,01
3	α Phœnicis	2	3 28 0,9	0 13 52	3,01	43 39 43,4 S	−20,00
4	δ Andromedæ	3	6 30 15,0	0 26 1	3,16	9 9 18,3 N	+20,01
5	α Cassiopeæ	3	6 3 4,8	0 6 8	3,30	55 9 37,9 N	+20,00
6	β Ceti	2	7 15 6,6	0 31	3,01	19 1 51,1 S	−19,85
7	γ Cassiopeæ		10 7 14,	0 41 49	3,49	59 21 19, N	+19,70
	α Polaris	2	0 10 56,0	0 42 4	10,05	5, 56 2,4 N	+19,69
9	β Phœnicis	3	13 13 1,	0 54 53	73	48 4 42,1 S	19,46
10	β Andromedæ		1 57 6	0 55 48	3,29	34 17 15,7 N	+19,43
11	η Ceti	4	14 0 16,1	0 56 1	3,00	11 30 48,8 S	−19,44
1	δ Cassiopeæ	3	17 5 14,5	1 9 41	3,71	58 55 27,1 N	+19,07
1	θ Ceti	4	17 53 9,1	1 11 33	3,01	9 28 49,5 S	−19,07
14	γ Phœnicis	3	19 2 10,	1 17 29	67	44 36 18,6 S	−18,90
1	α Eridani	1	2 5 42,	1 28 23	26	58 30 52,1 S	−18,56
16	ε Cassiopeæ	3	1 10 2,9	1 36 42	4,13	6 25 20,6 N	+18,
17	α Trianguli bor.	4	4 9,0	1 38 54	3,49	8 20 59,3 N	+18,19
13	γ Arietis	4	24 57 46,3	1 39 51	3 7	18 3 34,0 N	+18,16
19	β Arietis	3	5 13 7	1 40 52	3 8	19 34 34,1 N	+18,12
0	γ Andromedæ	2	7 9 56,5	1 48 40	61	41 7 0,5 N	+1,8
1	α Piscium	3	17 7,0	1 49 9	3,09	1 32 50,2 N	+17,0
	α Hydri	3	7 13 ,5	1 50 53	1,87	47 36,3 S	−17,
3	α Arietis	3	8 17 0	1 53 6	3,34	16 9,6 N	+17,6
4	β Trianguli bor.	1	41 15,0	1 54 45	3,46	33 47 28,2 N	+17,56
	γ [illegible] aus.	1	30 38 0,7		3,5	3 40 37,7 N	+17
6	ο Ceti	4	31 40 53,8	2 6 44	3,0	4 7 28,0 S	−17,05
	δ Ceti	3	36 40 35,4	2 26 42	3,08	0 45 50,5	−16,07
	ε Ceti	3	36 5 22,9	2 27 30	90	1 56 51,9	−16,03
9	γ Ceti	3	7 35 34,0	30 2	3,12	2 10 10,5 N	+15,67
30	Lilii bor.	4	38 10 0,0	2 33 4	3,53	8 1 6,3 N	+15,73
	Lilii aus.	4	8 49 39,5	2 35 19	3,50	26 12 41,4 N	+15,61
3	γ Persei	3	41 42 55,6	2 16 5	4 4	2 30 13,8 N	+14,96
33	θ Eridani		4 11 52,	48 46	2 3	41 19 0 S	−14,85
34	α Ceti		16 3,	2 49 1	3, 3	3 5 6 N	+14,81
35	β Medusæ	2	1 0 1,6	52 1	3,84	58 1,9 N	14,64

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STARS

Numb. of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Right Ascension in Time			Annual Variat. +	Declination			Annual Variation
			D	M	S	H	M	S	S	D	M	S	
36	Eridani	4	45	21	58,8	3	1	[illegible]8	[illegible],54	29	59	13,7S	−14,07
37	ζ Eridani	3	45	55	39,1	3	3	43	2,91	9	45	51,[illegible]S	−13,93
38	α Persei	[illegible]	46	39	2[illegible],5	3	6	37	4,19	48	56	46,6N	+13,74
39	ε Eridani	3	50	17	3[illegible],0	3	[illegible]1	10	2,89	10	19	12,0S	−1[illegible]
40	δ Persei	3	51	16	33,3	3	25	14	4,17	46	57	40,0N	+1[illegible],52
41	δ Eridani	3	52	49	30,1	3	31	18	2,88	10	37	3[illegible],0S	−1[illegible],10
42	η Pleiadum	3	53	10	1,6	3	32	40	3,54	23	18	35,8N	+1[illegible],01
43	ζ Persei	3	54	37	4,6	3	38	28	3,72	31	7	2,6N	+11,60
44	ε Persei	3	55	17	24,1	3	41	10	3,94	38	15	41,6N	+11,41
45	γ Eridani	3	56	35	48,3	3	46	23	2,79	14	14	[illegible]1,6S	11,02
46	γ Hydri	4	57	51	51,2	3	51	2[illegible]	−1 1	74	59	55,9S	−10,66
47	γ Hyadum	3	61	23	56,[illegible]	4	5	36	+3,39	15	0	6,6N	+9,59
48	δ Hyadum	4	6[illegible]	8	17,3	4	8	3[illegible]	3,44	16	56	2,8N	+9,36
49	α Reticuli	3	62	49	11,6	4	11	17	0,71	63	6	16,3S	−9,15
50	Tauri	3	63	[illegible]0	42,1	4	14	3	3,48	16	36	13,7N	+8,[illegible]
51	Aldebaran	1	65	24	4,0	4	[illegible]1	36	3,43	15	59	3,2N	+8,33
52	α Doradûs	3	67	9	19,[illegible]	4	28	37	1,28	55	34	1[illegible],9S	−7,[illegible]8
53	β Eridani	3	73	53	44,0	4	55	3[illegible]	2,95	5	25	49,2S	−5,56
54	α Capræ	1	74	33	56,5	4	58	16	4,40	4[illegible]	[illegible]	36,7N	+5,33
55	Rigel	1	75	38	12,2	5	[illegible]	33	2,89	8	30	37,4S	−[illegible]97
56	β Tauri	2	77	37	27,2	5	10	30	3,78	28	22	6,2N	+4 [illegible]9
57	γ Orionis	2	77	56	2,2	5	11	44	3,[illegible]2	6	5	55,7N	+4,19
58	η Orionis	3	77	58	48,0	5	11	55	3,0[illegible]	2	38	56,9[illegible]	−4,17
59	β Leporis	4	79	23	11,8	5	17	33	2,58	20	59	43,1S	−[illegible],69
60	δ Orionis	2	79	48	54,4	5	19	16	3,07	0	30	22,8[illegible]	−[illegible]51
61	α Leporis	3	80	25	49,7	5	[illegible]1	4[illegible]	2,65	18	1	19,6S	3,[illegible]4
62	ζ Tauri	3	80	40	42,9	5	2[illegible]	43	3,58	0	57	5[illegible],7N	+3,[illegible]4
63	ε Orionis	2	80	53	11,1	5	23	33	3,05	1	23	1[illegible]2S	−3,1[illegible]
64	ζ Orionis	[illegible]	8[illegible]	2	3[illegible],5	5	[illegible]8	10	3,04	2	5[illegible]	18,9S	−2,[illegible]
65	α Columbæ	2	82	39	1[illegible],2	5	30	37	[illegible],20	34	13	22,7S	−[illegible]
	β Doradus	4	82	52	17,5	5	31	19	0,50	62	39	29,7S	−[illegible],4[illegible]
	γ Leporis	4	83	30	43,2	5	34	3	2,53	22	32	58,7[illegible]	[illegible]
66	κ Orionis	3	83	58	48,[illegible]	5	35	55	2,85	9	46	38,3	[illegible],10
69	δ Leporis	4	[illegible]5	6	46,0	5	40	35	2,57	20	55	16,1S	−1,6[illegible]
70	β Aurigæ	3	85	17	57,6	5	41	12	4,41	44	53	16,5N	+1,61

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STARS

Numb. of Stars	Names and Places of the Stars	Magn.	Right Ascension in Degrees D	′	″	Right Ascension in Time H	M	″	Annual Variat. +	Declination °	′	″	Annual Variation ″
71	α Orionis	1	85	24	46,2	5	41	39	3, 5	7	20	10,6 N	+ 1,60
72	β Columbæ	3	85	3	30,4	5	4	10	,11	35	52	40,3 S	− 1,6
73	θ Aurigæ	3	85	40	8,5	5	42	41	4,08	37	9	51,6 N	+ 1,51
74	η Castoris	4	89	56	36,3	5	59	48	3,63	[illegible]	33	15,5 N	+ 0,02
75	μ Pollucis	4	91	58	25,0	6	7	54	3,63	2	30	59, N	− 0,07
76	γ Canis maj	3	92	40	58,9	6	10	44	2,31	9	59	11,8 S	+ 0,94
77	ζ Canis maj	3	9	55	5,3	6	11	4	,63	1	51	11,4 S	+ 1,02
78	Canopus	1	94	36	7,6	6	18	5	1,34	52	34	8,9 S	+ 1,60
79	γ Pollucis	3	95	43	54,	6	23	16	3,48	16	35	16,9 N	− 2,02
80	ε Castoris	3	97	8	9,0	6	8	33	3,	5	21	9,4 N	− 2,0
81	Navis	3	97	31	44,4	6	30	7	1,84	42	59	25, S	+ ,62
82	Sirius	1	98	3	0,3	6	34	6	2,69	16	23	36,6 S	+ 2,97
83	Canis maj	3	10	1	7,	6	48	48	2,36	8	38	59,6 S	+ 4,
84	ζ Pollucis	3	102	18	49,4	6	49	1	3,58	20	54	4 1 N	− 4,27
85	δ Canis maj	2	104	33	26,8	6	58	14	2,45	26	0	56,1 S	+ 5,03
96	δ Pollucis	3	106	47	26,4	7	5	10	3,61	2	25	8,5 N	− 5,61
67	π Navis	3	107	4	44,0	7	8	19	,13	36	39	44,4 S	+ 5,88
88	β Canis min	3	108	23	46,5	7	13	35	3,27	8	46	20 N	− 6,32
89	η Canis maj	[illegible]	108	33	3,6	7	14	12	9	28	49	59,7 S	+ 6,37
90	α Castoris	[illegible]	109	39	,	7	18	36	3,87	3	4	34,5 N	− 6,3
91	σ Navis	3	110	19	40,5	7	21	19	1,94	4	48	2 ,6 S	+ 6,96
9	Procyon	1	111	3	59,4	7	6	1	3, 1	5	50	42,5 N	− ,37
93	β Pollucis	[illegible]	11	9	45,6	7	9	59	3,75	28	36	,5 N	− 7,60
94	[illegible]	[illegible]	118	4	4,5	7	5	46	,12	39	18	39,5	+ 9,6
95	[illegible] Navis	[illegible]	120	7	4 ,0	6	1	51	1,85	46	36	34,9 S	+ 0,16
96	[illegible]	4	1 0	44	4,3	8	[illegible]	56	3,15	9	56	7,0 N	− 10,
97	Navis	3	1 4	0	10,	8	17	21	1,26	58	42	5,5 S	+ 1
98	γ Aſini	4	1	11	42,	8	28	47	3,52	22	20	59,3 N	− 11
99	Aſini	4	1	16	43,	8	30	7	44	19	3	22,0 N	− 1,
100	Navis	[illegible]	1 9	[illegible]	5,0	8	37	46	1,61	3	48	,1 S	+ 1 7
101	ι Urſæ maj	3	130	29	7,9	8	41	57	4 5	48	59	58,7 N	− 13,00
10	α Cancri	3	1 0	[illegible]	1,3	8	4	9	3,31	12	48	33,5 N	− [illegible]
103	κ Urſæ maj	4	1 1	36	8,0	8	46	6	4, 0	48	7	16,6 N	− [illegible]
104	λ Navis	3	1 4	4	[illegible]	8	57	49	2,21	4	26	4,1 S	+ 14, 9
105	β Nav	1	13	[illegible]	0	9	10		0,75	68	41	26, S	+ 14,79

M. De la CAILLE's CATALOGUE OF THE PRINCIPAL STARS

Number of Stars	Names and Letters of the Stars	[illegible]	Right Ascension in Degrees			Right Ascension in Time			Annual Variation +	Declination			Annual Variation
			°	'	"					°	'	"	
106	Navis	3	137	36	3,3	9	10	[illegible]	1,65	[illegible]	14	[illegible]	+11,79
107	κ Luc Boreal	3	138	25	52,9	9	11	24	1,60		57	0,15	+15,03
108	α Hydrae	[illegible]	135	49	40,0	9	15	19	96	7	35	13,15	+15,08
109	θ Ursae maj	3	136	59	39,2	9	15	59	4	5[illegible]	47	4 [illegible]N	−15,11
110	Leonis	4	141	56	46,0	9	27	47	3,24	10	43	56,5N	−15,79
111	ε Leonis	3	14[illegible]	54	6,0	9	31	36	3,45	24	[illegible]	55,[illegible]N	−15,99
112	μ Leonis	3	144	37	21,0	9	38	29	3,47	27	10	6,5N	−16,33
113	ο Navis	3	145	12	41,2	9	40	51	1,50	[illegible]	55	6,75	+16,45
114	η Leonis	3	148	24	53,6	9	53	40	3,31	17	58	16,4N	−17,07
115	Regulus	1	148	45	26,0	9	55		4	13	10	43,7N	−17,13
116	ζ Leonis	3	150	40	55,0	10		41	3,34	4	9	4,0N	−17,47
117	γ Leonis	3	151	32	6,1	10	6	0	3,31	1	[illegible]	49,3N	−17,61
118	ρ Leonis	4	154	54	20,0	10	19	37	3,18	10	35	0,1N	−18,14
119	θ Navis	3	158	31	34,9	10	34	6	2,1	[illegible]		[illegible]	+15,64
120	η Navis	2	158	51	9,5	10	35	[illegible]	2,7	55		[illegible]	+15,68
121	μ Navis	3	159	1	11,4	10	36	5	2,55	13	[illegible]	15,05	+18,1
122	β Ursae maj	2	161	38	6,5	10	46	34	3,75	7	1	51,0N	−19,01
123	α Crateris	4	161	54	13,7	10	47	37	2,95	16	55	9,05	+19,05
124	α Ursae maj	2	162	0	45,1	10	48	3	3,85	[illegible]	5	33,9N	−19,05
125	δ Leonis	3	165	11	[illegible]7,2	11	0	46	[illegible]2	[illegible]	[illegible]	2[illegible],6N	−19,[illegible]
126	θ Leonis	3	165	16	13,1	11	1	5	3,18	1	47	3[illegible],5N	−19,30
127	β Leonis	2	174	4	17,4	11	36	17	3,12	15	55	[illegible]N	19,9[illegible]
128	β Virginis	3	174	25	5,0	11	37	0	3,0[illegible]	3	10	0[illegible]	−19,9[illegible]
129	γ Ursae maj	2	175	8	[illegible]1	11	40	3	[illegible]	55	5	0[illegible]	−19,98
130	δ Centauri	3	178	51	51,4	11	55	7	[illegible]	[illegible]	19	39[illegible]	[illegible]0,03
131	α Corvi	4	[illegible]	53	1[illegible]	11	55	1	[illegible]7	[illegible]	2	6[illegible]	[illegible]0
132	Corvi	4	179	1[illegible]	[illegible]6,1	11	57	19	3,0[illegible]	1	1	41[illegible]	+[illegible]04
133	δ Crucis	3	180	30	34,7	12			3,1	[illegible]	1	[illegible]	+20,04
134	δ Ursae maj	2	180	45	4	12	[illegible]	5	[illegible]0	[illegible]	5	[illegible]3N	[illegible]
135	γ Corvi	[illegible]	1[illegible]0	44	14,5	12		59		10	9	3,55	[illegible]0[illegible]
136	η Virginis	3	181	46	18,1	12	7	[illegible]	[illegible]	[illegible]	43	[illegible]1,1N	
137	α Crucis	1	183	13	43,7	12	1	[illegible]		61	4	17,1	[illegible]
138	δ Corvi	[illegible]	184	14	9,6	12	1[illegible]		[illegible]9	15	[illegible]	1,5	[illegible]
139	γ Crucis	[illegible]	184	29	[illegible]	12	17	[illegible]	[illegible]	[illegible]0	44	[illegible]	[illegible]9,9[illegible]
140	β Corvi	3	185	19	30,9	12	[illegible]	1[illegible]	3,11	[illegible]	0	[illegible]0	[illegible]9[illegible]

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Number of Stars	Names and Letters of the Stars	Magnitude	Right Ascension in Degrees	Right Ascension in Time	Annual Variation +	Declination	Annual Variation
			° ′ ″	h ′ ″		° ′ ″	
141	α Corvi	4	185 8 41,6	12 [illegible] 35	[illegible]	67 46 17,2 S	+19,94
14[illegible]	Centauri	[illegible]	186 57 53,6	1[illegible] 27 5	7	47 34 5[illegible] 9 S	+19,9
[illegible]	γ Virginis	3	187 1[illegible]	12 29 0	3,06	0 4 22,3 S	+19,68
144	β V[illegible]	4	187 16 53,4	[illegible] 31 16	3,5	66 44 6,1 S	+19,84
145	Crucis		188 19 40,1	1[illegible] 3 19	[illegible] 1	59 19 8,3 S	+19,8[illegible]
146	Ursae maj.	[illegible]	190 [illegible] 6	1[illegible] [illegible] 50	2,69	57 19 15,1 N	−19,69
147	Virginis	[illegible]	190 [illegible] 15,1	1[illegible] 43 1	3,06	[illegible] 45 43 N	−19,69
148	[illegible] Ursae m.	3	[illegible]	1[illegible] [illegible] 17	56	59 40 5,8 N	−19,66
149	Virginis	[illegible]	[illegible]	1[illegible] 4[illegible] 1[illegible]	3,01	1 18 34,3 N	−19,57
150	θ Virginis	4	[illegible]	1[illegible]	3,09	4 11 4[illegible] 2	+19,41
151	γ Hydrae	3	196 20 46,6	1[illegible] 5 [illegible]	3	1 50 43	+19
152	ι Centauri	3	196 39 [illegible] 6	13 6 36	[illegible] 4	35 [illegible] 6,6 S	+ 9,20
153	α Virginis	1	198 0 51	13 1 3	3,15	9 5[illegible] 51 1[illegible]	+19,00
154	ζ Ursae maj.	2	198 6 [illegible]	13 13 43	45	56 14 1 N	− 9 01
155	ζ Virginis	3	200 9 3[illegible]	13 1 58	3,06	0 41 7,5 N	−18,[illegible]7
156	Centauri	3	201 3 30,1	13 4 14	3,69	52 10 55 S	+19,70
157	η Ursae maj.	[illegible]	204 24 58,6	13 37 40	,41	50 34 5,4 N	−18,[illegible]4
158	ζ Centauri	3	20[illegible] 1 14,6	13 40 5	3,66	46 2 38,3 S	+18,15
159	η Bootis	3	205 41 34,1	13 4[illegible] 46	2,88	19 3[illegible] 45,0 N	−18,06
160	β Centauri	2	206 36 41	13 46 7	4,09	59 9 [illegible],0 S	+17,91
161	θ Centauri	3	207 59 57,5	13 52 0	3,5	35 [illegible] 34,9 S	+17,69
162	α Draconis	3	209 24 26	13 57 38	1,50	65 34 3 [illegible] N	−17,45
163	κ Virginis	4	209 53 57,6	13 59 36	[illegible] 19	9 5 13 [illegible] S	+17,[illegible]
164	Arcturus	1	211 [illegible] 1 [illegible]	14 4 16	,6	0 9 39 N	−1[illegible],16
165	ι Virginis	1	211 [illegible] 1[illegible]	14 5 37		1 1 [illegible]0	+1[illegible],0
166	η Centauri	3	21[illegible] [illegible]0 9,1	1[illegible] 19 [illegible]	3,[illegible]5	41 [illegible]0 S	+ 6,4
167	γ Trionum	3	215 29 55,5	14 2 0	2,43	[illegible]6 [illegible] 19,0 N	−16,31
168	Circini	3	215 0 45,4	14 [illegible] 39	4,68	6[illegible] 51 [illegible] 2	+16,26
169	α Centauri	1	215 [illegible] 31,1	14 [illegible] 5[illegible]	4,41	[illegible] 0,4 S	+16,6
170	Lupi		216 [illegible]1 48,[illegible]	14 25 [illegible]	3,09	46 1 40,1 S	+16,14
171	β Trionum		21[illegible] 16 1[illegible],6	14 [illegible]9 1[illegible]	[illegible]	14 46 5[illegible] N	−1[illegible],9
172	[illegible]		216 [illegible] 15	14 34 [illegible]	63	0 [illegible],0 N	−15,67
173	α Librae		219 16 0,0	14 37 5	3,[illegible]0	14 59 9,[illegible] S	+15,50
174	β Lupi		[illegible]0 [illegible]1 6,0	14 42 16	[illegible],06	42 6 1 9	+15
175	Centauri	2	[illegible] 45 1[illegible]	14 43 1	[illegible] 04	41 [illegible] [illegible],0 S	+ [illegible],1
176	γ Scorpionis	4	[illegible] 22 [illegible] 5	14 49 30	3 [illegible]	4 16 53,5 S	+14,[illegible]
177	β Ursae min.		[illegible] 51 4	14 51 4[illegible]	2	[illegible]5 1 1 N	14,6[illegible]

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Numb. of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Right Ascension in Time			Annual Variat. +	Declination			Annual Variation
			D	M	S	H	M	S		D	M	S	S
178	β Trionum	3	23	8	0,0	14	52	32	36,00	41	23	15,7	− 14,62
179	γ Trian. auſt.	3	224	0	2,8	14	56	0	5,[illegible]9	67	43	32,[illegible]	+ 14,40
180	β Libræ	2	225	53	50,2	15	3	35	3,[illegible]0	8	26	30,5 S	+ 13,94
181	δ Trionum	3	26	23	12,5	15	5	33	2,14	34	15	4,8 N	− 13,82
182	γ Lupi	3	229	38	36,2	15	16	34	3,94	40	18	4,6 S	+ 12,97
183	ι Draconis	4	229	51	2,1	15	19	24	1,31	59	50	54,3 N	− 12,91
184	γ Urſæ min.	3	230	19	58,5	15	21	20	22,00	72	43	27,9 N	− 12,79
185	γ Libræ	4	230	23	4,6	15	21	35	3,34	13	56	10,4 S	+ 12,77
186	δ Serpentis	3	230	43	9,0	15	22	53	2,87	11	23	32,0 N	− 12,66
187	α Coronæ	2	231	1	40,2	15	24	7	2,05	7	34	18,2 N	− 12,60
188	α Serpentis	3	232	59	39,8	15	31	59	3,01	7	1	50,5 N	− 12,06
189	β Trian. auſt.	3	233	20	42,1	15	33	23	5,06	62	7	34,2 S	+ 11,96
190	β Serpentis	3	233	39	52,0	15	34	39	2,76	16	3	16,0 N	− 11,67
191	ε Serpentis	4	234	35	30,2	15	38	22	2,98	5	14	56,[illegible] N	− 11,62
192	ρ Scorpionis	4	235	22	40,5	15	41	31	3,68	8	27	36,0 S	+ 11,38
193	π Scorpionis	4	235	56	38,8	15	43	47	3,60	5	22	1,7 S	+ 11,32
194	γ Serpentis	3	236	13	52,5	15	44	55	2,75	14	29	1,4 N	− 11,14
195	δ Scorpionis	3	236	23	52,2	15	45	35	3,52	1	53	7,6 S	+ 11,09
196	β Scorpionis	2	237	44	7,0	15	50	56	3,50	19	5	55,2 S	+ 10,69
197	γ Scorpionis	4	239	15	5,4	15	57	0	3,47	18	47	19,6 S	+ 10,24
198	θ Draconis	4	239	18	37,2	15	57	14	1,14	59	14	17,5 N	− 10,23
199	δ Ophiuchi	3	240	18	58,8	16	1	16	3,14	3		46,[illegible]	+ 9,92
200	ε Ophiuchi	3	241	16	51,2	16	5	7	3,16	4		30,0 S	+ 9,63
201	σ Scorpionis	4	241	30	40,7	16	6	3	3,62	4	[illegible]	2,9 S	+ 9,56
202	γ Herculis	3	242	43	30,8	16	10	51	2,66	19	5	29,6 N	− 9,18
203	Antares	1	43	31	52,8	16	14	8	3,66	5	1	6,4 S	+ 8,93
204	β Herculis	3	244	52	29,7	16	19	30	2,59	2	3	7,1 N	− 8,51
205	τ Scorpionis	4	245	5	32,3	16	20	22	3,70	7	40	14,7 S	+ 8,44
206	η Draconis	4	245	9	46,9	16	20	39	0,78	6	5	2,5 N	− 8,41
207	α Trian. auſt.	3	245	37	5,0	16	[illegible]	28	6,16	6	31	24,3 S	+ 8,26
208	ζ Ophiuchi	3	245	51	19,0	16	23	5	3,30	[illegible]	2	16,0 S	+ 8,20
209	ζ Herculis	3	47	58	6,2	16	31	52	2,30	2	4	19,4 N	− 7,54
210	ε Scorpionis	3	248	30	14,8	16	34	1	3,90	33	48	41,1 S	+ 7,34
211	η Herculis	4	248	34	59,6	16	34	20	2,07	39	24	46,9 N	− 7,31
212	μ Scorpionis	3	248	44	54,6	16	35	0	4,04	37	35	18,6 S	+ 7,26
213	ζ Scorpionis	3	249	15	51,0	16	37	3	4,20	41	54	3,6 S	+ 7,09

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Numb. of Stars	Names and Places of the Stars	Magn.	Right Ascension in Degrees			Right Ascension in Time			Annual Variat. +	Declination			Annual Variation
			D	M	S	H	M	S	S	D	M	S	
214	Herculis	3	252	40	5[illegible]	16	50	43	[illegible]29	31	18	41 6N	−5,96
[illegible]15	η Scorpionis	3	[illegible]5[illegible]	34	34, 1	16	54	18	4, 8	42	5	39,2S	+5[illegible]
216	η Ophiuchi	[illegible]	54	0	53, 9	16	56	4	3 44	15	2[illegible]	[illegible]8,4S	+5,5[illegible]
[illegible]17	α Herculis		255	48	49	17	3	15	[illegible],74	14	41	44,5N	−4,91
[illegible]18	δ Herculis	[illegible]	56	26	36, 4	17	5	46	2 47	[illegible]5	9	4,7N	−4,69
[illegible]19	[illegible] Ophiuchi	3	[illegible]56	40	11,	17	6	21	3 67	24	4[illegible]	20,6S	+4,61
[illegible]0	Aræ	3	258	8	[illegible]7, 0	17	12	34	4 61	49	38	31,8S	+4,1[illegible]
[illegible]21	Scorpionis	4	[illegible]5	[illegible]7	0 8	17	13	48	4 09	37	3	55,9S	+4,01
[illegible]	λ Scorpionis	3	259	9	59 4	17	16	40	4 08	36	53	26 6S	+3,77
[illegible]	θ Scorpionis	3	[illegible]59	50	56, 1	17	19	24	4,30	[illegible]	46	23,2S	+3,53
[illegible]24	α Ophiuchi	2	260	50	5, 3	17	23	20	[illegible],78	12	45	17,9N	−3,17
225	β Draconis	3	61	1[illegible]	6, 8	17	24	48	1,36	5[illegible]	[illegible]9	44,5N	−3,06
2[illegible]6	κ Scorpionis	[illegible]	261	18	19, 3	17	25	13	4,13	38	5[illegible]	14,6S	+3,03
2[illegible]	ι Scorpionis	3	262	31	53, 5	17	30	8	4,18	39	59	49 0S	+2,60
228	β Ophiuchi	3	[illegible]62	46	55, 0	17	31	8	[illegible],97	4	41	40,6N	−[illegible],52
229	γ Ophiuchi	3	263	50	35, 2	17	35	22	3,01	2	49	28,[illegible]N	−2,15
230	μ Herculis	4	264	10	9, 5	17	36	41	2 38	27	53	3,9N	−2,03
231	ζ Serpentis	4	266	49	26, 9	17	[illegible]7	18	3,16	3	38	56,3S	+1,11
232	θ Herculis	3	266	55	[illegible]5, 6	17	47	4[illegible]	2 06	37	17	56 4N	−1,08
233	[illegible] Sagitta	4	267	26	21, 5	17	49	45	3 87	30	53	23 7S	+0,89
234	γ Draconis	3	267	42	3 0	17	50	48	2,21	51	31	37 9N	−0,80
[illegible]	μ Sagittarii	4	269	42	12, 7	17	58	49	3,60	21	5	56,9S	−0,10
[illegible]	δ Sagittarii	3	271	14	31, 8	18	4	58	3,85	29	54	18 4S	−0,43
237	ε Sagittarii	3	[illegible]71	53	39, 7	18	7	35	4,00	34	28	17,4S	−0,67
238	[illegible] Sagittarii	3	273	8	5, 2	18	12	32	3,72	25	31	57 4S	−1,09
239	[illegible] Lyræ	1	277	7	7 0	18	[illegible]8	28	[illegible] 06	38	34	0,0N	[illegible] 48
240	φ Sagittarii		[illegible]77	30	26, 7	18	[illegible]0	2	3,77	27	13	13,0S	−2 61
41	σ Sagittarii		279	56	13, 4	18	39	45	3,74	26	31	51,0S	−3 45
[illegible]42	β Lyræ		280	12	[illegible]7, 3	18	40	51	2,22	33	5	23,4N	+3,55
43	θ Serpentis	4	280	56	51, [illegible]	18	43	47	3 00	3	54	6,0N	+3 80
44	δ Lyræ	3	[illegible]81	6	36, 3	18	45	46	[illegible],11	36	35	48,6N	+3,97
245	ζ Sagittarii	3	[illegible]1	40	16, 6	18	46	41	3,84	30	12	[illegible]6,0S	−4 06
24[illegible]	[illegible] Aquilæ	[illegible]	[illegible]2	4	17, [illegible]	18	48	17	2,73	14	44	56,1N	+[illegible]18
[illegible]	γ Lyræ	3	2[illegible]	[illegible]3	54, 3	[illegible]	[illegible]9	36	2,26	32	21	49,0N	+[illegible]9
248	ο Sagittarii	[illegible]	[illegible]	[illegible]5	19, 1	18	49	41	3 60	[illegible]2	5	0,8S	[illegible]4,31
249	τ Sagittarii		2[illegible]	49	3[illegible] 3	18	51	19	3 77	26	0	30,6S	−4,45

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Numb. of Stars	Names and Places of the Stars	Magnitud.	Right Ascension in Degrees D	M	S	Right Ascension in Time H	M	S	Annual Variat. + S	Declination D	M	S	Annual Variation S
285	β Cephei	4	321	19	34,4	21	25	18	0,0[illegible]	69	27	5[illegible],8 N	+15,64
86	γ Capricorni	3	321	27	12,5	21	25	49	3,35	17	46	4[illegible],0 S	−15,66
87	ε Pegasi	3	32[illegible]	58	21,6	21	31	53	[illegible],95	8	41	9,3 N	+16,00
88	μ Cygni	4	323	14	41,[illegible]	21	32	59	[illegible],64	27	37	[illegible]4,1 N	+16,05
89	δ Capricorni	3	3[illegible]	18	[illegible],6	21	33	13	3,3[illegible]	17	14	48,3 S	−16,06
290	γ Gruis	3	324	40	33,5	[illegible]1	38	4[illegible]	3,70	38	31	26,7 S	−16,35
291	α Gruis	2	3[illegible]8	5	7,7	[illegible]1	52	21	3,98	48	[illegible]	1,4 S	−1[illegible],00
92	α Aquarii	3	3[illegible]8	14	0,6	[illegible]1	5[illegible]	[illegible]6	[illegible],10	1	31	[illegible]5,[illegible] S	−1[illegible],03
93	α Tucanæ	3	330	16	46,0	[illegible]	[illegible]	[illegible]	[illegible],3[illegible]	[illegible]	9	40	−1[illegible],40
294	γ Aquarii	3	33[illegible]	11	5,6	[illegible]	[illegible]	44	3,[illegible]	[illegible]	[illegible]	13	[illegible]1[illegible],71
95	β Gruis	3	3[illegible]6	54	3,2	[illegible]2	27	[illegible]6	3,65	4[illegible]	10	5[illegible]	[illegible]43
296	ζ Pegasi	3	3[illegible]7	14	4[illegible],9	2[illegible]	[illegible]8	59	[illegible]	[illegible]	[illegible]	3 N	+1[illegible],40
97	η Pegasi	3	3[illegible]7	49	3[illegible]	22	31	16	2,60	[illegible]8	55	14,[illegible] N	+18,5[illegible]
298	λ Aquarii	4	3[illegible]9	5[illegible]	[illegible]7,6	[illegible]2	39	34	3,16	6	54	8,[illegible] S	−18,8[illegible]
299	δ Aquarii	3	340	20	14,6	[illegible]	41	[illegible]1	3,22	17	6	39,9 S	−18,67
300	Fomalhaut	1	3[illegible]	16	41,7	2[illegible]	43	47	3,34	30	56	24,8 S	−18,94
301	[illegible] Andromedæ	4	342	36	49,0	[illegible]	50	27	[illegible],7[illegible]	40	59	2[illegible],0 N	+19,12
302	β Pegasi	2	342	55	1[illegible],5	22	51	41	2,69	[illegible]6	43	5[illegible],7 N	+19,15
303	α Pegasi	2	3[illegible]3	4	57,3	[illegible]2	52	20	2,99	13	51	56,0 N	+19,17
304	φ Aquarii	4	[illegible]45	20	23,1	23	[illegible]1	22	3,13	7	[illegible]3	31,[illegible] S	−19,36
305	γ Cephei	4	35[illegible]	19	22,1	23	[illegible]9	1[illegible]	[illegible],31	76	14	11,0 N	+19,66
306	α Andromedæ	2	35[illegible]	52	44,1	23	55	31	3,06	27	42	3[illegible],1 N	+20,04
307	β Cassiopeæ	3	35[illegible]	59	41,4	23	55	59	3,04	56	46	7,0 N	+20,04

THESE Catalogues of M. de la CAILLE were taken from the *Ephemerides des Mouvemens Celeſtes*, from 1765 to 1775. The right aſcenſions in the ſecond Catalogue were determined by taking equal altitudes with a quadrant of three feet radius, but this method of determining the right aſcenſions is leſs exact than that by the tranſit inſtrument. The declinations in each Catalogue were deduced from the meridian zenith diſtances obſerved with a quadrant of ſix feet radius. The right aſcenſions in the firſt Catalogue were ſettled with a tranſit inſtrument, by compariſon with the ſtars in the *Fundamenta Aſtronomiæ*, but the right aſcenſions of the ſtars in the *Fundamenta Aſtronomiæ* having been ſettled by equal altitudes, the right aſcenſions of the ſtars compared with them muſt be ſubject to the ſame inaccuracy.

A

CATALOGUE

OF

381 PRINCIPAL FIXED STARS,

IN

RIGHT ASCENSION, AND DECLINATION,

WITH

Their MAGNITUDES, and ANNUAL VARIATIONS
in RIGHT ASCENSION

ADAPTED FOR THE BEGINNING OF THE YEAR 1800

BY

FRANCISCO DE ZACH

ZACH's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Time			Annual Var at +	Right Ascension in Degrees			Annual Variat +	Declination	
			H	M		s	D	M	S	s	D	M
1	88 γ *Pegaſi*	2	0	2	56,79	3,063	0	44	11 85	45,95	14	4 N
2	8 ι Ceti	3	0	9	13,51	3,059	[illegible]	18	22 66	45 89	9	57 S
3	15 κ Caſſiopeiæ	4	0	21	45,12	3,301	5	26	16 75	49,51	61	50 N
4	17 ζ Caſſiopeiæ	4	0	25	53,93	3,262	6	28	[illegible]9 01	46 93	52	49 N
5	31 δ Andromedæ	3	0	28	39,0[illegible]	3 161	7	9	45 [illegible]1	47,42	[illegible]9	45 N
6	18 α Caſſiopeiæ	3	0	29	14,40	3,311	7	18	35 95	49,66	55	26 N
7	16 β Ceti	2,3	0	33	31,83	3,001	8	2	57 40	45 01	19	5 S
8	24 η Caſſiopeiæ	4	0	37	1,44	3 389	9	15	21 64	50 83	56	46 N
9	63 δ Piſcium	4	0	38	19,08	3,093	9	34	46 15	46 [illegible]9	6	30 N
10	27 γ Caſſiopeiæ	3	0	44	44,75	3 505	11	11	11,[illegible]9	52 58	59	38 N
11	71 Piſcium	4	0	52	33,95	3,103	13	8	29,20	46,55	6	49 N
12	43 β Andromedæ	2	0	58	34,[illegible]3	3,297	14	38	33 38	49,46	34	33 N
13	33 θ Caſſiopeiæ	4	0	59	0,22	3,531	14	45	3 3[illegible]	52 96	53	3[illegible] N
14	86 ζ Piſcium	4	1	3	16,99	3,109	15	49	14 80	46,63	6	31 N
15	37 δ Caſſiopeiæ	3	1	12	50,56	3,761	16	1	38 70	56,42	[illegible]9	11 N
16	98 μ Piſcium	5	1	19	42,07	3 108	19	55	31 07	46 6[illegible]	5	[illegible] N
17	102 π Piſcium	5	1	30	30 70	3,164	2[illegible]	37	40 56	4[illegible] 46	11	7 N
18	106 ν Piſcium	4,5	1	31	1,77	3 107	[illegible]	45	26 6[illegible]	46 61	4	26 N
19	110 ο Piſcium	4,5	1	34	50,7[illegible]	3 144	23	4	40 84	47 16	6	9 N
20	45 Caſſiopeiæ	3	1	40	10 01	4,155	25	[illegible]	30 13	62 [illegible]3	6[illegible]	41 N
2[illegible]	55 ζ Ceti	3	1	41	36,69	[illegible],953	[illegible]5	24	10 33	44,[illegible]0	11	[illegible]0 S
2[illegible]	2 α Triang bor	3,4	1	41	42,74	3,379	25	25	41,15	50 68	8	36 N
23	5 γ 1 Arietis	4	1	42	34,5	3 258	[illegible]5	38	37,[illegible]3	48 87	18	19 N
[illegible]4	6 β Arietis	3	1	43	36,77	[illegible] 77	25	54	11 48	49 15	19	50 N
2[illegible]	9 λ Arietis	5	1	46	48,86	3,315	26	42	1 83	49 [illegible]		37 N
26	57 γ Andromedæ	2	1	51	41,05	3,615	27	55	15 76	54,2[illegible]	41	N
27	* præc α ♈			50	26,15		[illegible]7	36	[illegible] [illegible]5			
28	13 α *Arietis*	2	1	55	55 [illegible]7	3 335	26	58	49,05	50,02	[illegible]	31 N
29	* seq α ♈		1	59	38,13		9	54	31,95			
30	2[illegible] θ 1 Arietis	5 6		7	1,64	[illegible],[illegible]03	31	45	[illegible] 55	49 6[illegible]	18	[illegible]8 N
31	68 ο Ceti (var.)	0,	2	9	14,70	3,019	[illegible]	18	40 50	4[illegible] 9	3	54 S
32	42 π Arietis	6		38	9, 9	3,3 1	39	[illegible]	19 [illegible]	49,81	16	38 N
33	43 σ Arietis	6		40	6,09	3 295	40	7	1 [illegible]9	49 8	14	1[illegible] N
34	82 δ Ceti	3	[illegible]	9	14,17	3,060	37	16	[illegible]2 [illegible]4	4[illegible] 90	0	[illegible]3 S
35	83 ε Ceti	3	2	9	53,42	2,884	37	6	1 3[illegible]	43,[illegible]	1	44 [illegible]

ZACH's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Time			Annual Variat. +	Right Ascension in Degrees			Annual Variat. +	Declination	
			H	M	S	S	D	M	S	S		M
36	86 γ Ceti	3	2	32	57,18	3,102	38	14	17,76	46,53	2	23 N
37	89 π Ceti	3	2	34	36,02	2,849	38	39	0,29	42,74	14	43 S
38	39 ♈ Lilii bor	4	2	35	57,70	3,521	38	59	25,49	52,81	28	25 N
39	41 ♈ Lilii aust	4	2	38	14,43	3,489	39	33	36,49	52,34	26	26 N
40	40 ρ 2 Arietis	6	2	44	35,65	3,344	41	8	54,75	50,16	17	32 N
41	46 ρ 3 Arietis	5, 6	2	45	9,35	3,340	41	17	20,19	50,10	17	13 N
42	3 η Eridani	3	2	46	39,72	2,917	41	39	55,78	43,75	9	12 S
43	48 Arietis	5	2	47	48,12	3,401	41	57	1,80	51,01	20	32 N
44	23 γ Persei	3	2	50	24,42	4,250	42	36	6,25	63,75	52	43 N
45	9 α Ceti	2	2	51	50,07	3,119	42	57	31,06	46,66	3	16 N
46	* seq. α Ceti		2	51	54,61		42	58	39,15			
47	26 β Persei	2, 3	2	55	12,07	3,846	43	48	1,04	57,69	10	11 N
48	57 δ Arietis	4	3	0	12,71	3,393	45	3	10,59	50,89	18	58 N
49	58 ζ Arietis	5	3	3	25,98	3,422	45	51	29,77	51,33	20	18 N
50	13 ζ Eridani	3	3	6	7,48	2,904	46	31	52,20	43,56	9	34 S
51	61 τ 1 Arietis	7	3	9	42,36	3,433	47	25	35,39	51,49	20	25 N
52	33 α Persei	2	3	10	6,85	4,20[illegible]	47	31	42,77	63,0[illegible]	49	8 N
53	63 τ 2 Arietis	6	3	11	16,37	3,428	47	49	5,18	51,42	20	1 N
54	65 Arietis	7	3	12	55,52	3,430	48	13	52,74	51,45	20	5 N
55	5 f Tauri	5	3	19	50,65	3,289	49	57	39,78	49,33	12	15 N
56	18 Eridani	3, 4	3	23	31,52	2,883	50	52	52,84	43,24	10	9 N
57	39 δ Persei	3	3	28	44,93	4,203	52	11	13,91	63,05	47	8 N
58	25 η Luc. Pleiad.	3	3	35	37,17	3,535	53	54	17,56	53,03	23	29 N
59	44 ζ Persei	3	3	41	35,20	3,734	55	23	48,00	56,01	31	17 N
60	45 Persei	3	3	44	29,19	3,977	56	7	17,87	59,66	39	25 N
61	34 γ Eridani	2, 3	3	48	42,25	2,786	57	10	33,77	41,79	14	5 S
62	37 A Tauri	5	3	52	53,46	3,515	58	13	21,83	52,7[illegible]	21	32 N
63	54 γ Tauri	3	4	8	25,23	3,387	62	6	18,48	50,80	15	8 N
64	61 δ 1 Tauri	4	4	11	24,68	3,432	62	51	10,26	51,48	17	4 N
65	64 δ 2 Tauri	4	4	12	34,93	3,431	63	8	43,69	51,46	16	58 N
66	65 κ 1 Tauri	5	4	13	27,66	3,545	63	21	54,93	53,17	21	50 N
67	67 κ 2 Tauri	4, 5	4	13	31,13	3,543	63	22	47,02	53,14	21	44 N
68	74 Tauri	3, 4	4	16	56,98	3,475	64	14	14,76	52,12	18	14 N
69	77 θ 1 Tauri	5	4	17	9,30	3,401	64	17	19,53	51,02	15	31 N
70	78 θ 2 Tauri	5	4	17	19,53	3,399	64	19	52,96	50,99	15	25 N

ZACH's CATALOGUE OF THE PRINCIPAL STARS

Numbers of Stars	Names and Places of the Stars	Magnitud.	Right Ascension in Time			Annual Variation +	Right Ascension in Degrees			Annual Variation +	Declination	
			H		S	S	D	M	S		D	
71	* præc α ♉		4	22	12,56		65	33	8,40			
72	87 α Aldebaran	1	4	4	27,9	3,4[illegible]1	66	6	49,38	51,[illegible]1	16	6 N
73	seq α ♉		4	26	43,44		66	40	54,60			
74	91 σ 1 Tauri	6	4	27	44,78	3,406	66	56	11,77	51,09	15	24 N
75	52 2 Eridani	3 4	4	27	47,6	2,329	66	56	48,63	34,94	30	59 S
76	92 σ 2 Tauri	6	4	[illegible]	50,67	3,409	66	57	40,07	51,13	15	31 N
77	54 Eridani	3 4	4	31	43,15	[illegible],615	67	55	47,21	39,[illegible]3	20	4 S
78	102 Tauri	4	4	51	9,38	3,565	7[illegible]	47	20,75	53,47	[illegible]1	18 N
79	67 β Eridani	[illegible]	4	58	2,38	2,948	74	30	35,74	44,2[illegible]	5	21 S
80	69 λ Eridani	4	4	59	34,73	[illegible]863	74	5[illegible]	40,95	42,95	9	1 S
81	* præc α Aurig		5	1	39,44		75	24	51,60			
82	13 *Capella*	1	5	1	56,16	4,414	75	29	2,40	66,21	45	47 N
83	* seq α Aurig		5	3	14,28		75	48	34,20			
84	* præc β Orion		5	3	56,39		75	59	5,85			
85	19 *Rigel*	1	5	4	55,54	2,867	76	13	53,10	43,01	8	27 S
86	* seq β Orionis		5	8	24,57		77	6	8,55			
87	120 β *Tauri*	2	5	13	39,38	3,778	78	24	50,70	56,67	8	26 N
88	24 γ Orionis	2	5	14	24,54	3,209	78	36	8,16	48,13	6	9 N
89	9 β Leporis	3,4	5	19	41,09	[illegible],565	79	55	16,40	38,47	0	56 S
90	34 δ Orionis	2	5	21	47,58	3,057	80	[illegible]6	53,69	15,86	0	28 S
91	11 α Leporis	3	5	[illegible]3	54,94	2,639	80	58	44,13	39,59	17	59 S
92	123 ζ Tauri	3	5	25	42,41	3,575	81	[illegible]5	36,14	53,62	[illegible]1	0 N
93	46 ε Orionis	2	5	26	4,15	3,037	81	31	[illegible],19	45,55	1	20 S
94	50 ζ Orionis	2	5	30	40,57	3,020	8[illegible]	40	8,5[illegible]	45,30	[illegible]	4 S
95	α Columbæ	2	5	3[illegible]	25,03	2,167	83	6	15,45	32,50	34	[illegible]1 S
96	13 γ Leporis	3,4	5	36	9,02	2,517	84	2	15,34	37,75	22	31 S
97	53 κ Orionis	4	5	38	16,26	2,839	84	34	4,21	42,59	9	45 S
98	* præc α Orion		5	41	[illegible]7,74		85	21	56,10			
99	58 α *Orionis*	1	5	41	20,57	3,239	86	5	8,55	48,59	7	21 N
100	* seq α Orionis		5	47	53,59		86	58	23,85			
101	34 β Aurigæ	2,3	5	44	51,68	4,398	86	12	55,22	6[illegible],97	44	55 N
102	1 H Geminorum	4,5	5	51	57,72	3,642	87	59	[illegible]5,77	54,63	23	16 N
103	7 η Geminorum	3,4	6	2	48,29	3,623	90	42	4,34	54,34	22	33 N
104	13 μ Geminorum	3	6	10	51,44	3,624	92	42	51,64	54,36	2[illegible]	36 N
105	1 ζ Canis maj	3	6	12	39,08	[illegible],298	93	9	46,24	34,47	[illegible]9	59 S

ZACH's CATALOGUE OF THE PRINCIPAL STARS.

Number of Star	Names of the Stars	M.	Right Ascension in Time			Annual Variat. +	Right Ascension in Degrees			Annual Variat. +	Declination	
			h				°				D	
106	2 β Canis maj	2,3	6	13	55,76	2,638	93	28	[illegible]6,33	39,57	17	52 S
107	18 ν Geminorum	4	6	17	5,48	3,562	94	16	22,[illegible]	53,43	0	20 N
108	24 γ Geminorum	2,3	6	26	9,35	3,463	96	32	20,32	51,95	16	34 N
109	27 Geminorum	3	6	31	37,34	3,695	97	54	20,05	55,42	25	19 N
110	* prec α Can maj		6	29	41,80		97	25	27,00			
111	9 *Sirius*	1	6	36	19,91	2,647	99	4	58,65	39,71	16	26 S
112	* seq α Can maj		6	41	26,66		100	21	40,[illegible]			
113	21 ε Canis maj	2,3	6	50	46,21	2,354	102	41	33,20	35,31	28	43 S
114	43 ζ Geminorum	3,4	6	52	14,55	3,567	103	3	38,24	53,47	0	51 N
115	25 δ Canis maj	2,3	7	0	15,59	2,436	105	3	53,85	36,54	6	5 S
116	55 δ Geminorum	3	7	8	10,06	3,594	107	2	30,94	53,91	2	20 N
117	3 β Canis min	[illegible]	7	16	14,01	3,261	109	3	30,[illegible]1	48,92	8	41 N
118	* prec α Gemin		7	16	13,66		109	3	24,90			
119	66 *Castor*	1,2	7	21	40,61	3,855	110	[illegible]7	1[illegible],15	57,83	32	19 N
120	* seq α Gemin		7	27	4,84		111	46	12,60			
121	69 υ Geminorum	4,5	7	23	34,54	3,715	110	53	38,04	55,7	27	[illegible]1 N
122	* prec α Can min		7	26	40,27		111	40	4,05			
123	10 *Procyon*	1,2	7	28	49,10	3,137	112	12	16,50	47,06	5	44 N
124	* seq α Can min		7	30	27,12		112	36	46,80			
125	78 *Pollux*	2	7	33	3,18	3,687	113	15	47,70	55,31	8	30 N
126	* seq β Gemin		7	35	28,78		113	52	11,70			
127	10 μ 2 Cancri	5	7	55	57,64	3,545	118	59	24,55	53,18	22	9 N
128	14 ψ 2 Cancri	4	7	58	23,25	3,639	119	35	48,75	54,58	6	7 N
129	17 β Cancri	3,4	8	5	39,37	3,266	121	24	50,61	48,99	9	48 N
130	31 θ Cancri	5,6	8	20	10,35	3,441	125	2	35,31	51,61	18	46 N
131	33 η Cancri	6,7	8	21	7,79	3,491	125	16	56,87	52,36	21	6 N
132	4 δ Hydræ	4	8	27	3,04	3,189	126	45	45,53	47,83	6	23 N
133	43 γ Cancri	4	8	31	41,86	3,499	127	55	27,85	52,49	22	10 N
134	47 Cancri	4	8	33	18,11	3,428	128	19	31,70	51,42	18	53 N
135	11 ε Hydræ	4	8	36	10,14	3,199	129	2	32,03	47,98	7	8 N
136	16 ζ Hydræ	4,5	8	44	48,86	3,187	131	12	12,84	47,81	6	4[illegible] N
137	60 α 1 Cancri	4,5	8	44	59,39	3,290	131	14	50,81	49,35	12	24 N
138	65 α 2 Cancri	3,4	8	47	31,82	3,292	131	52	57,26	49,38	12	37 N
139	76 κ Cancri	4,5	8	56	54,33	3,263	134	13	34,92	48,95	11	28 N
140	66 ξ 1 Cancri	5,6	8	57	50,37	3,472	134	27	35,46	52,08	22	51 N

ZACH's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitud	Right Ascension in Time			Annual Var at +	Right Ascension in Degrees			Annual Var +	Declinat	
			H	M	S		D	M				
141	22 ϑ Hydræ	4	9	3	54,80	3 120	135	56	42 03	46 80	2	10 N
142	1 κ Leonis	4	9	1_	58,32	3,524	138	14	34 77	52 86	27	2 N
143	30 *Alphard*	2	9	17	44,97	2,935	139	26	14,55	44 03	7	48 S
144	* ſeq α Hydræ		9	23	9,19		140	47	17 85			
145	5 ξ Leonis	4	9	21	9 26	3,253	140	17	18 97	48 90	1_	11 N
146	14 ο Leonis	4	9	30	27 65	3, 24	142	37	4,8	48 36	10	48 N
147	17 ε Leonis	3	9	34	_8 29	3,434	143	37	4 31	51 51	4	41 N
148	24 μ Leonis	3	9	41	21,84	3,457	145	20	27 54	51 65	6	57 N
149	27 ν Leonis	4	9	47	26 92	3 _43	146	51	40,76	48 65	13	24 N
150	29 π Leonis	4	9	49	37,99	3,18_	147	24	9,89	47 75	9	0 N
151	30 η Leonis	3,4	9	56	24,60	3,289	149	6	9,04	49 33	17	44 N
152	32 *Regulus*	1	9	57	42,02	3,204	149	25	30 30	48 06	1	56 N
153	* ſeq α Leonis		10	4	28,58		151	7	8 70			
154	36 ζ Leonis	3	10	5	32,34	3,361	151	23	5,16	50,4	4	_5 N
155	41 γ 2 Leonis	2,3	10	8	55, 2	3,306	152	13	48,23	49,60	20	51 N
156	34 μ Urſæ maj	3	10	10	21,55	3,635	152	35	23,32	54 52	42	30 N
157	47 ρ Leonis	4	10	22	15,77	3,170	155	33	56 49	47,55	10	_0 N
158	48 β Urſæ maj	2	10	49	39,53	3,709	162	24	52 93	55,63	57	7 N
159	7 α Crateris	4	10	50	4,55	2,943	162	31	8, 5	44 14	17	14 S
160	50 α Urſæ maj	1,2	10	51	15,84	3 847	16_	48	57,61	57 70	62	50 N
161	11 β Crateris	3,4	11	1	50,66	2,933	165	27	30 97	44 02	31	44 S
16_	68 δ Leonis	2,3	11	3	26,39	3,199	165	51	35,91	47,98	21	37 N
163	70 ϑ Leonis	3	11	3	44,23	3 165	165	56	3,49	47 48	16	31 N
164	13 λ Crateris	5,6	11	13	28,15	2 981	168	22	2,21	44,7	17	17 S
165	78 ι Leonis	4	11	13	28,32	3,125	168	22	4 85	46 87	11	38 N
166	84 τ Leonis	4	11	17	39,49	3 085	169	24	52,34	46 28	3	57 N
167	91 υ Leonis	4	11	26	42,82	3 069	71	40	42 9	46,04	0	17 N
168	3 ν Virginis	5	11	35	34,45	3,087	173	53	51 70	46 31	7	39 N
169	* præc β Leonis		11	38	19,46		174	34	51,90			
170	94 *Denebola*	1,2	11	38	50,49	3,062	174	42	37,35	45,93	15	41 N
171	5 β *Virginis*	3	11	40	16,38	3 122	175	4	5 70	46 83		54 N
172	64 χ Urſæ maj	2	11	43	14,22	3,212	175	48	33 33	48 18	54	48 N
173	1 α Corvi	4	11	58	6,94	3 062	179	31	44,10	45 93	23	37 S
174	2 Corvi	4	11	59	51,63	3 067	179	57	54 47	46 00	21	30 N
175	69 δ Urſæ maj	3	1_	ſ	27,23	3,0_1	181	21	48 42	45 3	58	9 N

ZACH's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Time H	M	S	Annual Variat + S	Right Ascension in Degrees D	M	S	Annual Variat +	Declination D	M	
176	4 γ Corvi	3	12	5	32,31	3,077	181	23	4,58	46,16	16	26	S
177	15 η Virginis	3	12	9	40 74	3,067	182	25	11,13	46,01	0	27	N
178	9 β Corvi	3	12	23	54,39	3,124	185	58	35,92	46 86	22	17	S
179	5 κ Draconis	3	12	24	47 65	2,661	186	11	54,72	39,91	70	53	N
180	29 γ 1 Virginis	3	12	31	33 85	3,069	187	53	27,72	46,03	0	21	S
181	77 ε Ursæ maj	2,3	12	45	12,58	2,746	191	18	8,67	41,19	57	3	N
182	43 δ Virginis	3	12	45	33,66	3,047	191	23	24 93	45,71	4	29	N
183	47 ε Virginis	3	12	52	13,33	3,004	193	3	20 01	45 06	12	2	N
184	51 θ Virginis	3,4	12	59	36 46	3,095	194	54	6 97	46 42	4	26	S
185	2 γ Hydræ	3	13	8	4,31	3,225	197	1	4 64	48,38	2	7	S
186	* præc α Virg		13	9	13,00		197	18	15,00				
187	67 *Spica*	1	13	14	40 11	3,137	198	40	1,66	47,06	10	7	S
188	79 ζ Ursæ maj	3	13	15	49 62	2,425	198	57	24,26	36,37	55	59	N
189	99 ι Virginis	4	13	16	10 79	3,129	199	2	41 83	46,93	11	40	S
190	79 α Virginis	3	13	24	30,65	3,064	201	7	39,68	45,96	0	26	N
191	4 τ Bootis	4	13	37	46,36	2,884	204	26	35,35	43 26	18	27	N
19	85 η Ursæ maj	2,3	13	39	38,85	2,355	204	54	42,80	35,88	50	19	N
193	8 η Bootis	3	13	45	9,21	2,860	206	17	18,12	42 90	19	25	N
194	11 α Draconis	2,3	13	58	58,88	1,628	209	44	43,24	24,42	65	20	N
195	98 κ Virginis	4	14	2	14,87	3,179	210	33	43,04	47,68	9	20	S
196	16 *Arcturus*	1	14	6	32,21	2,722	211	38	3,16	40,83	20	15	N
197	* seq α Bootis		14	6	36,46		211	39	6,90				
198	100 λ Virginis	4	14	8	19,14	3,223	212	4	47 16	48,35	12	27	S
199	24 γ Bootis	3	14	24	1,50	2,428	216	0	22,54	36,42	39	11	N
200	30 ζ Bootis	3	14	31	35,56	2,854	217	53	53,42	42,81	14	36	N
201	36 Bootis	3	14	36	14,99	2,622	219	3	44,80	39,33	27	56	N
202	7 μ Libræ	5	14	38	22,95	3,268	219	35	44,22	49 02	13	18	S
203	8 α 1 *Libræ*	6	14	39	38,74	3,299	219	54	41,10	49,40	15	9	S
204	* præc α 2 ♎		14	39	38,77		219	54	41,55				
205	9 α 2 *Libræ*	2,3	14	39	49 97	3 289	19	57	29,55	49,34	15	12	S
206	7 β Ursæ min	3	14	51	27,55	—0,329	222	51	53 19	—4,94	74	59	N
207	20 γ Scorpii	3	14	52	24,35	3,482	23	6	5,22	52,23	24	29	S
208	42 β Bootis	3	14	54	24 99	2,262	223	36	14,85	33,93	41	11	N
209	43 ψ Bootis	5	14	55	52,50	2,580	223	58	7,57	38,79	27	44	N
210	27 β Libræ	,3	15	6	15,61	3,215	226	33	54,21	48,22	8	38	S

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Numb of Stars	Names [illegible] of the Stars	Magnitude	Right Ascension in Time			Annual Variat +	Right Ascension in Degrees			Annual Variat +	Declination	
			H	M		s		M	S		°	M
211	49 δ Bootis	3	15	7	26 62	2,409	226	51	39,28	36,13	34	4 N
212	1 ο Coron Bor	6	15	11	51 97	2,487	227	57	59 55	37,30	30	21 N
213	2 η Coron Bor	5	15	14	56 05	2,465	228	44	0 78	36,97	31	1 N
214	3 β Coron Bor	4	15	19	34 88	2,483	229	53	43 13	37,24	29	48 N
215	13 γ 2 Ursæ min	2 3	15	21	11 76	–0 209	230	17	56,39	–3,14	7[illegible]	33 N
216	35 ζ 4 Libræ	4	15	21	38 42	3,365	230	24	36 26	50 48	16	10 S
217	38 γ Libræ	3 4	15	24	21 22	3 328	231	5	18,34	49 92	14	7 S
218	13 δ Serpentis	3	15	25	15 84	2 861	231	18	57,61	42 91	11	13 N
219	5 *Comma*	[illegible]	15	26	13,29	2,543	231	33	19,35	38,15	27	24 N
220	43 κ Libræ	4	15	30	27,12	3,433	232	36	46,77	51,49	19	1 S
221	4 α *Serpentis*	2	15	34	25,21	2,936	233	36	18,00	44,04	7	4 N
222	* seq α Serpent		15	36	23,53		234	5	53,05			
223	28 β Serpentis	3	15	36	57,70	2,756	234	14	25,47	41,34	16	4 N
224	32 μ Serpentis	4	15	39	10,30	3,023	34	47	34,55	45,35	2	48 S
225	37 ε Serpentis	3,4	15	40	50,97	2,969	235	12	44,51	44,54	5	6 N
226	10 δ Coron Bor	4	15	41	12,45	2 515	235	18	6,71	37,73	26	42 N
227	45 λ Libræ	4	15	41	44,86	3,457	235	26	12,93	51,86	19	33 S
228	5 ρ Scorpii	3,4	15	44	33,34	3 671	236	8	20,06	55,06	28	37 S
229	6 π Scorpii	3	15	46	46,43	3,600	236	41	36,48	54,00	25	31 S
230	48 ψ Libræ	4	15	47	0,91	3,339	236	45	13,63	50,09	13	41 S
231	41 γ Serpentis	3	15	47	12,93	2,740	236	48	13,98	41,10	16	21 N
232	7 δ Scorpii	3	15	48	31,89	3,521	237	7	58,38	52,82	22	2 S
233	13 ε Coron Bor	4,5	15	49	18,54	2,483	237	19	38,04	37,24	27	8 N
234	44 π Serpentis	4	15	53	41,18	2,576	238	25	17,72	38,64	23	21 N
235	8 β Scorpii	2	15	53	49,71	3,465	238	27	25,65	51,97	19	15 S
236	13 θ Draconis	3 4	15	[illegible]	8,28	1,142	239	32	4,27	17,13	59	6 N
237	14 Scorpii	4	16	0	23 31	3,465	240	5	49,60	51,96	18	56 S
238	1 δ Ophiuchi	3	16	3	52,80	3,132	240	58	11,95	46,98	3	10 S
239	2 ε Ophiuchi	3 4	16	7	45,09	3,154	241	56	16,31	47,30	4	12 S
240	20 γ Herculis	3	16	13	5,83	2,642	243	16	27,41	39,63	19	38 N
241	21 *Antares*	1	16	17	9,69	3,645	244	17	25,35	54,68	25	58 S
242	* α Scorpii		16	19	6,66		244	46	39,90			
243	8 φ Ophiuchi	4 5	16	19	43,03	3,418	244	55	45,42	51,27	16	10 S
244	14 η Draconis	3,4	16	21	18,33	0,785	245	19	34,92	11 78	61	58 N
245	27 β Herculis	3	16	21	37,87	2,579	245	24	28,08	38,68	[illegible]1	56 N

ZACH's CATALOGUE OF THE PRINCIPAL STARS

Number of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Time			Annual Variat. +	Right Ascension in Degree			Annual Variat. +	Declination	
			H	M	S	S	D	M	S	S	D	M
246	23 τ Scorpii	4	16	23	26,96	3,709	245	51	44,36	55,64	27	47 S
247	13 ζ Ophiuchi	2,3	16	26	9,55	3,287	246	32	23,24	49,30	10	9 S
248	40 ζ Herculis	3,4	16	33	45,64	2,292	248	26	24,67	34,38	32	1 N
249	44 η Herculis	3,4	16	36	3,14	2,046	249	0	47,15	30,69	39	19 N
250	58 Herculis	3	16	5	38,68	2,29	253	9	40,16	34,38	31	16 N
251	35 η Ophiuchi	2,3	16	58	55,15	3,424	254	43	47,26	51,36	15	28 S
252	* præc. α Herc.		17	5	12,70		256	18	10,50			
253	64 α *Herculis*	2,3	17	5	31,76	2,726	256	22	56,40	40,89	14	4 N
254	65 δ Herculis	3,4	17	6	49,41	2,459	256	42	21,17	36,86	25	5 N
255	42 θ Ophiuchi	3	17	9	44,25	3,669	257	26	3,68	55,04	24	47 S
256	35 λ Scorpii	2	17	20	2,64	4,057	260	0	39,58	60,65	36	57 S
257	* præc. α Oph.		17	24	45,90		261	11	28,50			
258	55 α *Ophiuchi*	2	17	25	38,97	2,768	261	24	44,55	41,52	12	42 N
259	* seq. α Ophiuc.		17	29	11,02		262	17	45,32			
260	23 β Draconis	3	17	25	55,99	1,348	261	28	59,62	20,[illegible]	52	7 N
261	60 β Ophiuchi	3	17	33	35,77	2,959	263	23	56,54	44,39	4	40 N
262	62 γ Ophiuchi	3	17	37	52,04	3,003	264	28	0,56	45,05	2	48 N
263	57 ζ Serpentis	3,4	17	49	54,59	3,153	267	28	38,67	47,30	3	40 S
264	67 Oph. Tau. Pon.	4	17	50	37,47	2,999	267	39	21,99	44,99		7 N
265	33 γ Draconis	2,3	17	51	57,79	1,389	267	59	26,85	20,93	51	31 N
266	10 γ Sagittarii	3,4	17	52	58,05	3,851	268	14	30,76	57,77	30	25 S
267	b Taur. Poniat.		18	0	41,10	2,993	270	10	16,50	44,90	3	19 N
268	30 μ 1 Sagittarii	4	18	1	48,37	3,584	270	27	5,53	57,76	21	6 S
269	15 μ 2 Sagittarii	4,6	18	3	12,90	3,575	270	49	13,57	53,62	20	46 S
270	20 Sagittarii	2,3	18	10	53,67	3,984	272	43	25,11	59,76	34	28 S
271	22 λ Sagittarii	4	18	15	37,66	3,705	273	54	24,9	55,57	25	31 S
272	* præc. α Lyræ		18	28	40,12		277	10	1,68			
273	3 *Wega*	1	18	30	9,89	2,028	277	32	28,35	30,43	38	36 N
274	* seq. α Lyræ		18	31	40,00		277	54	50,00			
275	27 φ Sagittarii	3,4	18	33	9,39	3,747	278	17	20,84	56,[illegible]	27	11 S
276	4 Lyræ	5	18	37	42,87	1,983	279	25	43,05	29,74	39	28 N
277	32 ι Sagittarii	4,5	18	4	5,11	3,625	260	31	21,22	54,38	22	59 S
278	10 β Lyræ	3	18	42	41,66	2,211	280	40	27,89	[illegible]	33	9 N
279	34 σ Sagittarii	3	18	42	51,10	3,7[illegible]4	280	42	50,99	55,36	26	3[illegible] S
280	35 2 Sagittarii	4,5	18	43	1,13	3,6[illegible]3	280	45	16,99	54,5	2[illegible]	54 S

ZACH's CATALOGUE OF THE PRINCIPAL STARS.

Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Time			Annual Variat +	Right Ascension in Degrees			Annual Var +	Declination	
			H	M		S		M			°	
281	63 θ Serpent dup	3	18	46	[illegible]	2,977	281	34	[illegible]	44,66	3	57 N
282	12 δ Lyræ	3,4	18	47	31,16	2,095	281	52	47,43	31,42	36	39 N
283	47 ο Draconis	4	18	48	14,15	0,800	262	3	3,20	13,21	59	9 N
284	14 γ Lyræ	3	18	51	28,04	2,241	282	52	0,60	33,61	32	26 N
285	39 Sagittarii	4	18	52	41,18	3,595	283	10	17,72	53,92	2	1 S
286	40 τ Sagittarii	4	18	54	26,45	3,756	283	36	36,62	56,37	27	57 S
287	16 λ Antinoi	3,4	18	55	38,10	3,186	283	54	31,55	47,79	5	10 S
288	17 ζ Aquilæ	3	18	56	12,69	2,755	284	3	10,37	41,33	13	35 N
289	41 π Sagittarii	3,4	18	57	51,37	3,574	284	27	50,57	53,61	1	20 S
290	42 ψ Sagittarii	4,5	19	3	15,27	3,605	285	48	49,04	55,27	5	35 S
291	43 d Sagittarii	4,6	19	5	55,86	3,517	286	28	57,90	52,76	19	18 S
292	57 δ Draconis	3	19	12	27,95	0,033	288	6	59,21	0,49	67	19 N
293	1 κ Cygni	4	19	12	28,28	1,383	288	7	4,19	20,73	52	58 N
294	30 δ Aquilæ	3	19	15	24,12	3,006	288	51	1,79	45,12	2	44 N
295	6 β Cygni	3	19	2	38,53	2,415	290	39	37,97	36,29	27	35 N
296	10 Cygni	4,6	19	24	39,61	1,511	291	9	54,19	22,67	51	19 N
297	41 ι Antinoi	3,4	19	6	2,16	3,106	291	35	32,37	46,59	1	43 S
298	13 θ Cygni	4	19	31	5,16	1,645	292	46	17,40	24,68	49	46 N
299	5 α Sagittæ	4	19	31	9,25	2,678	292	47	18,74	40,17	17	34 N
300	56 f Sagittarii	6	19	34	41,43	3,520	293	40	21,39	52,80	0	14 S
301	* præc γ Aquilæ		19	35	1,50		293	48	7,50			
302	50 γ *Aquilæ*	3	19	36	41,50	2,837	294	11	7,50	42,59	10	8 N
303	* seq γ Aquilæ		19	39	0,81		294	46	12,16			
304	18 δ Cygni	3	19	38	43,09	1,869	294	40	46,34	28,02	44	39 N
305	* præc α Aquilæ		19	36	35,67		294	38	58,05			
306	53 α *Atair*	1,2	19	41	1,02	2,918	295	15	15,30	43,78	8	21 N
307	* seq μ Aquilæ		19	42	52,37		295	43	5,55			
308	55 η Antinoi	3,4	19	42	17,14	3,058	295	34	17,08	45,87	0	30 N
309	59 h Sagittarii	4,5	19	44	39,56	3,699	296	9	58,46	55,48	27	41 S
310	60 β *Aquilæ*	3,4	19	45	28,97	2,939	296	12	14,55	44,08	5	55 N
311	65 θ Aquilæ	3	20	0	58,52	3,097	300	14	37,79	46,45	1	24 S
312	5 α 1 *Capricorni*	3,4	20	6	32,79	3,330	301	38	11,88	49,95	13	7 S
313	* præc α 2 Capr		20	5	17,48		301	19	22,20			
314	6 α 2 *Capricorni*	3	20	6	56,48	3,331	301	44	7,20	49,96	13	9 S
315	* seq α 2 Capr		20	8	33,3[illegible]		302	21	19,80			

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Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Time			Annual Variat +	Right Ascension in Degree			Annual Variat +	Declination	
			H	M	S	S	D	M	S	S	D	M
316	1 r β ♑ 827 Mayeri		20	9	31,35	3,380	302	22	50,19	50,70	15	24 S
317	8 ν Capricorni	6	20	9	33 69	3 337	302	23	25,39	50,06	13	23 S
318	9 β Capricorni	3	20	9	45,50	3,380	302	26	22,54	50,70	15	24 S
319	37 γ Cygni	3	20	15	2,63	2 148	303	45	39,39	32,22	39	38 N
320	11 ρ Capricorni	6	20	17	26,45	3,438	304	21	36,72	51,57	18	28 S
321	4 ζ Delphini	4,5	20	25	57,50	2,801	306	29	22 44	42,01	14	0 N
322	6 β Delphini	3	20	28	10,32	2,804	307	2	34,74	42,06	13	55 N
323	9 α Delphini	3	20	30	20 76	2,780	307	35	11 37	41,70	15	13 N
324	50 *Deneb*	1,2	20	34	36,68	2 034	308	39	10,20	30 51	44	34 N
325	* ſeq α Cygni		20	40	28,55		310	7	8,25			
326	2 ε Aquarii	4,5	20	36	50,38	3 255	309	12	35 72	48,83	10	13 S
327	* præc γ Delph		20	37	21,94		309	20	29,08			
328	12 γ Delphini	3	20	37	22,96	2,783	309	20	44,34	41 75	15	25 N
329	53 ε Cygni	3	20	38	6,70	2,393	309	31	40,56	35,89	33	13 N
330	6 μ Aquarii	4,5	20	41	51,17	3,243	310	27	47,61	48,65	9	43 S
331	7 Aquarii	6	20	46	4,59	3,255	311	31	8,80	48,82	10	28 S
332	23 θ Capricorni	5,4	20	54	39,75	3,384	313	39	56,25	50,76	18	1 S
333	13 ν Aquarii	5	20	58	41,24	3,274	314	40	18,64	49,11	12	10 S
334	8 α Equulei	4	21	5	48 78	2,997	316	27	11,73	44,96	4	26 N
335	32 ι Capricorni	5	21	11	5,67	3,355	317	46	25,00	50,33	17	41 S
336	4 β Equulei	6	21	12	57,71	2,981	318	14	25,62	44,72	5	58 N
337	18 Aquarii	6	21	13	14,77	3,286	318	18	41,53	49,29	13	44 S
338	5 α Cephei	3	21	23	46,85	1,427	318	26	42,69	21,40	61	45 N
339	22 β Aquarii	3	21	21	1,12	3,165	320	15	16,74	47,48	6	27 S
340	39 ε Capricorni	4	21	25	52,29	3,379	321	28	4,34	50,68	20	21 S
341	8 β Cephei	3	21	26	1,18	0,821	321	38	17,76	12 32	69	41 N
342	40 γ Capricorni	3,4	21	28	59,14	3,329	322	14	47,15	49,93	17	33 S
343	43 κ Capricorni	5	21	31	27,98	3,360	322	5	59,74	50,40	19	46 S
344	8 ε Pegaſi	3	21	34	21,53	2,943	323	35	22,95	44 15	8	58 N
345	80 π 1 Cygni	4	21	34	59,63	2 116	323	44	54,38	31,74	50	17 N
346	49 δ Capricorni	3	21	35	58,68	3,310	323	59	40,25	49,65	17	1 S
347	* præc α Aquar		1	55	8,21		328	47	3 15			
348	34 α *Aquarii*	3	21	55	29,75	3,067	328	52	26,25	46,00	1	17 S
349	48 γ Aquarii	3	22	11	18,89	3,094	332	49	43,39	46,41	2	23 S
350	52 π Aquarii	4,5	22	15	4,00	3,065	333	45	59,98	45,97	0	22 N

ZACH's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitud	Right Ascension in Time			Annual Variat +	Right Ascension in Degrees			Annual Variat +	Declination	
			H	M	S		D	M	S	S	D	′
351	55 ζ Aquarii	4	22	18	31,68	3,079	334	37	55,26	46,18	1	2 S
352	57 σ Aquarii	5	22	20	2,99	3,186	335	0	44,84	47,79	11	42 S
353	7 Lacertæ	4	22	23	7,61	–,431	335	46	54,14	36,46	49	16 N
354	62 η Aquarii	4	22	25	4,59	3,079	336	16	8,78	46,19	1	9 S
355	63 κ Aquarii	5	22	27	23,38	3,117	336	50	50,67	46,76	5	14 S
356	42 ζ Pegaſi	3	22	31	29,06	2,981	337	52	15,89	44,72	9	48 N
357	44 η Pegaſi	3	22	33	37,87	2,792	338	24	27,91	41,88	29	11 N
358	69 τ 1 Aquarii	5	22	37	4,70	3,197	339	16	10,57	47,96	15	7 S
359	71 τ 2 Aquarii	5,6	22	38	59,29	3,190	339	44	49,41	47,85	14	39 S
360	73 λ Aquarii	4	22	42	10,52	3,137	340	32	37,87	47,05	8	38 S
361	32 ι Cephei	4	22	42	35,33	2,109	340	38	49,95	31,63	65	9 N
362	76 δ Aquarii	3	22	44	1,80	3,201	341	0	27,06	48,02	16	53 S
363	* præc α Piſc auſ		22	40	17,35		340	4	26,25			
364	24 *Fomalhaut*	1	22	46	33,60	3,330	341	38	24,00	49,95	30	41 S
365	* ſeq α Piſc auſ			48	38,94		342	9	30,60			
366	53 β Pegaſi	2	22	54	5,50	2,874	343	31	22,47	43,11	27	0 N
367	54 *Markab*	2	22	54	47,99	2,964	343	41	59,85	44,46	14	8 N
368	* ſeq α Pegaſi		22	55	35,64		343	53	54,60			
369	90 φ Aquarii	4,5	23	3	57,39	3,109	345	59	20,79	46,64	7	7 S
370	91 ψ 1 Aquarii	5	23	5	23,05	3,125	346	20	45,68	46,88	10	10 S
371	6 γ Piſcium	5	23	6	46,42	3,057	346	41	36,37	45,85	2	12 N
372	95 ψ 3 Aquarii	3	23	8	32,63	3,125	347	8	9,48	46,88	10	42 S
373	16 Piſcium	6	2	26	11,10	3,065	351	32	50,96	45,97	1	0 N
374	18 λ Piſcium	5	23	31	51,01	3,066	352	57	45,08	45,99	0	40 N
375	19 Piſcium	5,6	23	36	10,88	3,062	354	2	43,18	45,93	2	23 N
376	8 ω Piſcium	5	23	49	2,88	3,061	357	15	43,16	45,92	5	46 N
377	* præc α Androm		23	55	45,47		358	56	22,05			
378	* præc α Androm		23	56	15,28	3,060	359	3	49,19	45,90		
379	21 α *Andromedæ*		23	58	4,32	3,065	359	31	4,95	45,97	27	59 N
380	* ſeq α Androm		0	1	32,98		0	23	14,70			
381	11 β Caſſiopeiæ	2,3	23	58	34,32	3,051	359	38	34,75	45,76	58	3 N

A
CATALOGUE
OF
THE DECLINATIONS
OF
162 PRINCIPAL FIXED STARS,
WITH THEIR
ANNUAL VARIATIONS
ADAPTED TO THE BEGINNING OF THE YEAR 1800

BY

FRANCISCO DE ZACH

M [illegible] ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS

Numb. of Stars	Names and Places of the Stars	Declination for the Year 1800			Annual Variation
		D	M	S	S
1	α Polaris	88	14	25 N	+19,57
2	*Polaris*	88	14	[illegible]6 N	
3	α Ursæ majoris	50	19	4 N	−18, 0
4	[illegible]	49	8	10 N	+13,59
5	ι Ursæ majoris	48	49	9 N	+13,21
6	δ Persei	47	8	14 N	+12,35
7	*Capella*	45	46	50 N	+ 5,09
8	α Cygni	44	34	[illegible]0 N	+1[illegible],5[illegible]
9	α *Cygni*	44	34	19 N	
10	[illegible] Bootis	41	11	11 N	+14,54
11	α Lyræ	38	36	15 N	+ 2,59
12	α *Lyræ*	38	36	10 N	
13	ζ Herculis	32	58	1[illegible] N	− 7,40
14	Castor	3[illegible]	18	[illegible]4 N	− 6,95
15	*Castor*	32	18	[illegible] N	
16	*Pollux*	28	[illegible]9	47 N	− 7,46
17	β Tauri	28	[illegible]	25 N	+ 4,08
18	β *Tauri*	8	25	30 N	
19	ι Bootis	27	5[illegible]	32 N	−15,59
20	α Andromedæ	7	59	15 N	+ 0, 5
21	α *Andromedæ*	2[illegible]	59	11 N	
22	β Cygni	27	32	51 N	+ [illegible],04
23	Gemma	27	2[illegible]	[illegible]8 N	−1 50
24	*Gemma*	27	23	49 N	
25	μ Leonis	26	56	37 N	−16 16
26	β Pegasi	26	59	58 N	+ 19, 1
27	ι Geminorum	25	18	55 N	− 2,[illegible]2
28	*Geminorum*	25	18	56 N	
29	δ Herculis	25	5	4 N	− 4,56
30	ε Leonis	[illegible]	41	1[illegible] N	−16 10
31	ζ Leonis	24	4	[illegible] N	[illegible] 17,56
32	Alcione	23	26	34 N	+11,88
33	Electra	23	26	[illegible] N	+1[illegible],04
34	Atlas	23	25	54 N	+ [illegible],74
35	*Pleiones*	23	15	40 N	+ 0,75

ZACH's CATALOGUE OF THE DECLINATIONS OF TH
PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Declinations for the Year [illegible] D	M		A
36	τ Pegasi	[illegible]	38	51 N	+17 55
37	μ *Geminorum*	[illegible]	56	1	−5,83
38	η Geminorum	[illegible]	[illegible]	[illegible]	
39	η *Geminorum*	[illegible]	33	[illegible]	
40	α Arietis	[illegible]	30	[illegible]	
41	α *Arietis*	[illegible]	30	[illegible]	12,28
42	δ Geminorum	[illegible]	[illegible]	[illegible]	− 9 67
43	γ Cancri	[illegible]	10	[illegible]	− 8 38
44	μ Cancri	[illegible]	[illegible]	[illegible]	
45	β Herculis	[illegible]	[illegible]	[illegible]	
46	δ Leonis	[illegible]	37	[illegible] N	−19 4[illegible]
47	ζ Tauri	[illegible]	[illegible]	54 N	+ 3 05
48	γ Leonis	[illegible]	[illegible]	0 N	−1,12
49	ζ Geminorum	[illegible]	51	5 N	− 4 48
50	ζ *Geminorum*	[illegible]	[illegible]	[illegible]	
51	ν *Geminorum*	20	19	34 N	− 1 44
52	Arcturus	20	13	45 N	−19 1[illegible]
53	*Arcturus*	20	1[illegible]	45 N	−19,10
54	γ Herculis	19	3[illegible]	52 N	− 9,05
55	η Bootis	19	21	19 N	−16,00
56	δ Cancri	16	52	5[illegible] N	− 2,40
57	e Pegasi	18	57	14 N	+ 4,91
58	β Arietis	16	49	30 N	+16,05
59	γ Arietis	18	16	[illegible]9 N	[illegible]
60	δ Sagittæ	[illegible]	3	15 N	+ 1,73
		[illegible]	[illegible]	57 N	−17,16
61	η Leonis	17	43	[illegible] N	[illegible]
62	α Sagittæ	17	33	[illegible]	+ 9 19
63	1 δ Tauri	17	3	38 N	[illegible]
64	θ Leonis	16	31	13 N	[illegible]
65	γ Geminorum	16	33	27 N	− [illegible]
66	γ *Geminorum*	16	3[illegible]	28 N	
67	γ Serpentis	16	19	40 N	−[illegible]1,01
68	β Serpentis	16	3	21 N	−11,75
69	Aldebaran	16	5	43 N	+ [illegible]6
70	*Aldebaran*	16	5	45 N	

PIAZZI's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS

Number of Stars	Names and Places of the Stars	Declination for the Year 1800	Annual Variation
		D M	s
[illegible]	α *Hyd*[illegible]		
[illegible]	*Regulus*	15 41 30 N	−19,96
143	α Libræ	15 41 7 N	
[illegible]	λ Aquilæ	15 24 40 N	+12,68
[illegible]	α Sp[illegible]ini	15 12 48 N	+12 21
146	[illegible]i	15 8 1 N	+ 9,42
76	ζ Bootis	14 55 34 N	−15,85
77	α Herculis	14 37 38 N	− 4,75
78	α Pegasi	14 7 19 N	+19 22
79	*Pegasi*	14 7 57 N	+19 22
80	η Pegasi	14 4 16 N	+20,04
81	γ [illegible]	14 4 15 N	
82	β Delphini	13 54 31 N	+12,05
83	ζ Aquilæ	13 34 32 N	+ 4,83
84	Regulus	[illegible] 56 23 N	−17,24
85	*Regulus*	12 [illegible] [illegible] N	
86	α Cancri	[illegible] 27 30 N	−13,18
87	α Ophiuchi	1 41 55 N	− 3 05
88	α *Ophiuchi*	12 43 7 N	
89	ι Virginis	1 [illegible] [illegible] N	−19,54
90	δ Serpentis	11 1 56 N	−12,57
91	ο Leonis	10 47 40 N	−15,94
92	ι Delphini	10 37 50 N	+11 73
93	ρ Leonis	1[illegible] 19 5[illegible] N	−18 4
94	α Aquilæ	[illegible] 8 6 N	+ 8 17
95	γ *[illegible]*	10 8 10 N	
[illegible]	[illegible]	8 57 43 N	+16 10
[illegible]	[illegible] Can[illegible]	8 40 51 N	− 6 51
[illegible]	[illegible] Aqu[illegible]	8 20 58 N	+ 8,51
[illegible]	[illegible] *Aquilæ*	8 20 48 N	
[illegible]	α Orionis	7 21 27 N	+ 1,42
101	α *Orionis*	7 21 27 N	
102	ι Hydræ	7 8 37 N	+12 60
103	α Serpentis	7 3 55 N	−11,94
104	α *Serpentis*	7 [illegible] 50 N	
105	ο Hydræ	6 23 22 N	−11,97

ZACH's CATALOGUE OF THE DECLINATIONS OF TH[illegible] PRINCIPAL STARS

Numb of Stars	Names and Place of the Stars	Declination for the Year [illegible]			Annual Variation
		D	M	S	s
106	β Aquilæ	5	55	4 N	[illegible]
107	β *Aquilæ*	5	55	19 N	[illegible]
108	*Procyon*	5	44	11 N	[illegible]
109	β Ophiuchi	4	39	41 N	[illegible]
110	δ Virginis	4	29	13 N	[illegible]
111	ϑ Serpentis	3	57	12 N	+ 3,97
112	α Ceti	3	17	49 N	+14[illegible]
113	α *Ceti*	3	18	0 N	
114	β Virginis	2	53	25 N	−19,97
115	β *Virginis*	2	53	38 N	
116	γ Ophiuchi	2	47	4 N	− 1,97
117	δ Aquilæ	2	13	36 N	+ 6,44
118	γ Ceti	2	23	7 N	+15,77
119	α Piſcium	[illegible]	47	40 N	+17,73
120	η Antinoi	[illegible]	[illegible]	12 N	+ 8,61
121	δ Orionis	0	27	17 N	−3,38
122	ζ Virginis	0	25	48 N	−18,72
123	ι Hydræ	0	8	18 S	−15,66
124	γ Virginis	0	21	4 S	+19,86
125	δ Ceti	0	32	18 S	−15,97
126	α Aquarii	1	17	7 S	−17,15
127	α *Aquarii*	1	17	7 S	
128	ι Orionis	1	24	12 S	− 3,02
129	ϑ Antinoi	1	24	10 S	−10,05
130	ζ Orionis	2	3	33 [illegible]	− 2,[illegible]
131	γ Aquarii	2	[illegible]	31 S	−17,81
132	δ Ophiuchi	3	10	8 S	+ 9,77
133	ζ Serpentis	3	39	32 S	− [illegible]
134	ι Ophiuchi	4	11	37 S	+[illegible]
135	ϑ Virgini	4	28	4 S	+19,[illegible]
136	β Eridani	5	21	16 S	− 5,41
137	ι Orionis	6	3	0 S	− 3,04
138	β Aquarii	6	26	3[illegible] S	−15 39
139	φ Aquarii	7	7	25 S	−[illegible] 14
140	α Hydræ	7	47	56 S	+[illegible],21

ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS

Number of Stars	Names and Places of the Stars	Declination for the Year 1800			Annual Variation
		D	M	S	s
141	α *Hydræ*	7	47	53 S	+15,21
142	*Rigel*	8	26	35 S	− 4,81
143	β Libræ	8	38	14 S	+13,82
144	λ Aquarii	8	38	29 S	−18,89
145	α Spicæ	10	6	46 S	+19,01
146	[illegible]	10	6	45 S	
147	ζ Ophiuchi	10	9	1 S	+ 8,02
148	δ Eridani	10	27	5 S	−11,99
149	μ Ceti	11	22	48 S	+15,71
150	Virginis	12	26	26 S	+17,01
151	1 α Capricorni	13	6	58 S	−10,47
152	1 α *Capricorni*	13	7	0 S	
153	2 α Capricorni	13	9	15 S	−10,50
154	2 α *Capricorni*	13	9	17 S	
155	γ Libræ	14	6	42 S	+12,63
156	γ Eridani	14	5	3 S	−10,90
157	α Libræ	15	12	0 S	+15,40
158	α *Libræ*	15	12	0 S	
159	δ Corvi*	15	24	5 S	+19,98
160	β Capricorni	15	24	1[illegible] S	−10,71
161	γ Canis majoris	15	21	6 S	+ 4,69
162	η Ophiuchi	15	27	58 S	+ 5,33
163	ι Aquarii	15	48	54 S	−17,14
164	γ Corvi	16	25	47 S	+20,04
165	Sirius	16	27	7 S	+ 4,33
166	*Sirius*	16	27	5 S	
167	δ Aquarii	16	52	59 S	−18,85
168	δ Capricorni	17	1	38 S	−16,19
169	α Crateris	17	14	11 S	+19,11
170	[illegible] Capricorni	17	33	22 S	−15,82
171	ι Capricorni	17	39	58 S	−14,97
172	β Canis majoris	17	51	55 S	1,18
173	α Leporis	17	58	22 S	− 3,18
174	ϑ Capricorni	18	1	3 S	−13,81
175	ι Scorpii	18	55	46 S	+10,03

ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS

Number of Star	Names and Places of the Star	Declination for the Year 18 0			Annual Variation
			M	S	
176	β Ceti	19	5	9 S	−19,84
177	β Scorpii	19	14	46 S	+10,5
178	μ Sagittarii	21	5	50 S	− 0,09
179	π Sagittarii	21	19	45 S	4,95
180	ε Corvi	21	30	32 S	+ 0,05
181	δ Scorpii	22	2	30 S	19,92
182	ο Sagittarii	2	1	[illegible] S	− 4,51
183	β Corvi	22	17	17 S	+19,94
184	γ Leporis	22	31	15 S	[illegible]
185	α Corvi	23	36	50 S	[illegible]0,04
186	γ Scorpii	24	29	10 S	+14,67
187	θ Ophiuchi	24	47	17 S	+ 4,43
188	σ Scorpii	25	6	8 S	+ 9,36
189	π Scorpii	25	[illegible]	36 S	+11,06
190	Antares	25	58	38 S	+ 8,75
191	*Antares*	25	58	23 S	
192	δ Canis majoris	26	5	10 S	+ 5,18
193	Canis majoris	28	42	29 S	+ 4,38
194	ζ Canis majoris	29	58	50 S	+ 1,07
195	*Fomalhaut*	30	40	38 S	−19,01

IN theſe two Catalogues, the right aſcenſions and declinations of the ſtars denoted by *Italic* characters, were taken from Dr MASKELYNE's Catalogue of fundamental ſtars; by a compariſon with which, M. de ZACH found the right aſcenſions of the other ſtars, as given in the firſt Catalogue, by a tranſit inſtrument eight feet long, made by Mr RAMSDEN. In the ſecond Catalogue, the declinations of the ſtars which were not taken from Dr MASKELYNE's Catalogue were computed from obſervations made by M. CASSINI at Paris, from 1778 to 1790, by a quadrant of ſix feet radius. The variations in right aſcenſion and declination were taken from Mr WOLLASTON's Catalogue, except thoſe of *Arcturus* and *Sirius*, to which are added their proper motions in declination, as determined by Dr MASKELYNE.

A

CATALOGUE

OF

THE PRINCIPAL FIXED STARS,

IN

RIGHT ASCENSION, AND NORTH POLAR DISTANCE,

ADAPTED TO THE YEAR 1790

WITH

Their MAGNITUDES, and ANNUAL PRECESSIONS in RIGHT ASCENSION, and NORTH POLAR DISTANCE

BY

TOBIAS MAYER

7 7 L 2

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees D	M	S	Annual Precess. S	Right Ascension in Time H	M	S	Annual Precess. S	North polar Distance °	'	"	Annual Precess.
1	γ Pegaſi	2	0	36	35	46,07	0	2	26,3	3,078	75	59	0	−20,04
2	Piſcium	7	1	2	41	46,07	0	4	10,7	3,071	82	20	44	−20,04
3		7	1	27	1	46,06	0	5	48,1	3,0[illegible]	8[illegible]	55	3[illegible]	0,04
4		8	1	28	38	46,03	0	5	54,[illegible]	3,069	89	16	59	− 0,04
5		7	1	45	20	46,03	0	7	[illegible]1,3	3,069	89	[illegible]8	46	− 0,04
6	d Piſcium	6	2	27	10	46,12	0	9	48,7	3,075	[illegible]8	56	34	− 0,03
7		7	3	16	41	45,95	0	13	6,7	3,065	93	[illegible]	47	0,01
8	Piſcium	6,7	3	[illegible]9	48	46,04	0	14	39,[illegible]	3,069	89	13	7	− 0,01
9	Ceti	7	3	58	3	45,99	0	15	5,2	3,066	91	1[illegible]	47	− 0,00
10		8	4	8	43	46,06	0	16	34,9	3,071	88	21	54	− 0,00
11		6,7	4	50	10	46,18	0	19	20,7	3,079	84	30	16	−19,97
12	Piſcium	6,7	5	23	42	46,21	0	21	34,8	3,081	84	1[illegible]	1	−19,96
13		9	5	41	19	45,96	0	22	45,3	3,064	91	45	57	−19,95
14		6,7	6	11	40	45,96	0	26	46,7	3,064	91	39	3[illegible]	−19,93
15		7,8	6	50	21	45,95	0	27	21,4	3,063	91	39	40	−19,91
16		6	7	30	27	45,77	0	39	1,8	3,051	[illegible]5	[illegible]0	37	−19,88
17		8	8	35	33	45,97	0	34	22,2	3,065	90	53	31	−19,82
18		6	9	18	17	46,33	0	36	33,1	3,069	84	24	[illegible]3	−19,79
19		6,7	9	20	25	46,26	0	37	21,6	3,084	85	47	26	−19,78
20		7,8	9	20	58	46,37	0	37	[illegible]3,9	3,091	83	50	57	−19,[illegible]8
21	δ Piſcium	4	9	27	6	46,39	0	37	48,4	3,093	83	33	33	−19,77
22		7,8	10	7	42	46,16	0	40	30,7	3,077	87	45	16	−19,73
23		6	10	34	25	45,87	0	4[illegible]	17,7	3,058	92	17	10	−19,71
24		8	10	37	59	46,21	0	42	31,9	3,081	87	3	16	−19,70
25		8 9	11	6	49	46,40	0	44	27,[illegible]	3,093	84	17	10	−19,67
26		7	11	17	20	46,92	0	45	9,3	3,128	77	11	18	19,66
27		7	11	43	40	46,93	0	46	54,7	3,1[illegible]9	77	6	31	−1[illegible],63
28		7,8	11	51	45	46,43	0	47	[illegible]7,0	3,095	84	17	[illegible]8	−19,62
29		7	12	14	27	46,42	0	48	57,8	3,095	84	39	9	−19,59
30	ε Piſcium	4	13	1	1	46,55	0	52	4,1	3,103	83	14	34	−19,53
31		8	13	13	35	46,47	0	52	54,3	3,098	84	[illegible]1	53	−19,51
32	Piſcium	7,8	13	30	13	46,39	0	54	0,9	3,093	65	29	19	−19,49
33		7,8	13	44	43	46,33	0	54	58,6	3,089	86	12	41	− [illegible]9,4[illegible]
34		8	14	21	8	46,79	0	57	24,5	3,119	61	13	0	−19,42
35	e Piſcium	5	14	23	43	46,42	0	57	34,9	3,095	65	[illegible]7	45	−19,4[illegible]

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitud	Right Ascension Degrees			Annual Variat +	Right Ascension in Time			Annual Variat +	North Polar Distance			Annual Proces
			D	M	S		H	M	S	S	D	M	S	S
36		8	14	58	10	46,79	0	59	54,7	3,119	81	31	22	− 19,37
37		7,8	15	7	33	46,66	1	0	30,2	3,124	80	49	38	− 19,36
38	1 ζ Piſcium	6	15	41	42	46,63	1	2	46,8	3,109	83	32	17	− 19,30
39	2 ζ Piſcium	7	15	42	4	46,63	1	2	48,3	3,109	83	32	4	− 19,30
40	f Piſcium	6	16	44	49	46,27	1	6	59,3	3,085	87	29	40	− 19,20
41		8,9	17	8	16	46,24	1	8	33,1	3,083	87	48	57	− 19,16
42		6	17	16	23	45,85	1	9	5,5	3,057	91	36	54	− 19,14
43		7	17	57	22	46,09	1	11	49,5	3,073	89	22	[illegible]2	− 19,07
44		8	17	58	6	46,41	1	11	52,4	3,094	86	21	50	− 19,07
45		7,8	17	59	52	46,71	1	11	59,5	3,114	83	41	53	− 19,07
46	ρ Piſcium	5	18	44	24	48,12	1	14	57,6	3,[illegible]08	71	55	29	− 18,99
47		5	18	50	34	48,14	1	15	[illegible]2,2	3,209	71	51	6	− 18,98
48		7	19	16	53	47,92	1	17	[illegible],5	3,195	74	0	4[illegible]	− 18,93
49	μ Piſcium	5	19	47	53	46,6[illegible]	1	19	11,5	3,108	84	56	31	− 18,96
50	η Piſcium	4,3	20	4	0	47,77	1	0	16,0	3,185	75	41	[illegible]7	− 18,83
51		7,8	20	9	30	47,[illegible]1	1	0	39,0	3,147	80	11	51	− 18,92
52		7,8	20	34	46	46,90	1	22	19,1	3,127	8[illegible]	52	21	− 18,[illegible]7
53		8	20	58	21	46,93	1	3	53,4	3,129	8[illegible]	48	17	− 18,[illegible]2
54	π Piſcium	5,4	21	29	51	47,46	1	25	59,4	3,164	78	56	15	− 18,66
55		8,9	[illegible]1	37	50	47,46	1	26	31,3	3,164	78	59	50	− 18,64
56		7,8	22	23	9	47,05	1	9	3[illegible],6	3,037	82	18	32	− 18,54
57		9	22	35	47	47,10	1	30	[illegible]3,1	3,140	81	59	40	− 18,51
58	ν Piſcium	5	22	37	52	46,6[illegible]	1	30	31,5	3,108	85	34	46	− 18,51
59	Piſcium	5	23	34	53	47,16	1	34	19,5	3,144	81	54	15	− 18,38
60		8,9	[illegible]4	0	25	47,43	1	36	1,7	3,162	80	1	42	− 18,3[illegible]
61		7,8	24	56	13	47,51	1	39	44,9	3,167	60	0	5	− 16,13
62	1 γ Arietis	4	25	30	27	48,87	1	42	1,5	3,[illegible]56	71	44	19	− 18,09
63	2 γ Arietis		25	30	29	48,87	1	42	1,9	3,[illegible]58	71	44	8	− 16,09
64	β Arietis	3	25	45	54	49,15	1	43	3,[illegible]	3,[illegible]77	70	13	24	− 14,06
65	ι Arietis	5	26	28	27	46,71	1	45	53,6	3,247	73	12	49	− 17,95
66		8	27	3	9	47,84	1	48	1[illegible],6	3,169	78	43	47	− 1[illegible],65
67	γ Andromedæ	2	27	45	55	54,3	1	51	3,7	3,615	48	41	7	− 17,71
68	α Piſcium	3	27	48	4	46,30	1	51	12,3	3,087	88	15	[illegible]1	− 17,73
69	κ Arietis	6	28	42	36	49,84	1	54	50,4	3,323	68	2	[illegible]	− 17,55
70	α Arietis	2	28	50	21	50,02	1	55	21,4	3,335	67	32	11	− 17,56

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Number of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Dist.			Annual Preces.
			D	M		s	H	M			D			s
71	Piscium	8	9	3	56	49,00	1	56	15,7	3,267	72	58	36	−17,52
72		7	29	45	3	49,35	1	59	0,2	3,290	71	29	47	−17,40
73		9	29	54	15	49,73	1	59	37,0	3,315	69	37	12	−17,38
74	η Arietis	6	30	15	49	49,74	[illegible]	1	3,3	3,316	69	47	0	−17,32
75	1 ξ Ceti	4	30	28	29	47,42	2	1	53,9	3,161	82	8	39	−17,28
76	Arietis	7	31	24	17	48,68	2	5	37,1	3,–45	75	4–	46	−17,11
77	1 ϑ Arietis	6	31	36	58	49,6–	2	6	2–,9	3,30–	71	4	41	−17,06
78		7	32	17	3	45,03	2	9	6,–	3,00–	95	16	58	−16,95
79	ε Arietis	5	33	3	47	47,90	2	13	–5,1	3,193	80	–0	53	−16,74
80	2 ξ Ceti	4	34	15	18	47,51	2	17	1,–	3,167	8–	29	16	−16,57
81		7,8	34	43	13	49,93	2	18	52,9	3,329	71	5	5	−16,48
82	ν Ceti	4,5	36	13	11	46,99	2	–4	52,7	3,133	85	19	49	−16,18
83	ι Arietis	6	36	43	46	50,6–	2	26	55,1	3,375	68	57	18	−16,07
84	δ Ceti		37	11	10	45,90	2	28	44,6	3,060	90	35	3	−15,97
85	μ Arietis	6	37	38	10	50,26	2	30	32,7	3,351	70	53	26	−15,86
86	ο Arietis	6	38	14	56	49,21	[illegible]	3	59,7	3, 81	75	35	8	−15,75
87	μ Ceti	4	38	24	3	48,04	2	33	36,–	3,203	80	46	50	−15,71
88	π Arietis	6	39	23	53	49,61	2	37	35,5	3,321	73	25	4	−15,49
89	σ Arietis	6	39	58	45	49,28	2	39	55,0	3,285	75	47	–2	−15,37
90	2 ρ Arietis	6	41	0	28	50,16	2	44	1,9	3,344	72	31	32	−15,13
91	3 ρ Arietis		41	8	55	50,10	2	44	35,7	3,340	72	49	16	−15,10
92	ε Arietis	5	41	48	25	51,01	–	47	13,6	3,401	69	30	33	−14,95
93	λ Ceti	4	42	7	15	47,9–	2	46	29,9	3,195	81	56	19	−14,87
94	α Ceti	2	42	49	54	46,79	2	51	19,6	3,119	86	44	29	−14,70
95	β Persei	3	43	38	15	57,68	2	54	33,0	3,845	49	51	55	−14,51
96		8	44	19	44	51,10	–	57	18,9	3,407	70	2	55	−14,34
97		7	44	24	43	50,73	–	57	38,6	3,36–	71	25	55	−14,3
98	δ Arietis	4	44	54	33	50,87	2	59	30,1	3,391	71	4	42	−14,20
99	ζ Arietis	5	45	42	46	51,31	3	–	51,1	3,4–1	69	44	33	−14,00
100	1 τ Arietis	5	47	16	46	51,49	3	9	7,–	3,433	69	37	13	−13,60
101	α Persei	2	47	21	19	63,05	3	9	–5,3	4,203	40	51	3	−13,59
102	2 τ Arietis	6	47	40	20	51,41	3	10	41,3	3,427	70	1	11	−13,50
103		7	46	5	15	51,16	3	12	21,0	3,431	69	5–	15	−13,39
104	Tauri	4	48	23	6	48,20	3	13	32,4	3,213	81	43	1–	−13,31
105	ξ Tauri	4	48	57	10	46,41	3	15	48,6	3,2–7	81	0	37	−13,16

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Number of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees	Annual Variation +	Right Ascension in Time	Annual Variation +	North Polar Distance	Annual Differences
			D M S		H M S		D M S	S
106		7	49 17 21	48,89	3 17 9,4	3,[illegible]59	79 [illegible]0 45	−13,07
107		7,6	49 28 10	50,39	3 17 52,[illegible]	3,[illegible]59	73 59 [illegible]1	−13,03
108	s Tauri	6	49 41 18	48,88	3 18 5[illegible]	3,2[illegible]9	79 [illegible]3 43	−1[illegible],96
109	f Tauri	5	49 49 30	49,3[illegible]	3 19 18,0	3,289	77 47 37	−1[illegible],93
110	t Tauri	6	50 18 50	4[illegible],[illegible]7	3 21 15,3	3,2[illegible]9	61 [illegible]0 47	−12,60
111		[illegible]	50 33 25	50,79	3 [illegible] 13,6	3,353	72 51 48	−12,74
112		7	51 30 24	50,15	3 26 1,6	3,343	75 16 14	−12,48
113			51 53 38	50,58	3 [illegible]7 34,5	3,367	71 9 21	−12,39
114	g Pleiadum	6	53 5 15	53,03	3 3[illegible] 21,0	3,535	66 [illegible]2 59	−12,04
115	b Pleiadum	5	53 6 [illegible]9	52,97	3 [illegible] 25,9	3,531	66 [illegible] 33	−12,04
116		7	53 9 44	53,15	3 32 36,9	3,513	66 [illegible] 9	−12,02
117	e Pleiadum	5	53 11 1	53,10	3 3[illegible] 44,0	3,540	66 12 14	−12,02
118	c Pleiadum	6	53 20 21	53,08	3 33 [illegible]1,4	3,539	66 18 2	−11,97
119	k Pleiadum	7	53 21 20	53,14	3 33 [illegible]5,3	3,543	66 6 45	−11,96
120	l Pleiadum	7	53 23 29	53,14	3 33 33,9	3,543	66 8 23	−11,96
121	d Pleiadum	5	53 28 23	52,95	3 33 53,5	3,530	66 43 1	−11,94
122	p Pleiadum	7	53 43 2[illegible]	53,03	3 34 53,5	3,535	66 32 41	11,88
123	η Pleiadum	3	53 45 23	53,03	3 35 1,6	3,535	66 33 [illegible]3	−11,66
124	f Pleiadum	5	54 10 30	53,05	3 36 42,0	3,537	66 36 5	−11,74
125	h Pleiadum	6	54 10 43	53,06	3 36 42,9	3,539	66 31 1	−11,74
126	c Tauri	5	54 11 36	49,03	3 36 46,4	3,269	79 30 50	−11,73
127		7,8	54 1[illegible] 15	53,17	3 36 5[illegible],3	3,545	66 16 22	−11,73
128		7	55 17 35	50,96	3 41 10,3	3,397	73 18 38	−11,41
129			56 7 11	52,70	3 44 [illegible]8,7	3,513	68 8 16	−11,1[illegible]
130	λ Tauri	4	57 16 1	49,56	3 49 4,[illegible]	3,301	78 6 55	−10,85
131	a Tauri	5	58 4 30	52,72	3 52 18,0	3,515	66 [illegible]0 9	−10,60
132		7	58 13 47	52,71	3 5[illegible] 55,1	3,514	68 31 11	−10,55
133	ψ Tauri	5	58 30 43	55, 7	3 54 [illegible],9	3,665	61 31 50	−10,47
134	1 ω Tauri	6	59 14 4	51,97	3 56 56,2	3,465	70 57 32	−10,[illegible]5
135	p Tauri	6	59 31 0	54,42	3 58 4,0	3,6[illegible]6	64 4 51	−10,18
136	[illegible] ω Tauri	6	61 14 35	52,43	4 4 58,3	3,495	69 57 4	−9,65
137	φ Tauri	5	61 51 57	54,96	4 7 [illegible]7,6	3,664	63 9 54	−9,45
138	γ Tauri	3	61 57 47	50,60	4 7 57,1	3,387	74 53 34	−9,42
139	Tauri	6,7	62 2 4	50,28	4 8 8,3	3,354	76 29 3	−9,40
140	h Tauri		62 10 42	50,63	4 8 42,8	3,375	75 5 13	−9,36

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names in Letters of the Stars	Mag.	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Distance			Annual Preces.
			D	M	S	S	H	M	S		D			
141			62	16	13	50 23	4	9	4,9	3,349	76	39	5	−9 33
142	χ Tauri	5	62	27	17	54 35	4	9	49,1	3 623	64	52	48	−9 2,
143			62	33	31	50 31	4	10	14,1	3,354	76	25	58	−9 2ɔ
144	1 δ Tauri	4	62	42	33	51,48	4	10	50	3 432	7	57	48	−9,19
145			62	50	29	51 16	4	11	-1,9	3,411	73	54	43	−9 1ɔ
146	2 δ Tauri	5	63	0	3	51 46	4	12	0 2	3 431	7ɔ	3	22	−9,10
147	3 δ Tauri	6	63	20	31	51 65	4	13	2 1	3 443	7	33	58	−8 99
148	1 υ Tauri	5	63	26	15	53 38	4	13	45 0	3 559	67	40	31	−8 97
149			63	35	46	50 88	4	14	23,1	3,39	74	5	1ɔ	−8 91
150	2 υ Tauri	6	63	41	16	53 47	4	14	45,1	3,565	67	29	5	−8 88
151	ϖ Tauri	5	63	41	17	50,58	4	14	45 1	3 372	75	46	28	−8 88
152		7	63	41	50	51,20	4	14	47 3	3 413	7ɔ	54	55	−8,88
153	ε Tauri	3,4	64	5	29	52,12	4	16	21,9	3 475	71	17	56	−8,/6
154			64	7	36	50,60	4	16	30 4	3 373	75	44	3	−8 75
155	1 ϑ Tauri	5	64	8	55	51,02	4	16	35 4	3,401	74	30	59	−8,/5
156	2 ϑ Tauri	5	64	10	16	50 99	4	16	41,1	3 399	74	36	28	−8 74
157		8	64	29	56	52 38	4	17	59 7	3 492	70	37	52	−8 63
158			64	32	38	50,95	4	18	10 5	3,397	74	45	4	−8,62
159			64	38	18	51 12	4	18	33,	3 408	74	16	34	−8 61
160			64	40	14	50,95	4	18	40,9	3 ɔ97	74	46	38	−8 ɔ8
161		7	64	41	35	51,10	4	18	46,3	3,407	/4	19	16	−8 57
162		7,8	65	24	32	51,20	4	21	38 1	ɔ 413	74	7	59	−8,34
163	ρ Tauri	5	65	9	12	50,70	4	21	56,8	3 380	75	36	36	−8 31
164		8	65	44	3	51,29	4	2	56,2	3 419	73	55	18	−8, 4
165			65	51	19	52,48	4	23	25 3	ɔ 499	70	34	0	−8 20
166	α Tauri	1	65	58	15	51 30	4	-3	53,0	3 4 0	73	55	3-	−8 16
167			66	3	13	51,16	4	26	8,9	3 411	74	-4	4	−8,00
168	1 σ Tauri	7	66	47	43	51,09	4	27	10,8	ɔ 406	74	37	39	−/ 90
169	2 σ Tauri	6	66	49	9	51 13	4	27	16,6	3 409	74	30	43	−/ 69
170	τ Tauri	5	67	24	53	53,72	4	29	39,5	3 581	67	27	36	−7 70
171		7	68	30	-5	52 20	4	34	1,7	3,480	71	39	31	−7,35
172		8	69	6	9	52,23	4	36	24,6	3,48	71	ɔ9	6	−/,16
173	1 Tauri	6	69	46	29	52,ɔ0	4	39	5 9	3,487	71	31	54	−6 94
174	1 ο Orionis	4,5	70	10	1	50,68	4	40	40,1	3,379	/6	6	45	−6,61
175		9	70	28	50	51,68	4	41	55,3	ɔ 44ɔ	73	0	-	−6 71

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb. of Stars	Names and Places of the Stars	Magnitudes	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Distance			Annual Variat.
			D	M	S		H	M	S		D	M	S	
176		8	70	42	57	51,46	4	42	51,8	3,431	73	58	10	−6 63
177		8	70	48	50	51 55	4	43	15,3	3,437	73	4-	56	−6,60
178		6,7	71	16	17	54,32	4	45	5,1	3,621	66	23	39	−6,44
179		6	71	18	49	51 ,6	4	40	15 3	3,451	73	11	25	−6,43
180		6,7	71	46	19	50,84	4	47	5 3	3,389	75	47	-9	−6,-9
181		7	71	55	27	51,34	4	47	41,8	3,423	74	24	37	−6,2
18	ι Tauri	4	7-	38	19	53 47	4	50	33 3	3,565	66	43	30	−5,99
183			72	57	49	53,37	4	51	51,3	3,558	69	1	53	−5,87
184	1 y Orionis		73	8	42	51,19	4	52	34,8	3,413	74	54	4	−5,82
185			73	17	29	52,82	4	53	9 9	3,531	70	29	49	−5 76
186	m Tauri	6	73	45	22	5- 40	4	55	1 5	3,493	71	39	8	−5,61
187	1 l Tauri	6	73	50	54	53,09	4	55	23,6	3,539	69	5-	23	−5,58
188	2 y Orionis	6	74	25	21	51,31	4	57	11,4	3,421	74	41	6	−5,39
189	α Aurigæ	1	75	17	56	65 94	5	1	11,7	4,396	44	13	53	−5,09
190		7	75	42	42	53,88	5	2	50,8	3,59-	67	58	14	−4,94
191	β Orionis	1	76	6	50	43,13	5	4	27 3	2,875	98	27	24	−4,80
192	n Tauri	6	76	40	1	53 84	5	6	40,1	3,589	68	6	6	−4,62
193	2 l Tauri	7	76	42	21	53,08	5	6	49,4	3,539	70	6	2	−4,61
194		8	76	58	36	52,48	5	7	54,4	3,499	71	42	37	−4,52
195			77	8	3	52,97	5	8	3-,2	3,531	70	24	50	−4,47
196			77	13	41	56,30	5	8	54,7	3,753	62	16	22	−4,43
197		7,8	77	31	21	56,75	5	10	5,4	3,76-	61	16	45	−4,3-
198	β Tauri	2	78	15	23	56,64	5	13	1,5	3,776	61	35	6	−4,06
199	γ Orionis	2	78	28	15	48,13	5	13	53,0	3,209	83	51	16	−4,01
200			78	43	50	52 32	5	14	55 3	3,488	72	14	4	−3 9-
201	Tauri	5	78	45	25	53,86	5	15	1,7	3,591	66	15	29	−3,91
202	χ Aurigæ	5	79	46	2	58,36	5	19	4,1	3,891	57	56	50	−3,56
203	β Leporis	5	79	48	47	38,47	5	19	15,1	2,565	110	56	13	−3,55
204		7,6	80	17	32	53,33	5	21	10 1	3,555	69	41	23	−3,38
-05	δ Orionis	2	80	19	22	45,86	5	21	17 5	3,057	9-	28	6	−3,37
206	Orionis	6	80	30	4	51,01	5	22	0,3	3,401	75	50	5	−3,31
-07	α Leporis	3	80	52	7	39,59	5	23	3-,4	2,639	107	59	3	−3 18
208	ζ Tauri	3	81	16	29	53,63	5	25	5 9	3,575	69	--	3	−3 05
209	ι Orionis	2	81	-3	32	45,55	5	25	34,1	3,037	91	20	56	−3,00
210			81	40	58	55,59	5	26	43,9	3,706	64	14	11	−2,91

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb. of Stars	Names and Letters of the Stars	Magnitude	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Distance			Annual Precess.
			D	M	S		H	M	S	S	D	M	S	S
211	σ Orionis	4	82	3	15	45 07	5	–8	13,0	3,005	92	44	2	−2,77
212	ζ Orionis	–	82	32	35	45 30	5	30	10,3	3 020	92	3	59	−2,60
213			83	12	57	52 72	5	32	51,8	3,515	71	24	5	−–,57
214		7,8	83	37	7	55,31	5	34	28 4	3,556	69	49	0	−2,2
215		7,8	83	45	55	54 14	5	35	2,3	3,609	65	24	49	−2,17
216			83	47	53	52 35	5	35	11,5	3,490	72	21	53	−2,16
217	γ Leporis	4,3	83	55	55	37 75	5	35	40,7	2 517	112	31	26	−2,11
218			83	57	45	53 59	5	35	51 0	3,573	69	13	5	−2,10
219		6,7	84	2	0	55 10	5	36	8,0	3,673	65	31	11	−2,08
220		8	84	5	31	49,42	5	36	22,1	3,295	80	20	3	− 06
221	ϰ Orionis	3	84	27	1	4–,59	5	37	48,1	2,839	99	45	23	−1,93
222		7,8	85	12	45	53,36	5	40	51,0	3,559	69	45	57	−1,68
223	1 χ Orionis	5	85	29	19	53,38	5	41	57,3	3,559	69	47	37	−1,58
224	2 χ Orionis	7	85	37	38	53,17	5	4–	30,5	3,545	70	18	26	−1,53
225	α Orionis	1	85	57	9	48 60	5	43	48,6	3,240	8–	38	50	−1,41
226			86	14	28	55,74	5	14	57,5	3,716	64	5	20	−1,32
227		8	86	56	2	54,46	5	47	44 1	3,631	67	7	40	−1 07
228		7	87	15	11	54,–6	5	49	0 7	3,617	67	37	16	−0,96
229		6,7	87	45	3	52,45	5	51	0 –	3,497	72	11	55	−0,79
230	4 χ Orionis	5	87	45	17–	53 18	5	51	1,1	3,545	70	19	15	−0,79
231	h Geminorum	4	87	50	23	54,63	5	51	–1,5	3,642	66	44	24	−0,75
232	3 χ Orionis	7	87	51	42	53,36	5	51	26 8	3,557	69	52	19	−0,75
233			88	46	4	51,32	5	55	4,3	3,621	67	29	42	−0,43
234		7,8	89	13	16	54,20	5	56	53,0	3,613	67	47	33	−0 27
235			89	14	45	54,58	5	56	59 0	3,639	66	52	5	−0, 7
236		7,8	89	26	37	54 54	5	57	46,5	3,636	66	58	37	−0,20
237		6	89	53	44	53,24	5	59	34,9	3,549	70	10	45	−0,04
238	Geminorum	7,8	89	53	49	54,50	5	59	35,3	3,633	67	3	34	−0,04
239	η Geminorum	4	90	33	1	54,34	6	2	1–,1	3,6–3	67	26	52	+0 19
240		6	90	37	–0	53,01	6	2	28,3	3,534	70	47	11	+0,22
241			90	52	16	54,95	6	–	29,1	3,663	65	58	39	+0,30
242	μ Geminorum		91	–	25	54 84	6	4	9 7	3,656	66	14	21	+0,36
243			91	31	39	54,80	6	6	6 6	3,653	66	19	51	+0,53
244		4	91	38	4	54,74	6	6	32,3	3,649	66	27	50	+0,57
245			91	39	6	54,67	6	6	36,4	3,645	66	39	32	+0 58

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Number of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Annual Variation +	Right Ascension in Time			Annual Variation +	North polar Distance			Annual Preces.
			D	M		″				s	D	M	″	
246		8	92	10	11	53,79	6	8	40,7	3,566	68	47	22	+0,76
247		9	92	12	5	53,62	6	8	48,3	3,588	68	43	13	+0,77
248			92	14	52	54,87	6	8	59,5	3,658	66	9	24	+0,78
249	μ Geminorum	3	9[illegible]	33	39	54,36	6	10	14,6	3,624	67	23	34	+0,90
250	Geminorum		93	16	37	54,00	6	13	6,5	3,600	68	15	16	+1,14
251	Geminorum		93	48	52	53,66	6	15	15,6	3,577	69	5	44	+1,33
252			93	51	46	53,51	6	15	[illegible]7,1	3,569	69	23	36	+1,35
253	ν Geminorum	4	94	7	20	53,43	6	16	29,3	3,562	69	40	11	+1,44
254	Geminorum	8	95	0	25	52,18	6	[illegible]0	1,7	3,499	7[illegible]	5	16	+1,75
255		7,8	95	0	41	5[illegible],48	6	20	2,7	3,499	7[illegible]	5	0	+1,75
256		8	95	41	48	51,87	6	2[illegible]	47,2	3,458	73	38	47	+1,99
257	γ Geminorum	2,3	96	23	34	51,95	6	25	34,3	3,463	73	[illegible]6	8	+[illegible],23
258			96	29	27	51,95	6	25	57,8	[illegible],463	73	25	8	+[illegible],26
259	Geminorum	8	96	54	20	53,20	6	[illegible]7	37,3	3,547	70	9	48	+2,41
260		6,7	97	32	[illegible]9	52,41	6	30	9,9	3,494	72	9	48	+[illegible],63
261	ε Geminorum	3	97	45	2	55,42	6	31	0,1	3,695	64	40	37	+2,70
262	1 ξ Geminorum	5,4	98	2	10	50,76	6	32	8,7	3,384	76	34	26	+2,80
263	2 ξ Geminorum		98	22	35	50,64	6	33	30,3	3,376	76	53	25	+2,9[illegible]
264	α Canis	1	98	58	40	40,18	6	35	54,7	2,679	106	25	39	+3,13
265			99	28	48	50,82	6	37	55,2	3,386	76	21	30	+3,30
266	d Geminorum	6	99	44	17	54,00	6	38	57,1	3,600	68	2	23	+3,39
267	e Geminorum	6	100	41	51	50,72	6	42	47,4	3,381	76	34	10	+3,72
268		7,8	101	0	49	52,41	6	44	3,2	3,494	72	0	17	+3,[illegible]3
269			101	22	10	52,47	6	45	28,7	3,498	71	50	12	+3,96
270	Geminorum	7,8	101	27	33	55,75	6	45	50,2	3,717	63	39	29	+3,98
271		7	101	37	15	55,67	6	46	29,0	3,711	63	48	59	+4,04
272		9	101	58	53	54,63	6	47	55,5	3,6[illegible]2	66	16	57	+4,16
273		8,9	102	2	45	51,77	6	48	11,0	3,451	73	38	4[illegible]	+4,18
274	1 ω Geminorum	6	102	24	2	54,94	6	49	36,1	3,663	65	30	1	+4,31
275	ζ Geminorum	3	102	54	36	53,47	6	51	38,4	3,565	69	8	11	+4,46
276	2 ω Geminorum		103	9	46	54,[illegible]8	6	52	39,1	3,619	67	3	46	+4,56
277		8	103	31	10	52,37	6	54	4,6	3,491	71	57	5	+4,66
278	o Geminorum	7	104	4	42	51,69	6	56	18,8	3,446	73	44	47	+4,36
279	τ Geminorum	5	104	26	20	57,49	6	57	45,3	3,833	59	25	33	+4,99
280	m Geminorum	6	104	54	54	54,83	6	59	39,6	3,655	65	32	7	+5,16

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Annual Preces	Right Ascension in Time			Annual Preces	North polar Distance			Annual Pre
			D	M	S	S	H	M	S	S	D	M	S	S
281		5	105	19	32	51,75	7	1	18,1	3,450	73	29	55	+5,30
282	n Geminorum	5	105	27	33	55,12	7	1	50,-	3 675	64	45	54	+5,35
283		7,8	106	9	52	55,86	7	4	3,5	3 724	62	56	47	+5,53
284		7	106	10	16	51,72	7	4	41,1	3 418	73	-9	57	+5,59
285	λ Geminorum	5	106	30	14	51,86	7	6	0,9	3,457	73	5	41	+5,70
286	δ Geminorum	3	106	53	28	53 91	7	7	33,9	3,594	67	38	45	+5,82
287	q Geminorum	6,7	107	3	5	53,17	7	9	32 3	3,545	69	30	-9	+5,99
288	ι Geminorum	5 6	107	39	54	55,11	7	10	39,6	3,674	64	33	36	+6,07
289		7 8	107	50	40	51,47	7	11	2-,7	3 431	71	0	10	+6 14
290		7	107	52	-	56,18	7	11	24 1	3,745	61	58	22	+6 15
291	ι Geminorum	4,5	108	10	2	56 24	7	12	40,1	3,719	61	47	57	+6,25
292		7,8	108	36	15	53,69	7	14	25 0	3,579	68	1	32	+6,40
293	p Geminorum	6	108	48	53	53,64	7	15	15,5	3,576	68	8	20	+6,47
294	s Geminorum	5	108	53	33	57,96	7	15	34 2	3,864	57	48	58	+6,50
295	1 b Geminorum	6	109	3	35	56,32	7	16	14,3	3,755	61	27	50	+6,55
296	2 b Geminorum		109	10	58	56,23	7	16	43,9	3,749	61	39	56	+6,58
297	α Geminorum	1,2	110	17	35	57,93	7	21	10,3	3,862	57	40	0	+6,96
298	k Geminorum	6	110	24	8	51,50	7	21	36,3	3,433	73	44	9	+6,99
299	υ Geminorum	5	110	44	22	55,72	7	22	57,5	3,715	62	39	6	+7,10
300			111	18	53	5 ,59	7	25	15 5	3,506	70	37	13	+7,29
301		7	111	22	38	54,65	7	25	30,5	3 643	65	11	6	+7,31
302		7	111	37	21	54,59	7	26	29,4	3,639	65	16	58	+7,39
303	f Geminorum	6	111	49	59	52,1-	7	27	19,9	3,475	71	51	37	+7,45
304	α Canis min	1	11	4	57	47,89	7	28	19,8	3,193	84	14	22	+7,54
305		7,8	112	14	36	47 89	7	28	58,4	3,193	84	15	16	+7 58
306	σ Geminorum	5	112	32	24	56,45	7	30	9,5	3,763	60	37	15	+7,66
307		7	11	42	36	53,83	7	30	50,4	3,569	67	7	15	+7,73
308	c Geminorum	6	112	49	16	55,14	7	31	7 1	3,676	63	43	47	+7,75
309	κ Geminorum	4,5	112	56	14	54,59	7	31	44,9	3,639	65	6	49	+7,81
310	β Geminorum	[illegible]	113	6	59	56 04	7	32	27 9	3,736	61	28	48	+7,86
311	g Geminorum	6	113	9	15	52,35	7	38	57 0	3 490	70	59	-5	+7,96
312			114	55	49	52,58	7	39	43,3	3,505	70	9	4	+8,45
313	φ Geminorum	6	115	9	20	55,38	7	40	37 3	3,692	62	42	19	+8,52
314	l Geminorum	7	115	50	50	52,74	7	43	23,3	3,516	69	34	29	+8,74
315	11 ν Cancri	6	117	2	58	54 71	7	48	11,9	3,647	64	2	19	+9,11

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Number of Stars	Names and Place of the Stars	Mag.	Right Ascension in Degrees			Annual Var at +	Right Ascension in Time			Annual Variat +	North Polar Distance			Annual Preces.
			D	M	S		H	M	S		D	M		S
316	Cancri	5	117	38	54	55,61	7	50	35,6	3,707	61	37	5[illegible]	+9,30
317	Cancri		116	20	28	50,3	7	53	21,9	3,355	76	17	41	+9,51
318		7	118	27	7	50,44	7	53	48,5	3,[illegible]63	75	51	39	+9,55
319	2 μ Cancri	5	118	50	42	53,17	7	55	22,8	3,545	67	49	17	+9,67
320	1 ψ Cancri	7	119	22	16	54,71	7	57	29,2	3,647	63	33	3	+9,83
321	[illegible] ψ Cancri	4	119	26	37	54,58	7	57	46,5	3,639	63	51	56	+9,85
3[illegible]			119	29	0	49,61	7	57	56,0	3,[illegible]07	78	22	4[illegible]	+9,86
323	ψ Cancri	6	120	1	30	56,15	8	0	5,9	3,743	59	43	39	+10,03
3[illegible]4	1 ζ Cancri	5	120	2	11	51,75	8	0	8,7	3,450	71	43	53	+10,03
325	2 ζ Cancri	5	120	2	13	51,75	8	0	8,9	3,450	71	43	50	+10,03
326		7	120	12	10	49,21	8	0	48,7	3,281	79	34	5	+10,06
327			120	32	11	51,71	8	2	8,7	3,447	71	45	44	+10,18
328	β Cancri	4,3	121	16	46	49,99	8	5	7,1	3,[illegible]66	80	10	43	+10,40
329			121	32	10	48,88	8	6	8,7	3,259	60	29	44	+10,48
330	χ Cancri	6	121	49	9	55,03	8	7	16,5	3,660	62	6	42	+10,57
331	λ Cancri	6	122	0	20	53,831	8	8	1,3	3,589	65	19	43	+10,62
332	1 d Cancri	7	122	49	46	51,62	8	11	19,1	3,455	71	0	19	+10,87
333		6,7	123	24	52	55,12	8	13	39,5	3,675	61	8	39	+11,04
334	2 d Cancri	7	123	28	54	51,37	8	13	55,6	3,425	72	16	23	+11,06
335	2 φ Cancri		123	30	51	54,76	8	14	3,4	3,651	62	3	30	+11,07
336		7	123	32	5	53,76	8	14	8,3	3,584	65	8	17	+1,07
337	2 υ Cancri	6,7	124	2	1	53,70	8	16	8,1	3,560	65	10	13	+11,2[illegible]
338		7	124	17	38	53,75	8	17	10,5	[illegible],563	64	58	1	+11,30
339		8	124	17	47	54,41	8	17	11,1	3,627	63	7	12	+11,30
340	3 υ Cancri	6	124	45	55	53,62	8	19	3,7	3,575	65	13	[illegible]2	+11,43
341	ϑ Cancri	6,5	124	53	57	51,61	8	19	35,8	3,441	71	12	24	+11,47
342			124	54	14	51,90	8	19	36,9	3,460	70	16	58	+11,47
343	η Cancri	6,7	125	8	8	52,36	8	20	32,5	3,491	68	51	[illegible]5	+11,54
344	4 υ Cancri		1[illegible]5	8	30	53,59	8	20	34,0	[illegible],573	65	12	41	+11,51
345			125	31	30	50,08	8	22	6,0	3,339	76	2	9	+11,66
346	Cancri	8	125	48	24	52,03	8	23	13,6	[illegible],402	69	41	58	+11,73
347		6	125	54	37	53,09	8	23	38,5	3,539	69	30	50	+11,76
348		6	126	4	55	50,67	8	24	19,7	3,378	73	58	14	+11,81
349	1 c Cancri	6	126	25	24	48,97	8	26	41,6	3,265	79	37	35	+11,91
350			126	25	46	51,69	8	25	43,1	[illegible],459	70	0	46	+11,91

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Numb. of Stars	Names and Places of the Stars	Magn.	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Distance			Annual Preces.
			D	M	S		H	M	S	S	D	M	S	S
351	2 c Cancri		126	40	32	48,94	8	26	42,1	3,263	79	4	8	+11,96
352	Cancri	7	126	54	23	52,00	8	27	37,5	3,467	69	29	39	+12,03
353		7	126	56	37	51,93	8	27	46,5	3,462	69	43	50	+12,05
354			126	59	37	52,21	8	27	58,1	3,481	68	51	35	+12,07
355		7	127	0	2	52,08	8	28	0,1	3,472	69	15	42	+12,07
356		7	127	1	21	52,07	8	28	5,4	3,472	69	17	56	+12,06
357		7	127	4	22	51,97	8	28	17,4	3,465	69	25	36	+12,09
358		7	127	5	46	51,93	8	26	23,1	3,462	69	17	27	12,10
359	e Cancri	7	127	9	37	51,98	8	28	38,5	3,465	69	5	53	+12,11
360		7	127	13	8	51,93	8	26	5 5	3,462	69	41	12	+12,13
361		7	127	26	15	52,01	8	29	45,0	3,467	69	23	19	+12,19
362	γ Cancri	4	127	46	42	52,49	8	31	6,8	3,499	67	47	15	+12,28
363	1 η Cancri	6	127	54	17	49,80	8	31	37,1	3,320	70	34	38	+12,32
364	δ Cancri	4	128	10	55	51,42	8	32	43,7	3,428	71	4	57	+12,40
365	b Cancri	6	128	20	7	49,03	8	33	20,5	3,269	79	10	7	+12,44
366	Cancri	5	128	29	22	54,90	8	33	57,5	3,660	60	29	0	+12,48
367	2 a Cancri	6	128	51	7	49,59	8	35	24,5	3,306	77	[illegible]	51	+12,58
368		8	129	16	28	49,60	8	37	[illegible]	3,313	76	41	27	+12,70
369		9	129	[illegible]	12	51,28	8	38	44,8	3,419	71	10	1	+12,81
370		7 8	129	41	34	51,51	8	38	46,3	3,434	70	23	46	+12,81
371		6	129	49	41	50,47	8	39	18,7	3,365	73	52	54	+12,81
372		7,8	130	20	9	51,03	8	41	20,6	3,402	71	50	53	+12,98
373			130	28	11	51,80	8	41	52,7	3,453	69	14	57	+13,02
374		9	130	29	30	51,07	8	41	58,0	3,405	71	10	16	+13,0
375			130	30	59	50 16	8	42	3,9	3,344	74	48	3	+13,02
376		9	130	41	41	50,11	8	42	46,7	3,341	74	56	32	+13,07
377		9	130	47	52	48,43	8	43	11,5	3,229	80	57	15	+13,10
378		7	130	52	53	50 95	8	43	31,5	3 397	71	58	48	+13 12
379		8,9	130	59	26	50 07	8	43	57,7	3 338	75	1	27	+13 15
380	1 α Cancri	6,7	131	6	37	49 35	8	44	26 5	3,290	77	34	55	+13,15
381			131	19	36	50,90	8	45	18,4	3,393	72	3	29	+13,[illegible]
382	1 α Cancri	6	131	[illegible]	38	50,37	8	45	30,5	3,356	73	52	56	+13,25
383			131	24	54	50,98	8	45	39,6	3 399	71	44	45	+13,26
384	2 ο Cancri	6	131	27	32	50,44	8	45	50,1	3 363	73	37	24	+13,27
385	2 α Cancri	5,7	131	44	45	49 38	8	46	59,0	3,292	77	0	20	+13,34

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Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Distance			Annual Differ.
			D	M	S	S	H	M	S		D	M	S	
386		7	131	49	5	51,14	8	47	17,7	3,409	71	3	27	+13,37
387			131	58	18	49,70	8	47	53,2	3,313	76	7	6	+13,40
388	Cancri	6	132	36	25	52,96	8	50	25,7	3,532	64	43	55	+13,57
389		7,8	133	17	52	49,03	8	53	11,4	3,269	76	19	31	+13,75
390	κ Cancri	4,5	134	5	2[illegible]	48,95	8	56	2[illegible] 5	3,263	76	29	49	+13,95
391	ξ Cancri	5,6	134	18	53	52,08	8	57	5,5	3,47[illegible]	67	6	54	+14,01
392			134	33	50	52,04	8	58	15,3	3,469	67	9	42	+14,07
393			134	57	52	49,14	8	59	51,5	3,2[illegible]6	77	35	20	+14,17
394		7	135	54	14	49,97	9	3	36,9	3,331	74	11	49	+14,39
395		7	136	44	5	53,09	9	6	56,3	3,539	6[illegible]	45	[illegible]	+14,60
396		7	136	48	49	49,05	9	5	55,3	3,[illegible]70	77	37	37	+14,53
397		7,8	137	3	1	48,59	9	8	12,1	3,239	79	20	1	+14,67
398		8,9	137	11	47	50,97	9	8	47,2	3,398	70	1	44	+14,70
399		7	137	58	13	52,60	9	11	52,9	3,520	63	11	16	+14,89
400		7,8	138	4	[illegible]7	48,05	9	12	17,8	3,[illegible]3	81	23	34	+14,92
401	κ Leonis	4	138	5	50	[illegible],86	9	12	23,3	3,324	62	55	23	+14,92
402	ω Leonis	5	139	17	56	48,34	9	17	11,7	3,[illegible]21	80	2	14	+15,20
403	α Hydræ	2	139	19	4	44,24	9	17	16,2	2,949	97	45	21	+15,20
404	λ Leonis	4	139	55	35	51,74	9	19	4[illegible],3	3,449	66	6	53	+15,34
405	ξ Leonis	4	140	9	1	48,80	9	20	36,7	3,253	77	46	41	+15,39
406	h Leonis	6	140	10	1[illegible]	48,43	9	20	41,3	3,[illegible]29	79	21	59	+15,39
407		7,8	140	53	35	49,08	9	23	34,3	3,[illegible]72	76	25	1	+15,56
408	Leonis	6,7	141	21	28	49,94	9	[illegible]5	25,8	3,3[illegible]9	7[illegible]	37	5[illegible]	+15,66
409	Leonis	[illegible]	141	31	45	47,72	9	26	7,0	3,[illegible]81	6[illegible]	13	49	+15,69
410		[illegible]	141	57	58	49,16	9	27	51,9	3,277	75	41	58	+15,79
411	Leonis	5,4	142	29	0	48,36	9	29	56,0	3,224	79	9	35	+15,90
412	ψ Leonis	6	143	4	5	49,24	9	3[illegible]	16,3	3,283	75	1	30	+16,03
413	ε Leonis	3	143	28	[illegible]6	51,51	9	33	[illegible]3,7	3,434	65	16	1	+16,11
414		6,7	143	45	48	48,71	9	[illegible]5	3,2	3,247	77	13	9	+16,17
415		6	143	59	35	50,68	9	35	54,3	3,379	68	26	6	+16,[illegible]1
416		7,[illegible]	144	1	50	48,64	9	36	7,3	3,243	77	26	3	+16,23
417		6	144	4	24	48,61	9	36	17,6	3,241	77	36	6	+16,24
418		8	144	52	34	48,64	9	39	30,3	3,243	77	10	57	+16,40
419		7	144	54	44	48,90	9	39	38,9	3,260	75	57	32	+16,40
420		7,8	145	44	47	47,88	9	42	59,1	3,19[illegible]	80	56	29	+16,57

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb. of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees D M S	Annual Variat. +	Right Ascension in Time H M S	Annual Variat. +	North polar Distance D M S	Annual Preces.
421	Sextantis	6,7	146 19 22	47,97	9 45 17,5	3,198	80 4 44	+17 08
422	ν Leonis	4,5	146 43 36	48,65	9 46 54,4	3 243	76 33 39	16 76
423		7	146 44 48	47,87	9 46 59,2	3 188	80 41 22	+16 77
424	π Leonis	4	147 [illegible] 33	47,[illegible]5	9 49 [illegible],2	3,183	80 5 8	+16,87
425		8	148 [illegible] 3[illegible]	[illegible],06	9 52 22	3 04	79 4 32	+17 0[illegible]
426		8	148 13 4	48 39	9 52 52,3	3, 26	77 21 43	+17 04
427		8,9	148 29 31	47,68	9 53 58 1	3 179	80 59 45	+17 09
428		7,8	148 33 55	49,[illegible]	9 54 15,7	3, 78	73 13 49	+17 10
429	η Leonis	3,4	148 57 52	49 33	9 55 51,4	3 289	72 13 1	+17 18
430	a Leonis	5	149 11 14	48,02	9 56 44,9	3 201	78 58 45	+17 2
431	α Leonis	1	149 17 4	48 38	9 57 10 8	3 225	77 0 45	+17 4
432		8	149 35 32	47,92	9 58 22,1	3,195	79 23 1	+17 29
433		7,8	150 4 43	48 59	10 0 18,8	3, 39	75 36 48	+17 37
434		8	150 43 43	50,0[illegible]	10 2 54 9	3,331	67 47 30	+17 48
435	—	8	150 46 45	49,[illegible]5	10 3 7,0	3 270	72 49 31	+17 49
436			151 11 6	49,31	10 4 47,7	3 287	71 13 14	+17 56
437		6,7	151 20 48	48,56	10 5 23,2	3,237	75 13 47	+17 58
438		7,8	151 [illegible] 43	48,32	10 7 10,8	3,221	76 19 54	+17 66
439		6	152 4 15	49,54	10 [illegible] 17,0	3 303	69 28 2	+17 71
440	γ Leonis	2	152 5 16	[illegible]	10 [illegible] 21 1	3 397	69 6 1	+17 [illegible]
441		7	152 37 47	48 67	10 10 31 1	3 4[illegible]	7[illegible] 58 1[illegible]	+17 80
442		7,9	15[illegible] 47 18	47 64	10 11 [illegible]9 2	3 176	7[illegible] 58 3	+17,83
443		8	[illegible]53 17 49	47,88	10 13 [illegible]1,3	3 192	[illegible]8 1 13	+17 90
444		7	153 32 41	47,57	10 14 10,7	3 171	80 9 9	+17 95
445		7	154 8 9	47,09	10 1[illegible] [illegible]	3 170	79 10 18	+18 04
446		6	154 23 2[illegible]	48,41	10 17 3[illegible] 5	3 7	7[illegible] 35 [illegible]1	+18 08
447		7,8	154 31 12	47,73	10 1[illegible] 4,6	3 182	[illegible]8 46 [illegible]6	+18,10
448	1 Leonis	6	15[illegible] [illegible]4 31	48 30	10 [illegible]0 58 1	[illegible] 2 0	74 4[illegible] [illegible]3	+18 0
449	ρ Leonis	4	155 26 10	47 55	10 [illegible]1 44 7	[illegible] 170	79 [illegible]7 3	+18 [illegible]3
450	Leonis	7	155 57 36	47 17	10 [illegible]3 [illegible]0 4	3,14[illegible]	81 [illegible] 11	+18 [illegible]1
451		7	156 0 8	47 42	10 [illegible]4 0 5	3 161	80 16 [illegible]0	+18 [illegible]
452		9	156 37 10	48 66	10 26 8 7	3 44	[illegible]1 37 [illegible]0	+18 41
453		7	156 54 18	46,46	10 2[illegible] [illegible]7,[illegible]	3 31	[illegible] 47 [illegible]	+18 45
454		9	157 10 19	47,38	10 [illegible]8 41 [illegible]	2 159	80 4 5	+18 46
455	—	6	15[illegible] 6 43	4[illegible],79	10 32 26,9	[illegible] 119	84 9 16	+[illegible] 61

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Number of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees D M S	Annual Variat. + S	Right Ascension in Time H M S	Annual Variat. +	North Polar Dist. D M S	Annual Preces. S
456			158 38 48	4[illegible]	10 34 35,2	3,1[illegible]	81 3 2	+18,67
457	Sextantis	7,8	158 47 53	46,97	10 35 11,[illegible]	3,[illegible]31	82 31 2	+18,69
458	k Leonis	6	158 49 [illegible]	48,00	10 35 17,[illegible]	3,200	71 42 3	+18,70
459		8	159 5 55	46,96	10 36 23,[illegible]	3,131	8[illegible] 32 56	+18,73
460	l Leonis		159 30 5	47,47	10 [illegible]	3,165	78 20 48	+18,79
461		8,9	160 5[illegible] 55	46,82	10 43 51,7	3,121	82 2 17	+18,96
462	d Leonis	5	162 25 45	46,52	10 49 43,1	3,101	85 15 28	+19,11
463	c Leonis	5	162 27 56	46,79	10 49 51,[illegible]	3,[illegible]19	82 46 21	+19,12
464			163 17 18	46,50	10 53 9,2	3,100	85 13 56	+19,20
465	χ Leonis	4	163 32 54	46,87	10 54 11,6	3,125	81 31 55	
466	b Leonis	7,8	165 39 35	47,95	11 2 38,2	3,197	69 43 21	+19,43
467	δ Leonis	2,3	165 43 35	47,98	11 2 54,[illegible]	3,199	66 19 38	+19,43
468	ϑ Leonis	3	165 48 4	47,48	11 3 12,[illegible]	3,165	72 5 6	+19,43
469	n Leonis	6	166 12 55	47,5	11 4 51,7	3,1[illegible]	7[illegible] 32 57	+19,46
470	φ Leonis	4	166 30 5	45,62	11 0 0,2	3,[illegible]95	9 30 17	+19,49
471	σ Leonis	4,5	167 34 45	46,56	11 10 19,0	3,164	8 49 16	+19,57
472	ι Leonis	4	168 14 29	46,8[illegible]	11 1[illegible] 57,9	3,125	78 18 55	+19,62
473			168 19 8	46,20	11 1[illegible] [illegible]	[illegible]	67 26 6	+19,63
474		7	169 [illegible] 28	46,30	11 16 9,6	3,067	85 50 41	+19,68
475	τ Leonis	4	169 17 1[illegible]	46,[illegible]8	11 17 9,8	3,065	85 59 17	+19,70
476	e Leonis	4,5	169 53 59	45,91	11 19 35,9	3,061	91 50 44	+19,74
477		7,8	170 19 5	45,71	11 21 17,0	3,047	95 18 21	+19,76
478		7	170 31 9	45,63	11 [illegible] 7,9	3,[illegible]	96 40 9	+19,77
479	υ Leonis	4	17[illegible] 3[illegible] [illegible]	46,04	11 26 12,5	[illegible],069	[illegible]9 39 57	+19,63
480	ω Virginis	6	171 54 2[illegible]	46,[illegible]8	11 27 [illegible]7,9	[illegible],099	[illegible] 4[illegible] 14	+19,[illegible]
481		8	172 51 2	46,28	11 3[illegible] 24,1	3,06[illegible]	6[illegible] [illegible] 6	+19,69
482	1 ξ Virginis	5	173 36 50	46,39	11 34 [illegible]	3,093	80 34 [illegible]1	+19,93
483	ν Virginis	5	173 46 5	46,31	11 35 4,3	3,0[illegible]7	8[illegible] 17 34	+19,93
484	2 ξ Virginis		174 16 57	46,35	11 37 7,8	3,090	80 35 19	+19,95
485	β Leonis	1,2	174 35 22	46,55	11 38 21,4	3,103	74 15 12	+19,96
486	β Virginis	3	174 56 7	46,11	11 39 44,5	3,074	87 3 3	+19,97
487	a Virginis	6	176 4 3	46,25	11 44 16,2	3,083	80 23 26	+20,00
498		8	176 9 57	46,07	11 44 39,8	3,071	87 43 59	+20,06
489		7	176 52 10	46,11	11 47 28,7	3,074	85 1 10	+20,02
499			177 4 57	46,05	11 48 19,8	3,070	8[illegible] 18 8	+20,0[illegible]

MAYER CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names in the Place of the Stars	Magnitude	Right Ascension in Degrees	Annual Variat +	Right Ascension in Time	Annual Variat +	North Polar Distance	Annual Preces
			D M S	s	H M S		D M S	s
491	b Virginis	5,5	17[illegible] 18 2	46,10	11 49 12,1	3,073	85 10 28	+20,02
49	π Virginis	5	177 31 4[illegible]	46,14	[illegible] 50 6,7	3,076	82 12 50	+[illegible]0,02
493		7,8	[illegible] 5[illegible] 14	46,01	[illegible] [illegible] 20,9	3,067	90 35 50	+[illegible],03
494		7	176 [illegible]5 28	46,09	[illegible] 14 1,9	3,073	83 15 58	[illegible] 0,03
495	[illegible] Virginis	5	17[illegible] [illegible] 51	46,[illegible]	[illegible] 14 [illegible]	3,073	80 6 [illegible]	+20,04
496		[illegible],8	178 [illegible] [illegible]	47,01	[illegible]	3,067	91 57 34	+20,04
497			179 [illegible] [illegible]	46,03	[illegible]	3,069	88 12 [illegible]1	+ 0,04
498	r Virginis	6	179 44 11	46,0[illegible]	[illegible]	3,068	86 44 0	+20,05
499	s Virginis	6	180 [illegible]0 22	4[illegible]	[illegible]	3,069	83 1 31	+20,05
500		7,8	180 52 51	46,0[illegible]	[illegible]	3,069	94 33 19	+20,04
501	n Virginis	8	181 58 49	46,02	1[illegible] [illegible] 55,5	3,068	89 37 4	+20,03
502	ν Virginis	[illegible]	182 17 4	46,01	12 9 10,8	3,067	89 29 53	+20,03
503	c Virginis	5	182 [illegible]5 35	45,95	12 9 42,3	3,063	85 30 55	+20,03
504		7	182 58 2	45,90	12 11 52,1	3,060	83 31 27	+20,02
505		7	183 7 3	46,09	12 12 28,2	3,073	93 44 48	+20,02
506		8,9	183 23 14	4[illegible],21	1[illegible] 13 32,9	3,081	99 18 52	+20,01
507		7,8	184 16 36	46,1[illegible]	12 17 6,4	3,074	93 27 0	+19,99
508			184 4 9	45,87	12 17 36,6	3,058	84 26 21	+19,99
509		7	18[illegible] 4[illegible] 17	46,39	[illegible]2 19 17,1	3,093	102 13 32	+19,98
510			18[illegible] 1 15	46,11	1[illegible] [illegible]0 5,0	3,074	92 53 53	+19,97
[illegible]		7	18[illegible] 13 1	46,14	12 [illegible] 52,1	3,076	93 53 30	+19,96
512	q Virginis	6	185 44 [illegible]8	46,31	1[illegible] [illegible] 57,9	3,087	96 17 26	+19,95
513			18[illegible] 5[illegible] 40	46,03	12 23 30,7	3,069	[illegible]0 14 51	+19,94
514	f Virginis	6	186 29 54	46,20	1[illegible] [illegible]5 59,6	3,080	94 40 18	+19,92
515	χ Virginis	5	187 6 28	46,30	1[illegible] [illegible]8 [illegible]5,9	3,[illegible]87	96 50 11	+19,90
516			187 10 18	46,2[illegible]	[illegible] 48 41,1	3,0[illegible]3	94 56 30	+19,89
517	1 [illegible] Virginis	3	187 45 5[illegible]	46,03	12 31 3,3	3,069	90 17 41	+19,86
518	2 [illegible] Virginis	[illegible]	187 4[illegible] 54	46,03	12 31 3,5	3,069	[illegible]0 17 46	+19,86
519			18[illegible] 10 47	46,31	12 36 43,1	3,087	9[illegible] 9 0	+19,79
520			189 49 20	46,41	12 39 17,3	3,094	9[illegible] 29 12	+19,75
5[illegible]1		7	190 7 8	46,59	1[illegible] 40 28,5	3,106	99 [illegible] 5	+19,73
5[illegible]2	[illegible] Virginis	7	190 [illegible]6 58	[illegible]6,18	12 42 [illegible]7,9	3,079	92 [illegible] 27	+19,70
52[illegible]	ψ Virginis	5	190 51 50	46,58	12 43 [illegible]3	3,105	98 2[illegible] 38	+19,69
524	δ Virginis	3	191 1[illegible] 51	45,71	12 45 [illegible]4	3,047	85 27 9	+19,66
525	k Virginis	[illegible]	19[illegible] 13 1	46,22	12 48 52,1	3,[illegible]61	9[illegible] 10 [illegible]	+19,60

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb of Stars	Names and Places of the Stars	Magn	Right Ascension in Degrees			Annual Variat +	Right Ascension in Time			Annual Variat +	North Polar Distance			Annual [illegible]
			D	M	S	S	H	M	S	S	D	M	S	S
526	ι Virginis	3	192	56	1	45,06	[illegible]	51	44,1	[illegible]	77	54	30	+19,53
527		7	193	16	36	[illegible]	[illegible]	[illegible]	6,4	[illegible]	92	31	37	+19,51
528	g Virginis	5	194	13	47	[illegible]	12	56	[illegible]	[illegible]	99	36	46	+19,43
529	ϑ Virginis	4	194	16	[illegible]	46,41	12	59	6,3	3,094	94	[illegible]	46	+19,39
530	[illegible] Virginis		195	13	58	47,44	13	0	[illegible]	3,163	105	3	[illegible]	[illegible]
531			195	[illegible]	12	45,77	[illegible]	[illegible]	[illegible]	3,051	87	25	23	[illegible]
532			[illegible]	[illegible]	23	47,61	13	7	[illegible]	[illegible]	107	7	42	+19,18
533	γ Hydræ cont.	4	196	53	[illegible]	48,38	13	7	32,5	[illegible]	112	3	26	+19,18
534		9	197	45	[illegible]	47,27	[illegible]	11	3,7	[illegible]	101	[illegible]	23	+19,09
535	Virginis		197	57	2	[illegible]	[illegible]	[illegible]	40,1	3,191	106	37	43	[illegible]
536	α Virginis	1	198	32	21	47,15	13	14	9,4	[illegible]	100	[illegible]	[illegible]	[illegible]
537	ι Virginis	6	198	54	50	47,35	13	15	[illegible]	[illegible]	101	[illegible]	32	+18,97
538	Virgini		199	4	13	47,76	13	16	[illegible]	3,181	[illegible]	52	38	+18,95
539		7	199	37	[illegible]	46,05	13	18	29,8	3,070	90	16	4	+18,89
540	1 l Virginis	7	199	[illegible]	27	46,66	13	19	9,8	[illegible]	95	[illegible]	47	+18,86
541	2 l Virginis	6	[illegible]	16	9	46,65	[illegible]	21	[illegible]	[illegible]	95	9	50	+18,81
542	h Virginis	6	200	29	[illegible]	47,15	13	[illegible]	56,1	[illegible]	99	4	32	+18,78
543	ζ Virginis	3	201	0	[illegible]	45,96	13	24	1,7	[illegible]	89	30	59	+18,72
544	3 l Virginis	6	201	9	19	[illegible]	13	24	[illegible]	[illegible]	94	19	[illegible]	+19,70
545	m Virginis	6	[illegible]	39	18	47,05	13	30	37,2	[illegible]	97	38	11	+18,51
546		7,8	203	15	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6	1	[illegible]
547		7	203	[illegible]	43	[illegible]	[illegible]	33	[illegible]	3,192	[illegible]	[illegible]	[illegible]	+18,41
548			203	41	45	47,61	13	34	[illegible]	[illegible]	101	[illegible]	1	+18,36
549			204	0	[illegible]	[illegible]	13	36	1,6	[illegible]	106	47	56	[illegible]
550			[illegible]	37	[illegible]	[illegible]	13	38	29,6	3,239	107	4	16	+18,23
551	η Ursæ	2	[illegible]	48	58	[illegible]	[illegible]	39	15,9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
552	p Virginis	6	205	59	15	[illegible]	[illegible]	[illegible]	[illegible]	3,073	90	[illegible]	43	[illegible]
553		7	[illegible]	15	34	47,17	[illegible]	49	[illegible]	3,145	97	7	[illegible]	+17,82
554		7	208	18	59	47,40	13	53	[illegible]	3,160	[illegible]	14	[illegible]	[illegible]
555	ϑ Centauri	[illegible]	206	36	13	[illegible]	13	54	24,9	[illegible]	[illegible]	19	[illegible]	+17,60
556		7,8	208	48	7	47,36	13	55	12,5	3,157	97	52	[illegible]	+17,57
557		6,7	208	54	34	47,43	13	55	38,2	[illegible]	98	18	11	+17,55
558		8	206	57	[illegible]	48,65	13	55	49,4	3 43	105	10	33	+17,54
559		6,7	209	51	3	48,75	13	59	24,2	3,250	[illegible]	18	2	+17,38
560	κ Virginis	4	210	25	51	47,68	14	1	43,4	3 179	99	17	19	+17,29

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Number of Stars	Names and Places of the Star	Magnitude	Right Ascension in Degrees			Annual Variat +	Right Ascension in Time			Annual Variat +	North polar Distance			Annual Precess.
			D	'	"		H	'	"				"	
561			210	51	5[illegible]	[illegible] 91	14	3	27 5	3 127	94	57	48	+17,[illegible]
562	ι Virginis		211	1[illegible]	30	48,9[illegible]	14	5	2 0	3,129	94	59	8	+17,15
563	α Bootis		211	31	54	[illegible] 35	14	6	[illegible],6	2 809	69	41	[illegible]	+17,09
564	λ Virginis	4	2[illegible]	56	40	[illegible] 10	14	7	46,7	3,[illegible]33	10[illegible]	2[illegible]	41	+17,01
565		8	[illegible]12	1[illegible]	14				5[illegible],9	3 140	[illegible]	46	8	+16,96
566		8	2[illegible]	[illegible]	[illegible]	47,32	14	11	[illegible]	[illegible]55	96	47	35	+16,84
567	Libræ	8	213	[illegible]	2[illegible]	48 09	14	12	[illegible]7	3,[illegible]06	100	41	40	+16 81
568		8	[illegible]13	[illegible]9	17	48 45	14	1[illegible]	57,1	3,[illegible]30	10[illegible]	[illegible]3	34	+16,7[illegible]
569		[illegible]	214	6	0	48,49	14	16	24,0	3 233	102	4	19	+16 60
570		8	[illegible]14	[illegible]	8	47,82	14	17	20 5	3,188	99	3	5	+16,55
57[illegible]		7 8	[illegible]15	45	53	50,17	14	23	3,5	3 345	109	30	[illegible]5	+16,27
572		6	216	28	15	48 42	14	25	53,0	3,228	101	[illegible]4	[illegible]1	+16 12
573		[illegible]	[illegible]	[illegible]6	13	48 07	14	27	44,9	3,205	99	38	12	+16,02
574		[illegible]	[illegible]	[illegible]	7	48 48	14	30	40,5	3,232	101	19	33	+15,87
575	μ Virginis	[illegible]	[illegible]	[illegible]	[illegible]9	47,04	14	32	1,3	3,136	94	43	53	+15,80
576		7 8	21[illegible]	[illegible]4	38	[illegible]0,6[illegible]	[illegible]4	34	18,5	3,376	110	16	18	+15,67
577		9	[illegible]18	49	5[illegible]	50,[illegible]1	14	35	19 7	361	110	25	50	+15,62
578	μ Libræ		219	7	36	49 [illegible]	14	37	50,4	3, 68	103	15	37	+15,48
579	Libræ	[illegible]	219	[illegible]6	30	49 48	14	39	6 4	3,[illegible]99	105	6	40	+15,41
580	α Libræ		[illegible]9	49	36	4[illegible] 50	14	39	1,9	3,300	105	9	[illegible]4	+15,40
581	1 ξ Libræ	6	220	4[illegible]	10	46 57	1[illegible]	43	0 7	3, 38	101	1	46	+15,19
58[illegible]	ξ Libræ	6	221	[illegible]1	3	46,49	14	45	24 [illegible]	3,[illegible]33	100	33	1	+15,05
583			[illegible]	[illegible]	7	48,44	14	[illegible]	52,5	3,[illegible] 9	100	17	59	+14,97
584		6,7	[illegible]1	53	33	49 15	14	4[illegible]	34	[illegible] 30,[illegible]	100	17	13	+14,93
585	δ Libræ	[illegible],5	22[illegible]	26	47	47,64	14	4[illegible]	47,[illegible]	3 18[illegible]	97	40	26	+14,79
[illegible]		3	2[illegible]2	57	18	[illegible]	[illegible]	51	49,[illegible]	3,[illegible]82	114	26	33	+14,67
587	ι Libræ	5	2[illegible]3	44	[illegible]	49 54	14	54	56 8	3,32[illegible]	105	25	45	+14 48
588	Libræ	7 8	223	[illegible]	57	49 91	14	55	[illegible] 8	3 327	105	39	29	+14,47
589	Libræ	4	22[illegible]	4	18	50 90	15	0	17 2	3 393	[illegible]08	58	58	+14,17
590	α Libræ	[illegible]	2[illegible]5	20	52	50,89	15	1	23,5	3,393	[illegible]08	50	36	+14,09
591		7	[illegible]26	3	43	51,74	15	[illegible]	14,9	3,449	11[illegible]	[illegible]6	28	+1[illegible],92
592	β Libræ	2	226	26	7	48 21	15	5	44 4	3,214	9[illegible]	[illegible]5	44	+13 82
593		9	227	1	4	48 16	15	8	4,3	3,212	98	[illegible]1	57	[illegible] 13 6[illegible]
594	ι Libræ	7	2[illegible]7	19	40	49 91	15	9	16 7	3,327	104	[illegible]	41	+13,59
595		6	226	4	3	49 08	15	12	16 2	3,272	101	36	[illegible]	+13 39

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Numb. of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Distance			Annual Decreas.
			D	M	S	S	H	M			D			
596	Libræ	[illegible]	228	1	4	48,53	15	[illegible]	50,8	3,235	[illegible]9	33	9	+1[illegible],36
597	1 ζ Libræ	4	[illegible]29	6	38	50,3[illegible]	15	16	[illegible]	3,357	[illegible]5	58	13	+13,[illegible]2
598			229	[illegible]5	59	[illegible],59	[illegible]	17	43,9	3,[illegible]73	106	42	3	+13,94
599	3 ζ Libræ		229	42	51	[illegible]0,37	1[illegible]	[illegible]6	51,4	3,158	105	52	29	+1[illegible],97
600		7,8	229	55	2	51 43	15	[illegible]9	49,1	3,[illegible]9	109	25	57	+1[illegible],91
601		7	[illegible]	8	44	51 30	15	[illegible]	[illegible]	[illegible]20	108	56	25	+1[illegible],85
602	4 ζ Libræ		230	16	[illegible]8	50,48	15	2[illegible]	[illegible]5,2	3,305	106	7	[illegible]	+[illegible],82
603			230	28	56	54,01	15	21	55 9	3,601	117	[illegible]	24	+1[illegible],76
604	γ Libræ	3,4	230	57	4	49,[illegible]	15	23	48 3	3,3[illegible]8	10[illegible]	4	31	+12,63
605			231	4	45	54,1[illegible]	15	2[illegible]	[illegible]9,0	3 607	11[illegible]	25	[illegible]5	+12 60
606		8	231	13	40	53,50	15	24	54 7	3,567	[illegible]	15	34	[illegible]
607		9	231	34	29	49,67	15	26	17 9	3,3[illegible]5	[illegible]	[illegible]	[illegible]	+12,47
608		8	231	42	8	49,89	15	26	4[illegible]	3 184	[illegible]	[illegible]	[illegible]	+12 13
609			231	42	41	51,31	15	26	[illegible]9,6	3 070	[illegible]	35	39	+12,43
610	Libræ	7	231	58	33	5 76	15	27	[illegible]29 8	3 11[illegible]	[illegible]3	[illegible]	12	+12,36
611	κ Libræ	4	232	28	14	51,4[illegible]	15	[illegible]9	52,9	3,433	108	58	56	+12,2[illegible]
612		6	232	44	54	50 39	1[illegible]	30	59,6	3,359	105	19	29	+12,14
613	υ Libræ	4	233	4	21	50,31	[illegible]5	32	17,4	3,354	104	59	19	+12,05
614	α Serpentis		233	29	5	44,01	15	33	56 3	[illegible],934	82	54	7	+11,9[illegible]
615	b Scorpii	6	234	35	44	53,67	15	38	[illegible]2,9	3 [illegible]	115	5	48	+11,6
616	1 a Scorpii	5	235	15	32	[illegible]	15	41	2,[illegible]	3,[illegible]5	114	41	0	+1[illegible],42
617	λ Libræ	4	235	17	34	[illegible],86	15	41	10,3	3 457	109	[illegible]	[illegible]	+[illegible],41
618	θ Libræ	4	235	28	21	50 79	15	41	53 [illegible]	[illegible]86	106	5	56	+11,36
619	2 a Scorpii	7,	235	31	9	53,58	15	42	4,6	3 57[illegible]	114	36	16	+11,35
620		8	235	[illegible]	6	[illegible]1 65	15	43	24,4	3,443	[illegible]8	44	47	+11,[illegible]5
621	ε Scorpii	4,3	[illegible]	59	15	[illegible]	[illegible]	[illegible]	57,0	3,670	1[illegible]8	33	1	+1[illegible],21
622	π Scorpii	3	[illegible]36	32	38	54,00	[illegible]	[illegible]	[illegible]0,5	3 600	1[illegible]5	29	[illegible]	+11,06
623	ψ Libræ		2[illegible]6	36	51	50,09	15	46	2[illegible]	3,339	10[illegible]	[illegible]9	3[illegible]	+11,04
624	δ Scorpii	[illegible],2	236	59	11	5[illegible],82	15	47	56,7	3,[illegible]21	112	0	[illegible]7	+10 9[illegible]
625		7	237	40	17	54,01	15	50	41 1	3,601	[illegible]5	15	45	+10 72
626		4,5	238	12	44	49,26	15	52	50,9	3 284	100	46	43	+10,56
627	1 β Scorpii	4	238	18	46	51,97	15	53	15,1	3,465	109	12	52	+10,53
628	2 β Scorpii	8	238	18	55	51,97	15	53	15 7	3,465	109	12	40	+10 53
629	1 ω Scorpii	5	238	38	17	52,28	15	54	33 1	3,485	110	5	3	+10,43
630	2 ω Scorpii	5	238	46	47	52,36	15	55	7,1	3,491	110	17	4	+10,39

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Numb. of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Distance			Annual Preces.
			D	M	S		H	M	S		D	M	S	S
631	1 c Scorpii	[illegible]	239	40	[illegible]	55,17	15	59	19,1	3,678	117	51	10	+ [illegible],07
632	2 c Scorpii	6,5	239	[illegible]	57	54,[illegible]9	[illegible]	59	[illegible]3,8	3,666	117	[illegible]1	48	+10,07
633		8	[illegible]	56	58	54 96	15	59	47,9	3,464	108	53	0	+10,04
634	υ Scorpii	4	[illegible]	57	17	51,96	16	0	49,1	3 464	108	[illegible]	57	+10,03
635		8	[illegible]	32	40	54,13	16	[illegible]	11,2	3 600	[illegible]4	58	38	+ 9,86
636	[illegible] Scorpii	6,	[illegible]	41	3	55,40	16	5	[illegible]	3,693	118	4	[illegible]1	+ 9,6[illegible]
637	Scorpii		[illegible]	42	0 9	53 77	16	8	1,9	3,58[illegible]	113	38	47	+ 9,41
638	σ Scorpii	4	4[illegible]	6	47	54 31	16	8	27,1	3 6[illegible]1	11[illegible]	4	16	+ 9,[illegible]8
639		6	24[illegible]	[illegible]5	25	55 99	16	11	[illegible]9,[illegible]	3 7[illegible]3	119	11	37	+ 9,14
640	[illegible] Ophiuchi	5	242	57	37	5[illegible] 55	16	11	50,5	3 490	109	31	4[illegible]	+ 9 11
641	[illegible]	[illegible]	243	15	19	53 60	16	13	1,3	3,573	112	56	45	+ 9 02
642	χ Ophiuchi	[illegible]	[illegible]	43	10	51 85	16	14	52,7	3,457	107	5[illegible]	49	+ 8,88
643	α Scorpii	[illegible]	[illegible]	[illegible]	21	54,80	16	16	33,4	3 653	115	[illegible]6	55	+ 8 7[illegible]
644		[illegible]	[illegible]	[illegible]	47	54,88	16	18	31,1	3,659	116	3	53	+ 8,59
645	φ Ophiuchi	[illegible]	[illegible]	[illegible]	1	51,27	16	19	8,7	3,418	106	8	17	+ 8 54
646	ω Ophiuchi	5	[illegible]44	55	4[illegible]	[illegible] 99	16	19	42,8	3 33	111	0	0	+ 8,4[illegible]
647	τ Scorpii	4	245	4	31	55 64	16	[illegible]	50,1	3 [illegible]09	11[illegible]	45	4	+ 8,[illegible]
648		8	246	31	48	51,9[illegible]	16	26	19,1	[illegible] 461	10[illegible]	44	42	+ [illegible]9[illegible]
649		[illegible]	[illegible]47	3	16	52 73	16	28	13,1	3,[illegible]1[illegible]	109	[illegible]8	54	+ 7,81
650		8	247	17	1[illegible]	51,90	[illegible]	9	9,0	3 460	107	38	4	+ [illegible]74
651	m Scorpii	6	247	21	47	51,79	[illegible]	9	7,1	3 45[illegible]	10	19	6	+ [illegible],1
652		7	[illegible]	[illegible]3	38	52 58	16	[illegible]	34,5	[illegible] [illegible]0[illegible]	109	[illegible]0	3	+ 7 1
653		7	[illegible]47	58	27	55 94	16	[illegible]1	53,6	3 7[illegible]	11[illegible]	6	6	+ [illegible]
654	[illegible] Scorpii		[illegible]49	9	17	58 61	16	[illegible]6	3,1	3 99[illegible]	1[illegible]	[illegible]3	36	+ 7 14
655	Ophiuchi	8	249	14	35	54 47	16	36	[illegible]3	[illegible]31	114	1[illegible]	1	+ [illegible],11
656		8	24[illegible]	[illegible]	[illegible]8	[illegible]1 47	16	38	51,9	3[illegible]31	106	10	6	+ 6 96
657		7	250	[illegible]3	40	5[illegible],91	16	41	7	3,[illegible]2	110	[illegible]	48	+ 6,8
658	Scorpii	[illegible]	25[illegible]	53	1	58 28	16	43	3,1	3 [illegible]	1[illegible]	[illegible]4	[illegible]	+ 6,[illegible]
659		[illegible]	251	[illegible]	23	5[illegible] 99	16	44	9,5	[illegible] 59[illegible]	11	4	[illegible]	+ [illegible]
660		7,6	51	11	11	52 63	16	44	44,7	3 [illegible]09	[illegible]0[illegible]	11	[illegible]	+ 6 4[illegible]
661		7	251	46	54	54 60	16	4[illegible]	[illegible]6	[illegible]	[illegible]	[illegible]	1	+ 6 [illegible]
662		6,7	251	49	42	54 77	16	4[illegible]	1[illegible],6	[illegible],1	[illegible]	[illegible]9	[illegible]	+ 6 6
663	Ophiuchi	7,8	252	23	54	5[illegible],41	16	4[illegible]	[illegible]	[illegible] 4[illegible]0	10[illegible]	[illegible]	[illegible]	+ 6 0[illegible]
664		8	52	46	33	55 11	16	51	0	3 6[illegible]	1[illegible]	[illegible]	0	+ [illegible]9[illegible]
665		8	52	5[illegible]	0	55 09	16	51	49,3	[illegible],3	1[illegible]	[illegible]	[illegible]	[illegible] [illegible]66

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Numb of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees D	M	S	Annual Variat +	Right Ascension in Time H	M	S	Annual Variat + S	North polar Distance D	M	S	Annual Increases S
666		7,8	253	5	7	53,07	16	52	20,5	3,538	110	10	58	+5,83
667	Scorpii	7,8	253	25	12	[illegible]	16	53	[illegible]	3,566	111	15	18	[illegible]
668		7,8	253	28	21	[illegible]	[illegible]	53	53,4	[illegible]	116	12	37	[illegible]
669		6,7	254	1	0	52,03	16	[illegible]	4,0	[illegible]	107	18	48	+5,52
670	Ophiuchi	[illegible]	254	35	18	51,36	16	[illegible]	21,2	[illegible]	105	26	55	[illegible]
671		5	255	[illegible]	31	[illegible]	17	[illegible]	[illegible]	[illegible]	109	14	[illegible]	+4,99
672	α Ophiuchi	5	255	37	[illegible]	55,60	17	[illegible]	28,4	3,707	116	[illegible]	18	[illegible]
673	Scorpii		[illegible]	50	19	55,59	17	3	21,2	3,706	116	[illegible]	51	+4,90
674	ι		[illegible]	18	19	[illegible]	17	5	13,3	3 647	[illegible]	2	23	+4,74
675	[illegible]	8	[illegible]	19	[illegible]	54,62	[illegible]	[illegible]	18	3,641	[illegible]	49	17	+4,73
676	ε Ophiuchi	4	257	6	22	53,47	17	8	25,4	3,565	110	52	[illegible]	+4,48
677	Scorpii	8	257	12	27	55,02	17	8	49 8	[illegible]	[illegible]	[illegible]	59	+4 44
678	θ Ophiuchi	3	257	16	55	55,04	17	9	[illegible]	3 184	[illegible]	46	15	+4 42
679	Ophiuchi	7	257	32	22	56,39	17	10	29,8	3 070	[illegible]	55	17	+4,33
680			258	2	25	53 64	17	12	[illegible]	[illegible]	[illegible]	13	37	+4,16
681	Scorpii	7	258	4	12	[illegible]	[illegible]	12	16 8	3,651	114	2	0	+4,15
682	b Ophiuchi	5	258	23	23	64,75	17	13	33 5	3,650	113	57	46	+4,04
683	c Ophiuchi	5	259	39	13	54,71	17	18	[illegible]	3 647	113	46	52	+3,60
684	λ Scorpii	2,3	[illegible]	50	34	60,65	17	19	22,3	[illegible]	126	55	54	+3 54
685			260	11	41	[illegible]	17	20	46,7	[illegible]	107	19	36	+3 41
686	Ophiuchi	8	260	40	20	[illegible]	17	22	41,3	[illegible]	111	[illegible]	1	[illegible]
687	α Ophiuchi	2	261	17	59	[illegible]	17	[illegible]	11,9	2,769	77	16	18	+3,04
688		7	261	23	31	51,49	17	25	34 0	[illegible]	105	25	32	+3,00
689	ξ Serpentis		261	23	40	51 42	17	[illegible]	34,6	[illegible]	[illegible]	14	55	+3,00
690	β Draconis	3	[illegible]	[illegible]	5	[illegible]	17	25	44 3	1,348	37	32	12	+2,99
691		7	[illegible]	39	24	[illegible]	[illegible]	[illegible]	[illegible]	3 134	105	26	[illegible]	+2,56
692	d Ophiuchi	6	[illegible]	43	14	53,88	17	30	[illegible]	3 592	111	33	[illegible]	+2,54
693	p Sagittarii	[illegible]	263	35	16	56,49	17	34	21 1	[illegible]	[illegible]	43	55	[illegible]
694			263	50	1	56,1	17	35	20 0	[illegible]	[illegible]	[illegible]	45	+2 15
695		7 8	264	35	52	54 12	17	38	23,5	3,628	[illegible]	59	11	+1 98
696		7 5	264	59	15	[illegible]	17	39	57 0	3 527	109	2	52	+1 75
697		6	[illegible]	21	0	49 64	17	41	24 0	3 323	100	49	42	+1 63
698		7 6	265	33	37	52 81	17	43	34 5	3,521	108	44	51	+1 44
699		6	265	36	34	50 01	17	42	26,3	[illegible]	101	16	31	+1 54
700		7	265	55	40	53,65	17	43	42 7	[illegible]	[illegible]	53	49	+1 43

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Name in Places of the St	Mag.	Right Ascension in Degrees	Annual Variat. +	Right Ascension in Time	Annual Var. +	North polar Distance	Annual Preces.
		M	s	h	s	D M s	
	6	266 3 4[illegible]	51,66	[illegible] 44 14 7	3,444	105 45 22	+1 38
Sagittarii	6	[illegible]6 44 41	54,84	17 [illegible] 58,7	3,656	113 46 40	+1,14
Sagittarii		[illegible] 49 48	55,0[illegible]	17 4[illegible] 19,8	3,669	114 14 48	+1,11
	7 8	6[illegible] 5[illegible] 4[illegible]	53 43	17 47 [illegible]0,8	3,56[illegible]	119 18 16	+1 09
	6,7	6[illegible] [illegible]7 48	54 42	17 49 1[illegible]	3,628	112 45 23	+0 95
gittarii		267 29 51	55,04	17 49 59 4	3,669	114 15 44	+0,88
	8	267 34 43	54,40	17 50 18,8	3 627	112 41 57	+0,85
agittarii		267 45 2	55,03	17 51 0 1	3,672	114 20 45	+0,79
Draconi	2	267 56 3	20,83	17 51 45 5	1,389	98 28 44	+0,72
agittarii	4	268 4 5	57,77	17 52 19,5	3 851	120 24 23	+0 68
	7,8	268 39 3	53,89	17 54 36,2	3,593	111 26 50	+0 47
	6	268 41 [illegible]5	56,88	17 54 47 0	3 792	118 27 35	+0 46
	[illegible],7	269 8 1[illegible]	57 94	17 56 33,2	3 863	120 44 31	+0,30
	[illegible]	269 43 33	54,83	17 58 54,2	3 655	113 43 28	+0,10
Sagittarii	4	[illegible] 18 41	[illegible],75	18 1 [illegible],7	3,583	111 5 44	−0 10
	7	270 24 45	54,02	18 1 38 9	3 601	111 44 59	−0,14
Sagittarii	4	270 40 23	53,62	18 2 41 5	3,575	110 46 20	−0, 3
	7	270 [illegible] 50	53,51	18 2 43,3	3,567	110 25 54	−0 23
agittarii	3	[illegible]71 53 8	57,54	18 7 32,5	3,836	119 53 51	−0,66
agittarii	4,3	272 33 30	59,76	18 10 14 0	3,984	124 27 47	−0 90
	6	273 12 41	53,56	18 1[illegible] 50,7	3 571	110 38 7	−1,12
gittarii	4	27[illegible] 45 8	55,57	18 15 0,5	3,705	115 31 2	−1,31
	7,8	[illegible]74 6 10	55,50	18 16 24,7	3,700	115 22 14	−1,43
	7	274 27 49	52,84	18 17 51,3	3,52[illegible]	108 50 37	−1,56
	6	274 45 7	[illegible]	18 19 0 5	3,5[illegible]	109 1 16	−1,66
	7	274 47 [illegible]	52,71	18 19 8 1	3 514	108 31 4	−1,68
	8	275 [illegible]	52,94	18 20 8,1	3,529	109 6 18	−1,76
	6,7	[illegible] 10 58	51,37	18 0 49,8	3 425	105 0 6	−1,81
		275 1[illegible] 40	52,70	18 20 50 7	3,513	10[illegible] 30 7	−1,82
	7	275 15 49	54,98	18 21 3 3	3,665	114 0 20	−1,84
	6,7	275 24 47	51 36	18 21 39 1	3,4[illegible]4	104 [illegible] 36	−1,89
	8	275 25 25	55,06	18 21 41,6	3,651	114 2 57	−1,90
	7	[illegible]75 45 7	53,05	18 23 0,5	3 537	109 24 51	−[illegible],01
	[illegible]	276 11 13	53,02	18 24 44,9	3,535	199 21 56	−2 16
		276 19 56	53,89	18 25 19,7	3,593	111 33 10	−2 20

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magn	Right Ascension in Degrees			Annual Variat +	Right Ascension in Time			Annual Variat +	North Polar Distance			Annual [illegible]
			D	M	S	s	H	M	S					
736		7,8	276	25	1	52,26	18	25	40,1	3,454	107	23	30	—,24
737		7	276	40	5	54,75	18	[illegible]	44,3	3,650	113	39	57	—2, 4
738		7	276	55	33	53,74	18	26	[illegible],2	3,583	111	11	11	—2,30
739	α Lyræ	1	277	27	[illegible]	30,16	18	9	48,5	2,011	51	4	17	—2,59
740	φ Sagittarii	4	278	7	55	56,21	18	32	31,7	3,[illegible]47	117	11	19	—2,84
741		7,8	278	[illegible]	17	53,16	18	33	37,1	3,544	109	48	29	—2,92
742		8	278	59	58	53,44	18	35	23,9	3,561	110	28	58	—3,09
743			279	18	6	53,43	18	37	12,4	3,562	120	32	15	—3,[illegible]4
744		8	279	13	18	54,16	18	38	13,2	3,611	112	23	6	—3,33
745		8	279	52	53	54,06	18	39	31,5	3,604	11[illegible]	[illegible]	54	—3,44
746			280	21	43	53,83	18	41	26,9	3,560	111	35	59	—3,60
747	1 ν Sagittarii	5	280	22	20	54,38	18	41	29,3	3,625	112	59	7	—3,61
748	σ Sagittarii	3	280	32	58	55,86	18	42	11,6	3,724	126	32	18	—3,67
749	2 ν Sagittarii	5	289	36	15	54,35	18	4	25,0	3,623	112	54	55	—3,69
750	1 ξ Sagittarii	7	281	12	53	53,53	18	44	51,5	3,59[illegible]	110	54	46	—3,90
751	2 ξ Sagittarii	4	281	17	58	53,71	18	45	11,9	3,582	111	21	50	—3,98
752		8	281	25	55	53,44	18	45	43,7	3,563	112	41	9	—3,98
753		6,7	282	14	33	54,32	18	48	58,[illegible]	3,621	112	58	[illegible]5	—4,25
754	ζ Sagittarii	3	282	18	36	57,40	18	49	14,4	3,827	120	9	42	—4,27
755		7	282	40	4	54,19	18	50	40,2	3,616	115	6	55	—4,40
756	ο Sagittarii	4	283	1	23	53,92	18	52	5,5	3,595	112	[illegible]	52	—4,52
757	τ Sagittarii	4	283	27	20	56,36	18	53	49,[illegible]	3,757	177	57	24	—4,66
758		7	283	34	9	56,79	18	54	16,6	3,786	116	56	2[illegible]	—4,70
759		8	283	35	12	54, 1	18	54	20,1	3,614	11[illegible]	48	6	—4,71
760		7,8	283	[illegible]	[illegible]	55,08	18	55	[illegible]4,3	3,67[illegible]	114	57	57	—4,79
761	π Sagittarii	3	284	19	4	53,61	18	57	16,[illegible]	3,574	111	0	23	—4,95
762		7,8	284	21	11	53,18	18	57	2[illegible]	3,5[illegible]	110	7	40	—4,96
763		8	284	28	42	53,84	18	59	54,9	3,589	111	59	20	—5,16
764		8	284	59	45	51,18	18	59	57,7	3,412	[illegible]4	54	59	—5,19
765		6,7	285	4	26	55,56	19	0	17,3	3,704	116	14	0	—5,21
766	ψ Sagittarii	6	285	30	49	55,[illegible]7	19	2	39,3	3,685	115	36	1	—5,41
767		7	285	41	12	54,82	19	2	44,8	3,655	114	31	4	—5,42
768		9	285	55	4	52,17	19	3	40,3	3,478	107	41	27	—5,50
769	d Sagittarii	6	286	20	11	52,76	19	5	20,7	3,517	109	18	31	—5,64
770		8,9	286	28	36	52,72	19	5	54,0	3,515	109	13	2	—5,68

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb. of Stars	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North polar Distance			Annual Preces.
			D	M	S	S	H	M	S	S	D	M	S	S
771		6	266	45	25	51,48	19	7	1,7	3,432	105	52	52	−5,78
772		7	267	0	40	54,0	19	8	2,7	3,605	11	46	33	−5,86
773	1 ρ Sagittarii	5	87	22	0	52,32	19	9	9,3	3,488	108	13	3	−5,98
774	2 ρ Sagittarii	6	87	23	49	52,49	19	9	35,3	3,499	108	40	49	−5,99
775	υ Sagittarii	6	287	25	27	51,63	19	9	41,8	3,442	10	19	56	−6,0
776	1 χ Sagittarii	5	268	7	16	54,86	19	1	9,	3,657	114	53	51	−6,24
777	3 χ Sagittarii	5	268	11	9	54,64	19	12	46,6	3,643	114	21	16	−6,7
778		7	288	7	2	53,77	19	1	48,1	3,565	112	10	30	−6,3
779		7	288	33	3	51,29	19	14	13,5	3,41	105	30	41	−6,39
780		7,8	88	42	58	53,76	19	14	51,9	3,584	112	10	7	−6,43
781		7,8	289	9	36	51,30	19	16	38,4	3,420	105	30	47	−6,58
782	q Sagittarii		289	12	57	55,83	19	16	51,8	3,7 2	117	23	45	−6,59
783		8	289	23	55	51,36	19	17	35,7	3,424	105	46	26	−6,66
764		8	269	36	27	53,55	19	16	25,8	3,570	111	43	52	−6,73
785		7,6	269	42	2	53,54	19	18	49,5	3,569	111	43	51	−6,76
786		6	290	28	1	54,50	19	21	52,8	3,633	114	17	44	−7,01
787	1 h Sagittarii	5	290	48	59	54,82	19	23	15,9	3,655	115	9	41	−7,12
788	2 h Sagittarii	5	290	56	44	54,88	19	23	54,9	3,659	115	19	49	−7,18
789		6	291	2	33	5 ,57	19	24	10,2	3,505	109	17	55	−7,20
790		8	291	12	57	52,34	19	24	51,8	3,489	108	40	53	−7, 6
791		8	291	47	50	54,27	19	27	11,3	3,618	113	53	18	−7,44
792		7,6	291	52	15	54 6	19	27	29,0	3,617	113	53	28	−7,47
793	1 e Sagittarii	6	292	10	14	51,61	19	28	40,9	3,441	106	45	30	−7,56
794	e Sagittarii	6,5	292	37	25	51,54	19	30	29,7	3,436	106	36	1	−7,71
795		6	292	53	30	51,30	19	31	14,0	3,420	105	56	21	−7,79
796	f Sagittarii	6	293	31	39	52,60	19	34	6,6	3,5 9	110	14	56	−8,00
797		8	293	54	13	53,22	19	35	36,9	3,548	111	27	26	−8,12
798		7,8	294	3	41	50,65	19	36	14,7	3,377	104	12	15	−8,17
799			294	17	14	50,18	19	3	6,9	3,345	102	49	31	−8,24
800	α Aquilæ	1	95	7	46	4 ,37	19	40	31,1	2,891	81	40	36	−8,51
801	ω Sagittarii	5	295	44	14	55,16	19	42	56,9	3,677	116	0	24	−8,70
802	b Sagittarii	5	296	0	38	55,48	19	44	,5	3,699	117	42	34	−8,79
803	g Sagittarii	5	296	30	6	51,17	19	46	1,7	3,411	106	1	55	−8,94
804	a Sagittarii	5	296	3	8	55,06	19	46	8,5	3,671	116	14	1	−8,96
805		6	97	5	51	55,59	19	49	43,4	3,706	118	16	40	−9,23

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Number of Stars	Name and Letters of the Stars	Magnitude	Right Ascension in Degrees D	M		Annual Variat. + s	Right Ascension in Time H	M		Annual Variat. + s	North polar Distance D	M		Annual Precess.
806	Sagittarii	7	297	32	57	50,52	19	50	11,8	3,368	104	12	19	— 9,7
807		8	297	49	13	53,61	19	51	16,9	3,574	113	10	9	— 9,36
808		8,9	297	54	47	51,09	19	51	39,1	3,406	105	59	7	— 9,38
809			298	8	44	53,12	19	52	34,9	3,541	111	53	9	— 9,45
810	Sagittarii	8	298	22	16	49,83	19	53	29,1	3,3	102	10	45	— 9,52
811	Sagittarii	7	29	6	44	50,17	19	53	44,9	3,345	103	14	37	— 9,54
812		8	299	1	1	52,19	19	56	4,1	3,479	109	3	47	— 9,73
813		8	299	9	19	50,91	19	56	37,3	3,394	105	37	0	— 9,77
814		7,8	299	15	32	49,31	19	57	2,1	3,287	100	39	21	— 9,60
815			299	19	0	52,79	19	57	16,0	3,519	111	11	1	— 9,62
816		9	299	33	38	[illegible]	19	58	14,5	3,491	109	58	41	— 9,89
817	Capricorni	6	300	4	45	50,03	20	0	10,0	3,335	103	0	6	— 10,04
818	ξ Capricorni	6	300	10	41	50,09	20	0	42,7	3,339	103	13	1	— 10,07
819	1 α Capricorni	4	301	11	8	49,97	20	4	44,5	3,331	102	57	55	— 10,38
820	2 α Capricorni		301	29	58	50,01	20	5	55,9	3,334	103	8	34	— 10,47
821	3 α Capricorni	3	301	35	50	50,02	20	6	23,3	3,335	103	10	54	— 10,50
822	σ Capricorni	6	301	48	57	52,14	20	7	15,8	3,476	109	45	31	— 10,56
823		6,7	302	14	26	50,69	20	8	57,7	3,380	105	25	59	— 10,69
824	ι Capricorni	6	302	15	3	50,06	20	9	0,2	3,337	103	24	21	— 10,69
825	β Capricorni	3	302	17	59	50,70	20	9	11,9	3,380	105	5	47	— 10,71
826		7,8	303	9	53	50,49	20	12	39,5	3,366	104	54	52	— 10,97
827		7,8	303	22	24	50,44	20	13	29,6	3,363	104	46	29	— 11,03
828		8	303	3	27	49,69	20	13	33,8	3,313	102	22	21	— 11,04
829			303	30	1	52,14	20	14	0,1	3,476	110	5	51	— 11,07
830	π Capricorni	6	303	49	18	51,72	20	15	17,	3,448	108	53	8	— 11,16
831	ρ Capricorni	6	304	13	1	51,57	20	16	52,	3,438	108	29	57	— 11,27
832		8	304	15	1	51,58	20	17	0,1	3,4[illegible]9	108	32	59	— 11,28
833		8	304	15	6	51,44	20	17	0,4	3,429	108	6	57	— 11,8
834	1 Capricorni *	8	304	7	16	51,85	20	17	49,1	3,457	109	25	56	— 11,34
835	2 Capricorni	6	304	27	40	51,85	20	17	50,6	3,457	109	25	49	— 11,34
836		8,9	305	5	15	53,89	20	20	21,0	3,593	115	38	17	— 11,53
837		8	305	9	13	51,13	20	20	36,9	3,409	107	18	22	— 11,54
838		6	305	13	31	49,05	20	20	54,1	3,270	100	29	55	— 11,57
839		7	305	37	16	50,21	20	22	29,1	3,347	104	24	47	— 11,68
840		7,8	305	54	23	51,06	20	23	37,5	3,404	107	13	56	— 11,76

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* There appears to be a miſtake of 10 in the polar diſtance of this ſtar; it ſhould probably be 109 15 56.

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitud	Right Ascension in Degrees	Annual Variat +	Right Ascension in Time	Annual Variat +	North polar Distance	Annual Preces
			D M	s	M		D M s	
841		8	306 3 59	52 34	20 24 15,9	3 489	111 17 48	−11,81
842		8	306 23 22	50 60	20 25 33,5	3,373	105 51 35	−11,90
843		8	306 28 31	51,21	20 25 54 1	3 414	107 50 20	−11 93
844	τ Capricorni	6	306 52 43	50 55	20 27 30,9	3,3[illegible]0	105 40 39	−12 04
845	υ Capricorni	6	307 1 12	51,49	20 28 4,8	3,433	108 51 50	−12 07
846		7	307 10 51	50 86	20 28 43,4	3,391	106 51 28	−12,12
847	ψ Capricorni	5	308 24 35	53,68	20 33 38,3	3,579	116 0 36	−12 46
848	α Cygni	1	308 33 57	30,59	20 34 15 8	039	45 27 44	−12 51
849	Aquarii	5	309 4 25	46 83	20 36 17 7	3 255	100 15 6	−12,64
850		7	309 37 1	51,28	20 38 28,1	3,419	108 47 45	−1[illegible],9
851	ω Capricorni	6	309 48 54	54,10	20 39 15 6	3 607	117 41 6	−12 84
852		8	310 13 39	49,83	20 40 54 6	3 322	103 58 51	−12,95
853	μ Aquarii	4,5	310 19 41	48,65	20 41 18 7	3 243	99 45 30	−12,98
854		7	310 23 51	49,36	20 41 35 4	3 91	102 21 19	−13,00
855		7	310 43 44	51,16	20 42 54 9	3,411	108 42 24	−13,08
856		7,8	311 4 14	53,75	20 44 16,9	3 583	117 4 55	−13,17
857		7	311 28 28	50,56	20 45 53 9	3 371	106 49 39	−13,28
858		7,8	311 54 46	51,39	20 47 39,1	3,426	109 50 12	−13,39
859		7	312 15 5	50 94	20 49 1,7	3 396	106 20 4	−13 48
860	Aquarii	7,8	312 23 20	49,80	20 49 33,3	3,320	104 20 19	−13 52
861		7,8	312 37 50	49 29	20 50 31 3	3 286	102 30 2	−13,58
862	η Capricorni	5	313 6 33	51 54	20 52 26,2	3,436	110 40 19	−13,70
863	ϑ Capricorni	5	313 31 50	50,75	20 54 7 3	3,383	108 3 17	−13 81
864	a Capricorni	6	313 42 20	53,03	20 54 49,3	3 535	115 49 55	−13,85
865	1 χ Capricorni	6	314 7 41	51,84	20 56 30,7	3 456	112 1 27	−13,97
866	3 χ Capricorni	6	314 22 46	51,63	20 57 31 1	3,442	111 23 8	−14 03
867	ν Aquarii	5	314 32 4	49 11	20 58 8,3	3,274	102 12 37	−14 06
868		7,8	315 1 7	49 90	21 0 4,5	3,327	105 19 7	−14 18
869		7,8	315 29 6	52,01	21 1 56 4	3 467	113 3 54	−14,30
870	φ Capricorni	6	315 54 54	51,52	21 3 39,6	3,435	111 30 40	−14 40
871		6	316 32 23	50,73	21 6 9,5	3 382	108 51 12	−14 55
872		7 8	316 37 40	51 36	21 6 30 7	3 424	111 12 10	−14 5[illegible]
873		7,8	316 53 27	50,22	21 7 33,8	3,348	107 3 1	−14 63
874		8,9	317 16 2	51,43	21 9 4,1	3,429	111 41 40	−14 7
875	ι Capricorni	5	317 38 5	50 33	21 10 32,3	3,355	107 43 1	−14,81

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb. of Stars	Names and Place of the Stars	Magn.	Right Ascension in Degrees			Annual Variat. +	Right Ascension in Time			Annual Variat. +	North Polar Distance –			Annual Preces.
			D	M				M		s	D	M		
876		8	318	1	23	51 89	21	12	5,5	3,459	113	35	1	−14 90
877		7	318	3	31	51,36	21	12	14,1	3,424	111	41	53	−14,91
878		6	318	10	28	49 29	21	12	41,9	3,266	103	46	7	−14,94
879	ζ Capricorni	5	318	39	5-	51 72	21	14	39,5	3,448	113	16	1	−14,95
880	b Capricorni	6	319	10	51	51 50	21	16	43,4	3,433	11	4-	3-	−15 17
881		7	319	12	39	48,91	21	16	50,6	3 261	10	28	2	15 16
882		7,8	319	32	46	50 77	21	18	11,1	3 385	110	3	8	−15 25
883		8	319	47	16	49 54	21	19	9,2	3,303	105	11	57	−15 31
884		9	319	54	11	50 76	21	19	36,7	3,384	110	6	50	−15,34
885	β Aquarii	3	320	7	28	47,48	21	20	9 9	3,165	96	29	5	−15,39
886		9	320	30	44	49 94	21	22	2,9	3,329	107	6	37	−15,47
887		6,7	320	45	35	50,89	21	23	2 3	3,393	111	0	34	−15,53
888			320	46	41	50 93	21	23	6,7	3,395	111	10	14	−15,53
889	Capricorni	4	321	19	37	50,66	21	25	18 4	3,379	110	23	14	−15,65
890	ξ Aquarii	5	321	38	6	47,94	21	26	33,7	3,196	98	47	8	−15,72
891	γ Capricorni	4,3	322	6	30	49,93	21	28	26,0	3,329	107	36	0	−15,82
892	1 d Capricorni		322	31	47	49 28	21	30	7,8	3,285	104	58	15	−15,91
893	κ Capricorni	5	322	43	38	50,39	21	30	54 6	3 359	109	48	47	−15,95
894		7,8	322	51	52	50 56	21	31	27 5	3,371	110	34	6	−15,98
895	1 c Capricorni	6	323	27	0	48 13	21	33	48 0	3,209	100	2	13	−16,10
896	λ Capricorni	5	323	48	18	48,61	21	35	13,2	3 241	102	19	28	−16,18
897	δ Capricorni	3	323	51	25	49,65	21	35	25,7	3,310	107	4	2	−16,19
898		7,8	324	34	54	48,85	21	38	19,6	3,257	103	41	33	−16,34
899		8	324	39	45	49 75	21	38	36,9	3,317	107	48	49	−16,35
900		8	325	0	12	50,11	21	40	0 8	3,341	109	35	26	−16,42
901	μ Capricorni	5	325	7	20	48,96	21	41	49,3	3,264	104	31	51	−16,51
902		6	325	53	14	49,29	21	43	3 ,9	3,286	106	14	21	−16,60
903		8,9	326	17	45	49 8-	21	45	11,0	3,321	108	53	7	−16 68
904		9	326	35	1-	49,21	21	46	20,8	3 281	106	6	49	−16 74
905		7	326	44	58	50,50	21	46	59,9	3 367	112	10	34	−16,77
906		8	326	46	2	46 69	21	47	4,1	3,246	103	39	43	−16,77
907		8	327	39	19	49,69	21	50	37,3	3,313	108	54	4	−16,94
908	Aquarii	5	328	6	47	46,60	21	52	21 1	3,107	93	9	39	−17,02
909	α Aquarii	3	328	45	3	46 26	21	55	0 2	3,084	91	19	55	−17,14
910	ι Aquarii	4	326	46	13	48,78	21	55	4,9	3 252	104	52	44	−17,14

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb. of Stars	Names & Places of the Stars	Mag	Right Ascension in Degrees	Annual Variat. +	Right Ascension in Time	Annual Variat. +	North polar Distance	Annual Preces.
			M		H M		D M	
911	e Aquarii	6	3-9 50 49	48 27	21 59 23,ɔ	3,218	102 35 -0	— 1/,33
912		7,8	330 3- 58	48,-9	22 2 11,9	3,-19	102 57 19	— 17 45
914	ϑ Aquarii	4	331 26 8	47,51	22 5 44,5	3 167	98 49 15	— 17,60
914		7	331 -6 43	47,7-	22 5 46 8	ɔ 181	100 4 4	— 17 60
915	ϱ Aquarii	5,6	332 17 10	47,17	-2 9 8 7	3 165	98 5- 4	— 17,74
916	γ Aquarii	3	332 42 8	46,41	2- 10 48 5	3,094	92 26 21	— 17,81
917		6	333 16 37	46 95	22 13 6 5	3,130	95 53 29	— 17 90
918		7	333 26 23	46,37	22 13 45,5	3,091	92 14 46	— 17,93
919		7,6	333 5- 56	47 94	22 15 31 7	3,196	10- 17 16	— 17,99
920	ζ Aquarii	4	334 30 17	46,18	-- 18 1,1	3,0/9	91 5 13	— 16,09
921	σ Aquarii	5	334 52 5-	47,79	22 19 31,5	3 186	101 44 4	— 18,15
92		7,8	335 8 8	47 81	22 20 32 5	3,187	101 58 30	— 18,19
923	η Aquarii	4	336 8 3-	46,19	22 24 34,1	3,079	91 11 33	— 18,34
924	κ Aquarii	[illegible]	336 43 12	46,75	22 26 5- 8	3,117	95 17 12	— 18,42
925		8	337 15 35	47,45	22 -9 2,3	3,163	100 26 55	— 18,49
926		8	337 16 47	4/ 05	22 29 15 1	3,137	97 37 13	— 16,50
927		8	337 59 3-	47,51	22 31 58,1	3,167	101 11 46	— 18,59
928		7	338 0 45	47,26	2- 3- 3 0	3,151	99 21 1	— 18,59
929		8 9	338 3 18	47,11	-2 32 13,1	3,141	98 18 32	— 18,60
930		7	338 4 2	47,08	22 3- 16,1	3,139	98 3 -2	— 16,60
931		9	338 34 31	47,41	22 34 18,1	3,161	100 44 19	— 16,66
932	1 τ Aquarii	6	339 8 11	47,96	22 36 3 ,7	3,197	105 10 23	— 18,74
933	2 τ Aquarii	6	339 36 56	47,85	22 38 27,7	3,190	104 41 40	— 16,79
934			339 57 20	47,04	-2 39 49,3	3,136	95 25 [illegible]	— 16,83
935			339 57 20	47,04	22 39 49 3	3,136	98 25 4	18,83
936	λ Aquarii	4	340 24 50	47,05	22 41 39,3	3,137	98 41 30	— 16,89
937	Aquarii	3	340 52 26	48,02	2- 43 9,7	3,201	106 55 53	— 18,95
938	α Piſcis Auſt	1	341 30 1	49,81	22 46 0,1	3,321	120 13 39	— 19 01
939		7	342 20 -2	47,09	22 49 -1,5	3 139	100 0 1	— 19 10
940		6,7	342 37 7	46,88	22 50 28,5	3,125	98 10 0	— 19 13
941	β Piſcium	4	343 18 0	45,75	22 53 12,0	3,050	87 18 22	— 19 20
942	β Pegaſi	2	343 24 2	43 11	-- 53 36,1	2,874	63 3 11	— 19 21
943	1 h Aquarii		343 33 4	46 90	22 54 12,3	3,127	98 49 20	— 19 22
944	α Pegaſi	2	343 34 38	44,60	22 54 18,5	2,97ɔ	75 55 1-	— 19 23
945	2 h Aquarii	7	3+3 35 37	46,91	22 54 22,6	3,127	98 54 7	— 19,23

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb of Star	Names and Places of the Stars	Magnitude	Right Ascension in Degrees			Annual Variat +	Right Ascension in Time			Annual Variat +	North polar Distance			Annual Preces
			D	M	S	s	H	M	S		D	M	"	s
946	3 h Aquarii	6,7	343	41	10	46,92	22	54	56,7	3,128	99	3	57	−19,24
947	4 h Aquarii	9	344	4	9	46,88	22	56	10,6	3,125	96	50	12	−19,28
948	a Piſcium	6	344	28	56	45,95	22	57	55,7	3,062	89	0	44	−19,32
949		7	344	39	52	45,93	22	58	39,5	3,062	66	59	28	−19,33
950		8	344	57	8	46,67	22	59	48,5	3,111	97	5	46	−19,36
951	φ Aquarii	5	345	51	41	46,64	23	3	26,7	3,109	97	10	29	−19,44
952	1 ψ Aquarii	5	346	13	6	46,88	23	4	52,4	3,125	100	13	37	−19,47
953	χ Aquarii	6	346	29	29	46,75	23	5	57,9	3,117	98	52	3	−19,49
954	γ Piſcium	4	346	33	59	45,65	23	6	15,9	3,057	87	51	40	−19,50
955	2 ψ Aquarii	5	346	44	48	46,86	23	6	59,2	3,124	100	19	28	−19,51
956	3 ψ Aquarii	5	347	0	30	46,88	23	8	2,0	3,125	100	45	13	−19,53
957	b Piſcium	5,6	347	24	47	45,70	23	9	39,1	3,047	85	45	36	−19,56
958		6,7	348	11	30	46,08	23	12	46,0	3,072	90	51	29	−19,62
959		7,8	348	13	7	46,71	23	12	52,5	3,114	99	36	29	−19,62
960	1 κ Piſcium	5	349	2	39	46,01	23	16	10,6	3,067	89	53	24	−19,68
961	2 κ Piſcium	6,7	349	7	51	46,02	23	16	30,1	3,068	90	1	28	−19,69
962	ϑ Piſcium	5	349	20	2	45,68	23	17	20,1	3,045	84	46	16	−19,70
963		8	349	42	14	46,16	23	18	48,9	3,077	92	11	18	−19,72
964		7	349	50	10	46,20	23	19	20,7	3,080	92	56	33	−19,73
965		7,8	350	10	8	46,33	23	20	40,5	3,089	95	13	46	−19,75
966		7	350	17	52	46,15	23	21	11,5	3,077	92	14	32	−19,76
967		6,7	350	50	23	46,15	23	23	21,5	3,077	92	24	14	−19,79
968		7,8	351	10	34	46,48	23	24	42,3	3,099	98	37	25	−19,81
969		7	351	11	22	46,01	23	24	45,4	3,067	89	50	35	−19,81
970		6,7	351	25	19	45,97	23	25	41,3	3,065	89	3	37	−19,82
971	Piſcium	5	352	17	15	45,81	23	29	9,0	3,054	85	30	16	−19,86
972	λ Piſcium	5	352	50	15	45,99	23	31	21,0	3,066	89	22	18	−19,89
973		8	353	31	43	45,78	23	34	6,9	3,052	83	58	14	−19,92
974		6,7	353	55	11	45,93	23	35	40,7	3,062	87	40	33	−19,94
975		6,7	354	6	50	46,49	23	36	27,3	3,099	103	4	11	−19,94
976		6,7	354	17	14	45,98	23	37	8,9	3,065	88	48	21	−19,95
977		7	354	26	18	46,28	23	37	45,2	3,085	97	31	97	−19,95
978		7	354	31	20	45,99	23	38	5,3	3,066	88	36	58	−19,96
979		6,7	354	40	49	46,02	23	38	43,2	3,068	90	5	16	−19,96
980		6	354	51	3	46,37	23	39	24,2	3,091	101	8	31	−19,97

MAYER's CATALOGUE OF THE PRINCIPAL STARS

Numb of Stars	Names and Places of the Stars	Magnitud	Right Ascension in Degrees			Annual Variat +	Right Ascension in Time			Annual Variat +	North polar Distance			Annual Preces
						s		M		s	D	M	s	s
981	Piſcium	7	355	32	1	46,14	23	42	8,8	3,076	94	19	9	− 19,99
982		6 7	356	0	40	46,05	23	44	2,7	3,070	91	3	27	− 20,00
983		6 7	356	5	55	45,88	23	44	23,7	3,059	84	5	40	− 20,00
984		6,7	356	59	1	46,12	23	47	56,1	3,075	94	43	8	− 20,02
985	ω Piſcium	5	357	8	5	45,92	23	48	32,3	3,061	84	17	50	− 20,02
986		5	357	46	5	46,12	23	51	4,3	3,075	94	11	40	− 20,03
987	Piſcium	5	357	47	55	46,12	23	51	11,7	3,075	97	10	44	− 20,03
988			357	49	16	46,07	23	51	17,1	3,071	93	56	1	− 20,03
989	2 c Piſcium	5,6	357	56	44	45,93	23	51	46,9	3,062	82	40	47	− 20,03
990	Piſcium		358	38	56	46,08	23	54	35,7	3,07	96	52	55	− 20,04
991	α Andromedæ	2	359	23	20	45,91	23	57	33,3	3,061	62	4	1	− 20,04
992		8	359	47	39	46,02	23	59	10,6	3,068	93	43	39	− 20,05

MAYER had no tranſit inſtrument to take the right aſcenſion of the ſtars but took them with the teleſcope of his quadrant He ſettled the places of ſeveral of the bright ſtars which were viſible in the day time by compariſon with the ſun in the ſame parallel But this could only be applied to thoſe ſtars whoſe declinations were leſs than 18 as the ſun s motion in declination would otherwiſe be too ſmall He compoſed his ſolar Tables by the help of theſe bright ſtars and by comparing the tranſits of the ſaid ſtars with thoſe of the ſun when it s declination was between 18 and 23½ taking the ſun s right aſcenſion anſwering to the longitude computed by his ſolar Tables he found the errors of the plane of his quadrant at thoſe altitudes and then by obſerving the zodiacal ſtars within thoſe limits he filled up his catalogue

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CORRIGENDA AND ADDENDA

TO

VOLUME II

PAGE 39 in the margin againſt the laſt line put FIG 201

41 l 1 *after Mn add* on *N*, deſcribe a ſemicircle *Nvw*, and

46 l 2 *for* 3 *read* ¼ *after* circular) *add* nearly line 8, *after* is *add* nearly, and at the end of line 12 *add* nearly

49 l 2 *for SEN read SEA* l 29 *for* paſs *read* paſſes laſt line for (908) read (909)

50 laſt line but three *for* (908) *read* (909)

55 l 18 *for* (908) *read* (909)

58 l 4 for *up* read *np*

59 l 23 *for* CABD *read* CADB

67, l 25 27 for *kv*, read *ku*

113 l 18 *for* in two, *read* into two,

147, l 5 7 9 11 for *mn* read *mp*

169 l 11 for m^2—, read *m* —1

181, l 14 15—22 23 *for* A B C D, F &c *read* A B′ C D, E &c

226 l 30 for *dt* read *dt*

236 to Art 1237 *add* Here the parallels of latitude are crowded together towards the poles

243 l 22 *for* 8 26 54 *read* 8° 24 18

283 to Art 1346 *add* After his death Dr SMITH Maſter of Trinity College the author of a celebrated Treatiſe on Optics and another on Harmonics publiſhed the works which he left behind him one containing his *Lectures on Hydroſtatics* and another, entitled *Harmonia Menſurarum*

335, l 1 *for* meridian *read* latitude

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